Adding bosons to an SYK-like tensor model

Sabine HARRIBEY

Internship M1 ENS Lyon - Perimeter Institute for theoretical physics

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ENS DE LYON

ÉCOLE POLYTECHNIQUE
UNIVERSITÉ PARIS-SACLAY

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

Introduction

- SYK model: N randomly interacting fermions, solvable at large N
- Generalization: Gurau-Witten models $\rightarrow$ SYK-like tensor models

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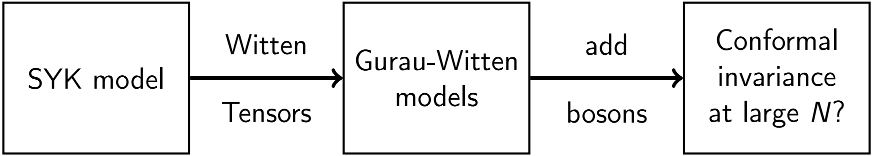

Tensor models

- $(D+1)$-colored model
- Correlation function

Gurau-Witten models

- SYK model
- Definition of the model
- Correlation functions

SYK-like tensor models with bosons and fermions

- Two-point function
- Four-point function

$D+1$-colored tensor model

Action:

$$S = \Psi^c_{abd}\overline{\Psi}^c_{abd} + \left(\frac{\lambda}{N^{3/2}}\Psi^0_{abc}\Psi^1_{ade}\Psi^2_{bdf}\Psi^3_{cef} + \frac{\overline{\lambda}}{N^{3/2}}\overline{\Psi}^0_{abc}\overline{\Psi}^1_{ade}\overline{\Psi}^2_{bdf}\overline{\Psi}^3_{cef}\right)$$

- $D+1$ fields, index c, N: size of the tensors
- $\langle\Psi^c_{abd}\overline{\Psi}^{c'}_{a'b'd'}\rangle = \delta^{cc'}\delta_{aa'}\delta_{bb'}\delta_{dd'}$

$D + 1$-colored tensor model

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- $D + 1$ fields, index c, N: size of the tensors
- $\langle \Psi^c_{abd}\overline{\Psi}^{c'}_{a'b'd'}\rangle = \delta^{cc'}\delta_{aa'}\delta_{bb'}\delta_{dd'}$
- Normalized partition function:

$$Z = \frac{1}{Z_0}\int d\Psi d\overline{\Psi} e^{-S(\Psi,\overline{\Psi})} = \sum_{\mathcal{G}} (\lambda\overline{\lambda})^{k(\mathcal{G})}\mathcal{A}(\mathcal{G})$$

with $k(\mathcal{G})$ the number of white vertices of $\mathcal{G}$ [Gurau, '10s]

Feynman graphs

- $Z = \frac{1}{Z_0} \int d\Psi d\overline{\Psi} e^{-S_0} (1 + \frac{\lambda\overline{\lambda}}{N^3} \Psi^0_{abc} \Psi^1_{ade} \Psi^2_{bdf} \Psi^3_{cef} \overline{\Psi}^0_{a'b'c'} \overline{\Psi}^1_{a'd'e'} \overline{\Psi}^2_{b'd'f'} \overline{\Psi}^3_{c'e'f'} + \dots)$

Feynman graphs

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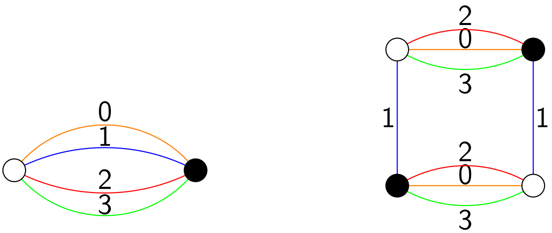
- Vertices: $\Psi \to$ White, $\overline{\Psi} \to$ Black
- Edge of color c : identification of the indices of Ψ^c and $\overline{\Psi}^c$
- Factor N per face: $\mathcal{A}(\mathcal{G}) = N^{F-3k(\mathcal{G})}$
- Large N expansion

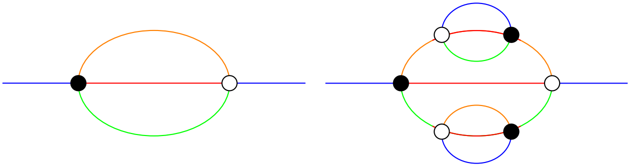


(a) First order in $\lambda\overline{\lambda}$: $\mathcal{A} = N^3$ (b) Second order in $\lambda\overline{\lambda}$: $\mathcal{A} = N^3$

Two-point function

$$\langle \Psi^{c_1}_{abc} \overline{\Psi}^{c_2}_{def} \rangle_c = \delta^{c_1 c_2} \delta_{ad} \delta_{be} \delta_{cf} + \ldots$$

- Two-point function: melon graphs dominate the large N expansion



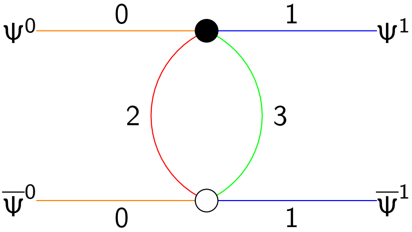
(a) Prime melonic graph (b) Melonic graph

- Melonic graphs built recursively: iterative adding of prime melons on the edges

Four-point function: first term

- Computation of the first term

$$\langle \Psi^0_{ijk} \overline{\Psi}^0_{\alpha\beta\gamma} \Psi^1_{lmp} \overline{\Psi}^1_{\lambda\mu\nu} \rangle_c = \frac{\lambda\overline{\lambda}}{N^2} \delta_{il}\delta_{j\beta}\delta_{k\gamma}\delta_{\alpha\lambda}\delta_{m\mu}\delta_{p\nu} + \dots$$



Four-point function

- Ladder diagrams dominate the large N expansion

[Gurau, '10s]

SYK model

- N Majorana fermions interacting randomly

$$S = \int d\tau \left(\frac{1}{2} \sum_{i=1}^{N} \psi_i \frac{d}{d\tau} \psi_i + \frac{i^{\frac{q}{2}}}{q!} \sum_{i_1,\ldots,i_q=1}^{N} j_{i_1\ldots i_q} \psi_{i_1} \ldots \psi_{i_q} \right)$$

- $j_{i_1\ldots i_q}$: totally antisymmetric, random gaussian variable

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- $j_{i_1 \dots i_q}$: totally antisymmetric, random gaussian variable
- Conformal symmetry at strong coupling, $J \gg 1$
- Solvable at large N, dominance of melonic graphs $\rightarrow$ tensor models

[Maldacena,Stanford, '16]

Complex Gurau-Witten model

- Tensorial colored fields

- Time dependence and anti-commuting variables

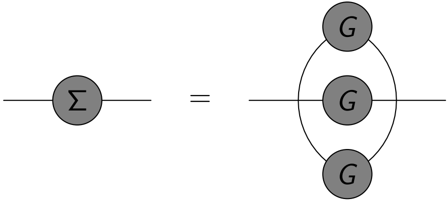
$$
\begin{aligned}
S_0 &= \frac{1}{2}\int d\tau \overline{\Psi}^i_{abc}\frac{d}{d\tau}\Psi^i_{abc} \\
S_I &= \frac{J}{N^{3/2}}\int d\tau \left(\Psi^0_{abc}\Psi^1_{ade}\Psi^2_{bdf}\Psi^3_{cef} + \overline{\Psi}^3_{cef}\overline{\Psi}^2_{bdf}\overline{\Psi}^1_{ade}\overline{\Psi}^0_{abc}\right)
\end{aligned}
$$

- Everything for $D = 3$: can be generalized for $D \geq 3$ odd

[Witten, '16],[Gurau, '16]

Two-point function and Schwinger-Dyson equation

- One-particle irreducible function: computed knowing that the melonic graphs dominate at large N



- Strong coupling limit: simplification of the equation on the two-point function

$$
\begin{aligned}
G^{-1}(\omega) &= \widehat{C}_\psi(\omega)^{-1} - \Sigma(\omega) \\
&= -i\omega - \Sigma(\omega) \\
&\approx -\Sigma(\omega)
\end{aligned}
$$

Conformal limit

$$-\delta(t_1 - t_2) = J^2 \int dt G(t_1 - t) G^{q-1}(t - t_2)$$

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- Invariance under reparametrization:

$$G(t, t') \to [f'(t) f'(t')]^{\Delta} G(f(t), f(t'))$$

with $\Delta = \frac{1}{q}$ and $D = q - 1$;

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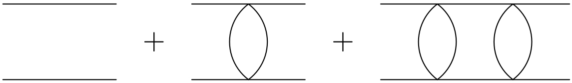
- Equation for the constant b

$$J^2 b^q \pi = (\frac{1}{2} - \frac{1}{q}) \tan(\frac{\pi}{q})$$

Four-point function

- Sum over ladder diagrams with edges decorated by the two-point function.



$$+ \cdots = \frac{1}{1-K} G_0$$

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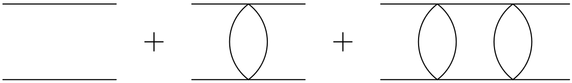
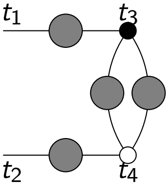


$$+ \cdots = \frac{1}{1-K} G_0$$

- Kernel : operator that adds a single rung $\rightarrow$ determined by the two-point function

$$K(t_1, t_2, t_3, t_4) = -J^2 G(t_1, t_3) G(t_2, t_4) G(t_3, t_4)^{D-1}$$



Diagonalisation

- Compute $\langle \Psi_{abc}(t_1)\overline{\Psi}_{abc}(t_2)\mathcal{O}_n(t_0)\rangle = v_n(t_0, t_1, t_2)$
- Dimension h of primary operators: diagonalisation of the kernel

$$\int dt_3 dt_4 K(t_1, t_2, t_3, t_4) v_n(t_0, t_3, t_4) = v_n(t_0, t_1, t_2)$$

$$v_n(t_0, t_1, t_2) = \frac{\mathrm{sgn}(t_1 - t_2)}{|t_0 - t_1|^{h_n} |t_0 - t_2|^{h_n} |t_1 - t_2|^{2\Delta - h_n}}$$

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- $h_n = 2, 3.58, 5.42, \ldots$ for $q = 6$ [Gross,Rosenhaus, '17]
- $h_n \to 2n + 1 + 2\Delta$ for $n \to \infty$

Modifications entailed by adding bosons

- Adding bosons: p bosons ϕ^i, q fermions ψ^j, $p+q-1=D$
- Bosons : fields Ψ^i with colors from 0 to $p-1$
- Fermions : fields Ψ^i with colors from p to D

Modifications entailed by adding bosons

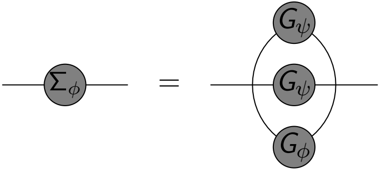
- Adding bosons: p bosons ϕ^i, q fermions ψ^j, $p+q-1=D$
- Bosons : fields Ψ^i with colors from 0 to $p-1$
- Fermions : fields Ψ^i with colors from p to D
- New action (for $D=3$):

$$S_0 = \sum_{i=0}^{p-1} \int d\tau \overline{\Psi}^i_{abc} \Psi^i_{abc} + \sum_{i=p}^{3} \int d\tau \overline{\Psi}^i_{abc} \frac{d}{d\tau} \Psi^i_{abc}$$

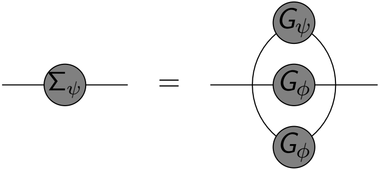
$$S_I = \frac{J}{N^{3/2}} \int d\tau \left(\Psi^0_{abc} \Psi^1_{ade} \Psi^2_{bdf} \Psi^3_{cef} + \overline{\Psi}^3_{cef} \overline{\Psi}^2_{bdf} \overline{\Psi}^1_{ade} \overline{\Psi}^0_{abc} \right)$$

Two-point function

- Same method but two different propagators and two-point functions



(a) Bosonic 1PI



(b) Fermionic 1PI

- Schwinger-Dyson equations

$$G_\phi(\omega)^{-1} = 1 - \Sigma_\phi(\omega), \qquad G_\psi(\omega)^{-1} = -i\omega - \Sigma_\psi(\omega) \tag{3.1}$$

Two-point function

- Strong coupling limit

$$-\delta(\tau_1-\tau_2)=\int d\tau\, G_\psi(\tau_1,\tau)J^2 G_\phi(\tau,\tau_2)^p G_\psi(\tau,\tau_2)^{q-1}$$

$$-\delta(\tau_1-\tau_2)=\int d\tau\, G_\phi(\tau_1,\tau)J^2 G_\phi(\tau,\tau_2)^{p-1} G_\psi(\tau,\tau_2)^{q}$$

Two-point function

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$$-\delta(\tau_1-\tau_2)=\int d\tau\, G_\phi(\tau_1,\tau)J^2 G_\phi(\tau,\tau_2)^{p-1} G_\psi(\tau,\tau_2)^{q}$$

- Still a conformal invariance
- Two conformal dimensions

$$p\Delta_\phi+q\Delta_\psi=1$$

Two-point function

- Ansatz for the two different two-point functions

$$G_\psi(\tau) = b_\psi \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_\psi}}, \quad G_\phi(\tau) = b_\phi \frac{1}{|\tau|^{2\Delta_\phi}}$$

Two-point function

- Ansatz for the two different two-point functions
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- Another constraint by injecting those ansatz in the Schwinger-Dyson equations

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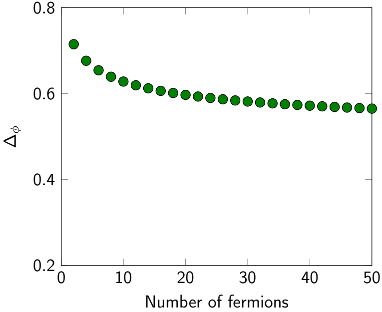
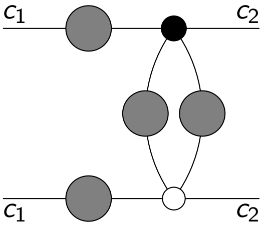
- Another constraint by injecting those ansatz in the Schwinger-Dyson equations
- Conformal limit : $p = 1$ and $\frac{1}{2} < \Delta_\phi < 1$



Four-point function

- Large N expansion dominated by ladder diagrams
- Bosons and fermions : different kernels depending on the colors propagating
- Kernel with two indices: $K^{c_1 c_2}$



Kernels

- 5 possibilities :

$$K^{c_1 c_2} = \begin{cases} 0 \text{ if } c_1 = c_2 \\ K_{bb} \text{ if } c_1 \neq c_2 \text{ bosons} \\ K_{ff} \text{ if } c_1 \neq c_2 \text{ fermions} \\ K_{bf} \text{ if } c_1 \text{ boson}, c_2 \text{ fermion} \\ K_{fb} \text{ if } c_1 \text{ fermion}, c_2 \text{ boson} \end{cases} \quad (3.2)$$

For example,

- $K_{bf} = J^2 G_\phi(t_1 - t_3) G_\phi(t_2 - t_4) G_\phi(t_3 - t_4)^{p-1} G_\psi(t_3 - t_4)^{q-1}$

Dimension of the primary operators

- Equivalent to the diagonalisation of the kernels with $g(h) = 1$:

$$g(h)v^a(\tau_0;\tau_1,\tau_2) = \sum_{c=0}^{D} \int d\tau_3 d\tau_4 K^{ac}(\tau_1,\tau_2,\tau_3,\tau_4) v^c(\tau_0;\tau_3,\tau_4) \tag{3.3}$$

with

$$v^a(\tau_0;\tau_1,\tau_2) = c_a \frac{\mathrm{sgn}(\tau_1-\tau_2)}{|\tau_1-\tau_2|^{2\Delta_\psi - h}}, \quad \text{if } a \in [p, D] \tag{3.4}$$

$$v^a(\tau_0;\tau_1,\tau_2) = c_a \frac{1}{|\tau_1-\tau_2|^{2\Delta_\phi - h}}, \quad \text{if } a \in [0, p-1] \tag{3.5}$$

Solutions and interpretation

Solutions for $p = 1$ and $q = 4$:

- $h = 2, 3.06, 3.68, 5.08, 5.57$
- $h_{2n+1} \to 2n + 1 + 2\Delta_\psi$
  $h_{2n} \to 2n + 2\Delta_\phi$ when $n \to \infty$.

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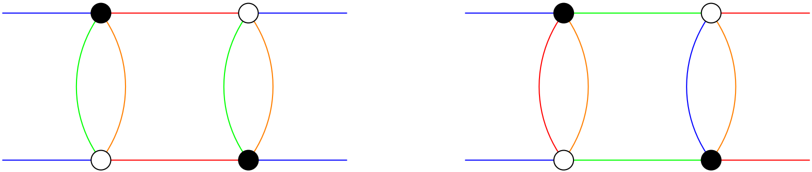
- $h = 2, 3.06, 3.68, 5.08, 5.57$
- $h_{2n+1} \to 2n + 1 + 2\Delta_\psi$
  $h_{2n} \to 2n + 2\Delta_\phi$ when $n \to \infty$.
- We found again an operator of dimension 2
- [Maldacena,Stanford, '16] : $h = 2$ in the basis used to invert $1 - K$
- Divergence in the four-point function ?
- Maximal chaos ?

Towards the computation of the full four-point function

- Unbroken ladder : a face propagates along the whole ladder.
- Unbroken ladder with different external colors : odd number of rungs.



(a) Unbroken ladder (b) Broken ladder

Towards the computation of the full four-point function

- For example : unbroken ladder for different fermions entering and exiting
- No choice for the face propagating

$$G_{U,\neq f} = G_1 \sum_{n\geq 0} K_{ff}^{2n} = \frac{G_1}{1-K_{ff}^2} \tag{3.6}$$

with

$$G_1(t_1,t_2,t_3,t_4) = \int dtdt' K_{ff}(t_1,t_2;t,t') G_\psi(t-t_3) G_\psi(t'-t_4) \tag{3.7}$$

- Effective kernel

$$K_{U,\neq f} = K_{ff}^2 \tag{3.8}$$

Conclusion

- SYK model : solvable at large N, conformal symmetry at strong coupling
- Gurau-Witten models: SYK-like tensor models, same behavior at large N
- Tensor model with bosons and fermions : same form for the large N expansion
- Still able to compute the conformal dimensions
- Next step : compute the full four-point function