On the large N limit of the Schwinger-Dyson equation of tensor field theory

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Based on:

R. Pascalie, C. I. Pérez-Sánchez, and R. Wulkenhaar. arXiv:1706.07358.

R. Pascalie, C. I. Pérez-Sánchez, A. Tanasa and R. Wulkenhaar. arXiv:1810.09867.

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Outline

The model

Complex rank-3 bosonic tensor field theory: $\mathrm{U}(N)^3$-invariant "pillow" interactions

$$\mathcal{S}[\varphi,\bar\varphi] = \mathcal{S}_0[\varphi,\bar\varphi] + \mathcal{S}_{\mathrm{int}}[\varphi,\bar\varphi] \tag{1}$$
$$= \sum_{\mathbf{x}} \bar\varphi^{\mathbf{x}} |\mathbf{x}|^2 \varphi^{\mathbf{x}} + \lambda \sum_{c=1}^{3} \sum_{\mathbf{a},\mathbf{b}} \bar\varphi^{\mathbf{a}} \varphi^{\mathbf{b}_{\hat c} a_c} \bar\varphi^{\mathbf{a}_{\hat c} b_c} \varphi^{\mathbf{a}},$$

with $\mathbf{x} = (x_1, x_2, x_3) \in \{\frac{1}{N}, \frac{2}{N}, \dots, 1\}^3$, $|\mathbf{x}|^2 = x_1^2 + x_2^2 + x_3^2$, $\lambda = N^{\delta}\tilde\lambda$,
$\mathbf{a}_{\hat c} b_c = (a_1, \dots, a_{c-1}, b_c, a_{c+1}, \dots, a_D)$ for D-tuple.

The kinetic term represents the discrete Laplacian in the Fourier transformed space of the tensor index space.

The generating functional of the model writes:

$$\mathrm{Z}[J,\bar J] = \int \mathcal{D}\varphi \mathcal{D}\bar\varphi \exp\left(-N^{\gamma} \mathcal{S}[\varphi,\bar\varphi] + N^{\beta} \sum_{\mathbf{x}} (\bar J_{\mathbf{x}} \varphi^{\mathbf{x}} + J_{\mathbf{x}} \bar\varphi^{\mathbf{x}}) \right). \tag{2}$$

Boundary graph

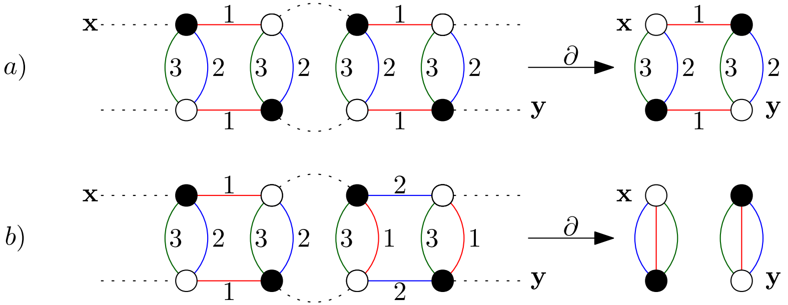

Figure: Two connected Feynman graphs and the associated boundary graphs. In the figure a) the boundary graph V_1 is connected and $\mathbb{J}(V_1)(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{x_1 y_2 y_3} \bar{J}_{y_1 x_2 x_3}$. In fig. b) the boundary graph $\mathrm{m}|\mathrm{m}$ is disconnected and $\mathbb{J}(\mathrm{m}|\mathrm{m})(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{\mathbf{x}} \bar{J}_{\mathbf{y}}$.

Free energy

The free energy is written as an expansion over boundary graphs:

$$\mathrm{W}[J,\bar{J}] = \sum_{k=1}^{\infty} \sum_{\substack{\mathcal{B}\in\partial_{\mathcal{S}_{\text{int}}} \\ V(\mathcal{B})=2k}} \sum_{\mathbf{X}} \frac{N^{\alpha(\mathcal{B})}}{|\mathrm{Aut}(\mathcal{B})|} \mathrm{G}^{(2k)}_{\mathcal{B}}(\mathbf{X}) \cdot \mathbb{J}(\mathcal{B})(\mathbf{X}), \qquad (3)$$

where $\partial_{\mathcal{S}_{\text{int}}}$ is the set of boundary graphs, $V(\mathcal{B})$ is the number of vertices of $\mathcal{B}$, $\mathrm{Aut}(\mathcal{B})$ is the symmetry group of the graph $\mathcal{B}$,
$\mathbf{X} = (\mathbf{x}^1, \dots, \mathbf{x}^k) \in \{\frac{1}{N}, \frac{2}{N}, \dots, 1\}^{3k}$, and $\mathbb{J}(\mathcal{B})(\mathbf{X}) = J_{\mathbf{x}^1} \dots J_{\mathbf{x}^k} \bar{J}_{\mathbf{p}^1} \dots \bar{J}_{\mathbf{p}^k}$.
Where $\mathbf{p}^i = \mathbf{p}^i(\mathbf{X}) \in \{\frac{1}{N}, \frac{2}{N}, \dots, 1\}^3$ is a momentum triplet determined by the boundary graph $\mathcal{B}$.

Green functions

A $2k$-point function with a boundary graph $\mathcal{B}$ is taken to be

$$\mathrm{G}_{\mathcal{B}}^{(2k)}(\mathbf{X}) = \frac{N^{-\alpha(\mathcal{B})}}{\mathrm{Z}_0} \prod_{i=1}^{k} \left(\frac{\delta}{\delta \bar{J}_{\mathbf{p}^i}} \frac{\delta}{\delta J_{\mathbf{x}^i}} \right) \mathrm{W}[J, \bar{J}] \bigg|_{J=\bar{J}=0} , \qquad (4)$$

where for all $c \in \{1, 2, 3\}$ and $(i, j) \in \{1, \ldots, k\}^2$, $x_c^i \neq x_c^j$.

The coefficient $\alpha(\mathcal{B})$ does not depend on the choice of colouring. For example, for the three pillow graphs $\alpha(V_1) = \alpha(V_2) = \alpha(V_3)$.

Ward-Takahashi Identity

The WTI for rank-D tensor field theory writes:

$$\sum_{\mathbf{q}_{\hat{a}}} \frac{\delta\, \mathrm{Z}[J,\bar{J}]}{\delta J_{\mathbf{q}_{\hat{a}} m_a} \delta \bar{J}_{\mathbf{q}_{\hat{a}} n_a}} - \delta_{m_a n_a} \mathrm{Y}^{(a)}_{m_a}[J,\bar{J}] \cdot \mathrm{Z}[J,\bar{J}] = \frac{N^{3\beta-2\gamma}}{m_a^2 - n_a^2} \sum_{\mathbf{q}_{\hat{a}}} \left(\bar{J}_{\mathbf{q}_{\hat{a}} m_a} \frac{\delta}{\delta \bar{J}_{\mathbf{q}_{\hat{a}} n_a}} - J_{\mathbf{q}_{\hat{a}} n_a} \frac{\delta}{\delta J_{\mathbf{q}_{\hat{a}} m_a}} \right) \mathrm{Z}[J,\bar{J}], \tag{5}$$

where $\mathbf{q}_{\hat{a}} = (q_1, \ldots, q_{a-1}, q_{a+1}, \ldots, q_D)$. The Y-term above is a functional given by

$$\mathrm{Y}^{(a)}_{m_a}[J,\bar{J}] = \delta_{m_a n_a} \sum_{\mathbf{q}_{\hat{a}}} \frac{\delta^2\, \mathrm{W}[J,\bar{J}]}{\delta J_{q_1 \ldots q_{a-1} m_a q_{a+1} \ldots q_D} \delta \bar{J}_{q_1 \ldots q_{a-1} n_a q_{a+1} \ldots q_D}} = \sum_{\mathcal{B} \in \partial \mathcal{S}_{\mathrm{int}}} \sum_{\mathbf{X}} \mathfrak{f}_a(\mathbf{X}; m_a; \mathcal{B}) \cdot \mathbb{J}(\mathcal{B})(\mathbf{X}), \tag{6}$$

Y-term

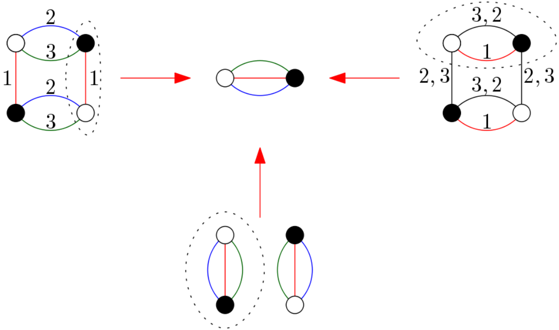

Figure: Contributions to the term $\mathfrak{f}_1(\mathbf{x}; m_1; \mathrm{m})$ in the boundary graph expansion of $\mathrm{Y}^{(1)}_{m_1}[J, \bar{J}]$.

$$\mathfrak{f}_1(\mathbf{x}; m_1; \mathrm{m}) = G_1^{(4)}(\mathbf{x}, m_1, x_2, x_3) + \sum_{q_3} G_2^{(4)}(\mathbf{x}; m_1, x_2, q_3) + \sum_{q_2} G_3^{(4)}(\mathbf{x}; m_1, q_2, x_3) + \sum_{q_2, q_3} G_{\mathrm{m}|\mathrm{m}}^{(4)}(\mathbf{x}, m_1, q_2, q_3). \quad (7)$$

Constraining the scalings

Aim: get from the SDE, a set of inequalities between the scaling coefficients α, β, γ and δ, such that:

- there is no divergent terms
- $\frac{1}{N}\sum \to \int$
- the higher point functions are decoupled

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- there is no divergent terms
- $\frac{1}{N}\sum \to \int$
- the higher point functions are decoupled

The 2-point function explicitly writes

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{N^{-\alpha}}{\mathrm{Z}_0} \frac{\delta^2 \mathrm{Z}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}}\bigg|_{J=\bar{J}=0} \tag{8}$$

$$= \frac{N^{2\beta-\gamma-\alpha}}{|\mathbf{x}|^2} - \frac{N^{2\beta-\gamma-\alpha}}{\mathrm{Z}_0} \frac{N^{\gamma}}{|\mathbf{x}|^2} \left(\bar{\varphi}^{\mathbf{x}} \frac{\partial \mathcal{S}_{\mathrm{int}}}{\partial \bar{\varphi}^{\mathbf{x}}}\right) \left[\frac{1}{N^{2\beta-\gamma}} \frac{\delta}{\delta J}, \frac{1}{N^{2\beta-\gamma}} \frac{\delta}{\delta \bar{J}}\right] \mathrm{Z}[J,\bar{J}]\bigg|_{J,\bar{J}=0}$$

In order for the free propagator to be dominant in the large N limit, one has:

$$\alpha = 2\beta - \gamma. \tag{9}$$

2-point function

Using the WTI we get:

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{|\mathbf{x}|^2} \tag{10}$$

2-point function

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$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{|\mathbf{x}|^2} - \frac{2\tilde{\lambda}}{|\mathbf{x}|^2} \sum_{a=1}^{3} \left(\frac{N^{3\gamma+2+\delta-4\beta}}{N^2} \sum_{\mathbf{q}_{\hat{a}}} \mathrm{G}^{(2)}(\mathbf{q}_{\hat{a}} x_a) \mathrm{G}^{(2)}(\mathbf{x}) \right. \tag{10}$$

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$$+ \frac{N^{\alpha(V_1)}}{N^{8\beta-5\gamma-\delta}} \mathrm{G}^{(4)}_a(\mathbf{x}, \mathbf{x}) + \frac{N^{\alpha(\mathrm{m}|\mathrm{m})+2}}{N^{8\beta-5\gamma-\delta}} \frac{1}{N^2} \sum_{\mathbf{q}_{\hat{a}}} \mathrm{G}^{(4)}_{m|m}(\mathbf{q}_{\hat{a}} x_a, \mathbf{x})$$
$$\left. + \frac{N^{\alpha(V_1)+1}}{N^{8\beta-5\gamma-\delta}} \frac{1}{N} \sum_{c \neq a} \sum_{q_b} \mathrm{G}^{(4)}_c(\mathbf{x}, \mathbf{x}_{\hat{b}} q_b) \right) \tag{10}$$

where $b \neq c$ and $b \neq a$.

2-point function

Using the WTI we get:

$$
\begin{aligned}
\mathrm{G}^{(2)}(\mathbf{x}) = {} & \frac{1}{|\mathbf{x}|^2} - \frac{2\tilde{\lambda}}{|\mathbf{x}|^2} \sum_{a=1}^{3} \Bigg(\frac{N^{3\gamma+2+\delta-4\beta}}{N^2} \sum_{\mathbf{q}_{\hat{a}}} \mathrm{G}^{(2)}(\mathbf{q}_{\hat{a}} x_a) \mathrm{G}^{(2)}(\mathbf{x}) \\
& + \frac{N^{\alpha(V_1)}}{N^{8\beta-5\gamma-\delta}} \mathrm{G}^{(4)}_a(\mathbf{x},\mathbf{x}) + \frac{N^{\alpha(\mathrm{m}|\mathrm{m})+2}}{N^{8\beta-5\gamma-\delta}} \frac{1}{N^2} \sum_{\mathbf{q}_{\hat{a}}} \mathrm{G}^{(4)}_{m|m}(\mathbf{q}_{\hat{a}} x_a, \mathbf{x}) \\
& + \frac{N^{\alpha(V_1)+1}}{N^{8\beta-5\gamma-\delta}} \frac{1}{N} \sum_{c\neq a} \sum_{q_b} \mathrm{G}^{(4)}_c(\mathbf{x}, \mathbf{x}_{\hat{b}} q_b) \\
& + \frac{1}{N} \sum_{q_a} \frac{N^{2\gamma+\delta+1-3\beta}}{x_a^2 - q_a^2} \left(\mathrm{G}^{(2)}(\mathbf{x}_{\hat{a}} q_a) - \mathrm{G}^{(2)}(\mathbf{x}) \right) \Bigg),
\end{aligned}
\tag{10}
$$

where $b \neq c$ and $b \neq a$.

$2k$-point function with connected boundary graph

The SDE for a $2k$-point function with a connected boundary graph:

$$G_{\mathcal{B}}^{(2k)}(\mathbf{X}) = -\frac{2\tilde{\lambda}}{|\mathbf{p}^1|^2} \sum_a \left\{ \frac{N^{3\gamma+2+\delta-4\beta}}{N^2} \sum_{\mathbf{q}_{\hat{a}}} G^{(2)}(\mathbf{q}_{\hat{a}} p_a^1) G_{\mathcal{B}}^{(2k)}(\mathbf{X}) \right. \tag{11}$$

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$$+\frac{N^{4\gamma+\delta-6\beta}}{N^{\alpha(\mathcal{B})}}\mathfrak{f}_a\left(\mathbf{X}; p_a^1; \mathcal{B}\right)$$

; (11)

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$2k$-point function with connected boundary graph

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$$+\frac{N^{4\gamma+\delta-6\beta}}{N^{\alpha(\mathcal{B})}}\mathfrak{f}_a\left(\mathbf{X};p^1_a;\mathcal{B}\right)+\frac{1}{N}\sum_{b_a}\frac{N^{2\gamma+\delta+1-3\beta}}{b_a^2-(x_a^\gamma)^2}\left(G^{(2k)}_{\mathcal{B}}(\mathbf{X})-G^{(2k)}_{\mathcal{B}}(\mathbf{X}|_{x_a^\gamma\to b_a})\right)$$
$$+\frac{N^{2\gamma+\delta-3\beta}}{N^{\alpha(\mathcal{B})}}\sum_{\rho=2}^{k}\frac{1}{(p_a^\rho)^2-(p_a^1)^2}\frac{1}{Z_0}\left[\frac{\partial Z[J,\bar{J}]}{\partial\zeta_a(\mathcal{B};1,\rho)}(\mathbf{X})-\frac{\partial Z[J,\bar{J}]}{\partial\zeta_a(\mathcal{B};1,\rho)}(\mathbf{X}|_{x_a^\gamma\to p_a^\rho})\right]\Bigg\}, \tag{11}$$

where $\mathbf{x}^\gamma$ corresponds to the only white vertex such that $x_a^\gamma = s_a$ and $\zeta_a(\mathcal{B};1,\rho)$ is the graph obtained by swapping the a-coloured lines between $\mathbf{p}^1$ and $\mathbf{p}^\rho$.

Swapping 1

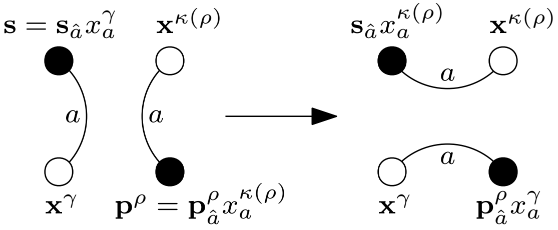

Figure: This figure shows the result of a swapping of the a-coloured lines between $\mathbf{s}$ and $\mathbf{p}^{\rho}$ in a graph $\mathcal{B}$. For $\mathbf{s} = \mathbf{p}^1$, it corresponds to the graph $\zeta_a(\mathcal{B}; 1, \rho)$. The white vertex $\mathbf{x}^{\gamma}$ corresponds to the only white vertex such that $x_a^{\gamma} = s_a$, similarly $\mathbf{x}^{\kappa(\rho)}$ corresponds to the only white vertex such that $x_a^{\kappa(\rho)} = p_a^{\rho}$.

Swapping 2

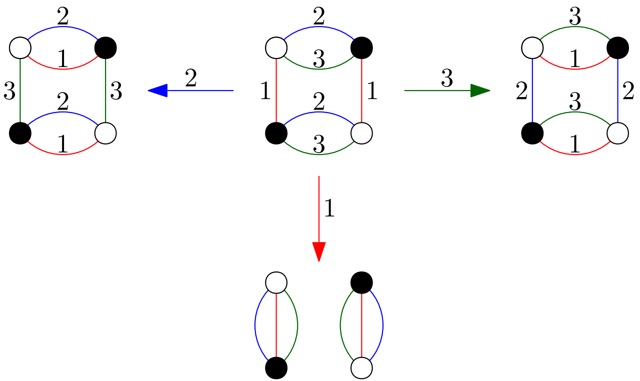

Figure: The result of the swapping of the three different colours starting from the pillow graph V_1. For the colours 2 and resp. 3, the swapping gives the graphs V_3 and resp. V_2; for colour 1, the swapping gives the disconnected graph $\mathrm{m}|\mathrm{m}$.

4-point function with disconnected boundary graph

The 4-point function with a disconnected boundary graph writes

$$\mathrm{G}^{(4)}_{\mathrm{m}|\mathrm{m}}(\mathbf{x},\mathbf{y}) = \frac{1}{N^{\alpha(\mathrm{m}|\mathrm{m})}} \frac{\delta^4 \mathrm{W}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}} \delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}}\bigg|_{J=\bar{J}=0}, \tag{12}$$

where

$$\frac{\delta^4 \mathrm{W}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}} \delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}} = -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{\delta^2}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{1}{\mathrm{Z}[J,\bar{J}]} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} \mathrm{Z}[J,\bar{J}] \right)$$

(13)

(14)

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where

$$\begin{aligned}\frac{\delta^4 W[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}} \delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}} &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{\delta^2}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{1}{Z[J,\bar{J}]} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} Z[J,\bar{J}] \right) \\ &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{1}{Z[J,\bar{J}]} \frac{\delta^3}{\delta J_{\mathbf{x}} \delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} Z[J,\bar{J}]\end{aligned} \tag{13}$$

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where

$$\begin{aligned}\frac{\delta^4 \mathrm{W}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}} \delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}} &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{\delta^2}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{1}{\mathrm{Z}[J,\bar{J}]} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}}\right)^{\partial} \mathrm{Z}[J,\bar{J}] \right) \\ &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{1}{\mathrm{Z}[J,\bar{J}]} \frac{\delta^3}{\delta J_{\mathbf{x}} \delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}}\right)^{\partial} \mathrm{Z}[J,\bar{J}] \\ &\quad + \frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{1}{\mathrm{Z}^2[J,\bar{J}]} \frac{\delta^2 \mathrm{Z}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}}\right)^{\partial} \mathrm{Z}[J,\bar{J}]. \qquad (13)\end{aligned}$$

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where

$$\begin{aligned}\frac{\delta^4 W[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}} \delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}} &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{\delta^2}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{1}{Z[J,\bar{J}]} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} Z[J,\bar{J}] \right) \\ &= -\frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{1}{Z[J,\bar{J}]} \frac{\delta^3}{\delta J_{\mathbf{x}} \delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} Z[J,\bar{J}] \\ &+ \frac{N^{2\beta}}{|\mathbf{x}|^2} \frac{1}{Z^2[J,\bar{J}]} \frac{\delta^2 Z[J,\bar{J}]}{\delta \bar{J}_{\mathbf{y}} \delta J_{\mathbf{y}}} \frac{\delta}{\delta J_{\mathbf{x}}} \left(\frac{\delta \mathcal{S}_{\mathrm{int}}}{\delta \bar{\varphi}^{\mathbf{x}}} \right)^{\partial} Z[J,\bar{J}]. \end{aligned} \tag{13}$$

The two produces "disconnected" term which must cancel each other.
This gives the constraint

$$2\beta = \gamma. \tag{14}$$

Large N limit

$$\mathcal{S}[\varphi,\bar\varphi] = \sum_{\mathbf{x}} \bar\varphi^{\mathbf{x}} |\mathbf{x}|^2 \varphi^{\mathbf{x}} + \frac{\tilde\lambda}{N^2} \sum_{c=1}^{3} \sum_{\mathbf{a},\mathbf{b}} \bar\varphi^{\mathbf{a}} \varphi^{\mathbf{b}_{\hat c} a_c} \bar\varphi^{\mathbf{a}_{\hat c} b_c} \varphi^{\mathbf{a}}, \tag{15}$$

$$\mathrm{G}^{(2k)}_{\mathcal{B}}(\mathbf{X}) = \frac{N^{-\alpha(\mathcal{B})}}{\mathrm{Z}_0} \prod_{i=1}^{k} \left(\frac{\delta}{\delta \bar J_{\mathbf{p}^i}} \frac{\delta}{\delta J_{\mathbf{x}^i}} \right) \mathrm{W}[J,\bar J] \Big|_{J=\bar J=0}, \tag{16}$$

with the conjecture for the scaling

$$\alpha(\mathcal{B}) = 3 - B - 2g - 2k, \tag{17}$$

where $2k$ is the number of vertices of $\mathcal{B}$, B its number of connected components and g its genus.
In the case of matrix model

$$\alpha(\mathcal{B}) = 2 - B - 2g, \tag{18}$$

see H. Grosse, R. Wulkenhaar, arXiv:1402.1041.

2- and 4-point functions

$$G^{(2)}(\mathbf{x}) = \left(|\mathbf{x}|^2 + 2\tilde{\lambda} \sum_{a=1}^{3} \int d\mathbf{q}_{\hat{a}} G^{(2)}(\mathbf{q}_{\hat{a}} x_a) \right)^{-1}, \tag{19}$$

$$G_1^{(4)}(\mathbf{x},\mathbf{y}) = -2\tilde{\lambda} G^{(2)}(x_1, y_2, y_3) G^{(2)}(\mathbf{y}) \frac{G^{(2)}(\mathbf{x}) - G^{(2)}(y_1, x_2, x_3)}{y_1^2 - x_1^2}, \tag{20}$$

$$G_{\mathrm{m}|\mathrm{m}}^{(4)}(\mathbf{x},\mathbf{y}) = -2\tilde{\lambda} G^{(2)}(\mathbf{x}) \sum_{a=1}^{3} \left\{ \sum_{c \neq a} \int dq_b G_c^{(4)}(x_a, q_b, y_c, \mathbf{y}) + \int d\mathbf{q}_{\hat{a}} G_{\mathrm{m}|\mathrm{m}}^{(4)}(\mathbf{q}_{\hat{a}} x_a, \mathbf{y}) \right\}, \tag{21}$$

where we used the SDE for the 2-point function to rewrite the SDE for the 4-point functions and where $d\mathbf{q}_{\hat{a}} = dq_b dq_c$ for $a \neq b, c$.

Perspectives

- Proving of the conjecture, using the SDE for disconnected boundary graphs.
- Solving the SDE in the large N limit, as it was done in arXiv:1807.02945 by E. Panzer and R. Wulkenhaar in the case of non-commutative quantum field theory.
- Implementing the methods for the study of SYK-like tensor models.

A solvable tensor field theory (in progress)

In the model with only the 1st pillow interaction, the renormalized SDE for the 2-point function

$$\mathrm{G}^{(2)}(\mathbf{x}) = \Big(1 + |\mathbf{x}|^2 + 2\lambda \int \mathrm{d}\mathbf{q}_{\hat{1}} \big(G(\mathbf{q}_{\hat{c}} x_1) - -\frac{1}{1 + |\mathbf{q}_{\hat{1}}|^2} \big)\Big)^{-1}, \qquad (22)$$

is solved by

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{1 + |\mathbf{x}|^2 + g(x_1, z)}, \qquad (23)$$

where

$$g(x_1, z) = zW\left(\frac{1}{z} e^{\frac{1+x_1^2}{z}}\right) - 1 - x_1^2, \qquad (24)$$

with $z = \frac{\pi}{2}\lambda$ and $W(z)$ the Lambert function (which is defined by $z = W(ze^z)$).