Tensor models from a functional Renormalization Group perspective

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based on work with Tim Koslowski, Johannes Lumma and Antonio Pereira

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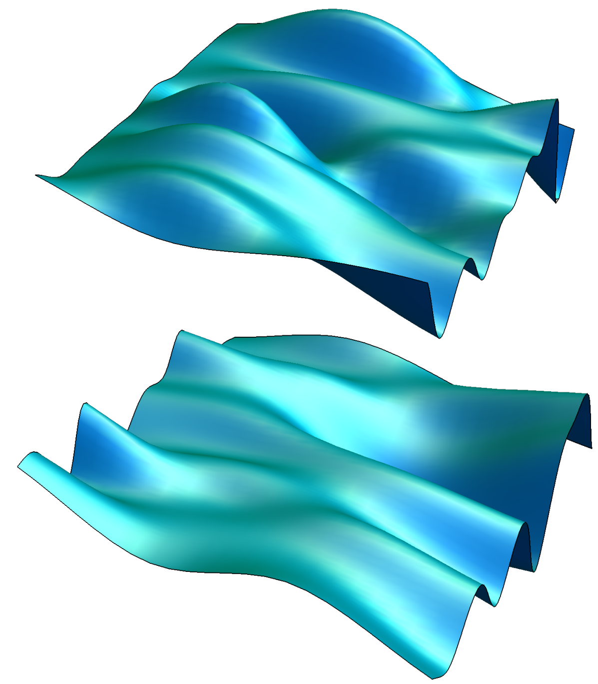
DFG Deutsche Forschungsgemeinschaft

Motivation: the path integral for quantum gravity

$$\int_{\text{spacetimes}} e^{iS[\text{spacetime}]}$$



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- **small fluctuations around a flat/ cosmological background**

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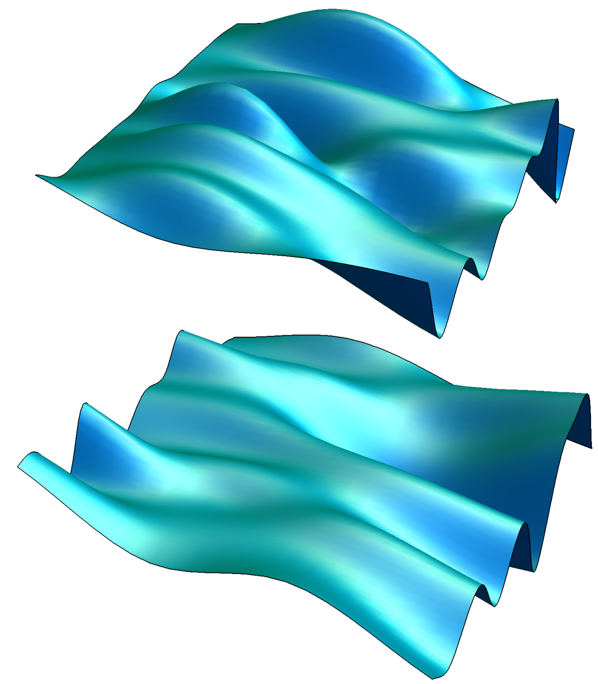

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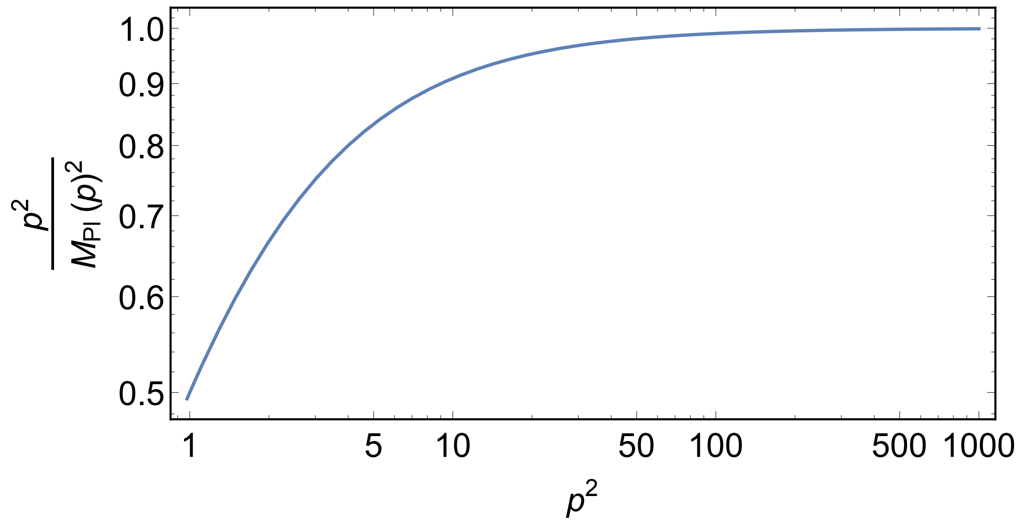

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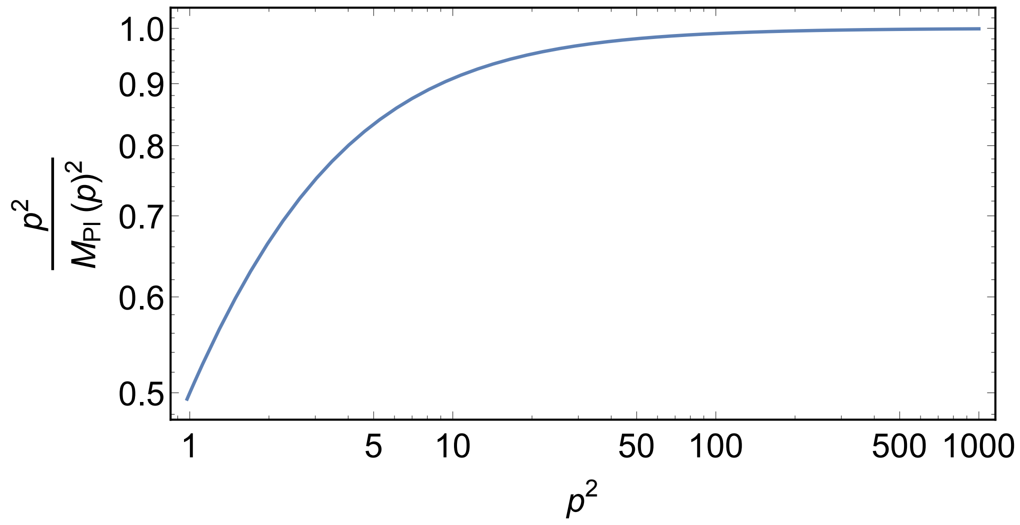

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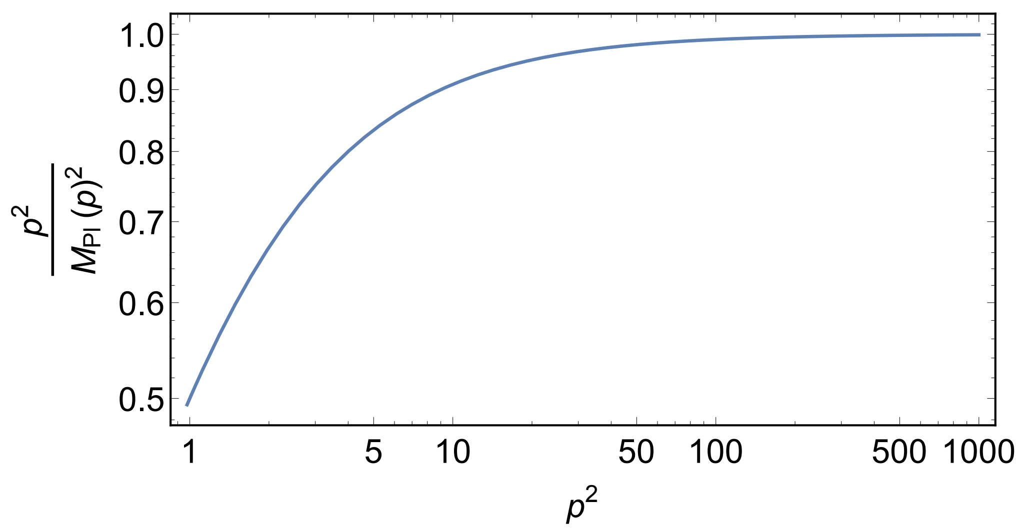

  - perfect action
    (lattice action provides continuum dynamics even at finite lattice spacing)

  - kinematical discreteness (at the level of individual configurations) need not persist at the dynamical level
    (path integral:
    sum over discrete config.'s yields continuum effective geometry)

Continuum limit: predictivity at all scales

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Models defined with a simple cutoff lose predictivity at the cutoff scale:

example: Effective field theory for gravity

$$S[g_{\mu\nu}] = \int d^4x \sqrt{g} \left(\frac{\Lambda}{16\pi G_N} - \frac{R}{16\pi G_N} + a_1 R^2 + b_1 R_{\mu\nu} R^{\mu\nu} + ... \right)$$

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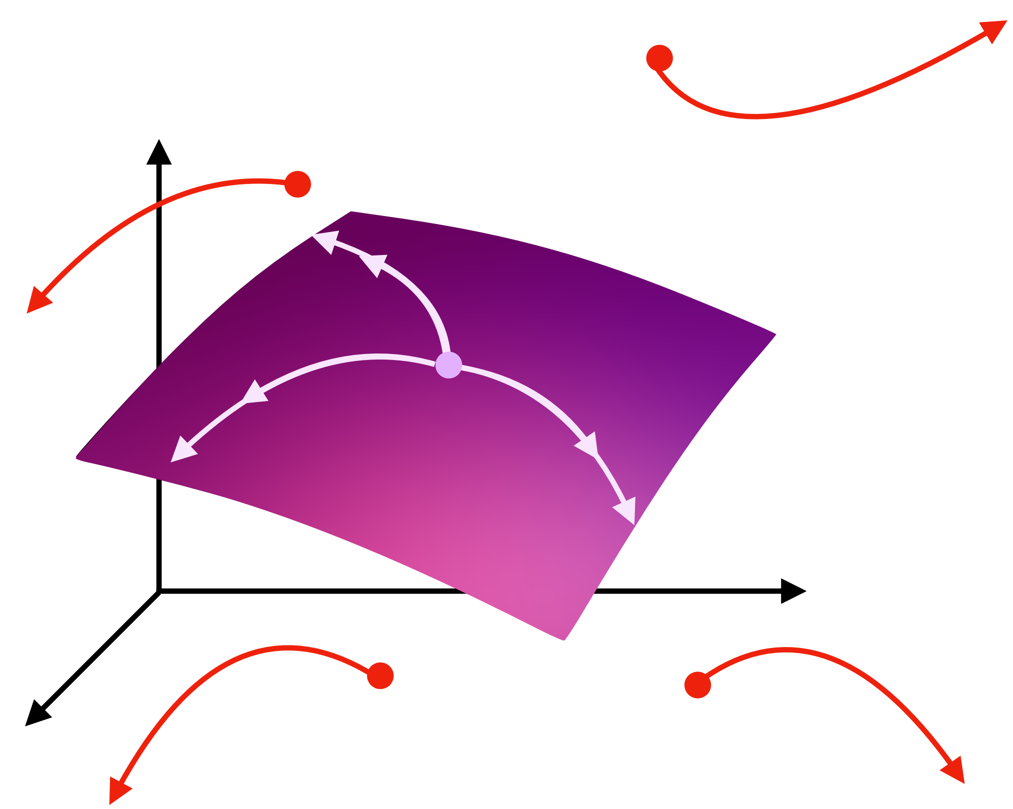

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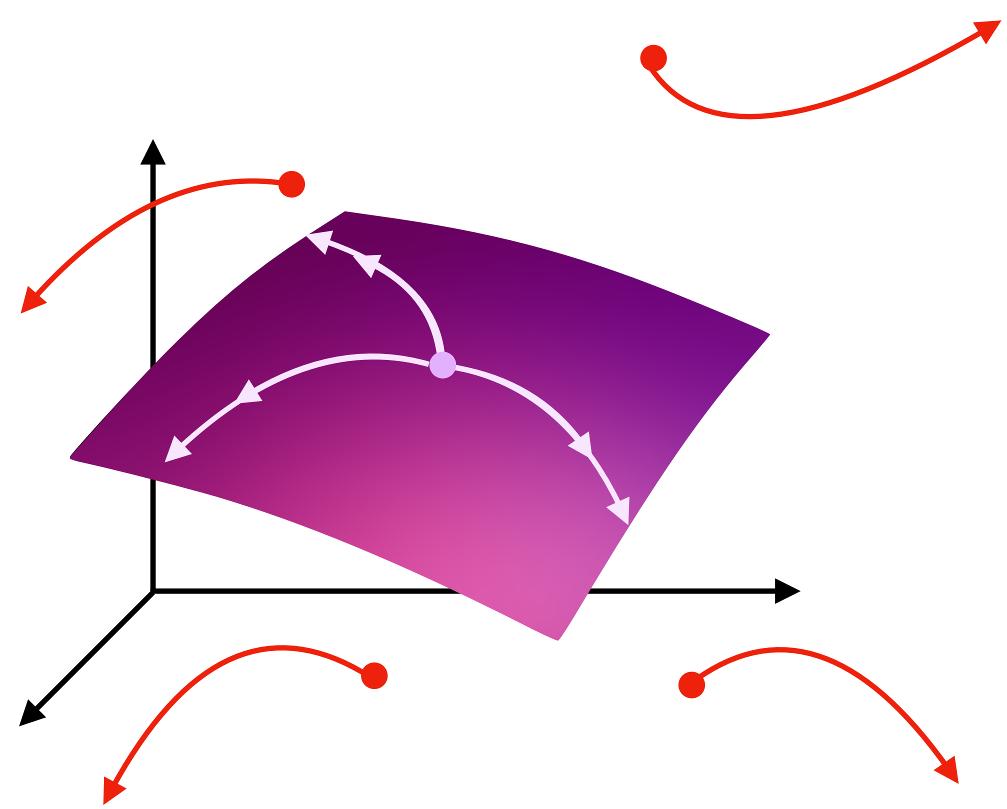
- **Continuum limit = mathematical phase transition:**
Physics at criticality determined by finitely many parameters



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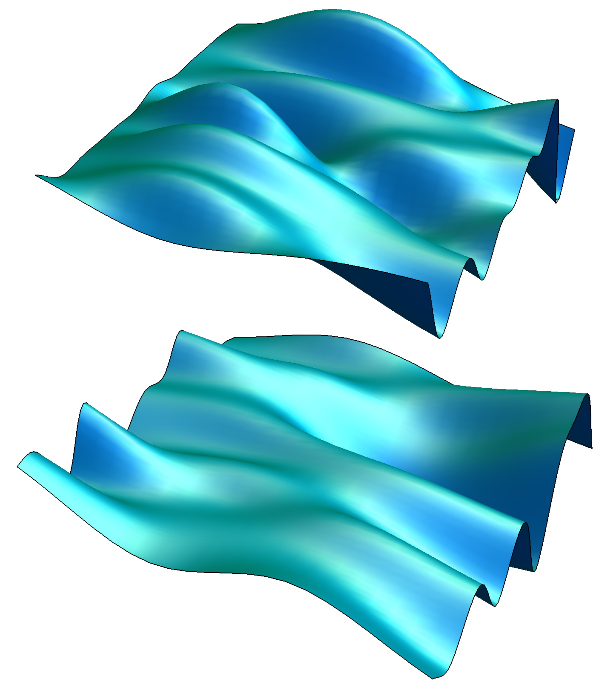

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asymptotically safe quantum gravity

tensor models

(causal) dynamical triangulations via Monte Carlo simulations

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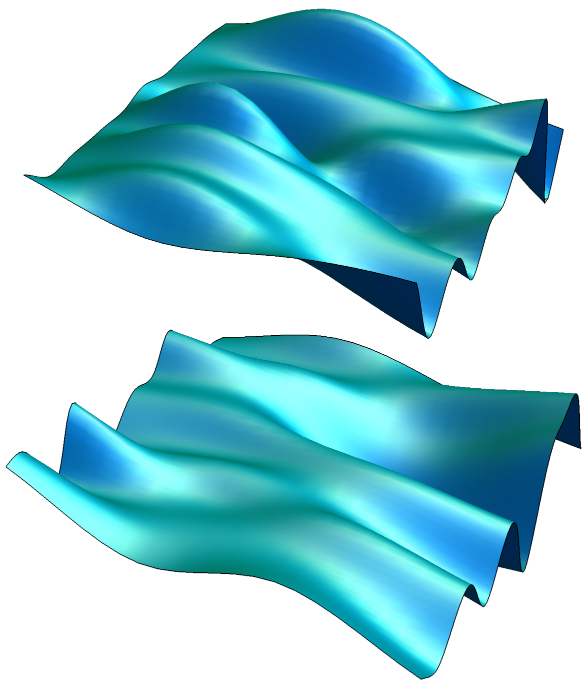

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trade-off?

- **background independence?**

indirect **asymptotically safe quantum gravity**

direct

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(causal) dynamical triangulations via Monte Carlo simulations

The continuum limit from local coarse graining

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$$\int \mathcal{D}\gamma_{\mu\nu}\, e^{iS[\gamma_{\mu\nu}]} \to \int \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]} \to \int_{\boxed{\text{“}p^2<k^2\text{”}}} \mathcal{D}\gamma_{\mu\nu}\, e^{-S[\gamma_{\mu\nu}]}$$

local coarse graining:
requires (auxiliary) background

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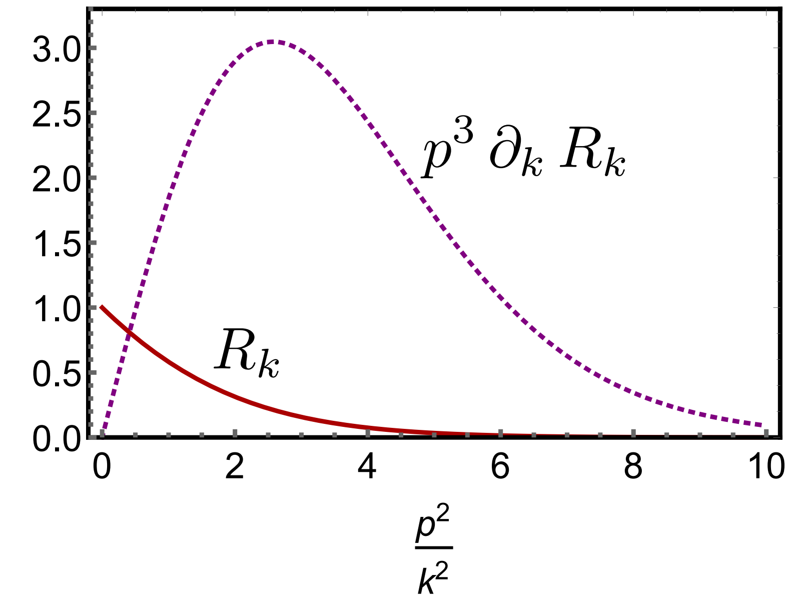
quadratic suppression term



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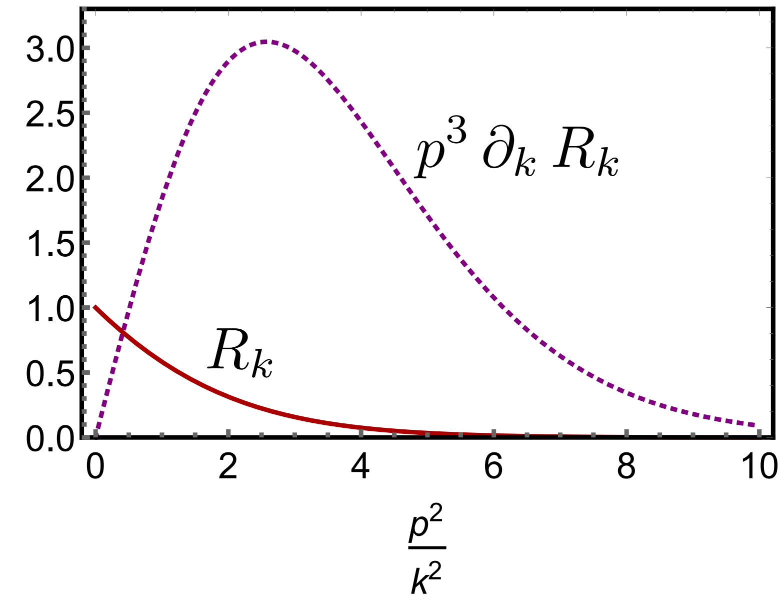

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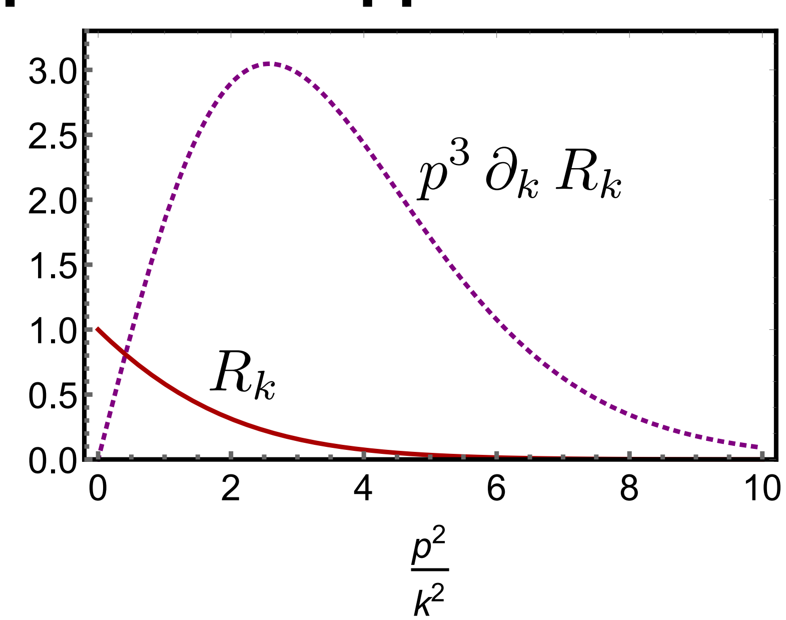
$$k\partial_k\, \Gamma_k[g,\bar{g}] = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k}{\delta g^2} + R_k\right)^{-1} k\partial_k\, R_k\right]$$



change of effective dynamics at *k*
driven by fluctuations at momenta *k*

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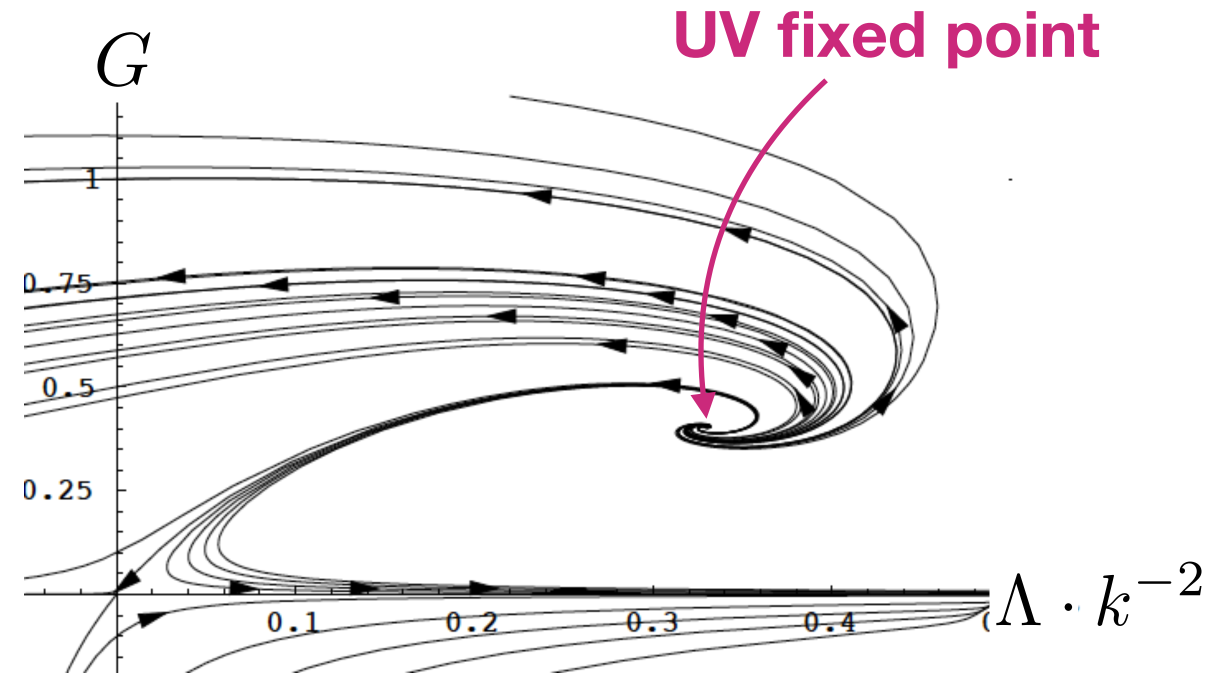
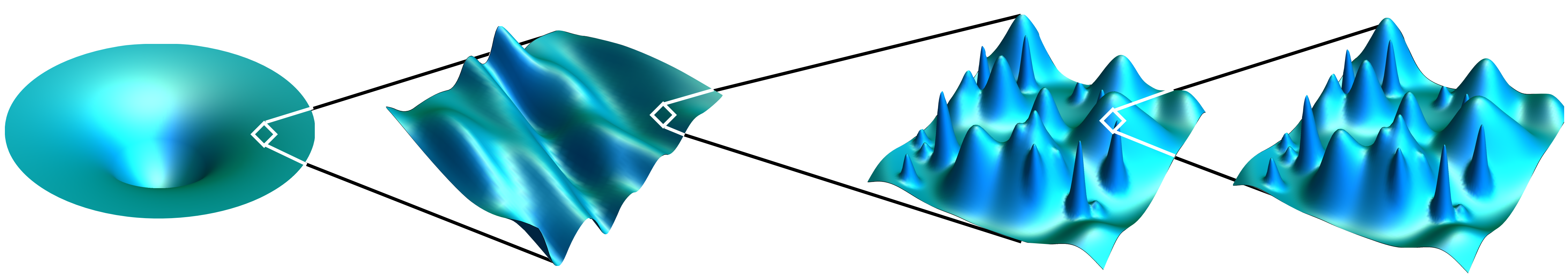
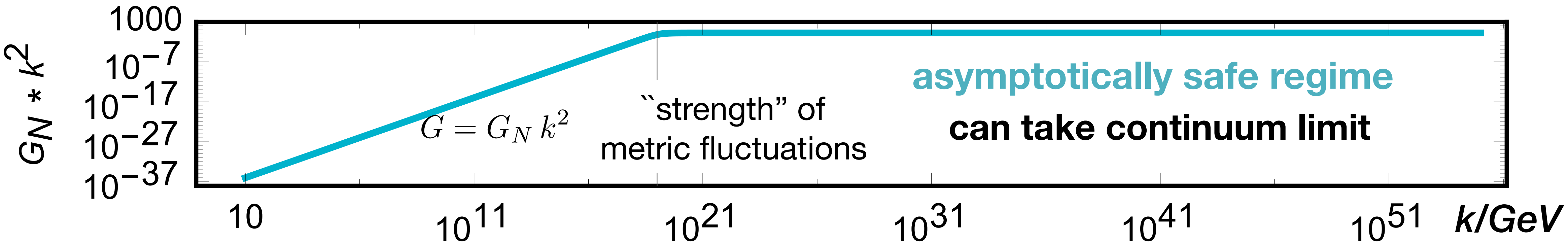


Compelling evidence for Reuter fixed point (with and without matter) such that continuum limit can be taken



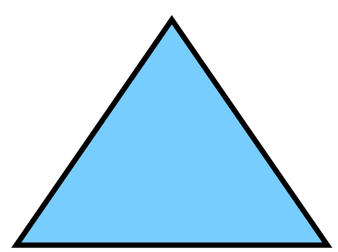


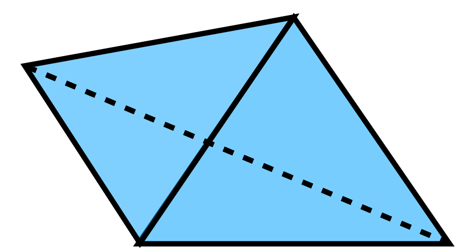
The continuum limit from non-local coarse graining

$$\int_{\text{spacetimes}} e^{-S} \xrightarrow{\sim} \sum_{\text{triangulations}} e^{-S_{\text{discrete}}}$$

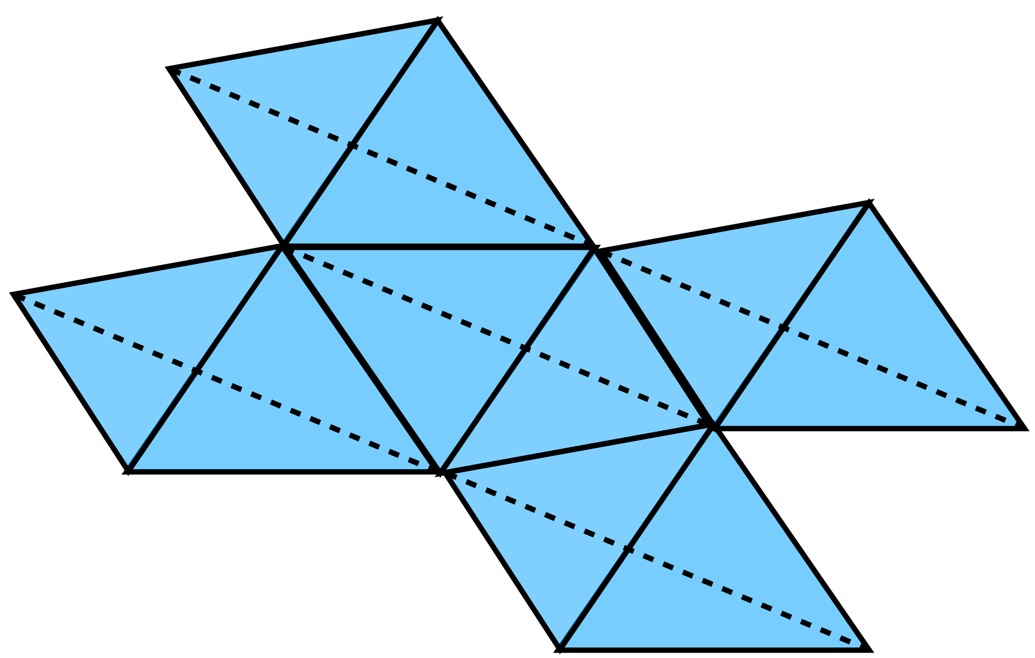
tensor models: rank D, N^D tensor, $O(N)^D$ invariance [Sasakura '91; Gross '91; Ambjorn, Durhuus, Jonsson '91; Gurau '11; Gurau, Rivasseau '11]

$$\xrightarrow{\sim} \int \mathcal{D}T\, e^{-T_{ijk}T_{ijk}+g_{4,1}^2 T_{ijk}T_{ilm}T_{nlm}T_{njk}+g_{4,1}\underbrace{T_{ijk}T_{ilm}T_{njm}T_{nlk}}}$$





Feynman diagrams: triangulations



The continuum limit from non-local coarse graining

example: rank 3; generalization to rank D straightforward

$$Z_N = \int \mathcal{D}T \, e^{-S[T] + \mathrm{Tr} T \cdot J - \frac{1}{2} T_{abc} R_N(a,b,c,d,e,f)^{abcdef} T_{def}}$$

suppresses ``infrared” indices

$$a, b, c, d, e, f < N$$

see also R. Toriumi, T. Krajewski for Polchinski equation

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main contribution to change of Γ_N at N tensor components at N

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formal ``trick": can transform any integral into differential equation
what is the physical meaning?

The continuum limit from non-local coarse graining

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How is this coarse graining?

Abstract view on RG flows:
RG flows go from many to few degrees of freedom (a-theorem)
(background dependent: local coarse graining; background independent: nonlocal ``coarse graining")

The continuum limit from non-local coarse graining

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search for universality at large N: fixed point of abstract RG flow

blueprint: matrix models for quantum gravity

double scaling limit $(g - g_{\mathrm{crit}})^{\frac{1}{\theta}} N = \mathrm{const}$

= continuum limit with contributions from all topologies

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blueprint: matrix models for quantum gravity

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= continuum limit with contributions from all topologies

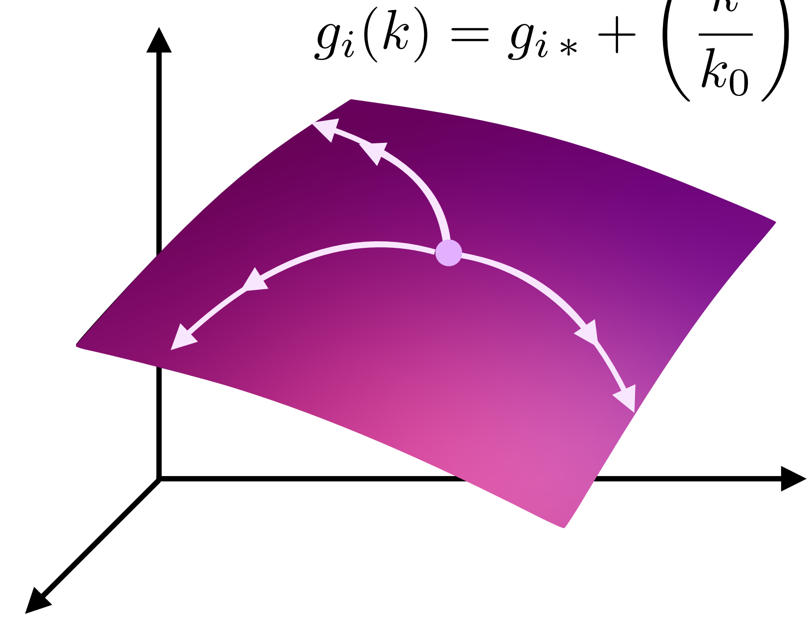
$$\longrightarrow \; g(N) = g_{\mathrm{crit}} + N^{-\theta}$$

linearized ``RG flow" in tensor size N

[Brezin, Zinn-Justin '92]

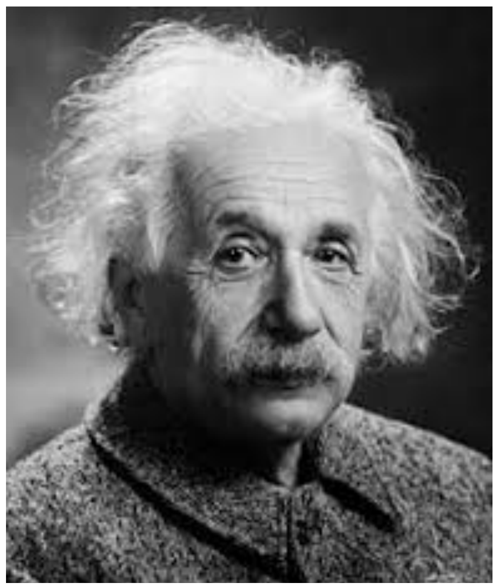
[AE, Koslowski '13]

$$g_i(k) = g_{i\,*} + \left(\frac{k}{k_0}\right)^{-\theta_i} + \ldots$$

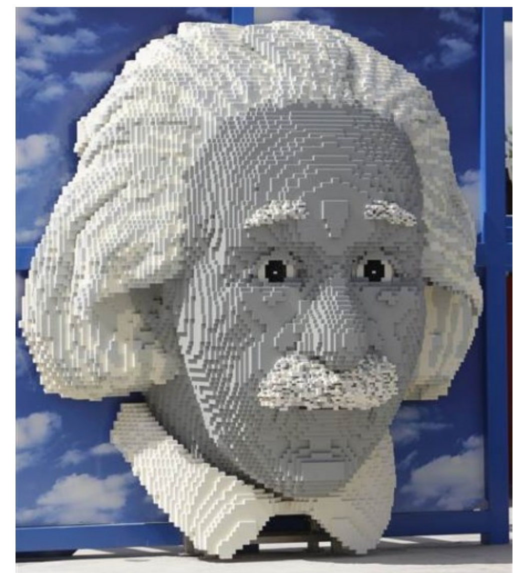


RG fixed point in tensor models

$$\int_{\text{spacetimes}} e^{-S} \xrightarrow{\simeq} \sum_{\text{triangulations}} e^{-S_{\text{discrete}}}$$

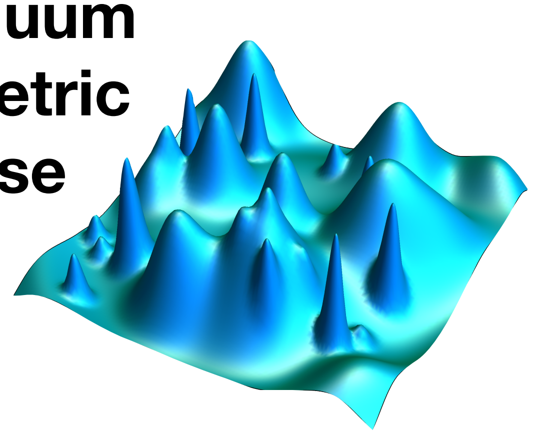


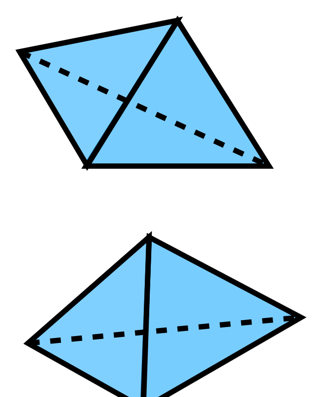
continuum



discrete

continuum geometric phase





discrete "pregeometric" phase

transition

interaction strength

RG fixed point in tensor models

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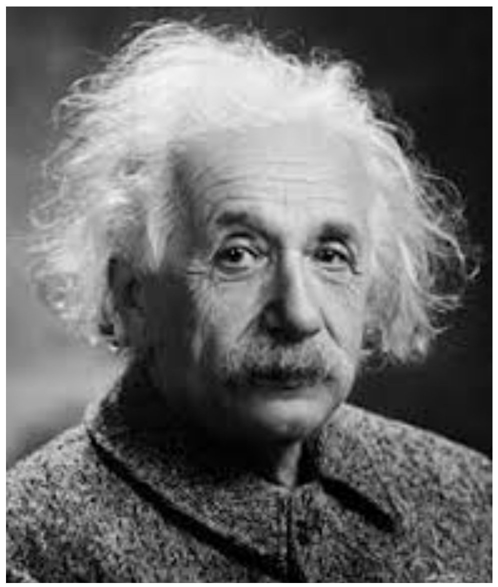

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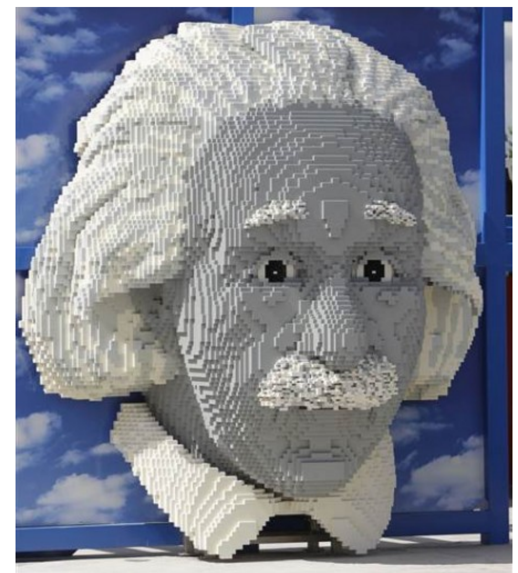


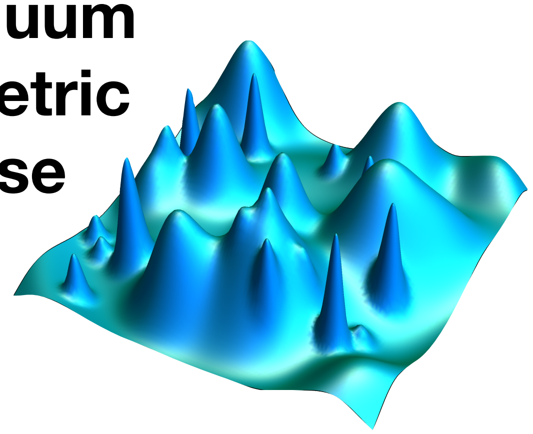
discrete

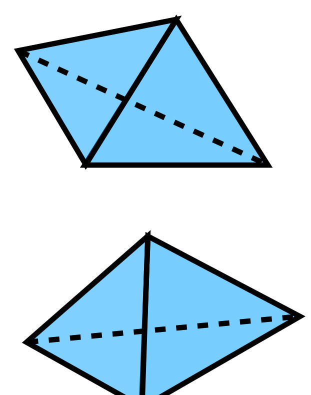
Asymptotic safety & dynamical triangulations: Two sides of the same (mathematical) phase transition?

(same configuration spaces?)

Testing: Comparison of universal scaling exponents

continuum geometric phase





discrete "pregeometric" phase

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interaction strength

FRG for tensor models in practice

$$\partial_N \Gamma_N = \frac{1}{2} \mathrm{Tr} \left[\left(\frac{\partial^2 \Gamma_N}{\partial T^2} + R_N \right)^{-1} \partial_N R_N \right]$$

- **background independence implies no mass-dimensionality**

  $\longrightarrow$ **rhs of flow equation determines ``canonical" scaling dimension**

- **rhs of flow equation generates all couplings compatible with symmetries**

  $\longrightarrow$ **truncations of theory space required in practice**

- **regulator breaks O(N) or U(N) symmetry** (no free lunch for coarse graining in QG?)

  $\longrightarrow$ **projection onto couplings not based on combinatorial structure of interactions alone**

  $\longrightarrow$ **Ward-identities imposed in enlarged theory space**

Canonical scaling dimension
in background-independent RG flow

local flows

background-independent flows

Canonical scaling dimension in background-independent RG flow

local flows

RG step:
local averaging
scale transformation

***canonical scaling* dimensions**
(at origin in coupling space)
defined by *mass* dimensionality

background-independent flows

RG step:
non-local averaging

no units, no scales,
no a priori scaling dimensions

Canonical scaling dimension in background-independent RG flow

local flows

RG step:
local averaging
scale transformation

canonical scaling **dimensions**
(at origin in coupling space)
defined by *mass* dimensionality

agrees with dimensions from
autonomous system of β functions

$$\beta_{\bar{G}} = -\# k^{(d-2)}\, \bar{G}^2 + ...$$

$$G = \bar{G} k^{d-2}$$

$$\beta_G = 2G - \# G^2 + ...$$

background-independent flows

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background-independent flows

RG step:
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no units, no scales,
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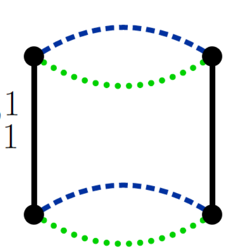
canonical dimensions from
autonomous system of β functions
in large N limit

$\bar{g}^{2,1}_{4,1}$



$$\beta_{\bar{g}^{2,1}_{4,1}} = 2\, N^2 \, \left(\bar{g}^{2,1}_{4,1}\right)^2 + ...$$

$$g^{2,1}_{4,1} = \bar{g}^{2,1}_{4,1} N^2$$

$$\beta_{g^{2,1}_{4,1}} = 2 g^{2,1}_{4,1} + 2 \left(g^{2,1}_{4,1}\right)^2 + ...$$

A bootstrap strategy to choose truncations

$$\partial_N \Gamma_N = \frac{1}{2} \mathrm{Tr} \left[\left(\frac{\partial^2 \Gamma_N}{\partial T^2} + R_N \right)^{-1} \partial_N R_N \right]$$

- **aim at quantitative determination of relevant critical exponents**

$\Longrightarrow$ **all relevant couplings at a fixed point should be included**

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- **aim at quantitative determination of relevant critical exponents**

$\Longrightarrow$ **all relevant couplings at a fixed point should be included**

1. **assume ordering principle in theory space (e.g., near-canonical)**
2. **include all couplings expected to be relevant & leading irrelevant ones**
3. **check ordering principle at fixed points in truncation**

A lightning review of FRG results in 2 and 3 dimensions

Benchmarks:

2d gravity

2d gravity with matter: multicritical points

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Benchmarks:

2d gravity

$\theta_1 = 0.8$ [Douglas, Shenker ’90; Brezin, Kazakov ’90; Gross, Migdal ’90]

TABLE I: Critical exponents θ_i and fixed point values at $\mathcal{O}(n)$ in the $\mathcal{P}^{-1}\mathcal{F}$ expansion.

n_{max}	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	g_4^*	$g_6^* \cdot 10^3$	$g_8^* \cdot 10^4$	$g_{10}^* \cdot 10^5$	$g_{12}^* \cdot 10^5$	$g_{14}^* \cdot 10^6$
2	1.066						-0.1005					
3	1.046	-1.080					-0.0722	-3.8				
4	1.036	-1.053	-2.14				-0.0563	-4.6	-3.6			
5	1.029	-1.037	-2.115	-3.171			-0.0461	-4.7	-5.4	-5		
6	1.025	-1.027	-2.10	-3.137	-4.197		-0.0390	-4.5	-6.2	-8	-1	
7	1.022	-1.020	-2.093	-3.110	-4.172	-5.213	-0.0338	-4.2	-6.5	-10	-1	-1.5

$$g_i \mathrm{Tr}(\phi^i)$$

[AE, Koslowski, ’13]

2d gravity with matter: multicritical points

[Kazakov, ’89] $\theta_{m-1} = \dfrac{2}{m+\frac{1}{2}}$

g_4	g_6	g_8	g_{10}	g_{12}	g_{22}	g_{24}	g_{26}	g_{28}	g_{44}	g_{46}	g_{222}	g_{224}	θ_1	θ_2	θ_3	θ_4	θ_5
$-\frac{1}{4x}$	0	0	0	0	0	0	0	0	0	0	0	0	1	-1/2	-1	-1	-3/2
$-\frac{8}{21x}$	$\frac{2}{63x^2}$	0	0	0	$\frac{1}{21x}$	0	0	0	0	0	0	0	0.82	0.82	-0.33	-0.64	-0.67
$-\frac{51}{116x}$	$\frac{27}{464x^2}$	$-\frac{9}{3712x^3}$	0	0	$\frac{15}{232x}$	$-\frac{3}{464x^2}$	0	0	0	0	0	0	0.80	0.80	0.61	-0.25	-0.5
$\frac{-16448}{34535x}$	$\frac{13392}{172675x^2}$	$\frac{-4608}{863375x^3}$	$\frac{576}{4316875x^4}$	0	$\frac{2634}{34535x}$	$\frac{-2112}{172675x^2}$	$\frac{288}{863375x^3}$	0	$\frac{108}{172675x^2}$	0	$\frac{144}{863375x^3}$	0	0.81	0.81	0.68	0.45	-0.2

$$g_{i,j,k} \mathrm{Tr}(\phi^i)\, \mathrm{Tr}(\phi^j)\, \mathrm{Tr}(\phi^k)$$

[AE, Koslowski, ’14]

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7	1.022	-1.020	-2.093	-3.110	-4.172	-5.213	-0.0338	-4.2	-6.5	-10	-1	-1.5

$g_i \mathrm{Tr}(\phi^i)$

reasonable results:
simple truncations sufficient

[AE, Koslowski, ’13]

2d gravity with matter: multicritical points

[Kazakov, ’89] $\theta_{m-1} = \frac{2}{m+\frac{1}{2}}$

g_4	g_6	g_8	g_{10}	g_{12}	g_{22}	g_{24}	g_{26}	g_{28}	g_{44}	g_{46}	g_{222}	g_{224}	θ_1	θ_2	θ_3	θ_4	θ_5
$-\frac{1}{4x}$	0	0	0	0	0	0	0	0	0	0	0	0	1	-1/2	-1	-1	-3/2
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$\frac{-16448}{34535x}$	$\frac{13392}{172675x^2}$	$\frac{-4608}{863375x^3}$	$\frac{576}{4316875x^4}$	0	$\frac{2634}{34535x}$	$\frac{-2112}{172675x^2}$	$\frac{288}{863375x^3}$	0	$\frac{108}{172675x^2}$	0	$\frac{144}{863375x^3}$	0	0.81	0.81	0.68	0.45	-0.2

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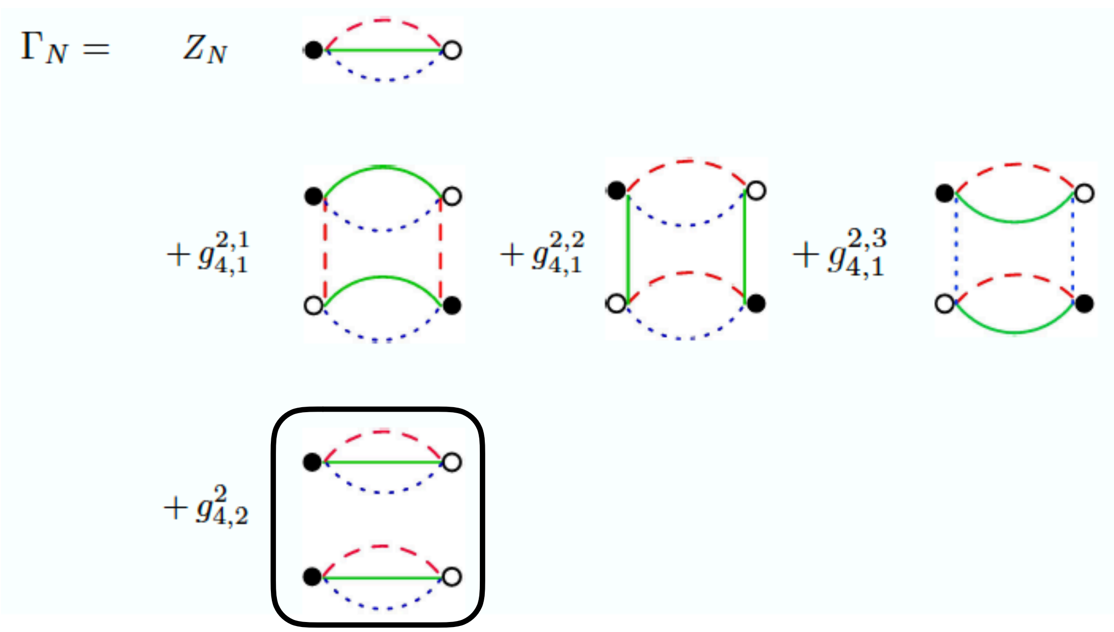
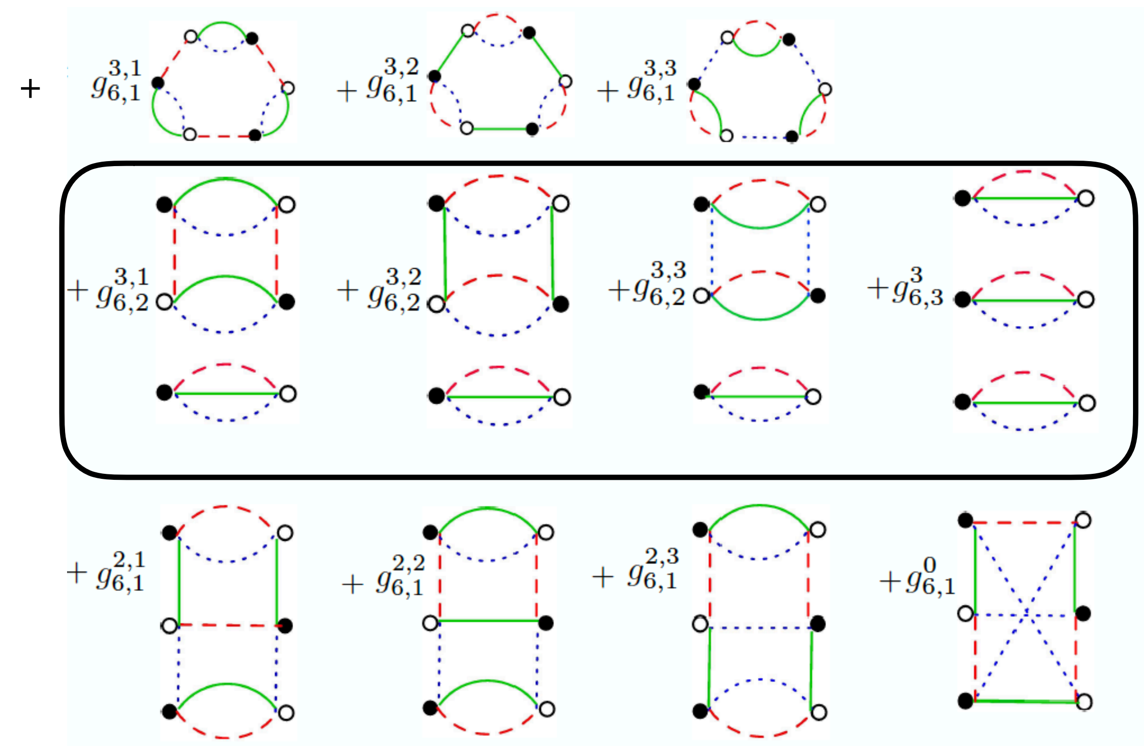
A lightning review of FRG results in 2 and 3 dimensions

rank 3 complex: [AE, Koslowski, '17]





dynamical decoupling unless additional symmetry

$g_{4,1}^{2,1} = g_{4,1}^{2,2} = g_{4,1}^{2,3}$ **is imposed**

$$\beta_{g_{4,2}^2} = (3+2\eta)g_{4,2}^2 + \frac{6-\eta}{15}\Big(\left(g_{4,2}^2\right)^2 + 2g_{4,2}^2\,\left(g_{4,1}^{2,1}+g_{4,1}^{2,2}+g_{4,1}^{2,3}\right) + \boxed{2\,g_{4,1}^{2,1}\,g_{4,1}^{2,2} + 2\,g_{4,1}^{2,1}\,g_{4,1}^{2,3} + 2\,g_{4,1}^{2,2}\,g_{4,1}^{2,3}}\Big) + \ldots$$

shifted Gaussian fixed point (= interacting)

A lightning review of FRG results in 2 and 3 dimensions

rank 3 complex: [AE, Koslowski, '17]

cyclic melonic

different approximations for anomalous dimension

	scheme	$g^{2,1}_{4,1*}$	$g^{2}_{4,2*}$	$g^{3,1}_{6,1*}$	$g^{2,1}_{6,1*}$	$g^{3,1}_{6,2*}$	$g^{3}_{6,3*}$
quartic	full	-1.94	0	–	–	–	–
	semi-pert	-2.14	0	–	–	–	–
	pert	-4.62	0	–	–	–	–
hexic	full	-1.37	0	-2.14	0	0	0
	semi-pert	-1.47	0	-2.46	0	0	0
	pert	-2.14	0	-6.12	0	0	0

	scheme	θ_1	θ_2	θ_3
quartic	full	2.21	-0.24	-0.93
	semi-pert	2	-0.21	-0.68
	pert	2	0.69	-2
hexic	full	2.14	-0.59	-1.26
	semi-pert	2	-0.58	-1.26
	pert	2	-0.35	-2

A lightning review of FRG results in 2 and 3 dimensions

rank 3 complex:

[AE, Koslowski, ’17]

cyclic melonic (dimensional reduction)

different approximations for anomalous dimension

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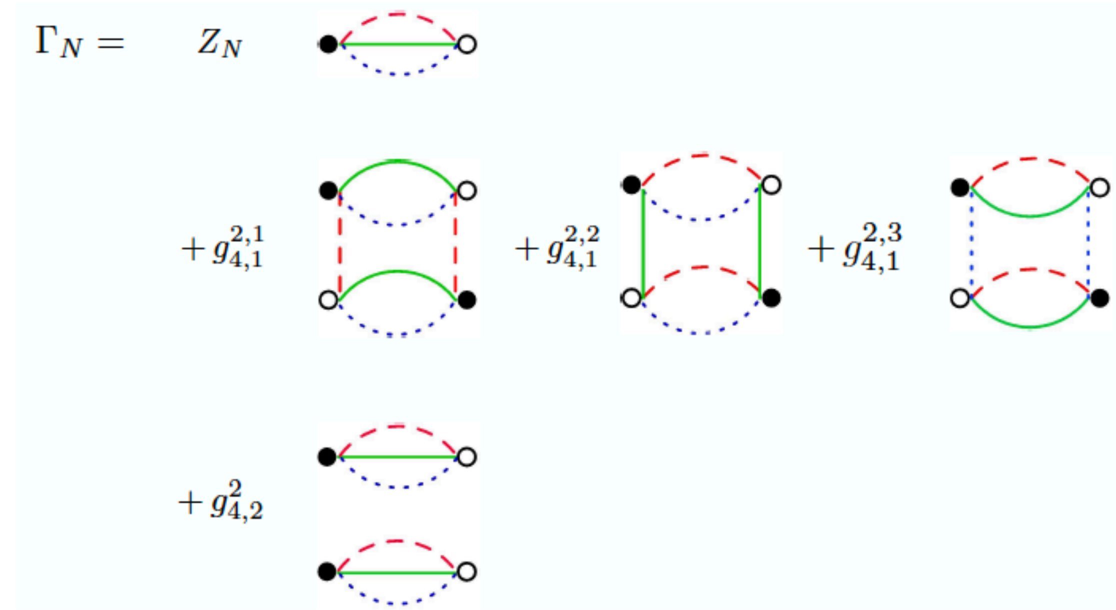

A lightning review of FRG results in 2 and 3 dimensions

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full	-1.94	0	–	–	–	–	quartic
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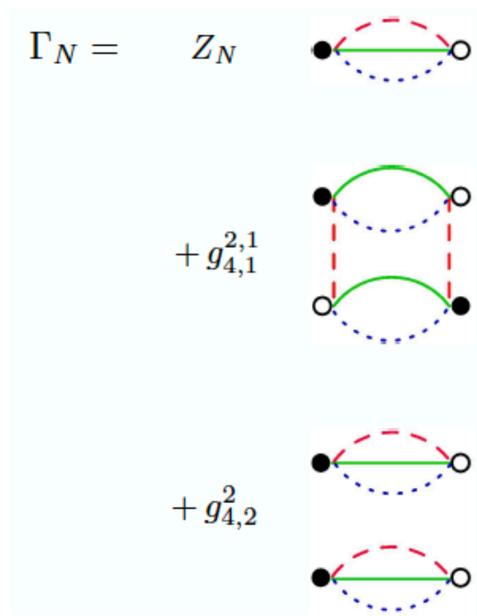

matrix-model like

(matrix-model critical exponent from tensor model: AE, Koslowski, Lumma, Pereira: to appear)

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[AE, Koslowski, '17]

cyclic melonic (dimensional reduction)

different approximations for anomalous dimension

	scheme	$g^{2,1}_{4,1*}$	$g^{2}_{4,2*}$	$g^{3,1}_{6,1*}$	$g^{2,1}_{6,1*}$	$g^{3,1}_{6,2*}$	$g^{3}_{6,3*}$
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cyclic melonic meets multitrace (dimensional reduction?)

scheme	$g^{2,1}_{4,1}$	$g^{2}_{4,2}$	$g^{3,1}_{6,1}$	$g^{2,1}_{6,1}$	$g^{3,1}_{6,2}$	$g^{3}_{6,3}$
full	-1.05	-1.33	–	–	–	–
semi-pert	-1.58	-1.05	–	–	–	–
pert	-4.62	1.73	–	–	–	–
full	-0.53	-2.11	-0.14	0	-0.39	–
semi-pert	-0.63	-2.35	-0.20	0	-0.57	–
pert	-2.01	-1.64	-4.43	0	-4.84	–
full	-1.04	-0.84	-0.97	0	-0.64	-0.99
semi-pert	-1.15	-0.89	-1.21	0	-0.78	-1.14
pert	-2.10	-0.54	-5.54	0	-1.68	-0.47

scheme	θ_1	θ_2	θ_3
full	2.64	0.14	-0.65
semi-pert	2.28	0.14	-0.68
pert	2	-0.69	-2
full	3.10	0.25	-0.48
semi-pert	2.68	0.26	-0.51
pert	2.11	0.34	-2
full	2.56	0.44	-0.97
semi-pert	2.30	0.44	-0.98
pert	2.04	0.37	-2

A lightning review of FRG results in 2 and 3 dimensions

rank 3 complex:

[AE, Koslowski, '17]

cyclic melonic (dimensional reduction)

different approximations for anomalous dimension

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quartic

hexic

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quartic

hexic

rank 3 real: talk by Johannes Lumma

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semi-pert	-1.58	-1.05	–	–	–	–
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full	-0.53	-2.11	-0.14	0	-0.39	–
semi-pert	-0.63	-2.35	-0.20	0	-0.57	–
pert	-2.01	-1.64	-4.43	0	-4.84	–
full	-1.04	-0.84	-0.97	0	-0.64	-0.99
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pert	2.04	0.37	-2

Summary

motivation: continuum limit generates predictivity

functional RG: tool to search for universal continuum limit

local coarse graining with background:	non-local coarse graining without background:
local QFT of the metric	tensor models

RG in background-independent case:

- determines canonical scaling dimensions in large N limit
- reproduces benchmark results in 2d models
- provides candidates for universality classes in rank 3

Outlook

- rank 4 (real and complex)
- connection to CDTs/EDTs
- emergent geometries?