The Thouless time for mass deformed SYK

Dario Rosa

Korea Institute for Advanced Study (KIAS)

Based on: 1804.09934 with Tomoki Nosaka and Junggi Yoon

(see also 1709.06498 with Chethan Krishnan and Pavan Kumar)

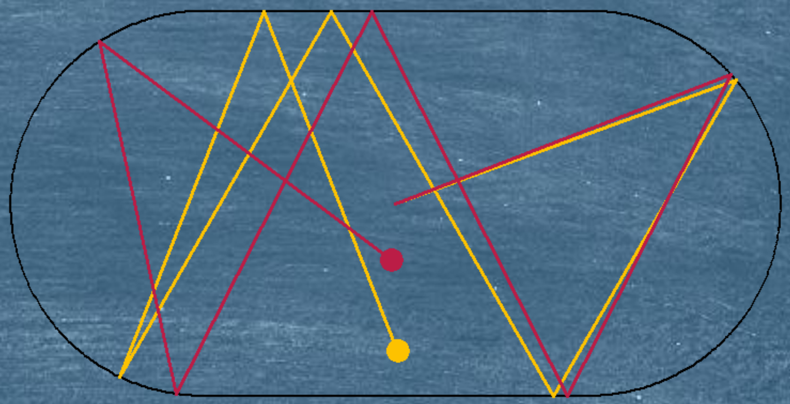
“Holographic Tensors”, OIST, 2-11-2018

Prelude: from classical to quantum chaos

Chaos in classical systems:

Systems which are highly sensitive

on initial conditions



Prelude: from classical to quantum chaos

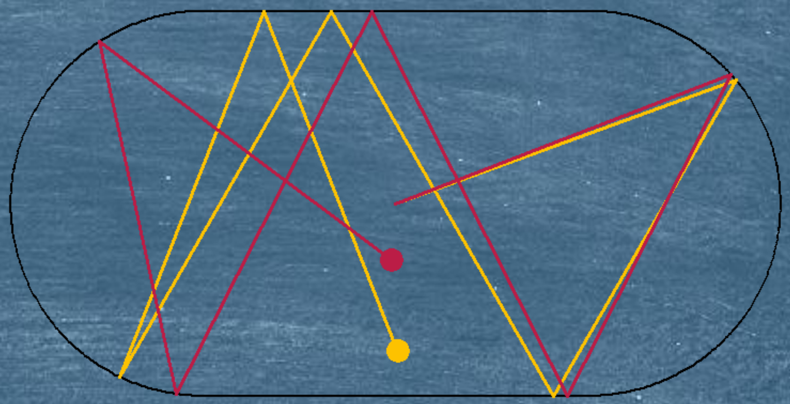
Chaos in classical systems:

$$\frac{\partial\, x(t)}{\partial\, x(0)} \propto e^{\mu_L\, t}$$

Lyapunov exponent

Systems which are highly sensitive

on initial conditions



Prelude: from classical to quantum chaos

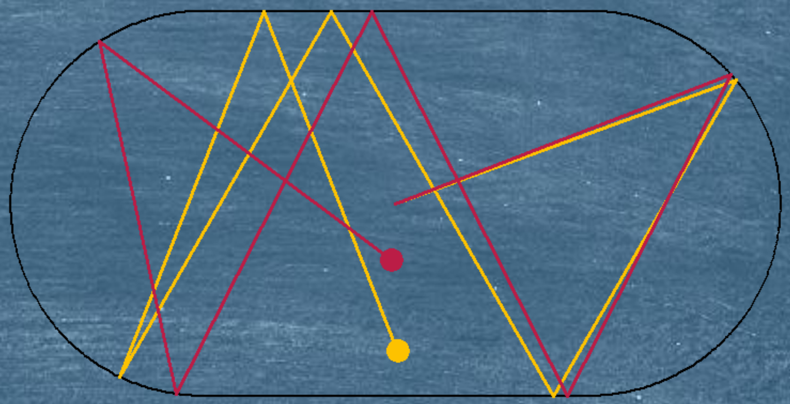
Chaos in classical systems:

$$\frac{\partial\, x(t)}{\partial\, x(0)} \propto e^{\mu_L\, t}$$

Lyapunov exponent

Systems which are highly sensitive

on initial conditions



Remark: This definition uses the notion of trajectory

The trouble with quantum chaos:

Classical notion of chaos relies on the concept of trajectory → Moving to the quantum regime is non-trivial

The trouble with quantum chaos:

Classical notion of chaos relies on the concept of trajectory → Moving to the quantum regime is non-trivial

1st approach: Semiclassical way [Larkin, Ovchinnikov '69]

Chaos in QM:

$$\frac{\partial\, x(t)}{\partial\, x(0)} = [x(t),\, p(0)]_{\text{pb}} \quad \longrightarrow \quad -\frac{i}{h}\,[\hat{q}(t),\, \hat{p}(0)] \propto e^{\mu_L\, t}$$

This definition can be extended to QFT

Semiclassical chaos in QFT [Kitaev, Shenker, Stanford, …]

Pick up 2 ''generic'' operators

(V, W) → Thermal correlator $$C(t) \equiv -\langle [W(t), V(0)]^2 \rangle_\beta$$

Semiclassical chaos in QFT [Kitaev, Shenker, Stanford, …]

Pick up 2 ''generic'' operators

(V, W) → Thermal correlator

$$C(t) \equiv -\langle [W(t),\, V(0)]^2 \rangle_\beta$$

1st chaos diagnostics:

- For chaotic systems we have: $C(t) \propto \hbar^2 e^{2\mu_L\, t} \qquad \hbar^2 \propto 1/N$

Semiclassical chaos in QFT [Kitaev, Shenker, Stanford, …]

Pick up 2 ''generic'' operators

(V, W) → Thermal correlator $C(t) \equiv -\langle [W(t), V(0)]^2 \rangle_\beta$

1st chaos diagnostics:

- For chaotic systems we have: $C(t) \propto \hbar^2 e^{2\mu_L\, t} \qquad \hbar^2 \propto 1/N$
- A first time-scale: "Scrambling time" $t_* \propto \frac{1}{\mu_L} \log N$

2^{nd} approach: Purely quantum [Bohighas-Giannoni-Schmit ‘84]

Based on fluctuations of the eigenvalues of the quantum Hamiltonian

2^{nd} approach: Purely quantum [Bohighas-Giannoni-Schmit '84]

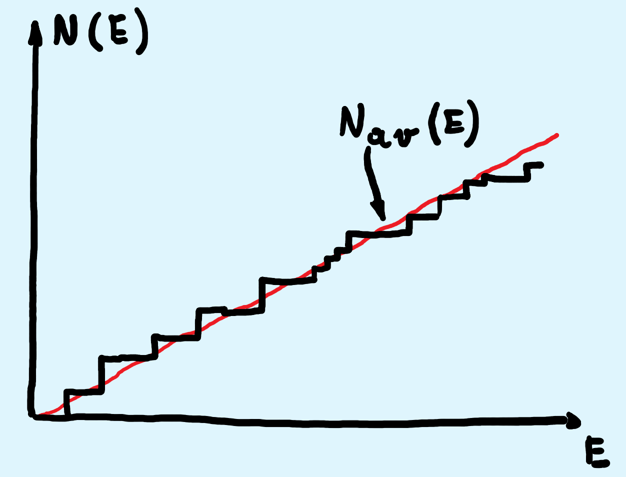
Based on fluctuations of the eigenvalues of the quantum Hamiltonian

Consider a certain quantum system with Hamiltonian $\mathcal{H}$ and energy levels $\{E_1, E_2, \ldots, E_N\}$

We construct the staircase function N(E): It counts the number of levels smaller than E

We separate N(E): average + fluctuations

$$N(E) = N_{\text{av}}(E) + N_{\text{fl}}(E)$$



2^{nd} approach: Purely quantum [Bohighas-Giannoni-Schmit ‘84]

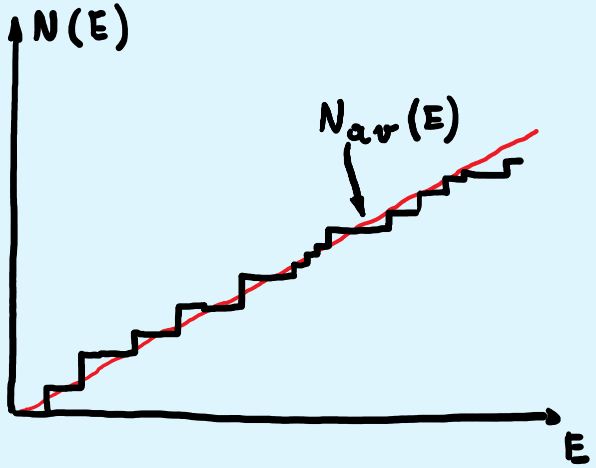
Based on fluctuations of the eigenvalues of the quantum Hamiltonian

Consider a certain quantum system with Hamiltonian $\mathcal{H}$ and energy levels $\{E_1, E_2, \ldots, E_N\}$

We construct the staircase function N(E): It counts the number of levels smaller than E

We separate N(E): average + fluctuations

$$N(E) = N_{\mathrm{av}}(E) + N_{\mathrm{fl}}(E)$$



Quantum chaos is defined in terms of the statistics of the fluctuation piece

Bohighas-Giannoni-Schmit's conjecture:

- Quantum chaotic systems show a distribution of the fluctuating piece, $N_{fl}(E)$, which reproduces the distribution of the eigenvalues obtained by Random Matrix Theory (RMT)

- They show Spectral Rigidity and Level Repulsion

Bohighas-Giannoni-Schmit's conjecture:

- Quantum chaotic systems show a distribution of the fluctuating piece, $N_{fl}(E)$, which reproduces the distribution of the eigenvalues obtained by Random Matrix Theory (RMT)
- They show Spectral Rigidity and Level Repulsion

How to compare and relate these two definitions of chaos?

Our strategy

- A variant of SYK: mass-deformed ⟹ chaos / integrable transition
- Quantify the chaos / integrable transition from both viewpoints:

  1) semiclassical: scrambling time ⟺ Lyapunov exponent

  2) RMT approach: time-scale? ⟹ Thouless time
- Matching ⟹ Connected unfolded spectral form factor in RMT

Plan of the talk

- Some basics on RMTs

- Mass deformed SYK: early time and late time chaos disagree?

- Solving the discrepancy: the connected unfolded SFF in RMT

Plan of the talk

- Some basics on RMTs
- Mass deformed SYK: early time and late time chaos disagree?
- Solving the discrepancy: the connected unfolded SFF in RMT

Outcome: in mass deformed SYK, the chaos / integrable transition affects the spectrum in-homogeneously. We need RMT observables focused on the low-lying modes

Some basics on RMTs

They are ensembles of random L x L matrices M_{ij}

3 principal examples: The Gaussian Ensembles, Hermitian matrices

The Gaussian Unitary Ensemble (GUE): M_{ij} are Complex

The Gaussian Orthogonal Ensemble (GOE): M_{ij} are Real

The Gaussian Symplectic Ensemble (GSE): M_{ij} are Quaternionic Real

Ensemble averages use a Gaussian weight $P(M) \propto \int d\, M_{ij}\, e^{-\frac{L}{2}\mathrm{Tr}(M^2)}$

It is convenient to consider the probability distribution of eigenvalues:

$$P(\{\Lambda\})\, d\{\Lambda\} \propto |\Delta(\{\Lambda\})|^{\alpha} \prod_k e^{-\frac{\alpha\, L\, \Lambda_k^2}{4}}\, d\lambda_k$$

Vandermonde determinant $\prod_{k>l}(\Lambda_k - \Lambda_l)$

$\alpha = 1 \rightarrow \mathrm{GOE}$

$\alpha = 2 \rightarrow \mathrm{GUE}$

$\alpha = 4 \rightarrow \mathrm{GSE}$

It is convenient to consider the probability distribution of eigenvalues:

$$P(\{\Lambda\})\, d\,\{\Lambda\} \propto |\Delta(\{\Lambda\})|^{\alpha} \prod_k e^{-\frac{\alpha\, L\, \Lambda_k^2}{4}}\, d\lambda_k$$

Vandermonde determinant $\prod_{k>l}(\Lambda_k - \Lambda_l)$

$\alpha = 1 \rightarrow \mathrm{GOE}$

$\alpha = 2 \rightarrow \mathrm{GUE}$

$\alpha = 4 \rightarrow \mathrm{GSE}$

We see a tension between the repulsive effect (Vandermonde) and the attractive effect (exponential):

Spectral Rigidity: This tension causes a rigid structure of the eigenvalues. Fluctuations (even at long range) are much more suppressed than in integrable models (Poisson like distributions)

Distinctive RMT features: the Level Repulsion

The Vandermonde makes very unlikely the presence of nearly degenerate eigenvalues

→

An RMT diagnostic:

Nearest Neighbor Distance Distribution (NNDD)

Distinctive RMT features: the Level Repulsion

The Vandermonde makes very unlikely the presence of nearly degenerate eigenvalues

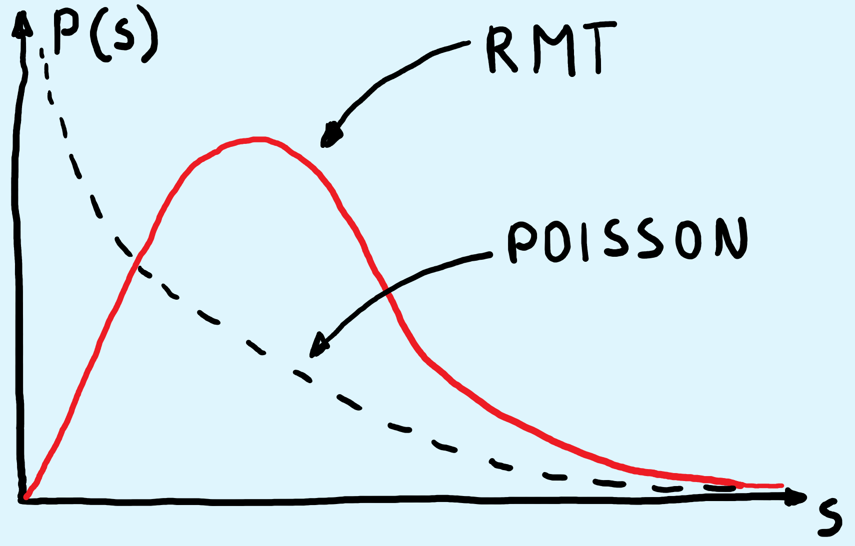
An RMT diagnostic:

Nearest Neighbor Distance Distribution (NNDD)

Ordered unfolded eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_L$

We construct the quantities $S_i = \lambda_i - \lambda_{i+1}$

The NNDD take a characteristic shape: the Wigner Surmise



Distinctive RMT features: the Level Repulsion

The Vandermonde makes very unlikely the presence of nearly degenerate eigenvalues

An RMT diagnostic:

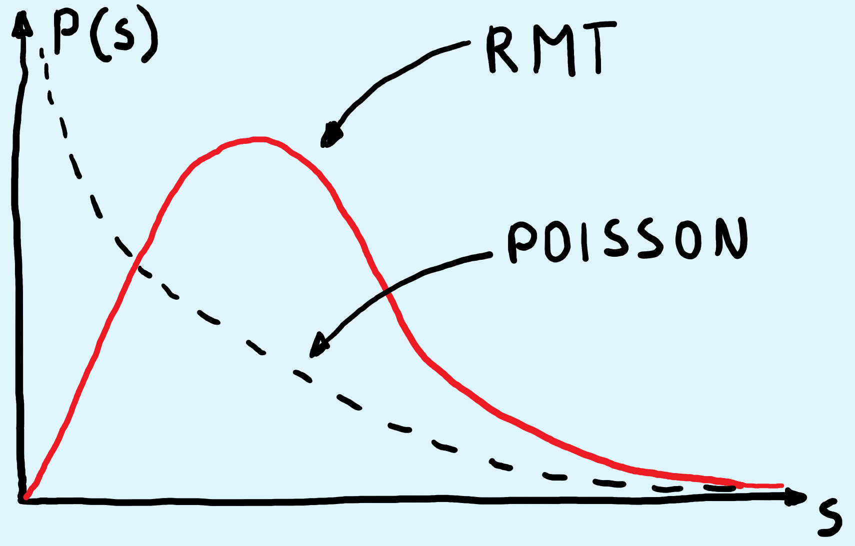
Nearest Neighbor Distance Distribution (NNDD)

Ordered unfolded eigenvalues $\lambda_1 > \lambda_2 > \cdots > \lambda_L$

We construct the quantities $S_i = \lambda_i - \lambda_{i+1}$

The level repulsion is very typical of RMT behavior: Rough but efficient test of chaos

The NNDD take a characteristic shape: the Wigner Surmise



Long range physics: the connected unfolded SFF

The NNDD statistics probes only very short-range aspects of the spectrum

We need long range: the connected unfolded Spectral Form Factor (CUSFF)

Long range physics: the connected unfolded SFF

The NNDD statistics probes only very short-range aspects of the spectrum

We need long range: the connected unfolded Spectral Form Factor (CUSFF)

1st step: single sample unfolded SFF

Inverse temperature:

UV cut-off

Unfolded eigenvalues

$$g(t,\,\beta) \equiv |Z(\beta,\,t)|^2 = \sum_{m,n} e^{-\beta(\lambda_m+\lambda_n)}\, e^{i\,(\lambda_m-\lambda_n)\,t}$$

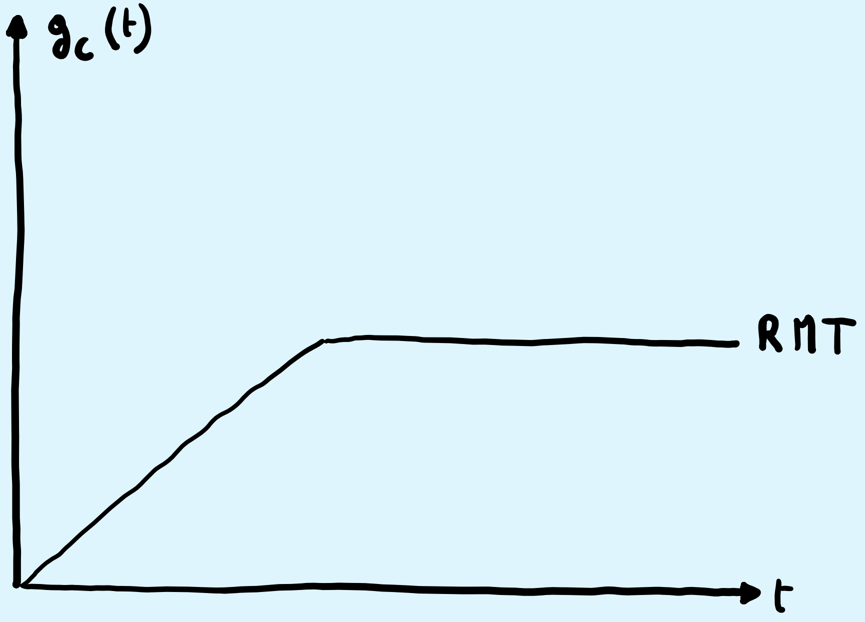
2nd step: ensemble average → connected unfolded SFF (CUFSS)

$$g_c(t,\, \beta) \equiv \frac{\langle | \sum_i e^{-(\beta - i\, t)\lambda_i} |^2 \rangle}{\langle | \sum_i e^{-\beta \lambda_i} |^2 \rangle} - \left| \frac{\langle \sum_i e^{-(\beta - i\, t)\lambda_i} \rangle}{\langle \sum_i e^{-\beta \lambda_i} \rangle} \right|^2$$

2nd step: ensemble average → connected unfolded SFF (CUFSS)

$$g_c(t,\,\beta) \equiv \frac{\langle |\sum_i e^{-(\beta - i\,t)\lambda_i}|^2\rangle}{\langle |\sum_i e^{-\beta\lambda_i}|^2\rangle} - \left|\frac{\langle \sum_i e^{-(\beta - i\,t)\lambda_i}\rangle}{\langle \sum_i e^{-\beta\lambda_i}\rangle}\right|^2$$

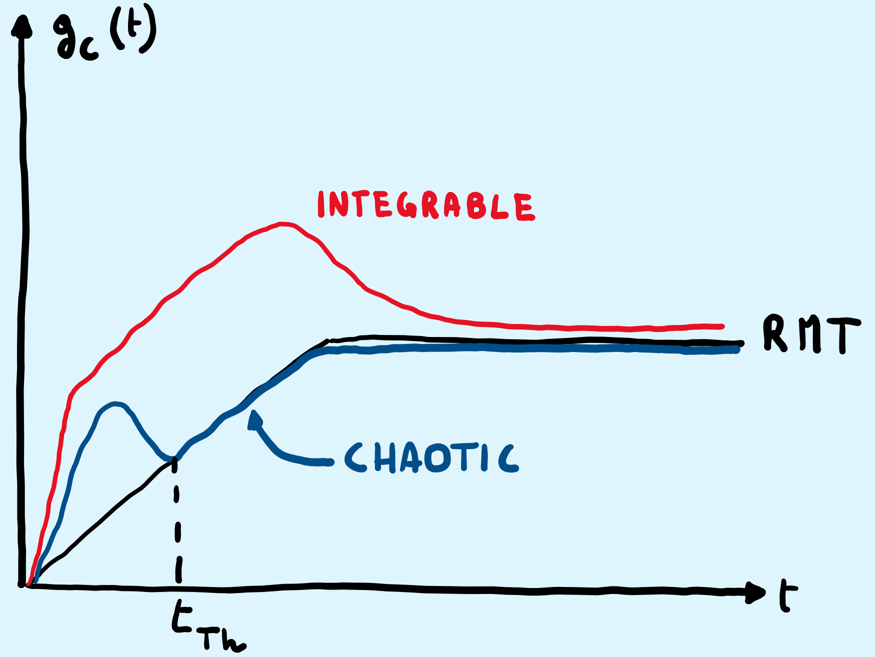
The CUFSS is a diagnostics of quantum chaos



2nd step: ensemble average → connected unfolded SFF (CUFSS)

$$g_c(t,\,\beta) \equiv \frac{\langle |\sum_i e^{-(\beta - i\,t)\lambda_i}|^2\rangle}{\langle |\sum_i e^{-\beta\lambda_i}|^2\rangle} - \left|\frac{\langle \sum_i e^{-(\beta - i\,t)\lambda_i}\rangle}{\langle \sum_i e^{-\beta\lambda_i}\rangle}\right|^2$$

The CUFSS is a diagnostics of quantum chaos



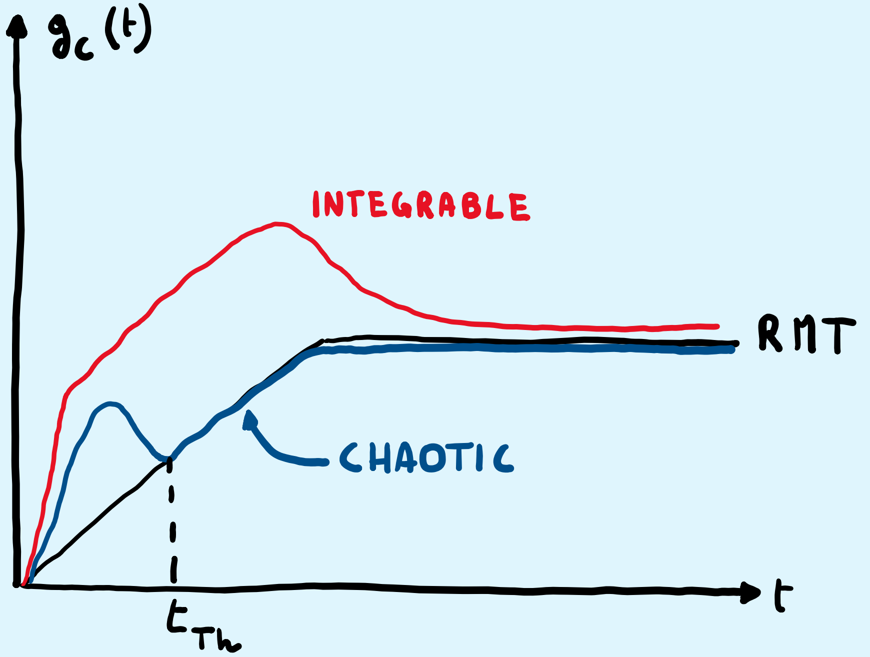
2nd step: ensemble average → connected unfolded SFF (CUFSS)

$$g_c(t,\,\beta) \equiv \frac{\langle|\sum_i e^{-(\beta-i\,t)\lambda_i}|^2\rangle}{\langle|\sum_i e^{-\beta\lambda_i}|^2\rangle} - \left|\frac{\langle\sum_i e^{-(\beta-i\,t)\lambda_i}\rangle}{\langle\sum_i e^{-\beta\lambda_i}\rangle}\right|^2$$

The CUFSS is a diagnostics of quantum chaos

New timescale, the Thouless time, t_{Th}:

RMT universality



The mass-deformed SYK: known facts [Garcia-Garcia et al. '17]

It is a SYK model deformed by a quadratic term, random mass term

Hamiltonian: $H = J_{ijkl}\,\psi^i\psi^j\psi^k\psi^l + i\kappa\,K_{ij}\,\psi^i\psi^j$ $\quad \{\psi^i,\psi^j\} = \delta^{ij} \quad i = 1,\dots,N$

Coupling constants: Random variables with Gaussian distribution

$$\langle J_{ijkl}J_{ijkl}\rangle = \frac{3!}{N^3} \qquad \langle K_{ij}K_{ij}\rangle = \frac{1}{N}$$

The mass-deformed SYK: known facts [Garcia-Garcia et al. '17]

It is a SYK model deformed by a quadratic term, random mass term

Hamiltonian: $H = J_{ijkl}\,\psi^i\psi^j\psi^k\psi^l + i\kappa\, K_{ij}\,\psi^i\psi^j \qquad \{\psi^i,\psi^j\} = \delta^{ij} \qquad i = 1,\ldots,N$

Coupling constants: Random variables with Gaussian distribution

$$\langle J_{ijkl}J_{ijkl}\rangle = \frac{3!}{N^3} \qquad \langle K_{ij}K_{ij}\rangle = \frac{1}{N}$$

Chaos / integrable parameter, κ:

Small values: the model collapses to standard SYK. It is highly chaotic

Large values: just the mass term is important. The model is non-chaotic

Chaos / integrable transition: A puzzle [Garcia-Garcia et al. '17]

- Early time chaos: Lyapunov exponents → Chaos / integrable transition at small κ: $\kappa \sim 1$

- RMT chaos: NNSD → Chaos / integrable transition at large κ: $\kappa \sim 10$

Chaos / integrable transition: A puzzle [Garcia-Garcia et al. '17]

- Early time chaos: Lyapunov exponents → Chaos / integrable transition at small κ: $\kappa \sim 1$

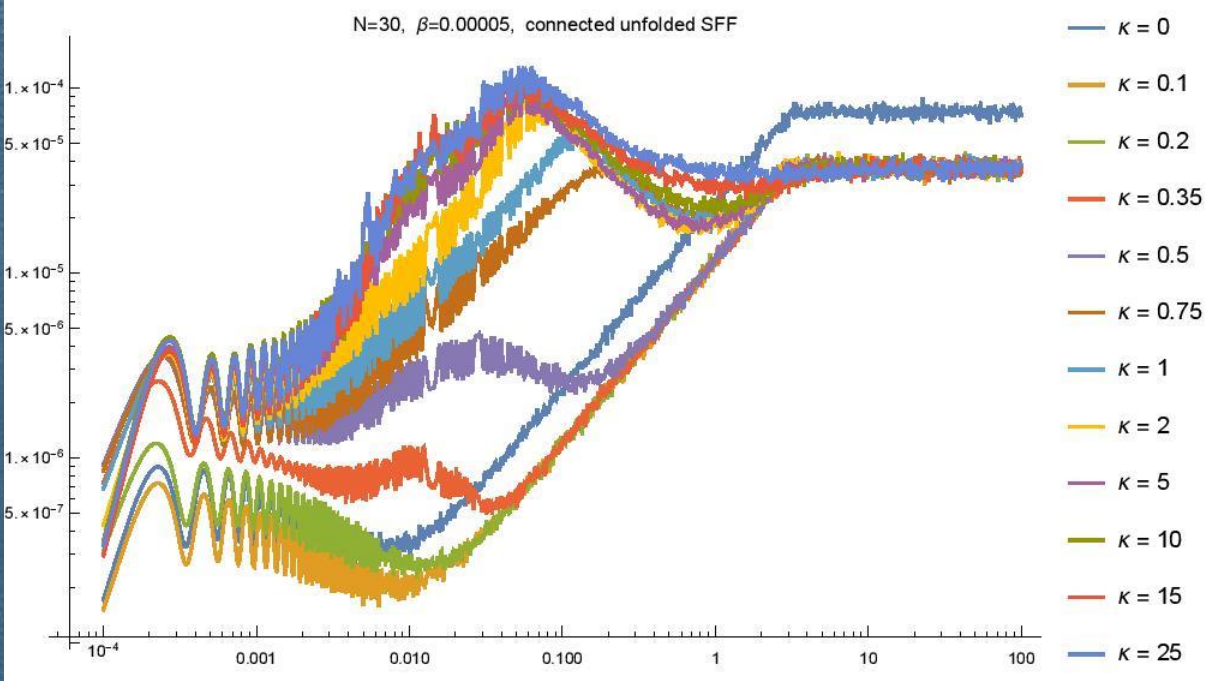
- RMT chaos: NNSD → Chaos / integrable transition at large κ: $\kappa \sim 10$

Why this discrepancy? How to reconcile?

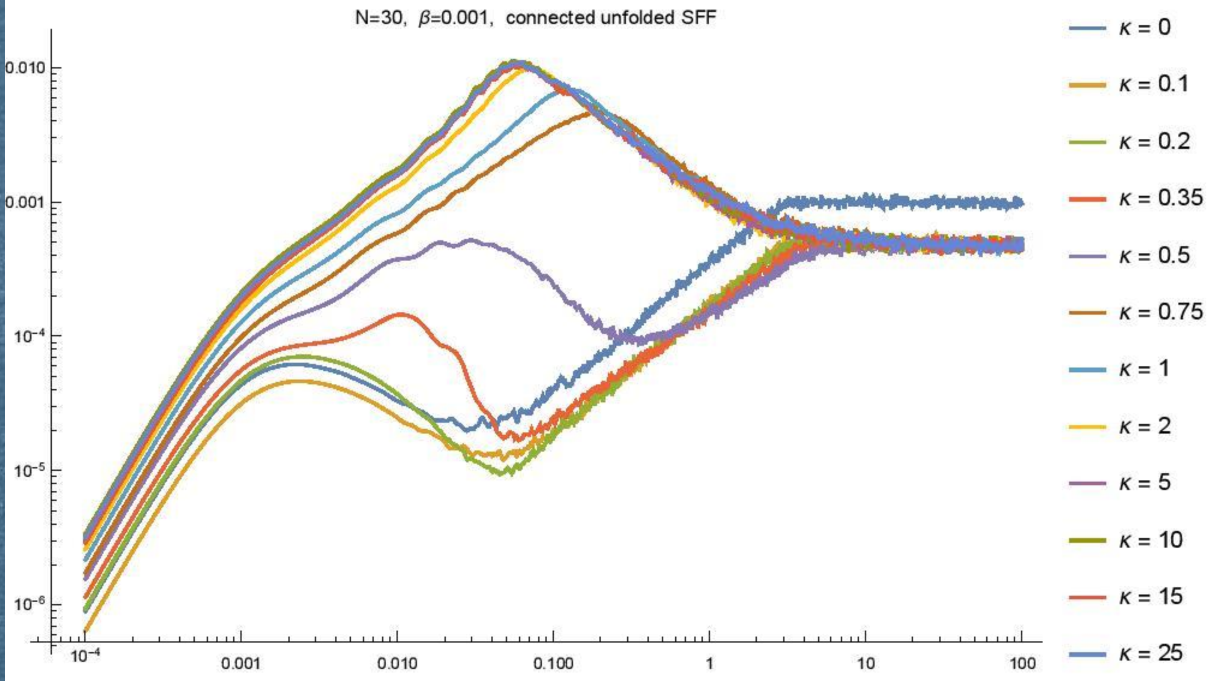
RMT chaos via the CUSFF

We studied chaos with the CUFSS at various values of the temperature:

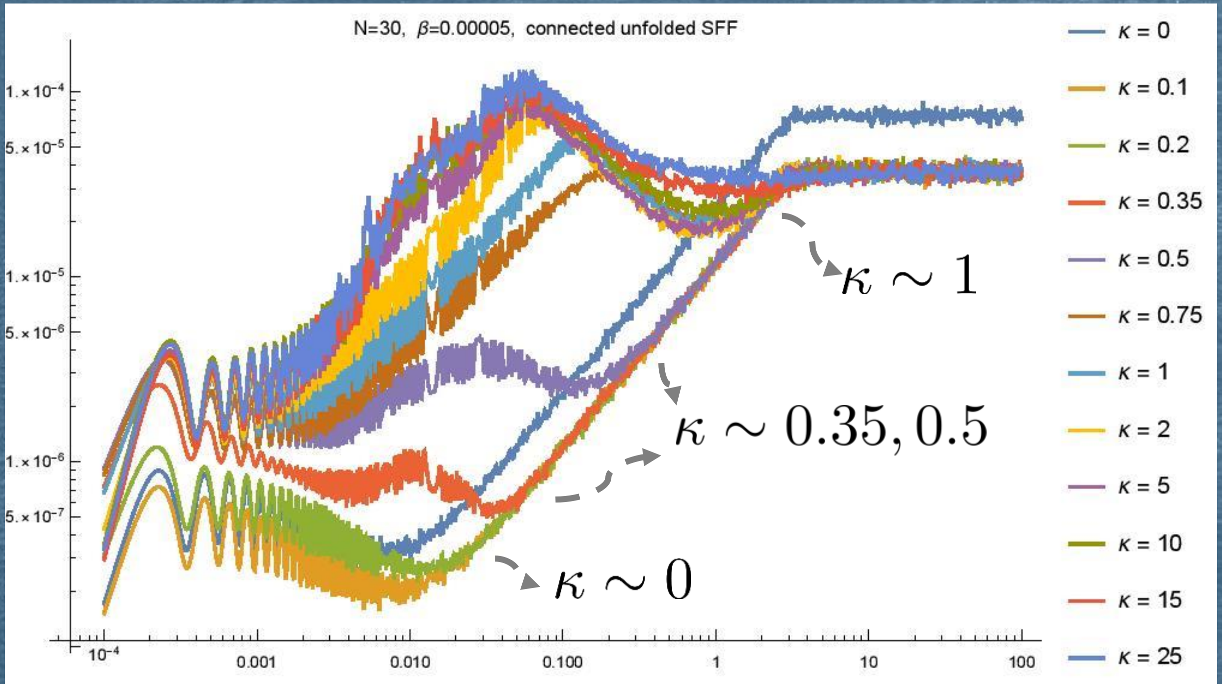
Large temperature:



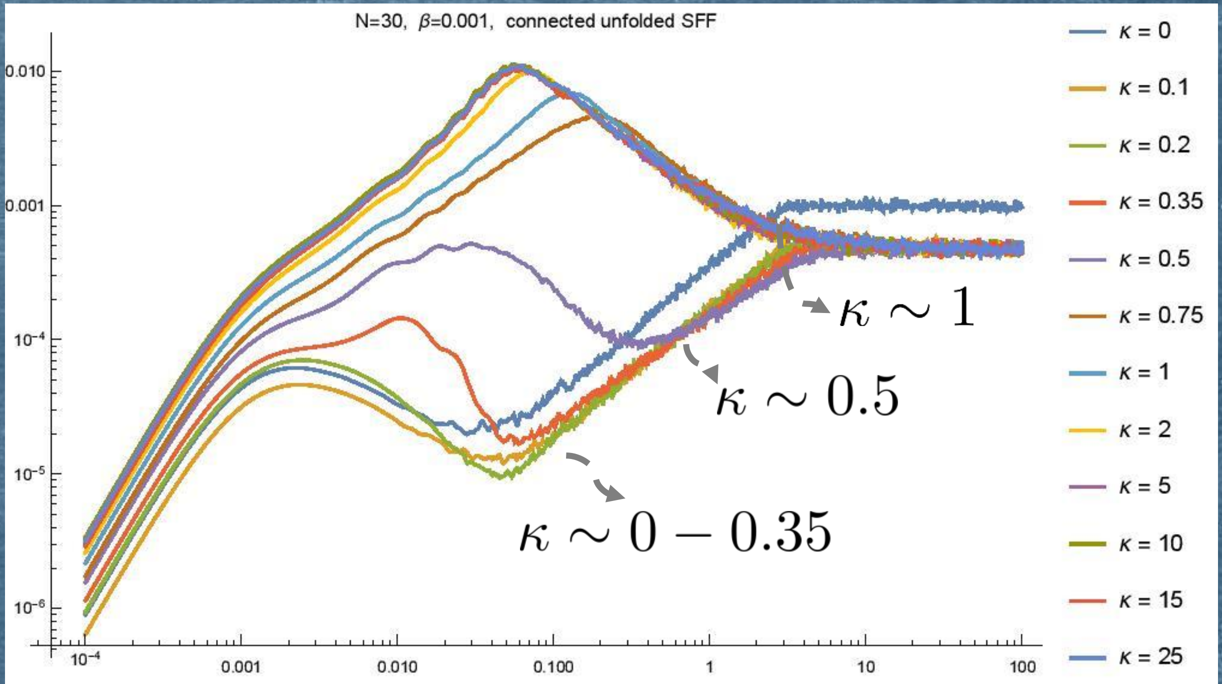
Small temperature:



Large temperature:



Small temperature:



- Large temperature: some remnants of chaos at $\kappa \sim 1$
- Low temperature: no chaos remains at $\kappa \sim 1$
- Chaos / integrable transition for $\kappa \sim 1$
- Qualitative agreement with the scrambling physics!

The CUFSS agrees with scrambling: why previous studies did not find agreement?

The CUFSS agrees with scrambling: why previous studies did not find agreement?

- RMT analysis is done on the entire spectrum

  → Highly excited states plays an important role

- Scrambling and the CUFSS are mostly controlled by the low-lying modes

- To agree: chaos / integrable transition must affect the spectrum homogeneously

The CUFSS agrees with scrambling: why previous studies did not find agreement?

- RMT analysis is done on the entire spectrum

  → Highly excited states plays an important role

- Scrambling and the CUFSS are mostly controlled by the low-lying modes

- To agree: chaos / integrable transition must affect the spectrum homogeneously

We checked that this is not the case for the mass-deformed SYK.
Low-lying modes migrate more easily to the integrable regime

Conclusion

- A new observable in RMT approach to chaos: CUSFF
- Its behavior is in qualitative agreement with the scrambling
- In the mass-deformed SYK model the chaos / integrable transition is not homogeneous
- Is this a general feature for many-body chaotic systems?
- More quantitative agreement?
- Other many-body quantum chaotic systems?