Phase Diagram and Fixed Points of Tensorial Gross-Neveu Models in Three Dimensions

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(based on [arXiv:1810.04583] with D. Benedetti)

Motivations

Tensor models originally developed for discretizing (piecewise-linear) $d \geq 3$ manifolds. Proper quantum theory, they appear now as a toy model for holography ($nAdS_2/nCFT_1$). Hence, they might have something to teach us for the gauge/gravity duality.

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(2) Study non-trivial tensorial CFTs. [Klebanov et al. '16 – '18]

(3) Extend the higher-spin/vector model duality. [Vasiliev '18]

Higher dimensional SYK-like models: a summary

$$\mathcal{L} \sim \Phi K \Phi + g \Phi^q$$

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[Ferrari et al '17]: large-D limit of D matrices X_μ, after planar limit (fermions, bosons, susy)

Setup

We consider a 3d generalization of Gross-Neveu model, with Dirac fields ψ_{abc} $(1 \leq a, b, c \leq N)$.
We opt for a 4 dimensional representation of 3d Clifford algebra with Euclidean signature:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi_i \text{ are 2 component complex spinors,}$$

$$\gamma^\mu = \begin{pmatrix} \tilde{\gamma}^\mu & 0 \\ 0 & -\tilde{\gamma}^\mu \end{pmatrix},$$

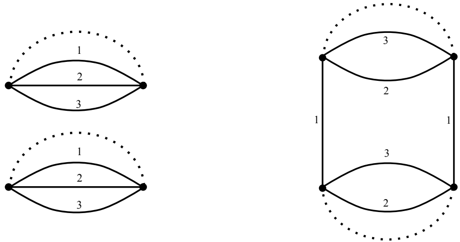
$$\tilde{\gamma}^1 = \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tilde{\gamma}^2 = \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tilde{\gamma}^3 = \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

which allows

$$\gamma^5 = \begin{pmatrix} & \mathbb{1} \\ \mathbb{1} & \end{pmatrix}, \quad \{\gamma^\mu, \gamma^5\} = 0, \quad (\gamma^5)^2 = \mathbb{1}.$$

Model I: single colored

$$S[\psi,\bar\psi] = \int d^3x\ \bar\psi_{abc}\partial\!\!\!/\psi_{abc} - \frac{\lambda}{N^3}\int d^3x\ (\bar\psi_{abc}\psi_{abc})^2 - \frac{\lambda_p}{N^2}\int d^3x\ \bar\psi_{abc}\psi_{a'bc}\bar\psi_{a'b'c'}\psi_{ab'c'}$$



Symmetries:

- continuous: $\psi_{abc} \to U^{(1)}_{a'a}U^{(2)}_{b'b}U^{(3)}_{c'c}\psi_{abc}$, with $U^{(i)} \in U(N)$
- discrete: $\psi \to \gamma^5\psi \quad \bar\psi \to -\bar\psi\gamma^5$.

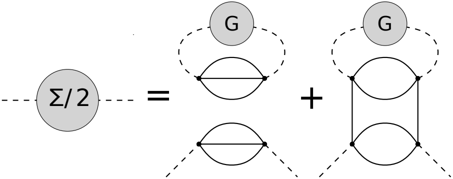
Model I: single colored

Assume unbroken $U(N)$ symmetry:

$$\langle\psi_{a_1a_2a_3}(x)\bar{\psi}_{b_1b_2b_3}(x')\rangle = G(x,x')\,\delta_{a_1b_1}\delta_{a_2b_2}\delta_{a_3b_3}\ ,$$

the large N SDE is

$$\hat{G}(p)^{-1} = i\not{p} - \Sigma(p) = i\not{p} + 2(\lambda+\lambda_p)\int \frac{\mathrm{d}^3 q}{(2\pi)^3}\,\mathrm{tr}\Big[\hat{G}(q)\Big]\mathbb{1}\ .$$



Introducing: $\Sigma = -m\mathbb{1}$, the SDE leads to

$$m = 2(\lambda+\lambda_p)\int_{|q|<\Lambda}\frac{\mathrm{d}^3 q}{(2\pi)^3}\,\frac{\mathrm{tr}\big(-i\not{q}+m\big)}{q^2+m^2}\ .$$

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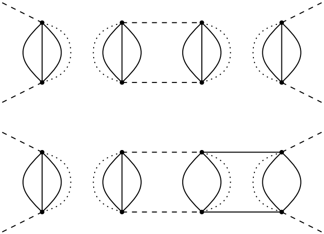
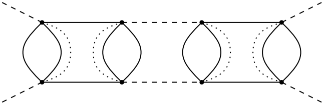
$$\beta = \tilde{\lambda} - \kappa\left(\tilde{\lambda}^2 + 2\tilde{\lambda}\tilde{\lambda}_p\right) \qquad \left(\kappa = 4/\pi^2\right)$$
$$\beta_p = \tilde{\lambda}_p - \kappa\tilde{\lambda}_p^2$$





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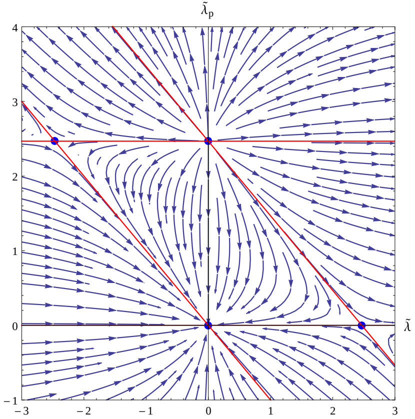

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Introduce *intermediate field* Hermitian matrix M_{ij}:

$$S_{\rm int}[\psi,\bar{\psi},M] = \frac{1}{2}\left[\mathrm{Tr}\left(M^2\right) + \frac{b}{(1-b)N}(\mathrm{Tr}\,M)^2\right] + \frac{\sqrt{2\lambda_p}}{N}\bar{\psi}_{ibc}\psi_{jbc}M_{ij},$$

$b \equiv \frac{-\lambda}{\lambda_p}$, integrate out the fermions:

$$S[M] = \int \frac{1}{2}\left[\mathrm{Tr}\left(M^2\right) + \frac{b}{(1-b)N}(\mathrm{Tr}\,M)^2\right] - N^2\,\mathrm{tr}\,\mathrm{Tr}\left[\ln\left(\partial\!\!\!/ + \frac{\sqrt{2\lambda_p}}{N}M\right)\right].$$

With dimensionless variables $\tilde{M} \equiv \frac{\sqrt{2\lambda_p}M}{\Lambda}, \tilde{\lambda} \equiv \lambda\Lambda, \tilde{\lambda}_p \equiv \lambda_p\Lambda$,
diagonalize M:

$$V_{\rm eff}[\{\mu_i\}] = \sum_i \left[\frac{1}{4\tilde{\lambda}_p}\mu_i^2 + \frac{1}{3\pi^2}\kappa(\mu_i)\right] - \frac{\tilde{\lambda}}{4\tilde{\lambda}_p(\tilde{\lambda}+\tilde{\lambda}_p)N}\left(\sum_i \mu_i\right)^2,$$

$$\kappa(\mu) = 2\mu^3\arctan\frac{1}{\mu} - 2\mu^2 - \log\left(1+\mu^2\right).$$

Model I: single colored

Analyze the Hessian close to the trivial vacuum

$$\frac{\partial^2 V}{\partial \tilde{M}_{ij} \partial \tilde{M}_{kl}} = \alpha(1-P)_{ij,kl} + \beta P_{ij,kl} \ ,$$

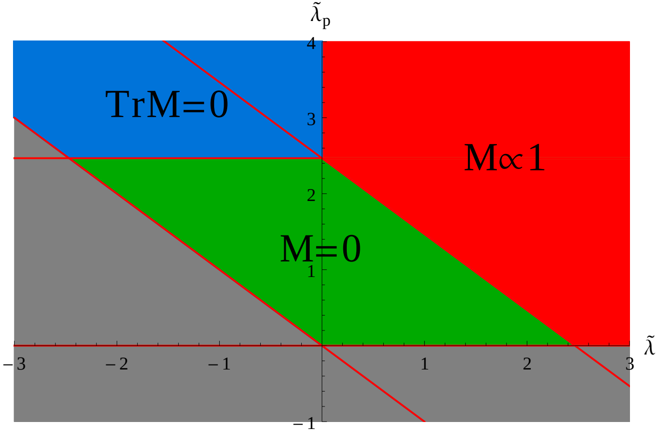
$$\alpha = \frac{1}{2}\left(\frac{1}{\tilde{\lambda}_p} - \frac{4}{\pi^2}\right) \ , \quad \beta = \frac{1}{2}\left(\frac{1}{\tilde{\lambda} + \tilde{\lambda}_p} - \frac{4}{\pi^2}\right) \ , \quad P_{ij,kl} \equiv \frac{\delta_{ij}\delta_{kl}}{N} \ ,$$

which diagonalizes for

- $\mathrm{tr}\, M = 0$, with eigenvalue α,
- $M \propto \mathbb{1}$, with eigenvalue β.

Model I: single colored

Phase diagram (after studying the equations of motion)



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- Phase with broken chiral and $U(N)$ symmetries.
- Phase transitions of first order between non-trivial phases, second order otherwise.
- SDE assuming broken $U(N)$ is compatible with above β functions.
- Anomalous dimension at the non-trivial fixed points:

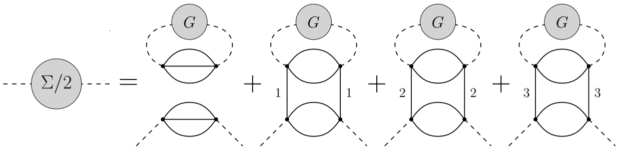
$$\langle M^*_{ij}(p) M_{kl}(-p) \rangle = \frac{4}{N^3 p} \delta_{ik} \delta_{jl} \ ,$$

same as usual Gross-Neveu.

Model II: color symmetric

$$S_{\text{int}}[\psi, \bar{\psi}] = -\frac{\lambda}{N^3} \int d^3x \, (\bar{\psi}_{abc}\psi_{abc})^2 - \frac{\lambda_p}{N^2} \sum_{\ell=1}^{3} \mathcal{P}_\ell[\psi, \bar{\psi}]$$

$$G^{-1}(p) = G_0^{-1}(p) + 2 \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \,\mathrm{tr} \left[\frac{\lambda}{N^3} \,\mathrm{Tr}\, G(q) + \frac{\lambda_p}{N^2} \left(\mathrm{Tr}_{\backslash 1} G(q) + \mathrm{Tr}_{\backslash 2} G(q) + \mathrm{Tr}_{\backslash 3} G(q) \right) \right]$$



Model II: color symmetric

1) Assuming $U(N)$ symmetry is unbroken, a diagonal ansatz $\hat{G}(p)^{-1} = i\not{p} + m$ leads to

$$m = 2(\lambda + 3\lambda_p) \int_{|q|<\Lambda} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{\mathrm{tr}\left(-i\not{q} + m\right)}{q^2 + m^2} \ .$$

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2) Assuming $U(N)^3$ breaking into $U(N/2)^2 \times U(N)^2$, the following ansatz:

$$M_1 = m(\mathbb{1}_N - 2\mathbb{P}_N) \; , \quad \mathbb{P}_N = \begin{pmatrix} \mathbb{0}_{N/2} & \mathbb{0}_{N/2} \\ \mathbb{0}_{N/2} & \mathbb{1}_{N/2} \end{pmatrix} ,$$

$$G^{-1}(p) = i\not{p}\, \mathbb{1}_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_3 + M_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_3 \; ,$$

leads to

$$m = 8\lambda_p \int_{|q|<\Lambda} \frac{\mathrm{d}^3 q}{(2\pi)^3} \frac{m}{p^2 + m^2} \; .$$

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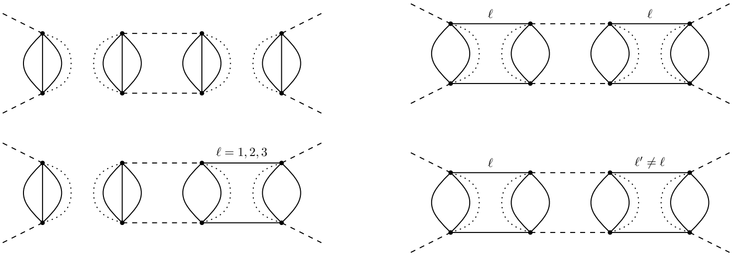

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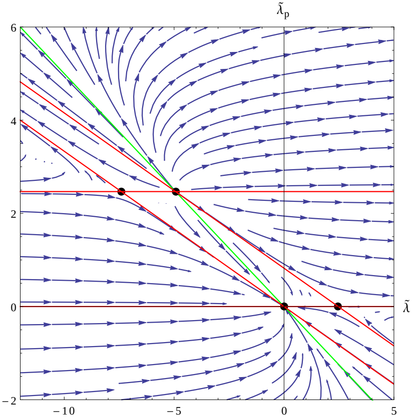

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Effective potential

$$S[M_1, M_2, M_3] = \int \frac{1}{2} \sum_{c=1,2,3} \left[\mathrm{Tr}\left(M_c^2\right) + \frac{b}{(1-b)N} (\mathrm{Tr}\, M_c)^2 \right]$$
$$- \mathrm{tr}\, \mathrm{Tr} \left[\ln \left(\partial\!\!\!/ + \frac{\sqrt{2\lambda_p}}{N} R \right) \right],$$

$$R \equiv (M_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_3) + (\mathbb{1}_1 \otimes M_2 \otimes \mathbb{1}_3) + (\mathbb{1}_1 \otimes \mathbb{1}_2 \otimes M_3) \,, \quad b = -\frac{\lambda}{3\lambda_p},$$

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$$V\left[\{\mu_{1,i}, \mu_{2,j}, \mu_{3,k}\}\right] \equiv \frac{S\left[\tilde{M}_1, \tilde{M}_2, \tilde{M}_3\right]\Big|_{\tilde{M}_i = \mathrm{const.}}}{N^3 \Lambda^3 \mathrm{Vol}}$$

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$$V\left[\{\mu_{1,i}, \mu_{2,j}, \mu_{3,k}\}\right] = \frac{1}{N} \sum_c \frac{1}{4\tilde{\lambda}_p} \left[\sum_i \mu_{c,i}^2 - \frac{\tilde{\lambda}}{(\tilde{\lambda} + 3\tilde{\lambda}_p)N} \left(\sum_i \mu_{c,i} \right)^2 \right]$$
$$+ \frac{1}{N^3} \sum_{1 \le i,j,k \le N} \frac{1}{3\pi^2} \kappa(\mu_{1,i}, \mu_{2,j}, \mu_{3,k})\,,$$

$$\kappa(\mu_{1,i}, \mu_{2,j}, \mu_{3,k}) = 2\mu^3 \arctan\frac{1}{\mu} - 2\mu^2 - \log\left(1 + \mu^2\right)\,, \quad \mu \equiv \mu_{1,i} + \mu_{2,j} + \mu_{3,k}\,.$$

Model II: color symmetric

Hessian in color space:

$$\frac{\partial^2 V}{\partial M_{c,ij}\partial M_{c',kl}} = \begin{pmatrix} \alpha(1-P_1)+\beta P_1 & C\frac{\Pi_1\Pi_2}{N} & C\frac{\Pi_1\Pi_3}{N} \\ C\frac{\Pi_2\Pi_1}{N} & \alpha(1-P_2)+\beta P_2 & C\frac{\Pi_2\Pi_3}{N} \\ C\frac{\Pi_3\Pi_1}{N} & C\frac{\Pi_3\Pi_2}{N} & \alpha(1-P_3)+\beta P_3 \end{pmatrix}_{ij,kl} ,$$

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$$P_{c;ij,kl} \equiv \frac{1}{N}\Pi_{c,ij}\Pi_{c,kl}, \quad \Pi_{c,ij} \equiv \delta^{(c)}_{ij}.$$

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is diagonal for:

- traceless matrices

$$E^1 = \begin{pmatrix} Q \\ 0 \\ 0 \end{pmatrix}, \quad E^2 = \begin{pmatrix} 0 \\ Q \\ 0 \end{pmatrix}, \quad E^3 = \begin{pmatrix} 0 \\ 0 \\ Q \end{pmatrix}, \quad \mathrm{Tr}\, Q = 0,$$

  with eigenvalue $\alpha = \frac{1}{2}\left(\frac{1}{\tilde{\lambda}_p} - \frac{4}{\pi^2}\right)$,

- matrices proportional to the identity, of the form

$$E^s = \gamma \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \text{with eigenvalue } \frac{3}{2}\left(\frac{1}{\tilde{\lambda}+3\tilde{\lambda}_p} - \frac{4}{\pi^2}\right) .$$

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[Those are local minima of V.]

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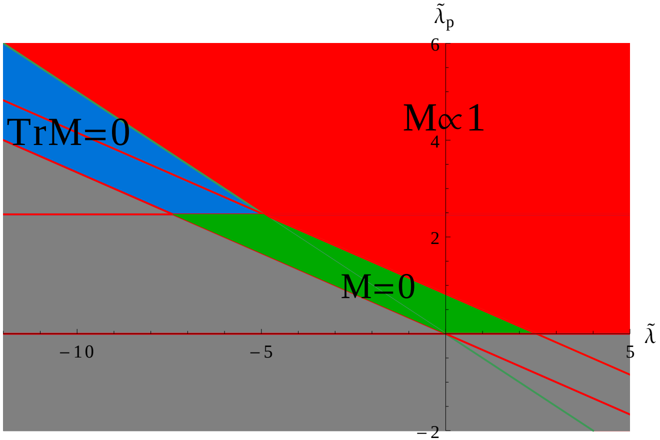
Phase diagram (after studying the equations of motion)



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- Next problem: gauge the $U(N)$ symmetry (differs from standard vector model analysis)
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- Include tetrahedron interaction?
  $\rightarrow$ 2PI effective action [Benedetti, Gurau '18],
  $\rightarrow$ $Sp(N)$ [Carrozza, Pozgay '18].