Invitation to random tensors and tensor field theories

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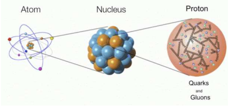
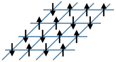
Fundamental interactions in nature (electroweak and strong interaction)



Condensed matter (Ising spins, Fermi liquids, etc.)



One of the best prediction in physics: electron anomalous magnetic moment
0.001 159 652 18(1)

Worst prediction in physics: overestimates the cosmological constant 10^{120} times

A QFT

Fields $\phi_A(x) \to$ random functions of $x \in \mathbb{R}^d$

Symmetries $\to$ group $G = \{g\}$, acts on the fields $\phi \to g\phi$

Action $S[\phi] \to$ functional of ϕ invariant under G, $S[g\phi] = S[\phi]$

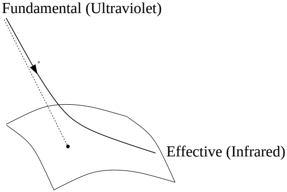
Observables $\to$ correlations (probabilities of joint detection)

$$\langle \phi_{A_1}(x_1) \ldots \phi_{A_n}(x_n) \rangle = \frac{1}{Z} \int [d\phi] \; e^{-S[\phi]} \; \phi_{A_1}(x_1) \ldots \phi_{A_n}(x_n)$$

RENORMALIZATION

Physics changes with the energy scale:

Renormalization Group



Flow in the space of theories (“time” $\sim$ energy scale) [Polchinski ’84; Berges, Tetradis, Wetterich ’02; ...]

Weak versus strong coupling

Weak coupling

Perturbation theory

$\rightarrow$ electron anomalous magnetic moment

Strong coupling

?

- derivative expansion: no small parameter
- $4 - \epsilon$ expansions: ϵ not small
- lattice (numerics): refinement limit

The goal

Solvable strongly coupled quantum field theories

Conformal Field Theories (CFT)

Solvable strongly coupled theories $\sim$ conformally invariant (1980s onward)

- fixed points of the renormalization group, phase transitions
- universal (same CFT for many physical systems)

Limited $\rightarrow$ perturbations **break conformal invariance**

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Do more!

Solvable strongly coupled theories **outside** the conformal limit

Beyond CFT

Vector with N components (spherical model, $O(N)$ model)

Matrix with N^2 entries (Quantum Chromodynamics $SU(N)$, $N = 3$)

Access the strong coupling regime in a $1/N$ expansion!

Vectors – large N limit solvable but too restrictive in any dimension

[Berlin, Kac '52; Stanley '68;...]

Matrices – planar limit very difficult in more than zero dimensions

['t Hooft '74;...]

How about tensor fields?

Proposals in the '90 [Sasakura '90; Ambjørn et al. '90; Boulatov '92; Ooguri '92,...]

New $1/N$ **expansion** and a new **melonic large** N **limit**

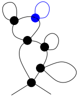
[Gurau '10 '11; Gurau, Rivasseau '11, Bonzom, Gurau, Riello, Rivasseau '11,...]

New family of strongly coupled CFTs in the infrared

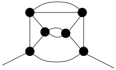
[Witten '16; Klebanov et al. '17,'18; Minwalla et al. '17; Tseytlin et al. '17; Ferrari et al. '17; Vasiliev '18...]

The surprise

Vectors – simple
"snails": local insertions



Matrices – complicated
planar



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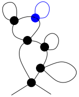

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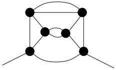
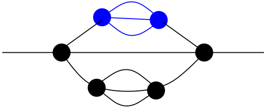


Tensors – in between
"melonic": recursive bilocal insertions



The vision

Tensor Field Theories:

Non trivial strongly coupled theories solvable order by order in $1/N$
outside the conformal limit

The field, the symmetry

- This talk - real, non symmetric, many fields
- We also have complex, with symmetries, with fewer fields, etc.

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- This talk - real, non symmetric, many fields
- We also have complex, with symmetries, with fewer fields, etc.

Fields $\to$ q real tensor fields $T^i_{A^i}(x)$ of rank $q-1$, $A^i = \{a^i_1, \dots a^i_q\} \setminus \{a^i_i\}$

$$\text{example } T^1_{a^1_2 a^1_3 a^1_4},\; T^2_{a^2_1 a^2_3 a^2_4},\; T^3_{a^3_1 a^3_2 a^3_4},\; T^4_{a^4_1 a^4_2 a^4_3}$$

Symmetry $\to$ global $O(N)^{\frac{q(q-1)}{2}}$, one for each pair of fields (ij).

$$T^i_{B^i}(x) = \sum_a O^{(i1)}_{b^i_1 a^i_1} \dots O^{(iq)}_{b^i_q a^i_q} T^i_{A^i}(x)$$

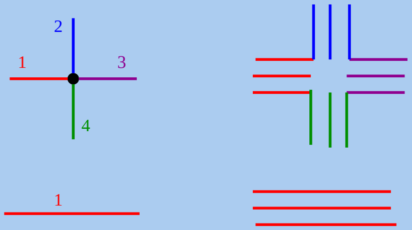
a^i_j (index j on T^i) and a^j_i (index i on T^j) turn with the same orthogonal

The action

a^i_j and a^j_i turn with the same orthogonal

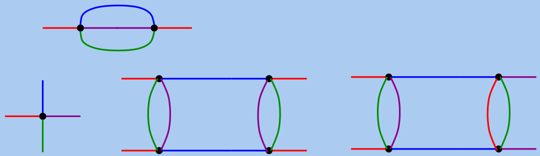
Action [Gurau '09; Witten '16] $\rightarrow$

$$S(T) = \frac{1}{2}\int_x T^i_{A^i}\underbrace{\left(-\Delta + m^2\right)}_{C^{-1}} T^i_{A^i} + \frac{\lambda}{N^{\frac{(q-1)(q-2)}{4}}}\int_x \prod_i T^i_{A^i}\prod_{ij}\delta_{a^i_j a^j_i}$$



$$\langle T^{i_1}_{A^{i_1}}(x_1)\dots T^{i_n}_{A^{i_n}}(x_n)\rangle = \frac{1}{Z}\int [dT]\; e^{-S[T]}\, T^{i_1}_{A^{i_1}}(x_1)\dots T^{i_n}_{A^{i_n}}(x_n)$$

Edge colored graphs



Two point function is diagonal $\langle T^i_{A^i} T^j_{A^j}\rangle = \delta^{ij}\delta_{A^iB^j} G$

Four point function either $q = 4$ and all different, or pairwise equal

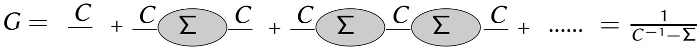
$$\langle T^1 T^2 T^3 T^4\rangle \neq 0 \;, \qquad \langle T^i T^i T^j T^j\rangle \neq 0$$

Schwinger Dyson equation (self energy Σ)

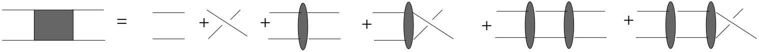
$$G = C + C\,\Sigma\,C + C\,\Sigma\,C\,\Sigma\,C + \ldots\ldots = \frac{1}{C^{-1} - \Sigma}$$

Correlation equations

Schwinger Dyson equation (self energy Σ)



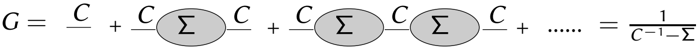
Four point Dyson equations (four point kernel $K = GG\frac{\delta\Sigma}{\delta G}$):



$$\langle T^i T^i T^j T^j \rangle_{i \to j} = 2GG + 2KGG + 2KGGKGG \cdots = 2\frac{1}{1-K}GG$$

Correlation equations

Schwinger Dyson equation (self energy Σ)



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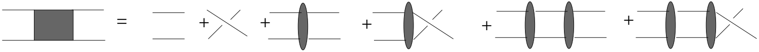

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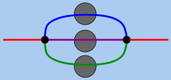
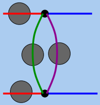
Bethe Salpeter equation for two particle bound state $\Phi(z) = T^j(\mathcal{O}T^j)(z)$

$$\langle T^i T^i \Phi \rangle_c = G(\mathcal{O}G) + (\mathcal{O}G)G + K\langle T^i T^i \Phi \rangle_c$$

Leading order



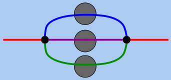
$$\Sigma(x,y) = \lambda^2 G(x,y)^{q-1}$$



$$K(xy;zt) = \lambda^2 G(x,z)G(y,t)G(z,t)^{q-2}$$

LARGE N LIMIT

Leading order



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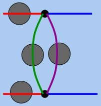

$$K(xy;zt) = \lambda^2 G(x,z)G(y,t)G(z,t)^{q-2}$$

Schwinger Dyson equation:

$$1 = C^{-1}G - G\left(\lambda^2 G^{q-1}\right)$$

Bethe Salpeter equation:

$$\left\langle T^i T^i \Phi \right\rangle_c = 2G\mathcal{O}G + \left(\lambda^2 GGG^{q-2}\right)\left\langle T^i T^i \Phi \right\rangle_c$$

The conformal limit (1)

CFT with primaries Φ_ν of dimension Δ_ν that is $\Phi_\nu(\lambda x) = \lambda^{-\Delta_\nu}\Phi_\nu(x)$

$$\langle \Phi_\mu(x)\Phi_\nu(y)\rangle \sim \frac{\delta_{\Delta_\mu \Delta_\nu}}{|x-y|^{\Delta_\mu+\Delta_\nu}}$$

The conformal limit (1)

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In the infrared the large N Schwinger Dyson equation admits a conformal solution:

$$1 = C^{-1}G - G\left(\lambda^2 G^{q-1}\right) \ , \qquad\qquad G \sim \frac{1}{|x-y|^{2\frac{d}{q}}}$$

T **primary operator with dimension** $\Delta = d/q$

THE CONFORMAL LIMIT (2)

$$\langle \Phi_\mu(x) \Phi_\nu(y) \Phi_\rho(z) \rangle \sim \frac{1}{|x-y|^{\Delta_\mu + \Delta_\nu - \Delta_\rho} |x-z|^{\Delta_\mu - \Delta_\nu + \Delta_\rho} |y-z|^{-\Delta_\mu + \Delta_\nu + \Delta_\rho}}$$

The conformal limit (2)

$$\langle \Phi_\mu(x)\Phi_\nu(y)\Phi_\rho(z)\rangle \sim \frac{1}{|x-y|^{\Delta_\mu+\Delta_\nu-\Delta_\rho}|x-z|^{\Delta_\mu-\Delta_\nu+\Delta_\rho}|y-z|^{-\Delta_\mu+\Delta_\nu+\Delta_\rho}}$$

In the infrared the large N Bethe Salpeter equation has conformal three point function solutions

$$\langle TT\Phi\rangle = 2G\mathcal{O}G + \left(\lambda^2 GGG^{q-2}\right)\langle TT\Phi\rangle \sim \frac{1}{|x-y|^{2\frac{d}{q}-h}|x-z|^{h}|y-z|^{h}}$$

with h solutions of (all real for q large enough and $d<4$)

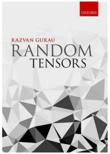
$$1 = -(q-1)\frac{\Gamma\left(\frac{d}{2}-\frac{d}{q}\right)\Gamma\left(d-\frac{d}{q}\right)\Gamma\left(-\frac{d}{2}+\frac{d}{q}+\frac{h}{2}\right)\Gamma\left(\frac{d}{q}-\frac{h}{2}\right)}{\Gamma\left(\frac{d}{q}-\frac{d}{2}\right)\Gamma\left(\frac{d}{q}\right)\Gamma\left(\frac{d}{2}-\frac{d}{q}+\frac{h}{2}\right)\Gamma\left(d-\frac{d}{q}-\frac{h}{2}\right)}$$

bilinear primary operators $\Phi_n = T\mathcal{O}_nT$ with dimensions h_n

The bottom line

Analytic framework to study random tensors and tensor field theories!

- $1/N$ expansion
- melonic large N limit
- universality, constructive results, classification of $1/N$ terms, algebra of constraints, double scaling, etc.



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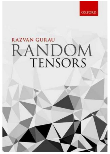

A new class of CFTs, melonic CFTs, waiting to be explored

Go beyond the conformal limit!