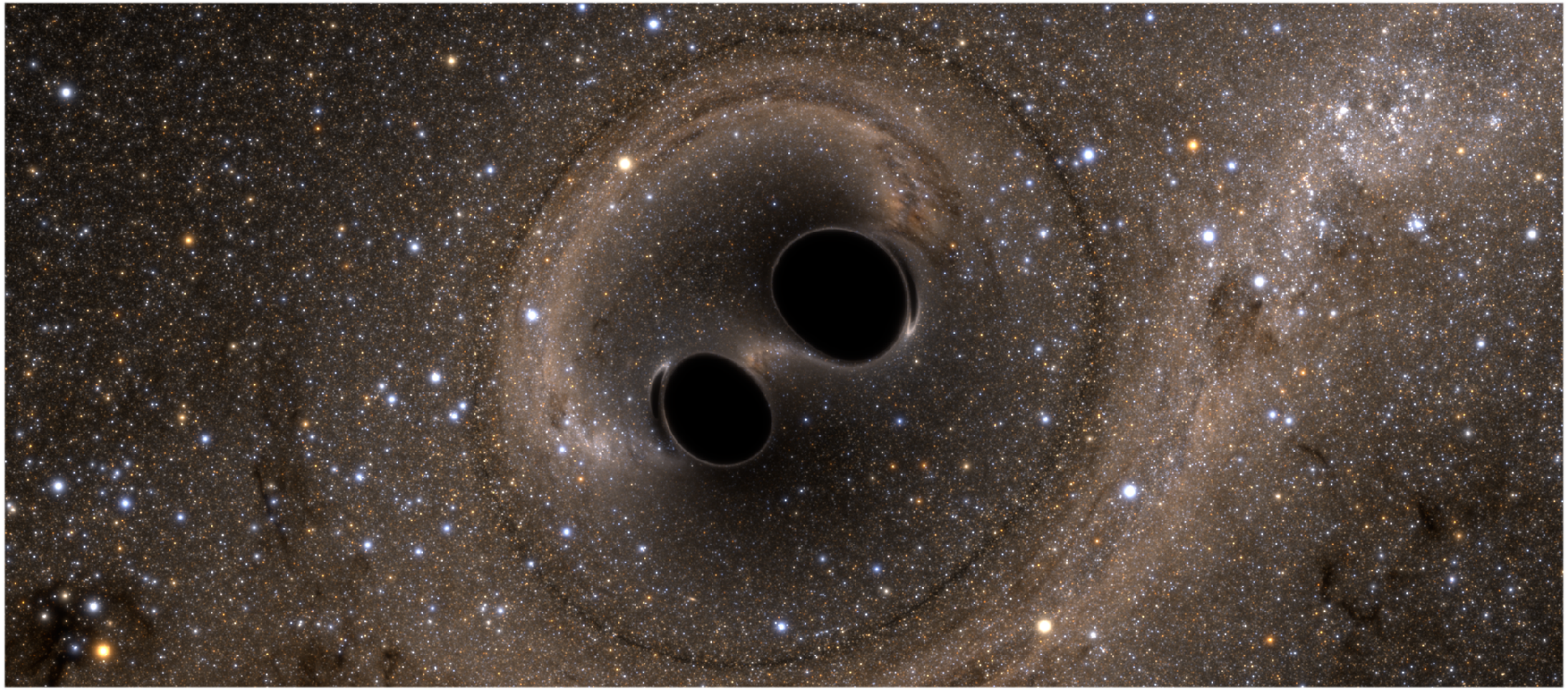
[SXS - black-holes.org]



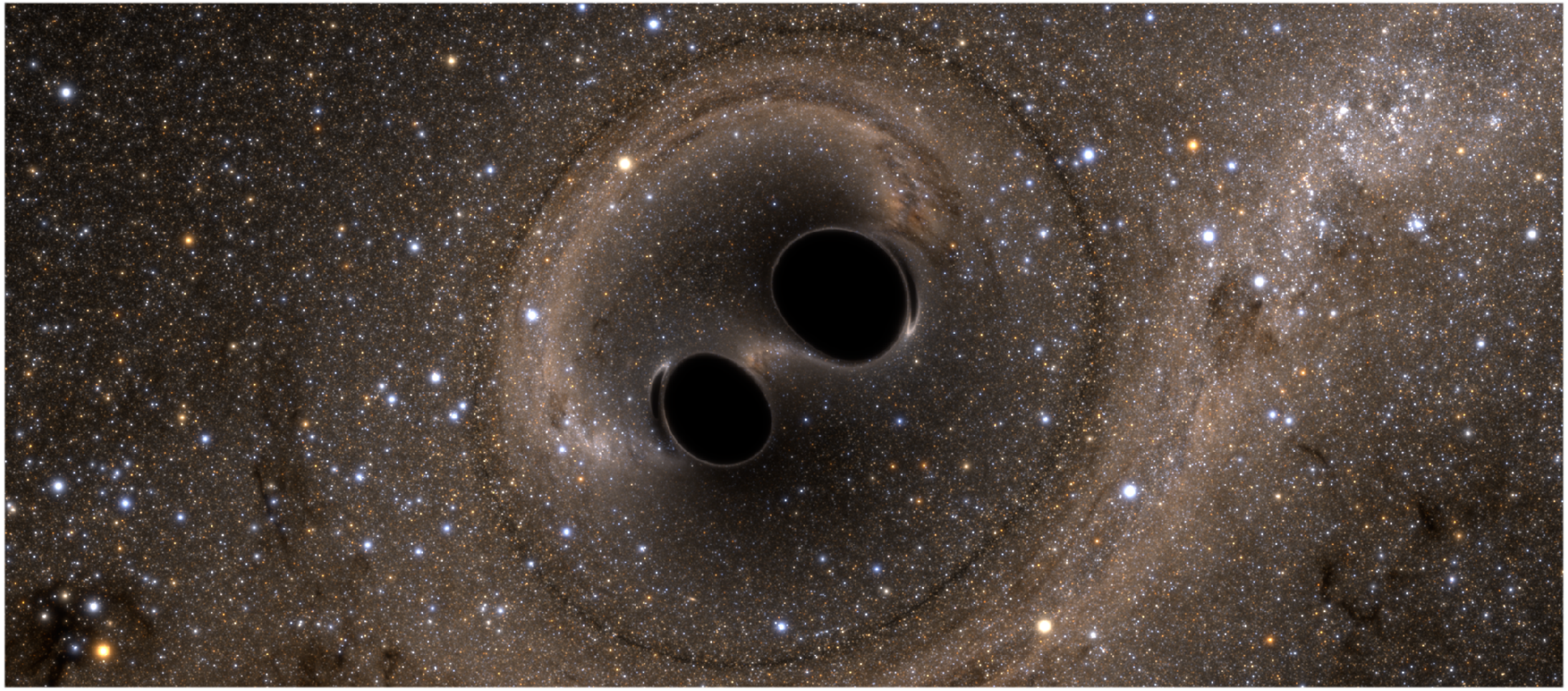
Quantum and Gravity in Okinawa - 07/24/2019 - OIST

Towards a non-perturbative study of quantum gravity effects

Enrico Rinaldi*

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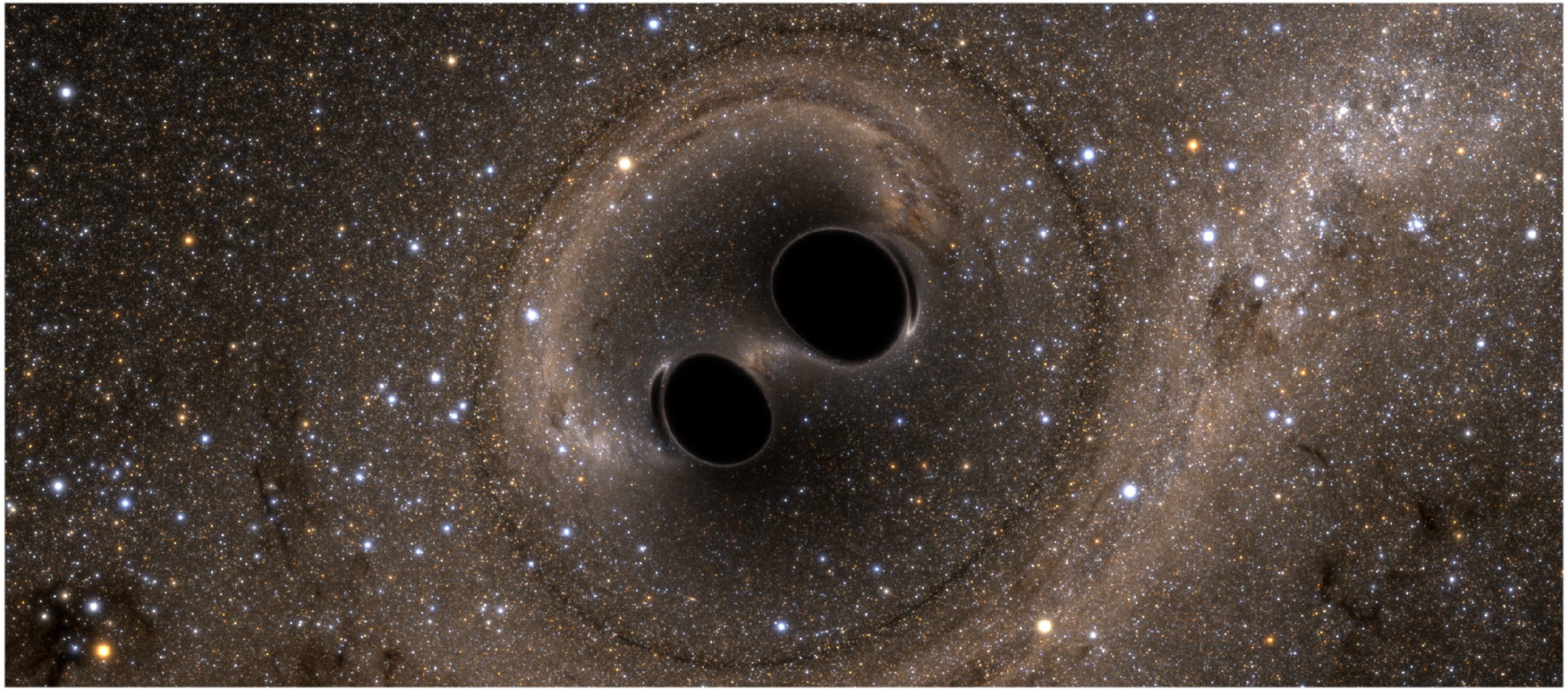
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https://mcsmc.github.io

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thanks to my collaborators and funding sources

Masanori Hanada

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Georg Bergner

Pavlos Vranas

Goro Ishiki

Andreas Schäfer

Norbert Bodendorfer

Lawrence Livermore National Laboratory

RIKEN BNL Research Center

RIKEN

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[Maldacena '97, Itzhaki,Maldacena,Sonnenschein,Yankielowic '98]

['t Hooft, Susskind, Maldacena …]

Gauge/Gravity duality

Gauge

supersymmetric SU(N) gauge theory



Gravity

Superstring/M-Theory

interaction strength

(quantum) string corrections

[Maldacena '97, Itzhaki,Maldacena,Sonnenschein,Yankielowic '98]

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Gauge/Gravity duality

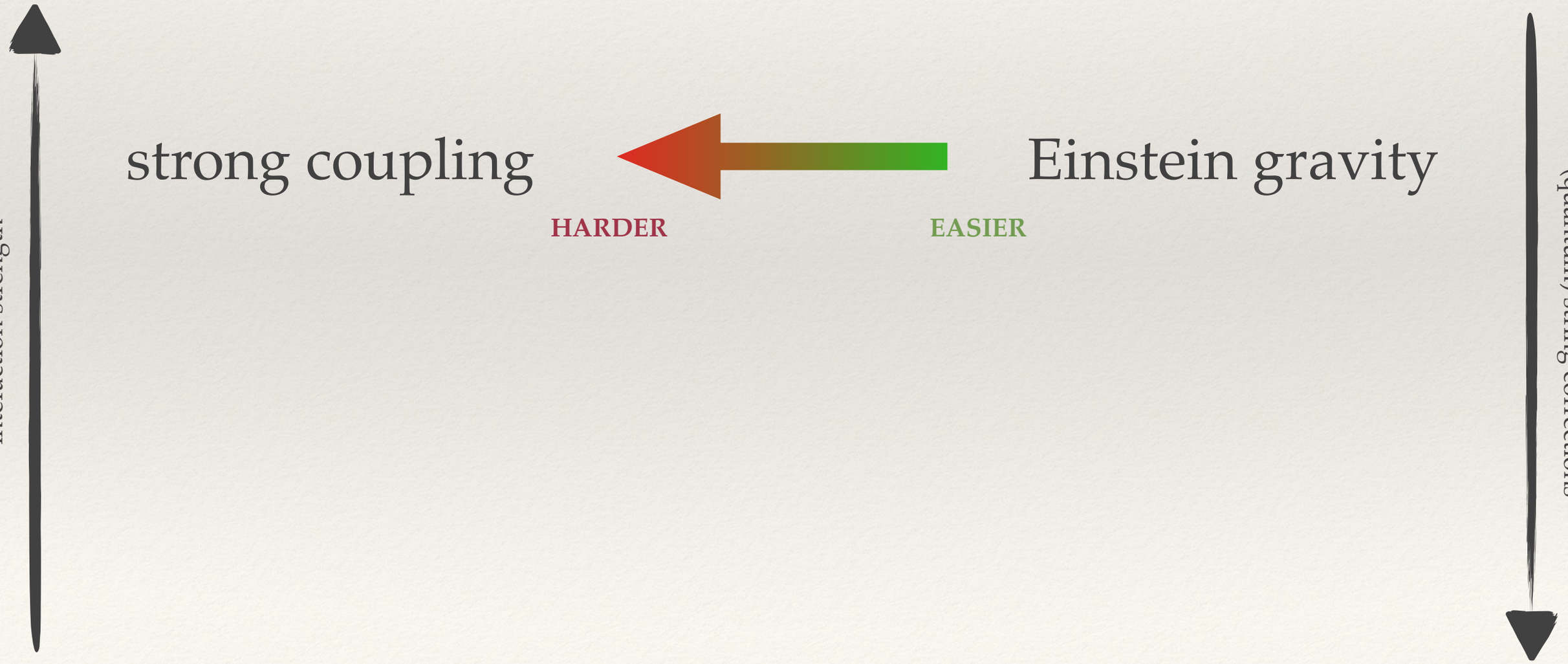
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Gauge/Gravity duality

Gauge ⟷ **Gravity**

supersymmetric SU(N) gauge theory

Superstring/M-Theory



strong coupling ← Einstein gravity

HARDER EASIER

finite coupling → string effects

EASIER HARDER



[Maldacena '97, Itzhaki,Maldacena,Sonnenschein,Yankielowic '98]

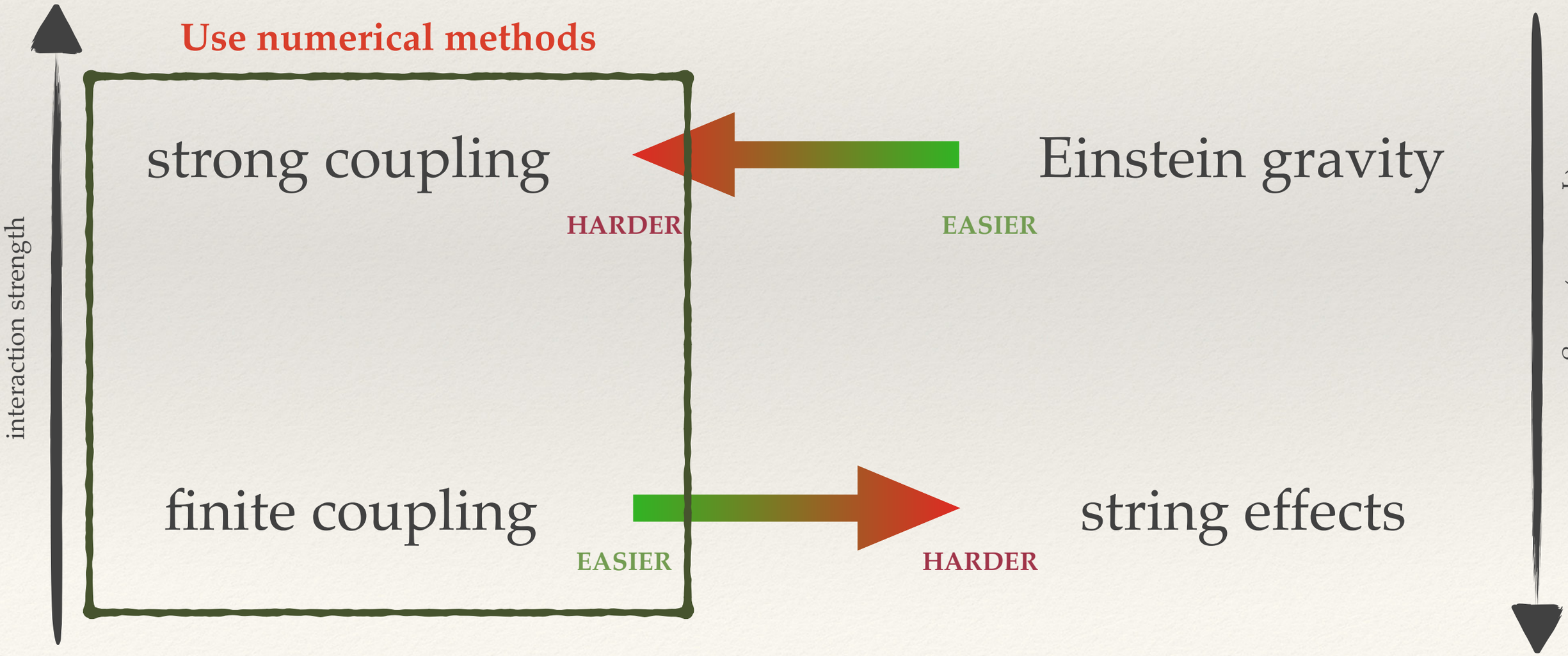
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Superstring/M-Theory



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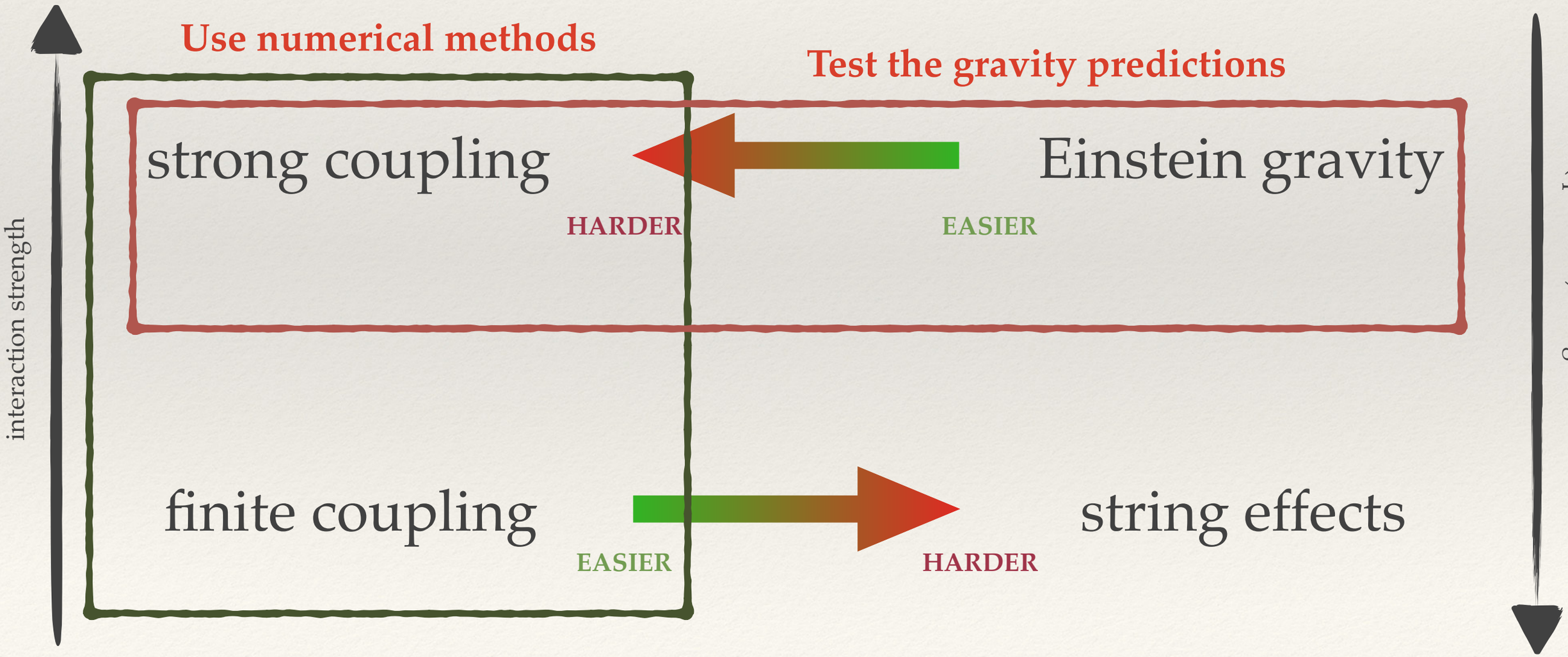
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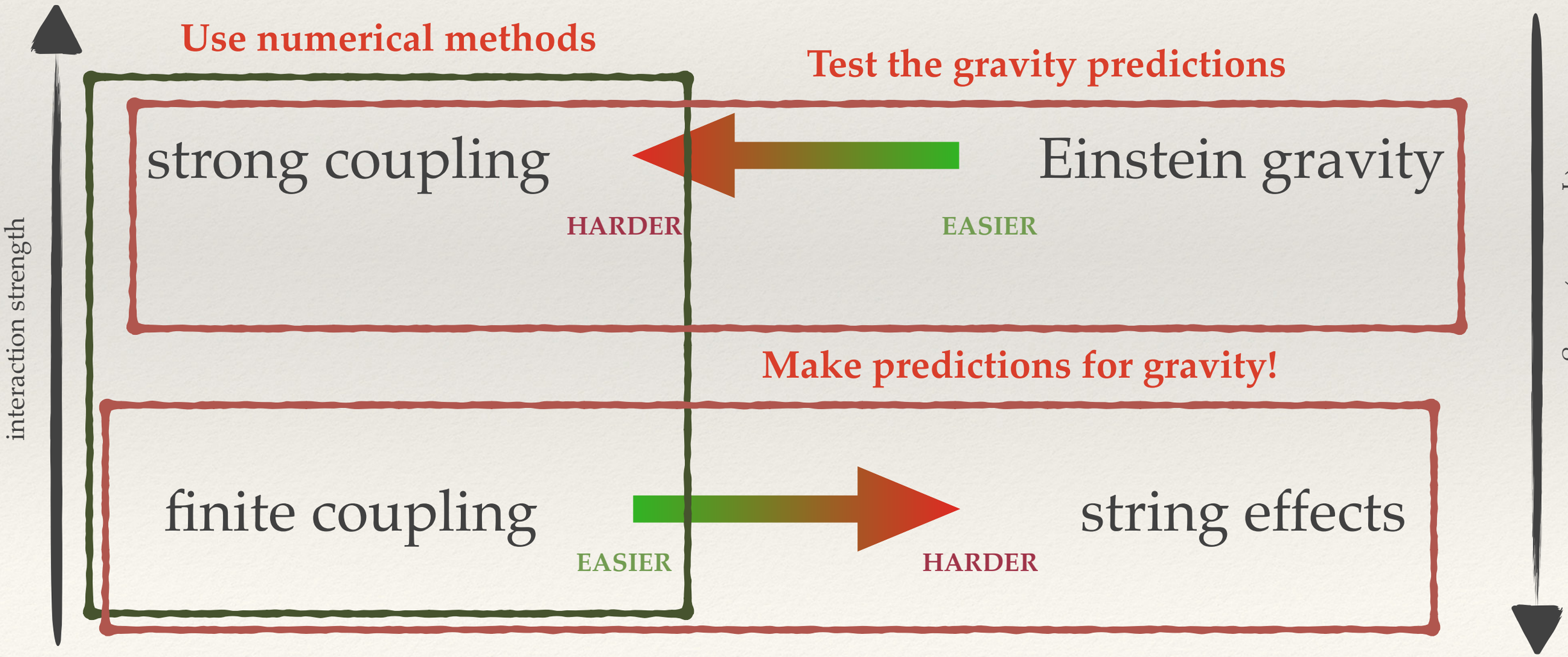
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Gauge/Gravity duality

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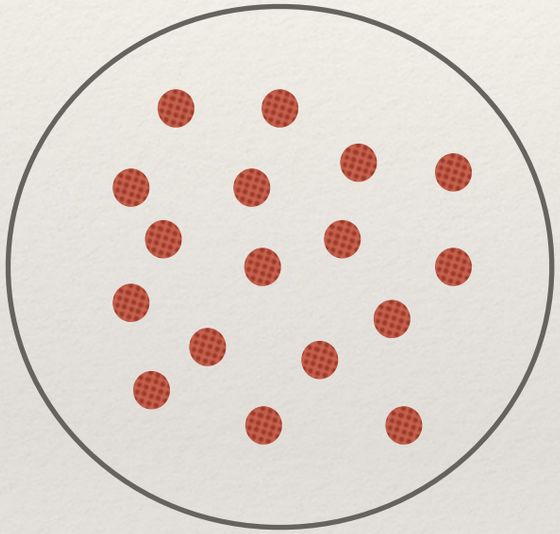
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Superstring/M-Theory

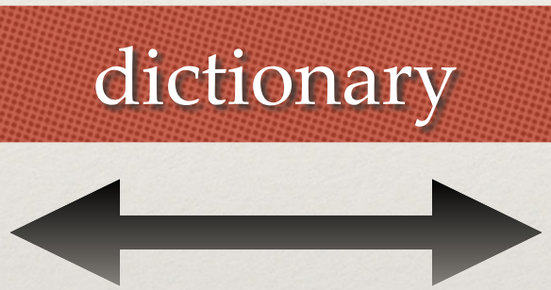


[’t Hooft, Susskind, Maldacena …]

Testing the gauge/gravity duality



dictionary



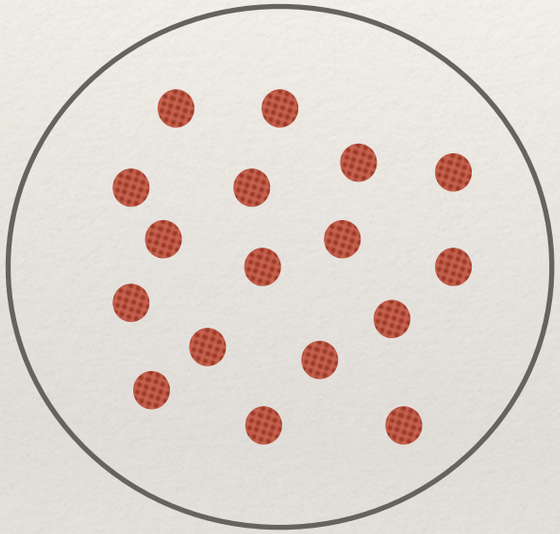


(0+1)-dim maximally supersymmetric gauge theory

Type IIA superstring theory
in the near horizon limit of
a black 0-brane geometry

[’t Hooft, Susskind, Maldacena …]

Testing the gauge/gravity duality



(0+1)-dim maximally supersymmetric gauge theory

dictionary



Type IIA superstring theory
in the near horizon limit of
a black 0-brane geometry

Compute observables that are *dual* to each other on both sides and compare in the *easy* case (0+1)d

[Witten '94, Banks, Fischler, Shenker, Susskind '96, but also Hoppe et al. '82-'85]

Supersymmetric quantum mechanics

$$L = \frac{1}{2g_{YM}^2}\mathrm{Tr}\left\{(D_t X_M)^2 - \frac{1}{2}[X_M, X_{M'}]^2 + i\bar{\psi}^\alpha D_t \psi_\alpha + \bar{\psi}^\alpha \gamma^M_{\alpha\beta}[X_M, \psi^\beta]\right\}$$

$X_M\,, M = 1, \ldots, 9\ (N \times N) \rightarrow$ hermitian scalars

$\psi^\alpha\,, \alpha = 1, \ldots, 16\ (N \times N) \rightarrow$ adjoint fermions

$D_t \cdot\ =\ \partial_t \cdot - i[A_t, \cdot]\ \rightarrow$ gauge covariant derivative

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obtained from $\mathcal{N}$=1 U(N) SYM in (9+1)d via dimensional reduction to (0+1)d
or equivalently from $\mathcal{N}$=4 U(N) SYM in (3+1)d: it is maximally supersymmetric

$$S = \int_0^{1/T} dt\, L$$

$\lambda = g^2_{\mathrm{YM}} N$ 't Hooft coupling

$X_M \, , M = 1, \ldots, 9 \; (N \times N) \; \rightarrow$ hermitian scalars

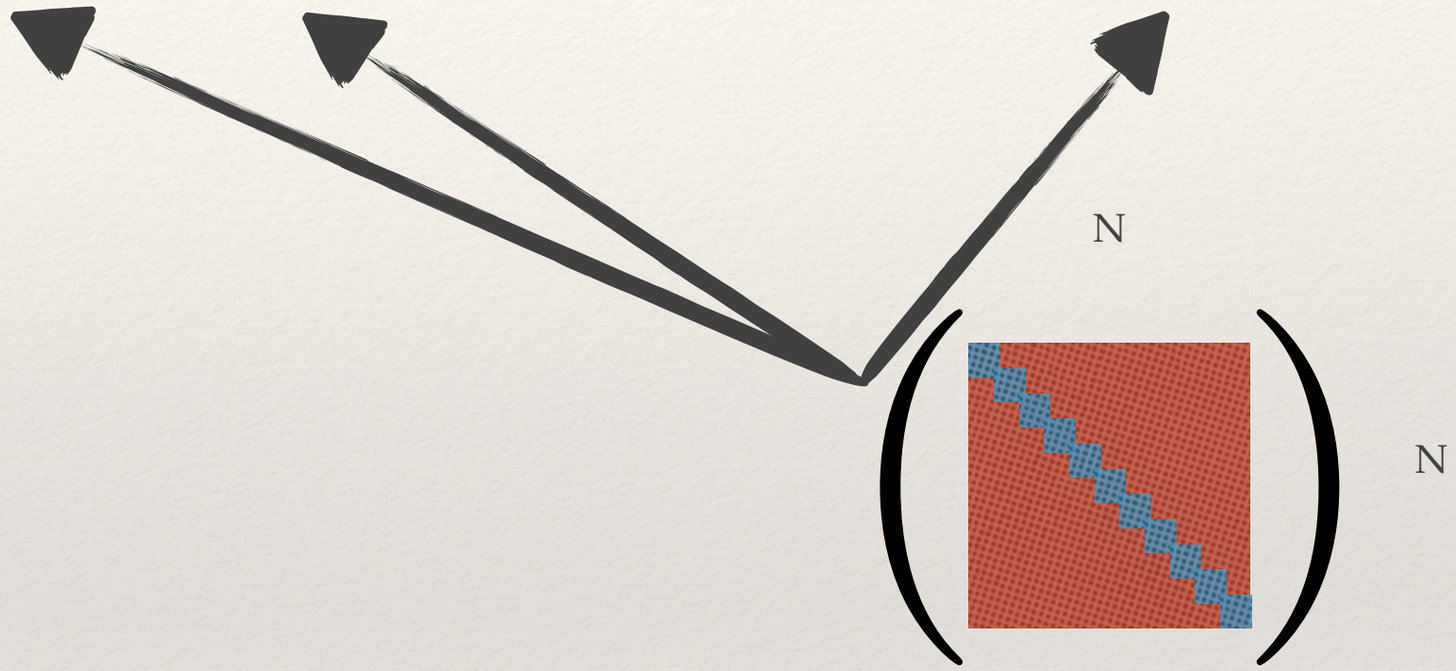
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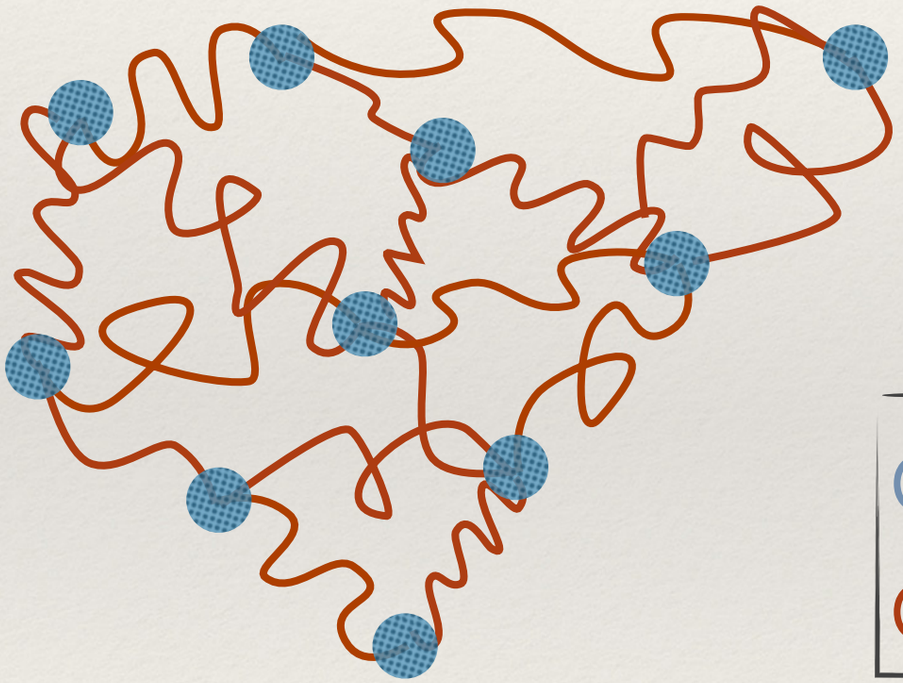
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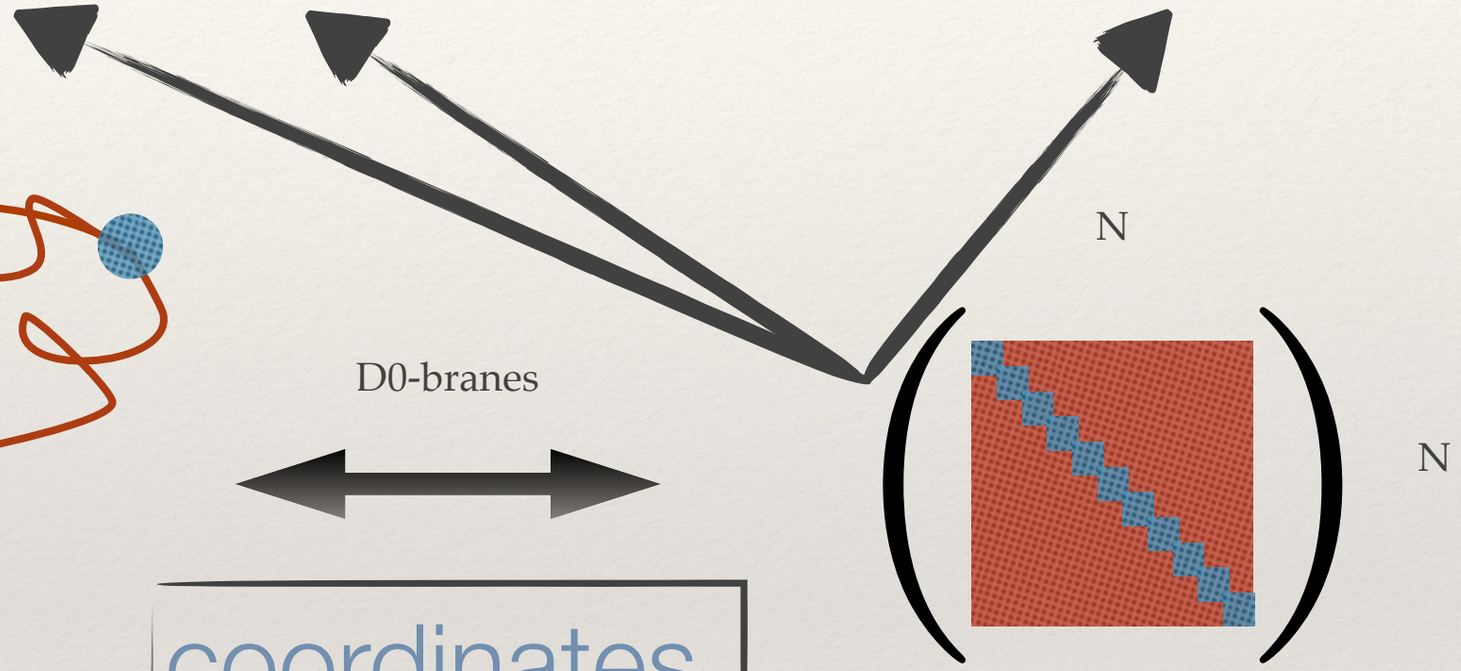
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coordinates

couplings

$X_M\,, M = 1, \ldots, 9\ (N \times N) \rightarrow$ hermitian scalars

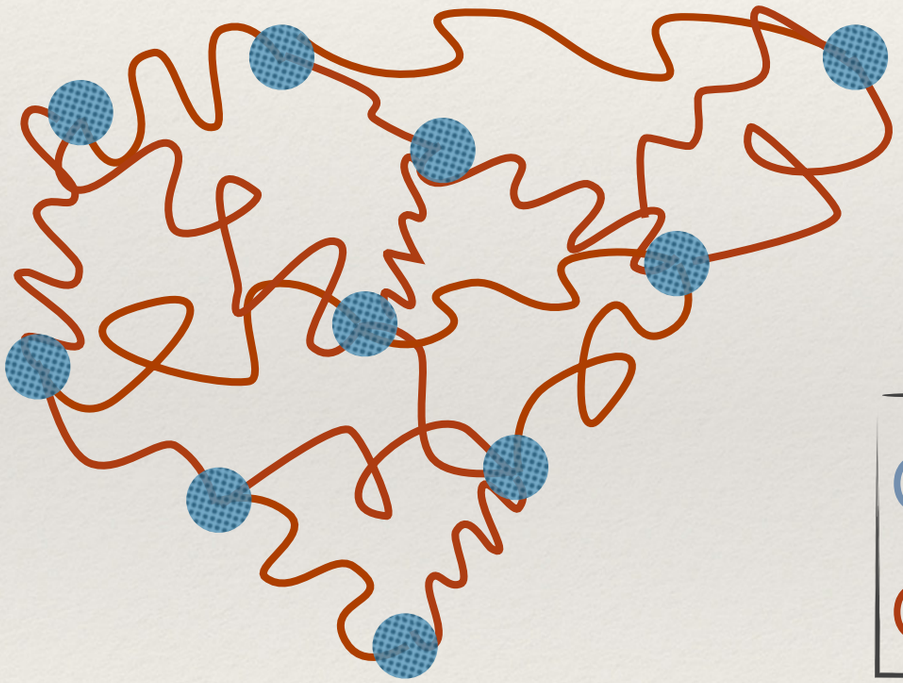
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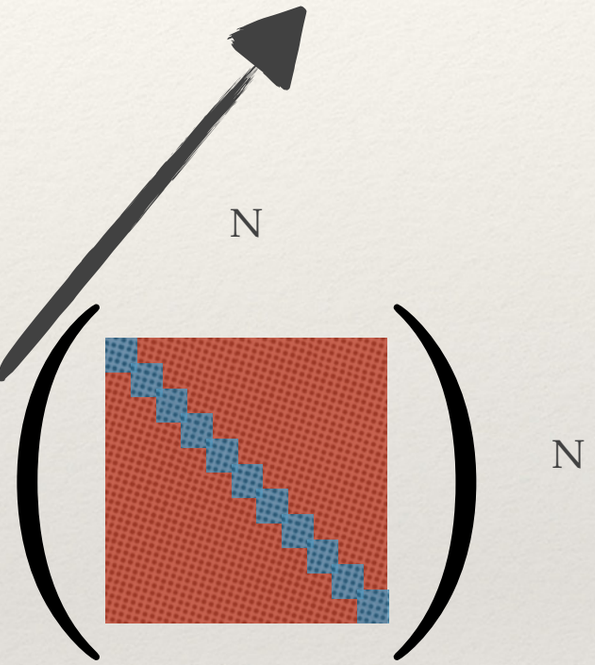
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D0-branes



coordinates
couplings

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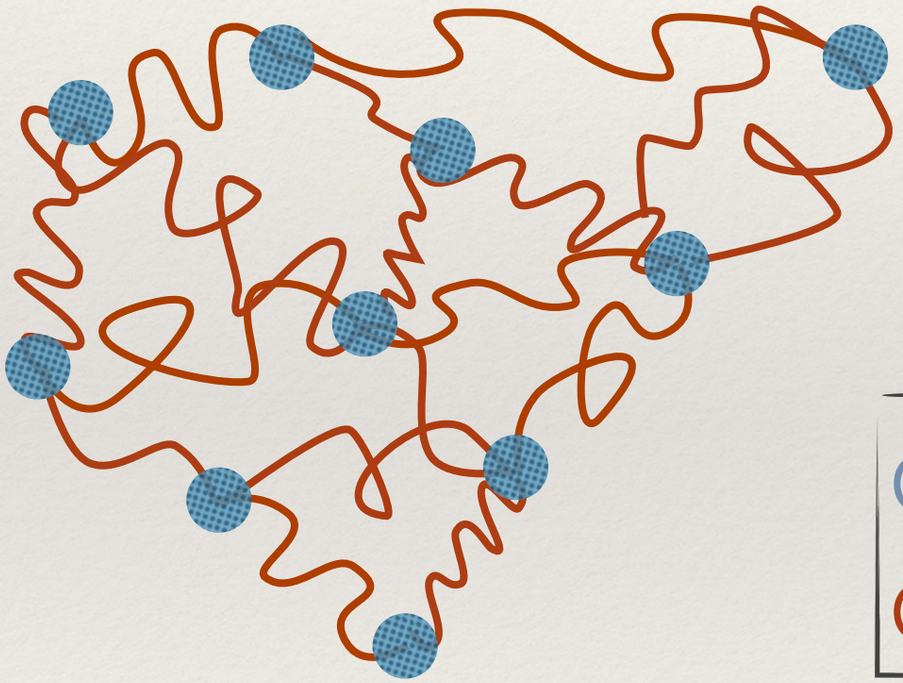
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conjectured to be equivalent to M-theory

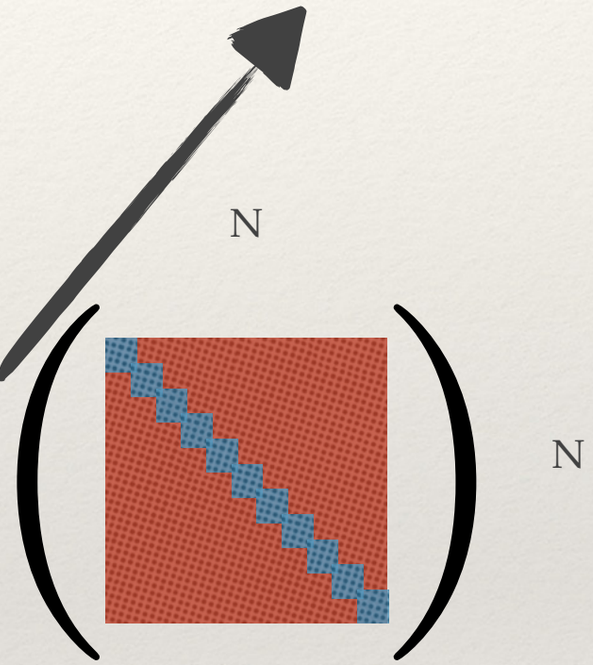
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D0-branes



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conj… …o be
eq… BFSS …to
M-theory

[Hoppe '82, Kawahara Nishimura Takeuchi 2007, Filev O'Connor 2016]

Matrix quantum mechanics

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it can be seen as the high-T limit of maximally supersymmetric ($\mathcal{N}$=8) (1+1)d U(N) **SYM** on a circle

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rich phase structure
confined at low-T

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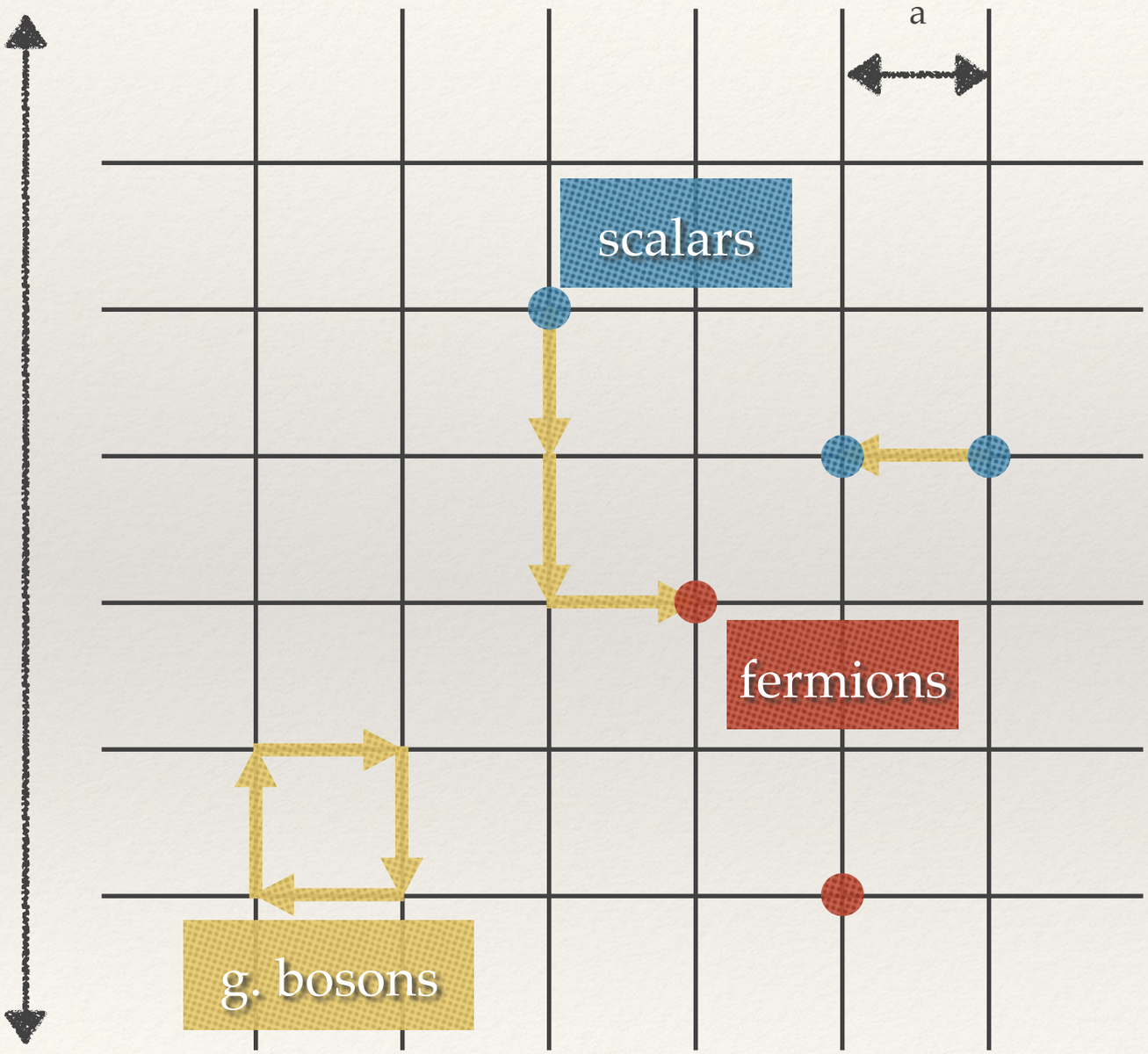
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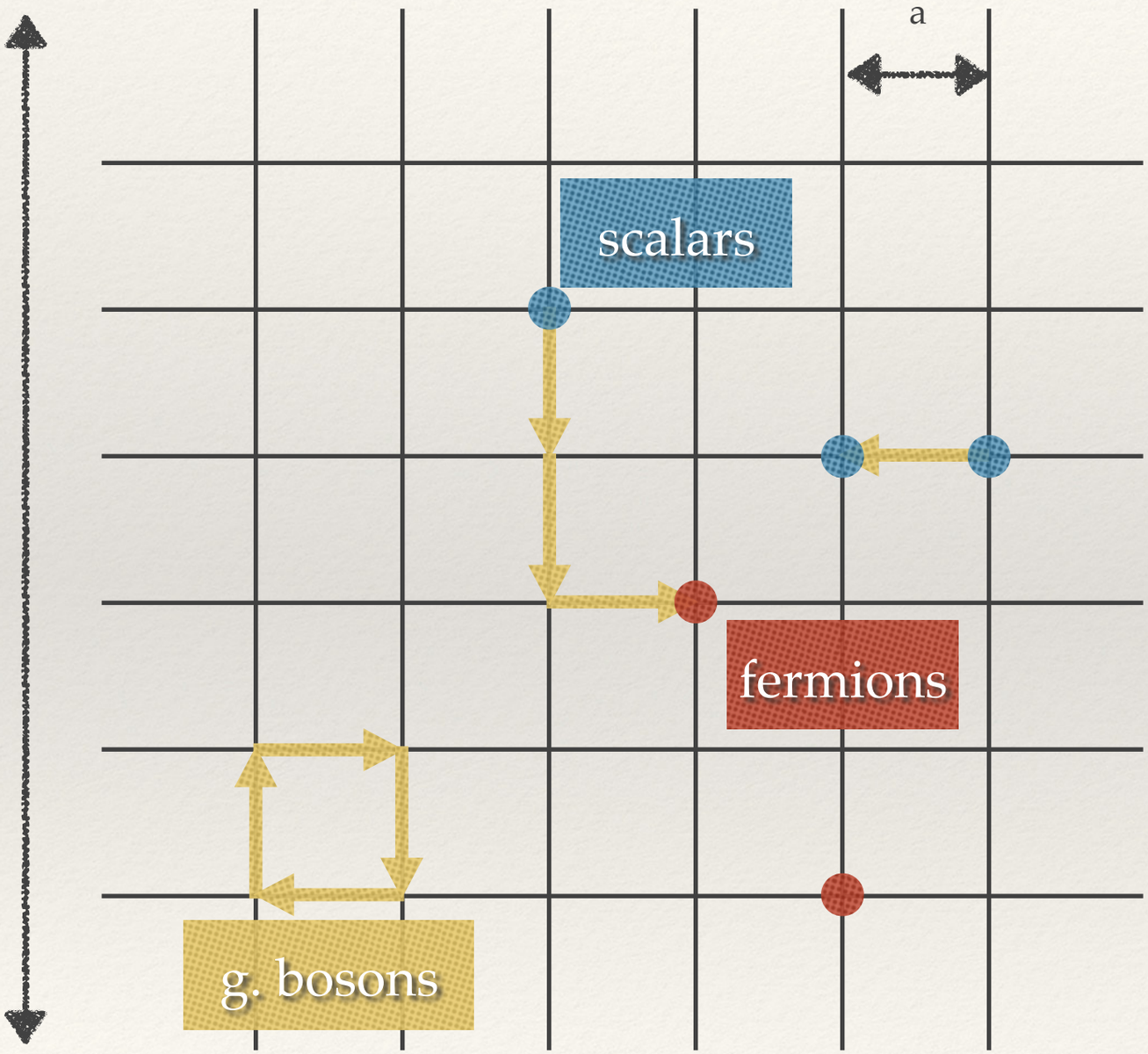
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Bosonic BFSS

Lattice Gauge Theory Primer

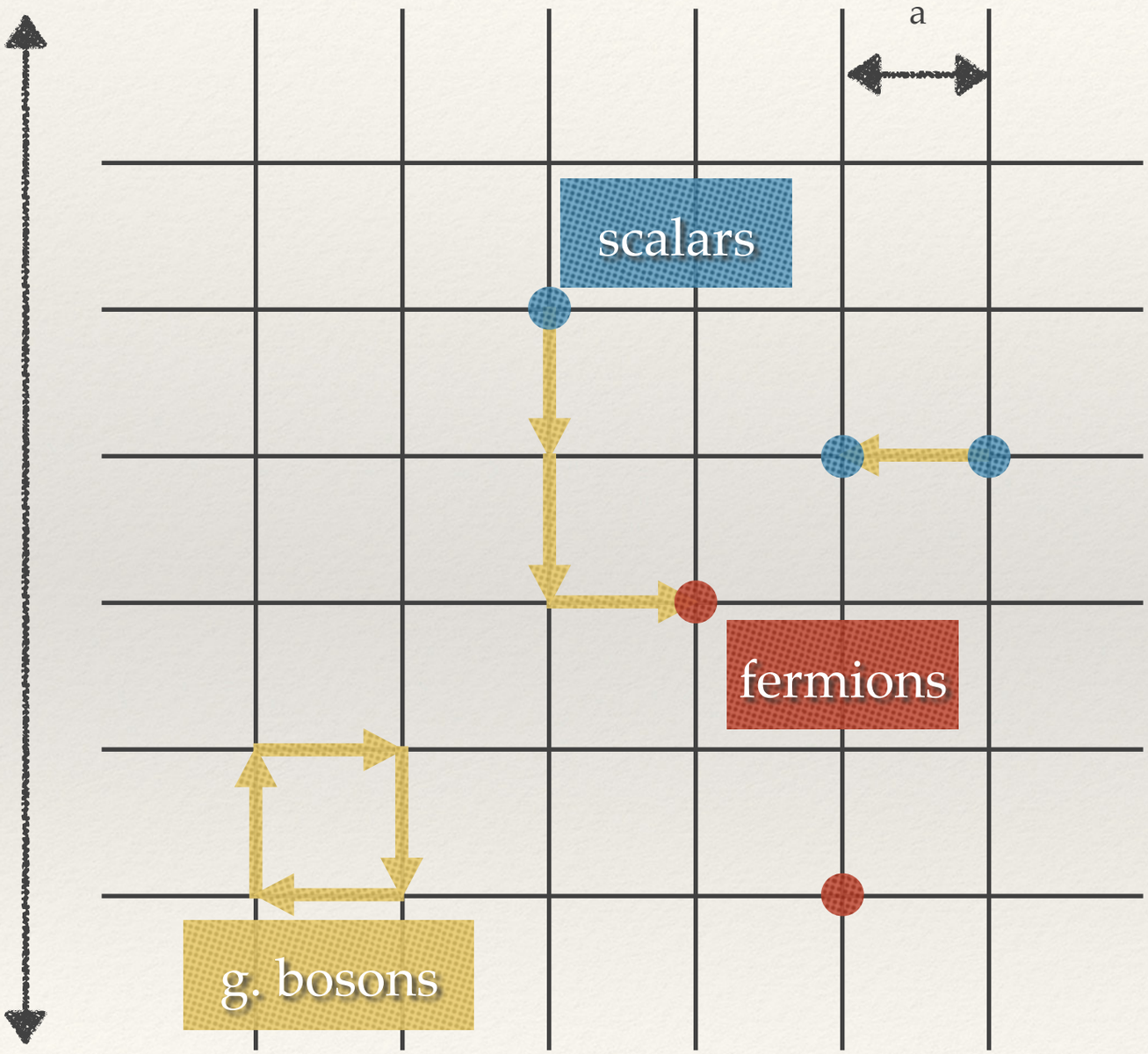


Lattice Gauge Theory Primer



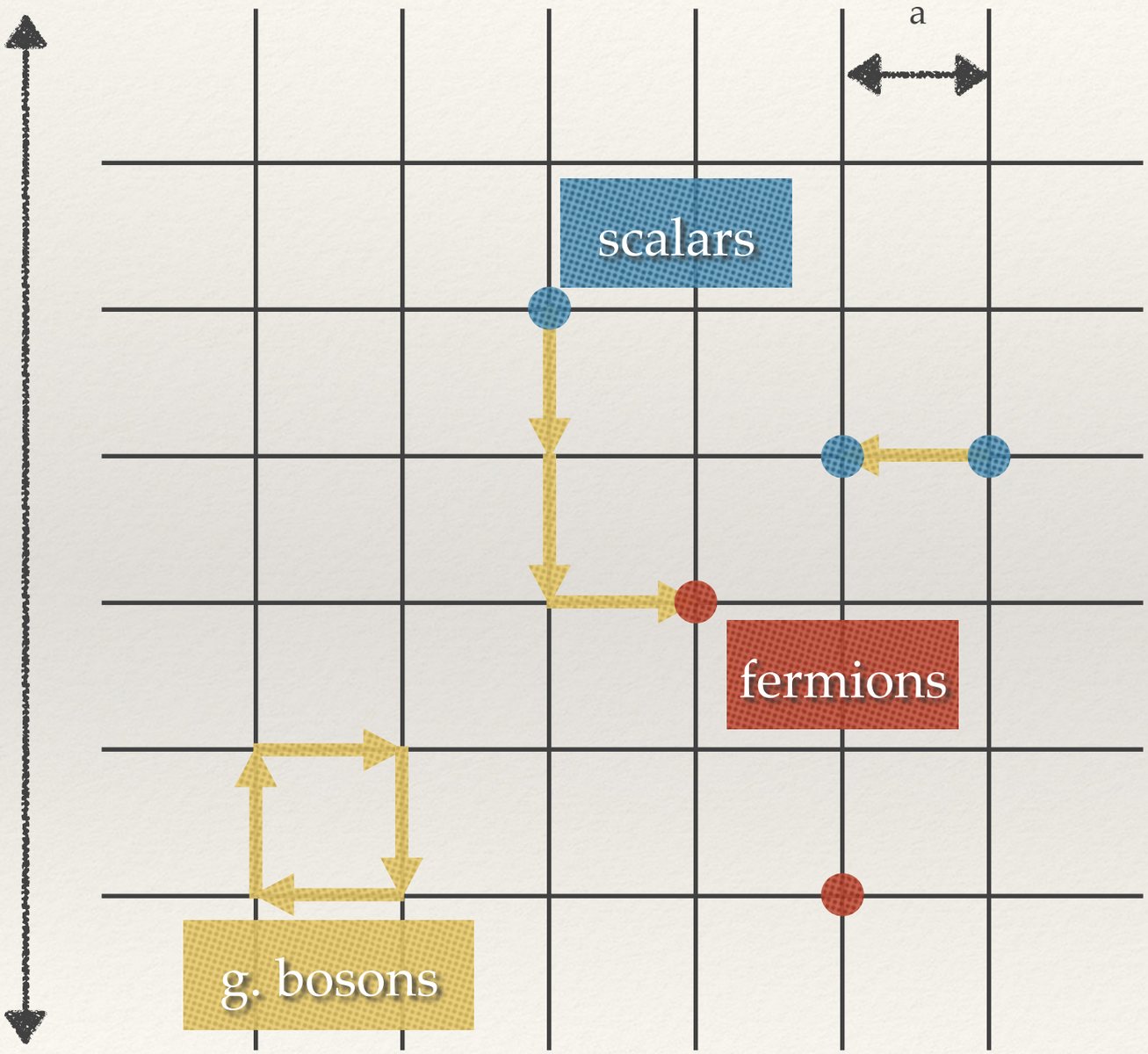
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  - lattice spacing “a”
  - lattice size “L”

Lattice Gauge Theory Primer



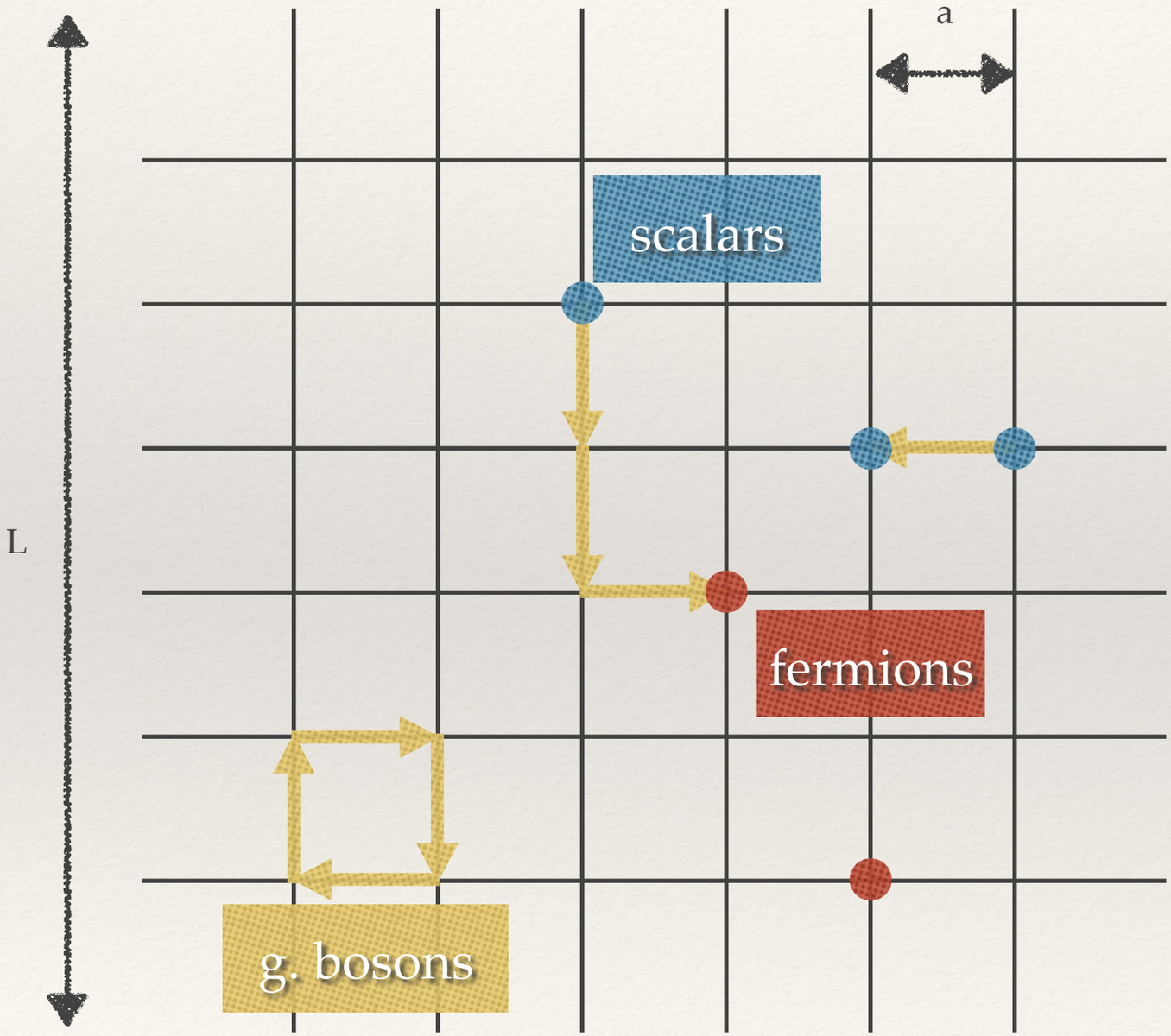
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  - no simplifications

Lattice Gauge Theory Primer



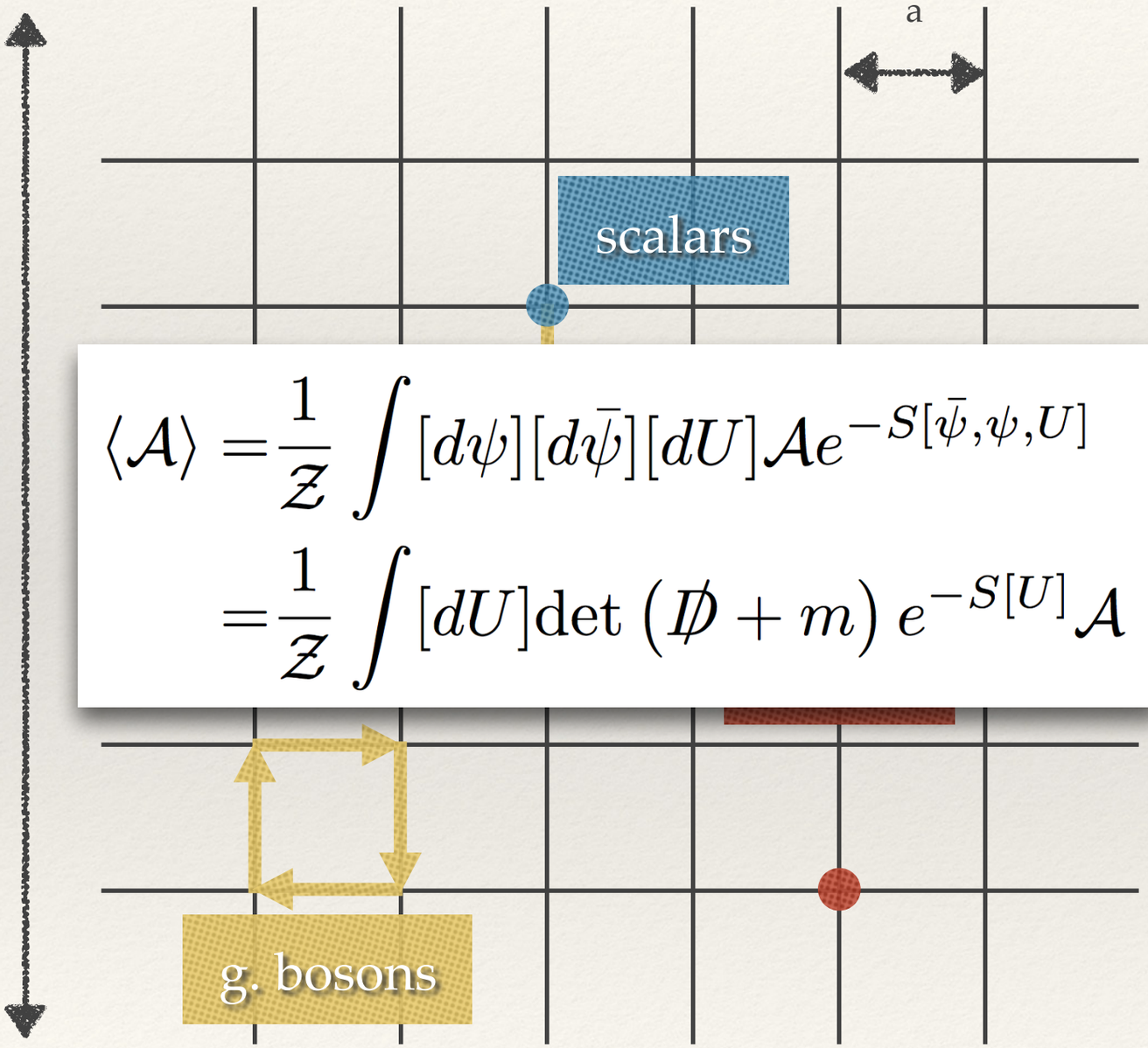
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- Amenable to numerical methods
  - Monte Carlo sampling
  - use supercomputers

Lattice Gauge Theory Primer



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- Precisely quantifiable and improvable errors
  - Systematic
  - Statistical

Lattice Gauge Theory Primer



$$\langle \mathcal{A} \rangle = \frac{1}{\mathcal{Z}} \int [d\psi][d\bar{\psi}][dU] \mathcal{A} e^{-S[\bar{\psi},\psi,U]}$$
$$= \frac{1}{\mathcal{Z}} \int [dU] \det\left(\not{D} + m\right) e^{-S[U]} \mathcal{A}$$

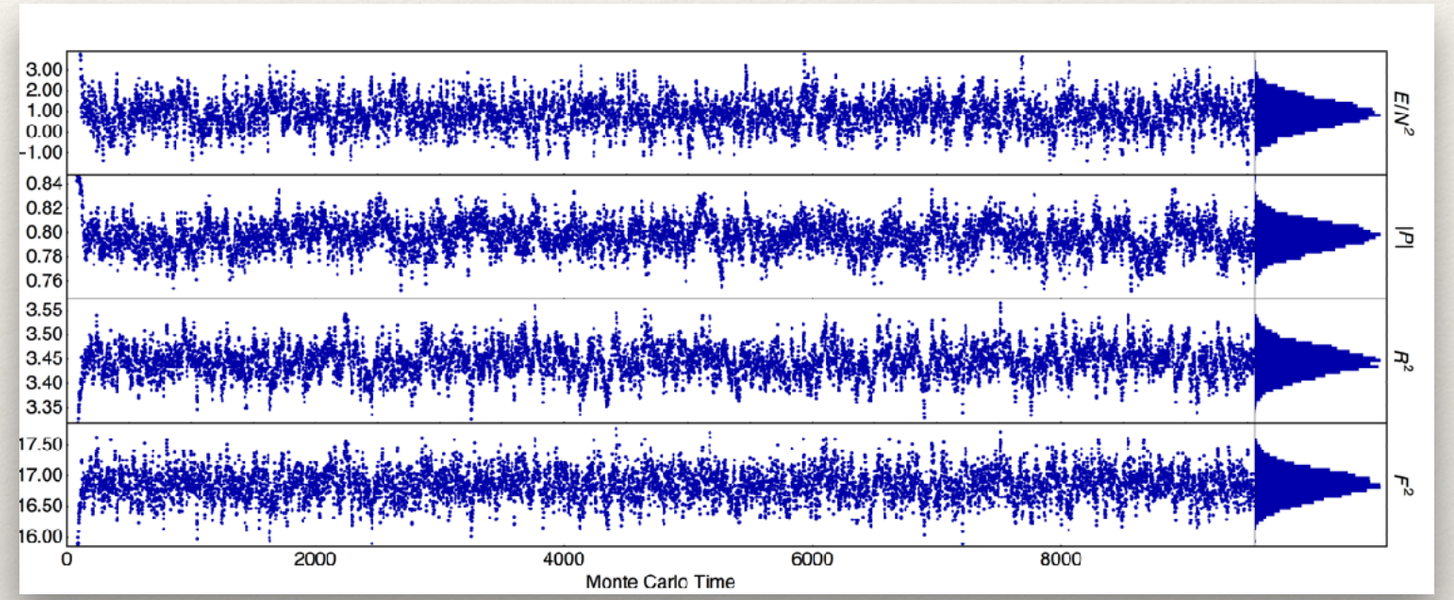
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  - Statistical

[MCSMC, Phys. Rev. D94 (2016) 094501]

Monte Carlo simulations

- Observables: E/N^2, $|P|$, $R^2=Tr[X^2]/N$, $F^2=Tr([X_i,X_j]^2)/N$
- Large statistics for all parameters (N,L,T) is needed
- Dedicated autocorrelation analysis is paramount due to long fluctuations
- All data is published in tables and can be used for future benchmarks!

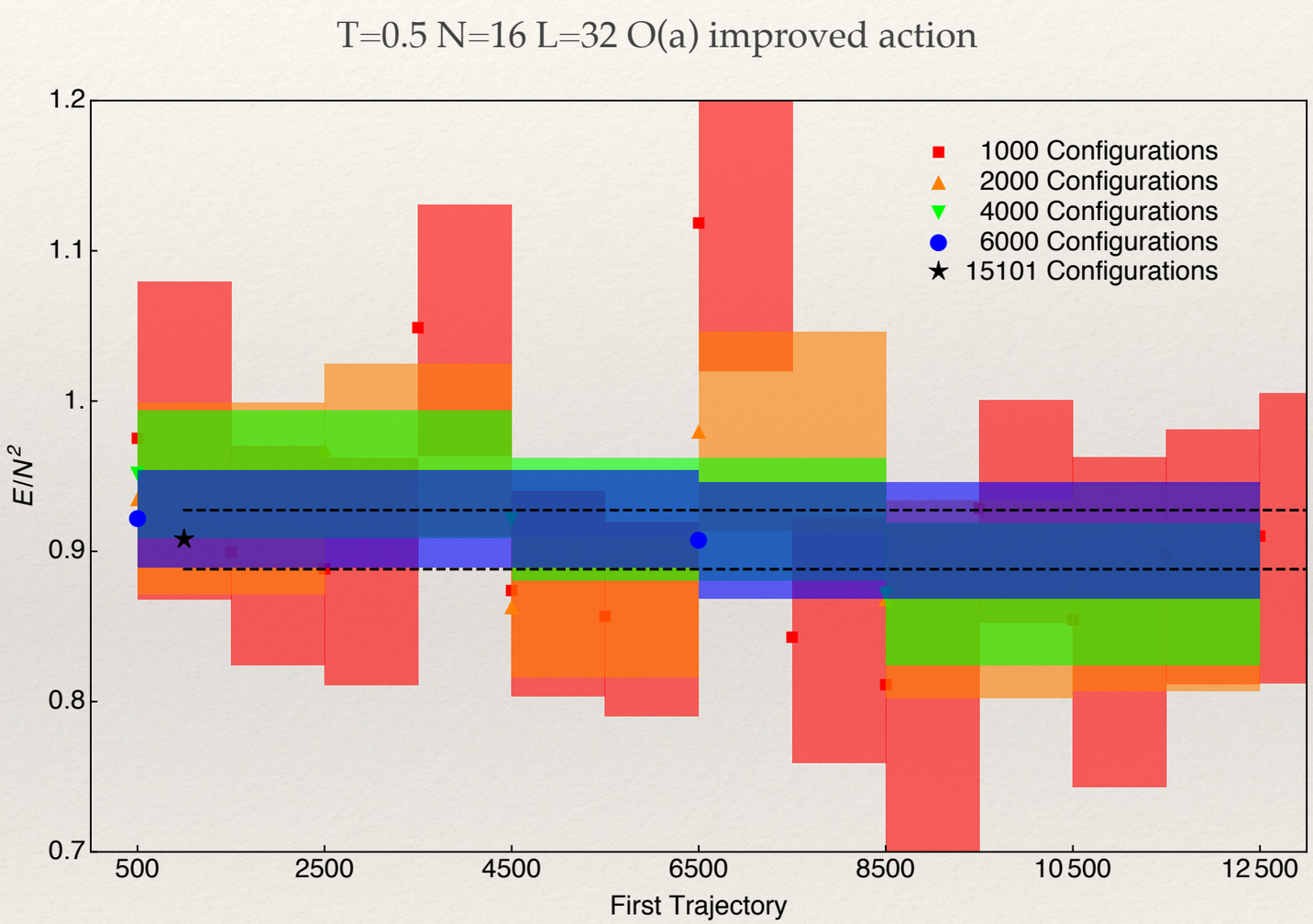
T=0.5 N=24 L=32 O(a) improved action



[MCSMC, Phys. Rev. D94 (2016) 094501]

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- Large statistics for all parameters (N,L,T) is needed
- Dedicated autocorrelation analysis is paramount due to long fluctuations
- All data is published in tables and can be used for future benchmarks!



[MCSMC, Phys. Rev. D94 (2016) 094501]

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$\tilde{T}$	N	L	# config	$\tilde{E}/N^2$	# bins	$\tilde{F}^2$	# bins	$\tilde{R}^2$	# bins
		24	77150	1.335(11)	653	16.6676(40)	637	3.45866(92)	258
		32	47098	1.284(12)	682	17.3021(38)	735	3.51756(88)	261
	32	8	81525	1.5790(28)	1772	13.4749(24)	627	3.1673(12)	119
		16	44800	1.4435(62)	711	15.5880(27)	640	3.34742(61)	296
		24	37995	1.3376(99)	436	16.6513(37)	391	3.44802(87)	132
		32	23365	1.282(18)	156	17.2927(54)	189	3.5079(11)	106
0.60	16	8	35489	1.9669(98)	865	14.280(11)	134	–	–
		12	94960	1.8965(92)	1396	15.5090(55)	797	3.3577(16)	218
		16	76700	1.8435(87)	2191	16.3184(41)	1420	3.4363(11)	511
		24	38633	1.726(17)	858	17.3404(70)	529	3.5351(18)	108
		32	60839	1.702(18)	965	17.8823(66)	602	3.5822(16)	224
		64	78466	1.612(22)	1318	18.8460(58)	861	3.6672(14)	288
	24	8	49303	1.9782(57)	1146	14.2312(39)	573	3.2155(10)	188
		16	43540	1.828(11)	649	16.2966(46)	544	3.41313(96)	235
		24	95005	1.750(11)	969	17.2897(41)	819	3.50643(75)	461
		32	58712	1.704(13)	863	17.8506(37)	1030	3.55674(70)	431
	32	8	85250	1.9894(33)	1813	14.2281(19)	1522	3.20939(45)	600
		16	54720	1.8519(66)	994	16.2872(29)	729	3.40559(55)	411
		24	52896	1.7386(98)	661	17.2848(36)	539	3.49982(67)	238
		32	31240	1.696(16)	240	17.8483(56)	226	3.5506(10)	127
0.70	16	8	47580	2.4294(99)	1106	15.0002(70)	587	3.2961(18)	214
		12	104350	2.3780(98)	1683	16.1766(54)	1054	3.4069(12)	366
		16	84565	2.3210(99)	2349	16.9525(43)	1691	3.48264(91)	735
		24	76014	2.199(14)	1900	17.9056(53)	1134	3.5734(11)	434
		32	72060	2.183(19)	1242	18.3763(66)	783	3.6114(13)	335

Perturbative treatment of type IIA superstring: type IIA supergravity + higher derivative corrections e.g. Hanada et al. arxiv:1603.00538

Analytical expectations at finite T

$$E/N^2 = \frac{\left(a_0 T^{2.8} + a_1 T^{4.6} + a_2 T^{5.8} + \cdots\right)}{N^0} + \frac{\left(b_0 T^{0.4} + b_1 T^{2.2} + \cdots\right)}{N^2} + \mathcal{O}\left(\frac{1}{N^4}\right)$$

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$\ell_s{}^6 \cdots$ Include effects of finite string length

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small T, infinite N, strong coupling $\quad a_0 = 7.41$

finite T, infinite N, finite coupling $\quad \ell_s{}^6 \cdots$ Include effects of finite string length

small T, finite N, strong coupling $\quad b_0 = -5.77$

Include effects of finite string coupling

small T, finite N, strong coupling $\quad \sim \ell_s{}^{12} g_s{}^2$

quantum string corrections

[MCSMC, Phys. Rev. D94 (2016) 094501]

Extract predictions on the gauge side

- LO
- NLO
- NNLO
- NNNLO

$$E_0 = a_0 T^{2.8}$$

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- NLOP1(fixed LO)
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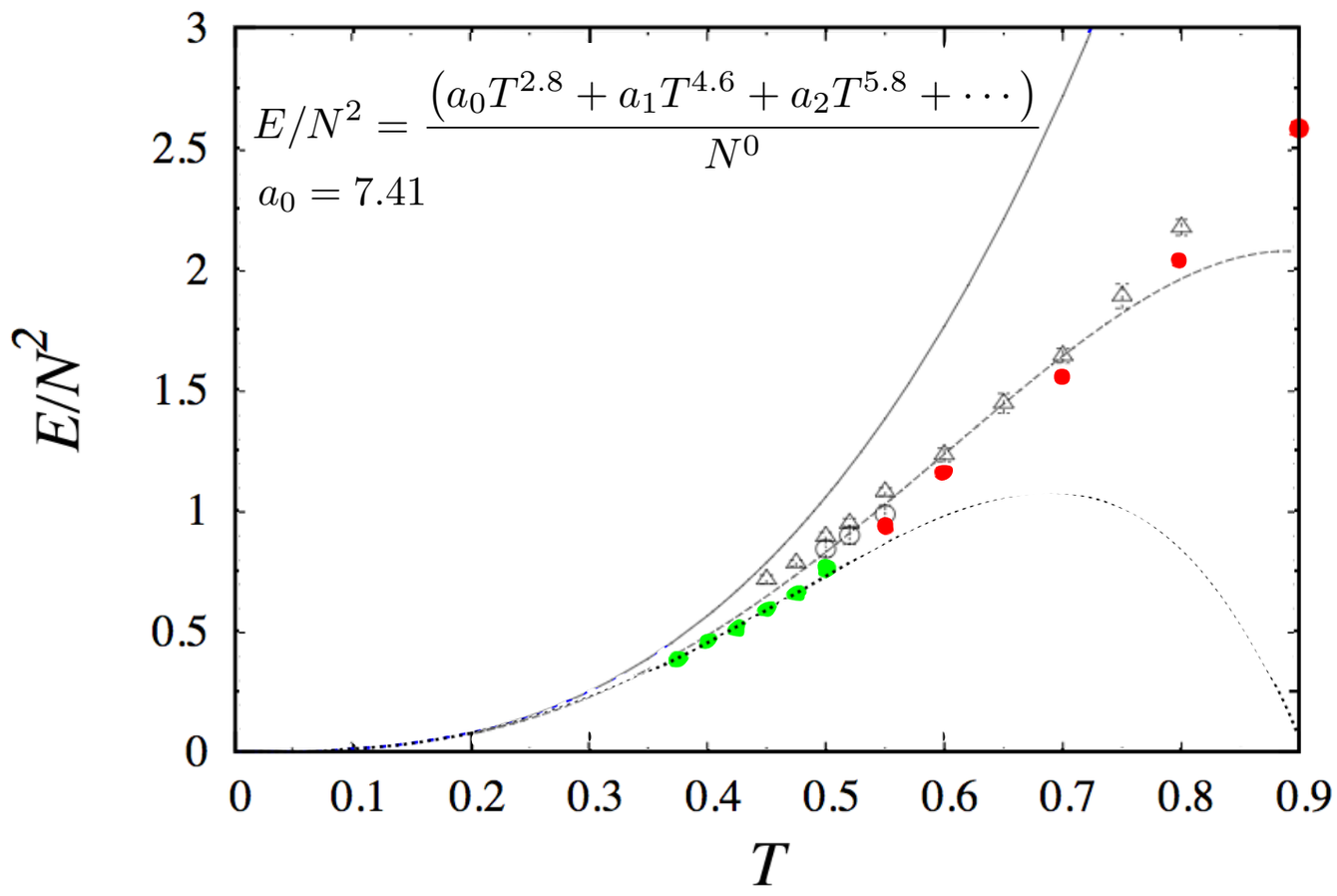
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TEST

Previous results

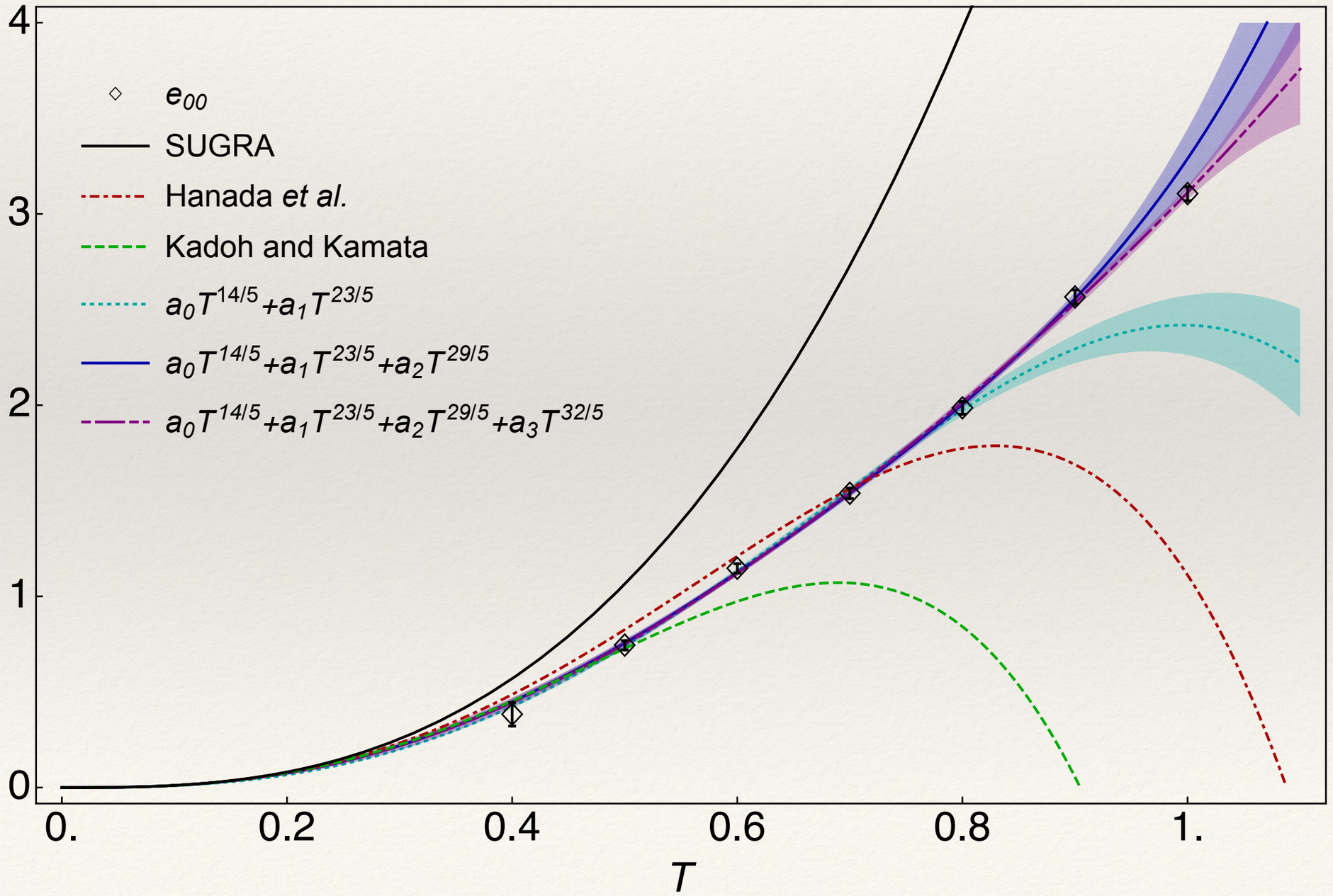
- Different cutoff regulator
- Different discretizations
- Finite N
- Finite cutoff
- Qualitative agreement
- Not enough precision for quantitative predictions

[Agnastopoulos et al. arxiv:0707:4454]
[Catteral, Wiseman arxiv:0803.4273]
[Hanada et al. arxiv:0811.3102]
[Hanada et al. arxiv:1311.5603]
[Kadoh, Kamata arxiv:1503.08499]
[Filev, O'Connor arxiv:1506.01366]
…



Test of duality in BFSS

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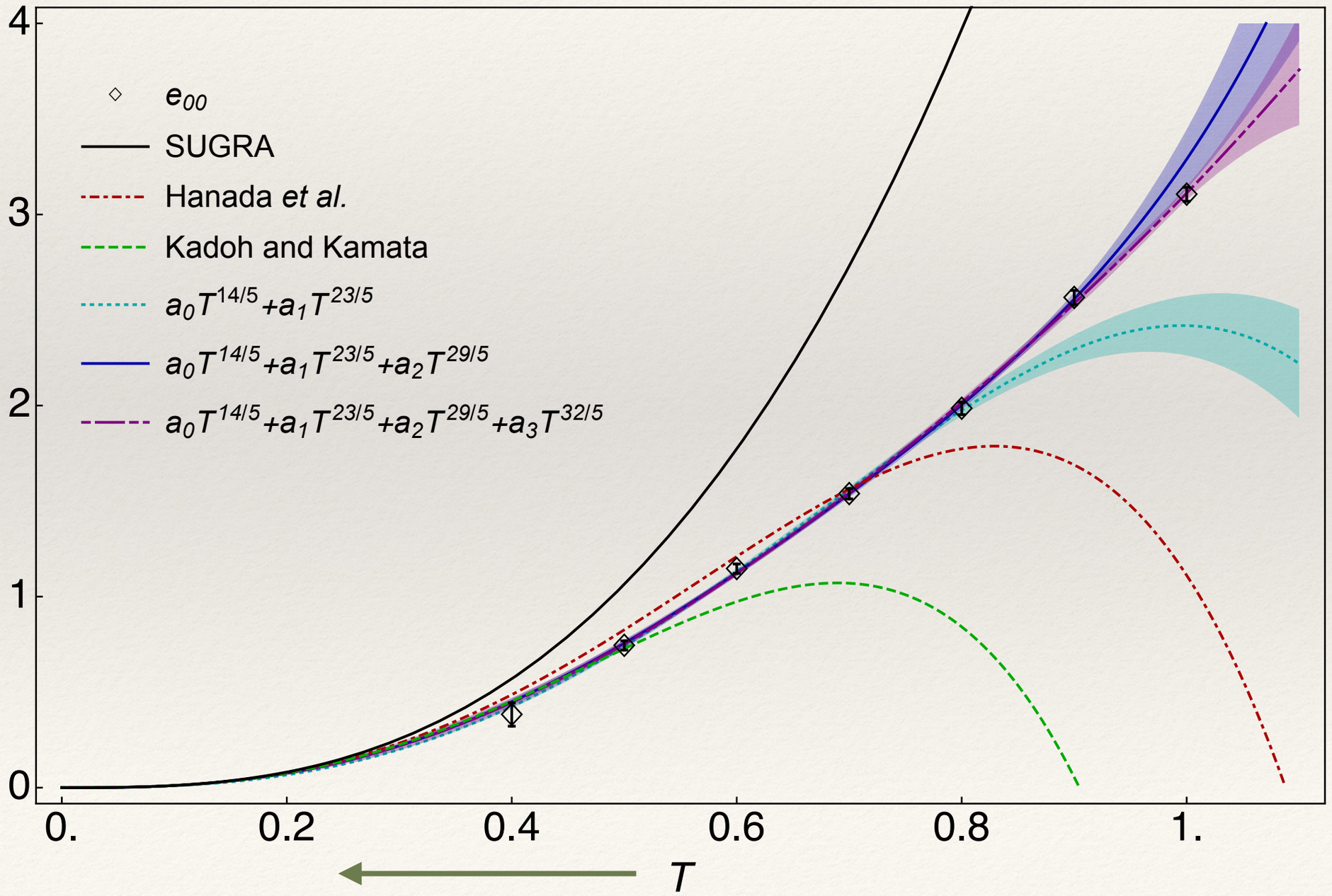


[Hanada et al. arxiv:0811.3102, Kadoh and Kamata arxiv:1503.08499] [MCSMC, Phys. Rev. D94 (2016) 094501]

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$a_0 = 7.41$ Supergravity



Gravity EASY

Gauge HARD

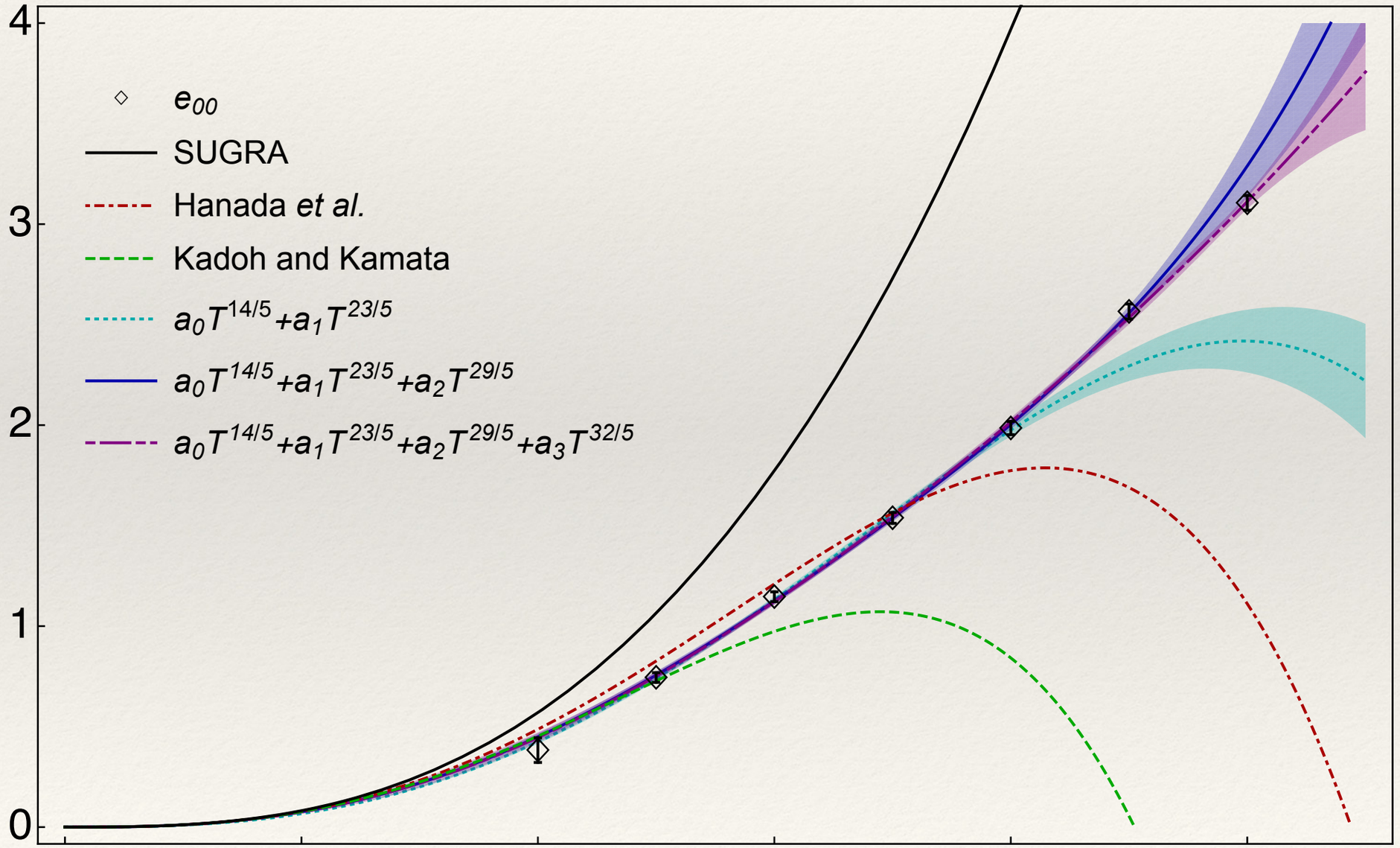
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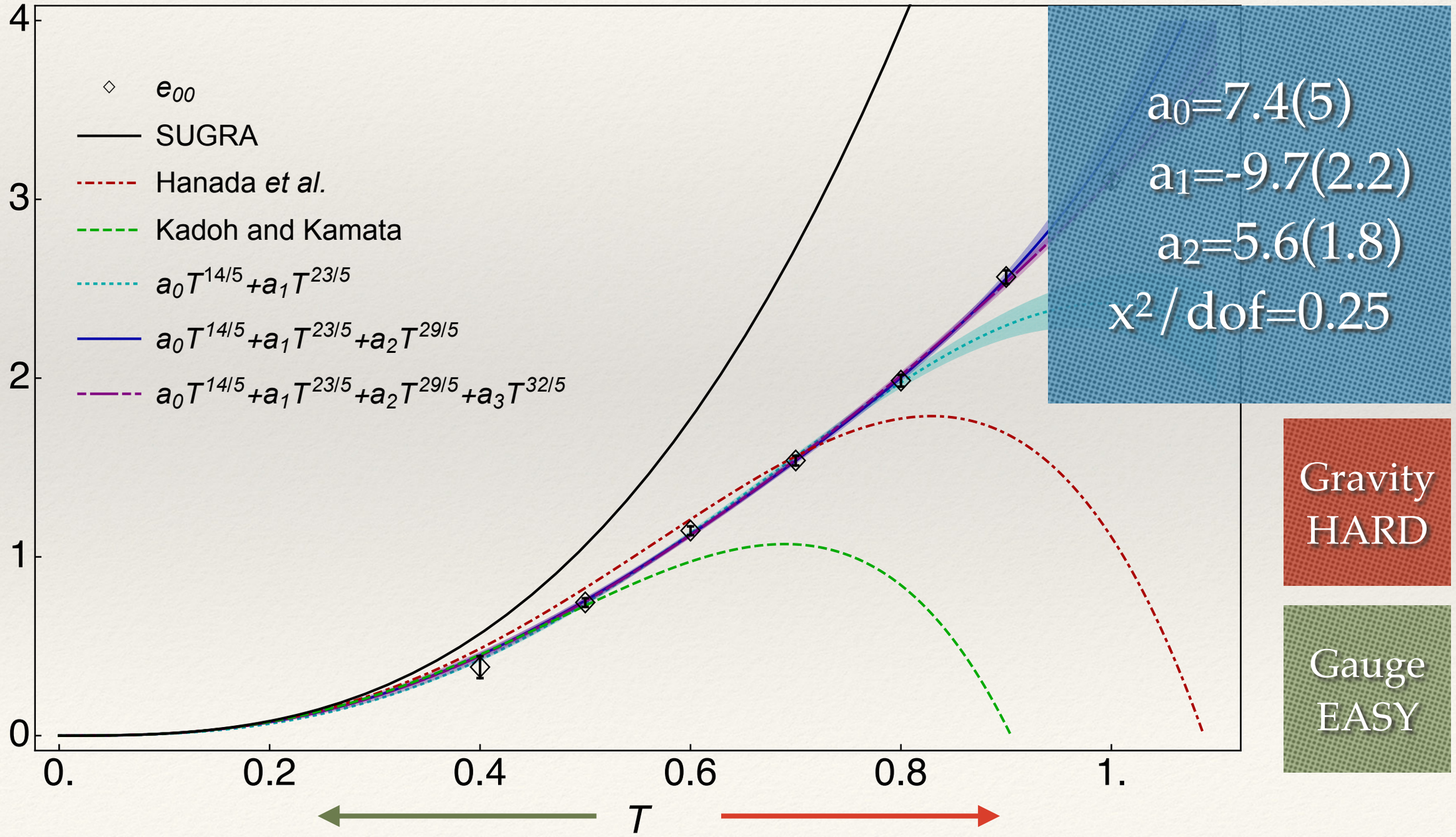
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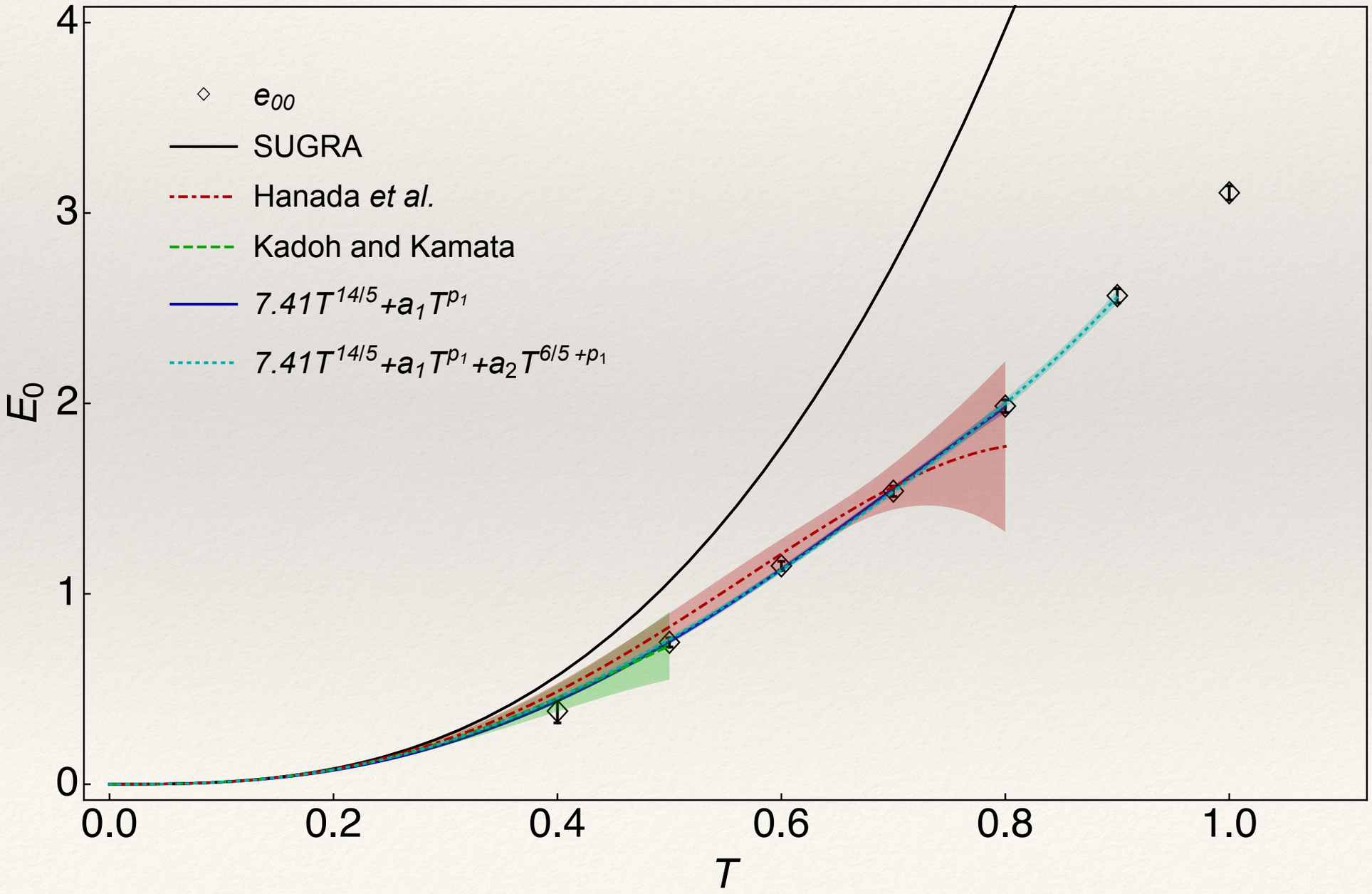
Gauge HARD

Gravity HARD

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Test string effects

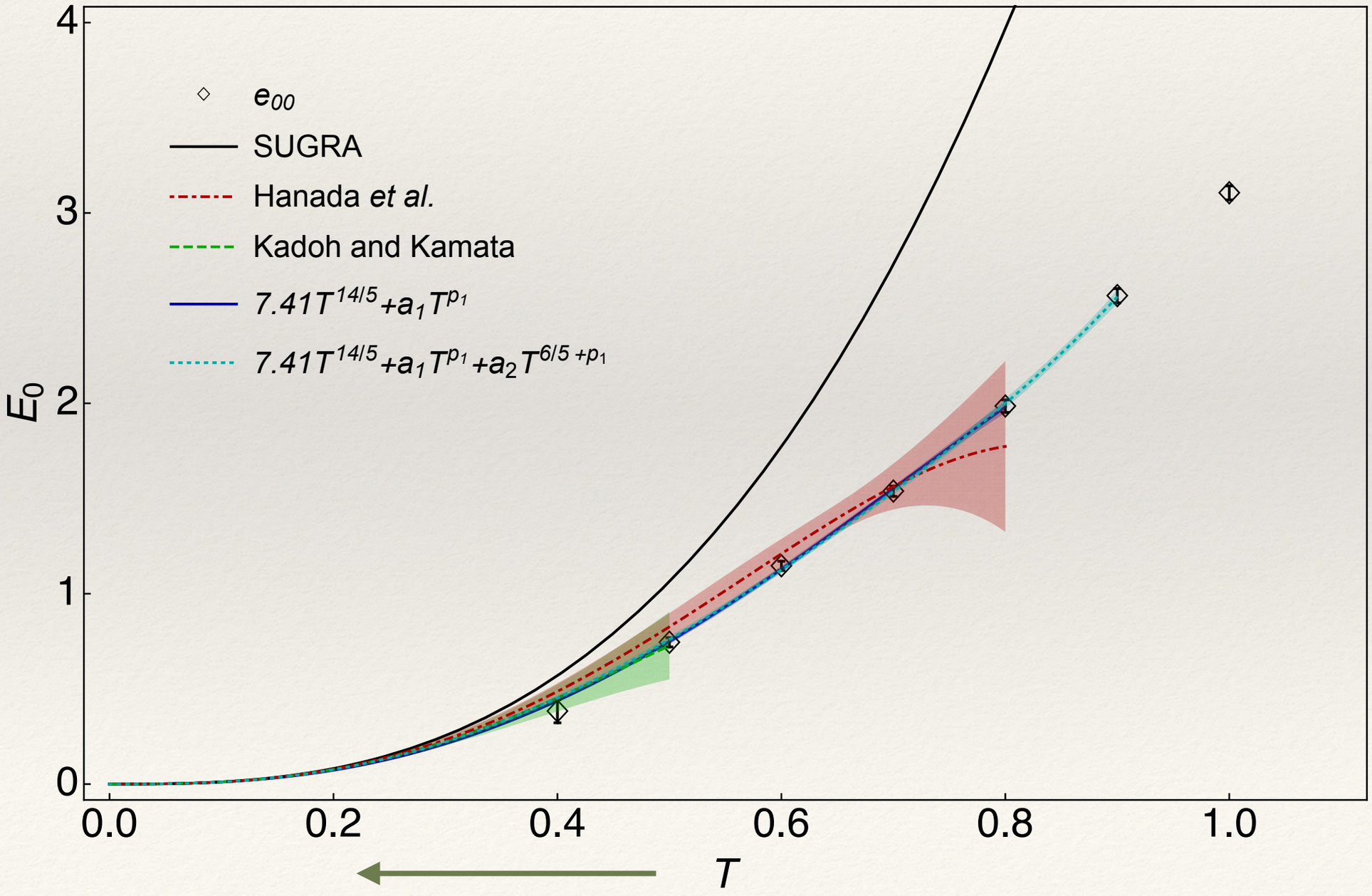
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Test string effects

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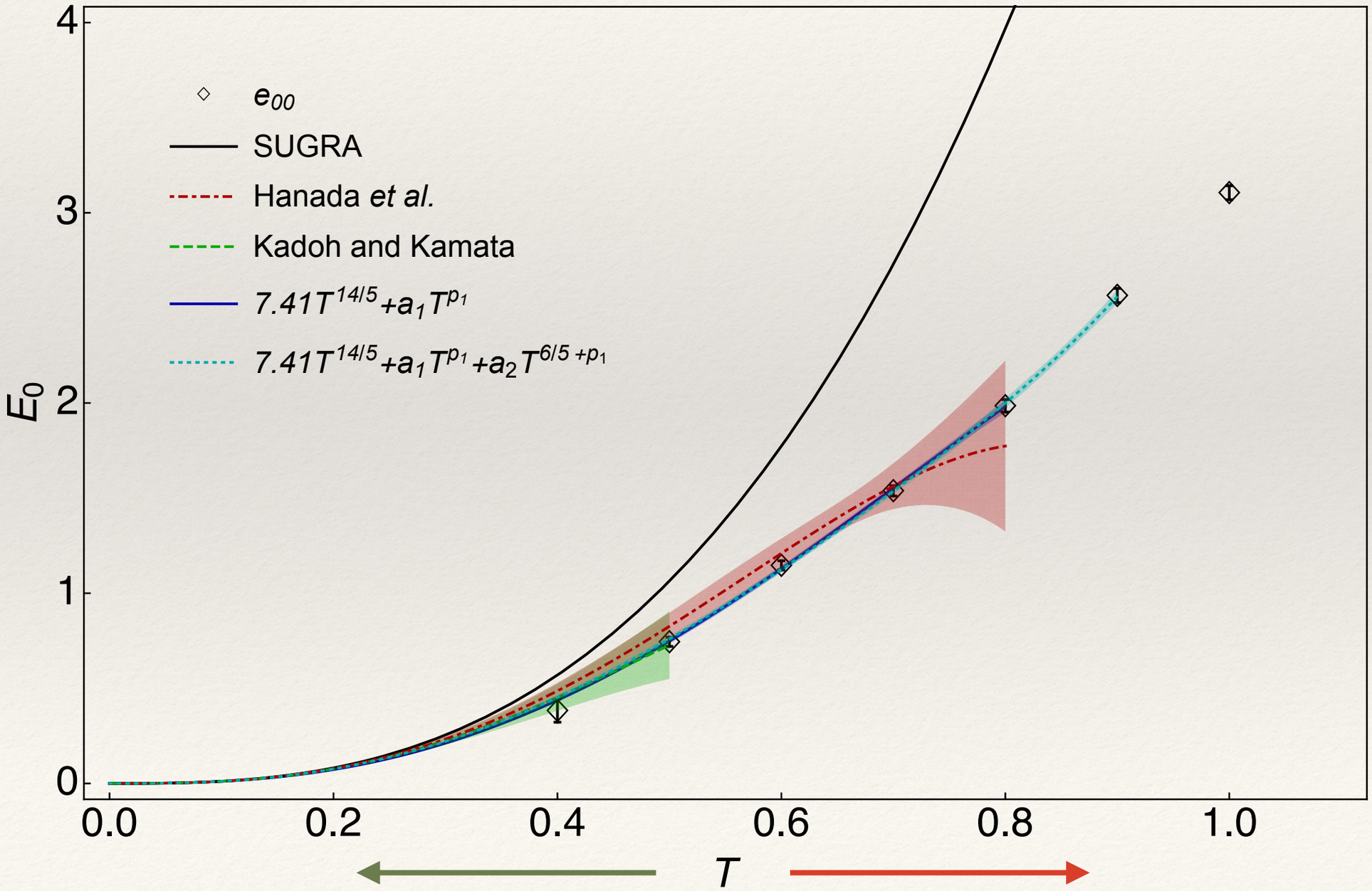
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Gravity EASY

Gauge HARD

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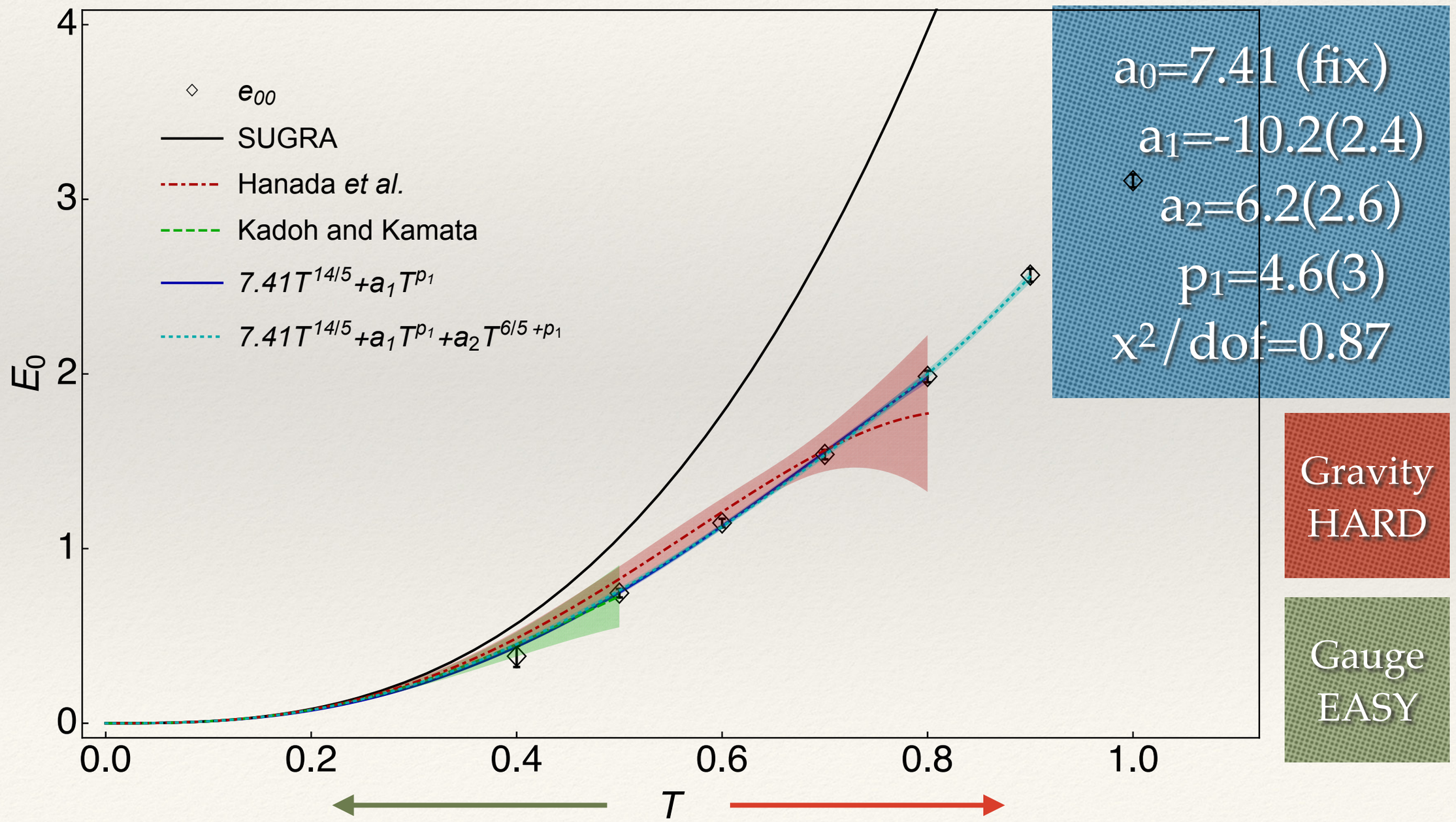
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$a_1 =? \; p_1 = 4.6$ String effects



a_0=7.41 (fix)
a_1=-10.2(2.4)
a_2=6.2(2.6)
p_1=4.6(3)
x^2 / dof=0.87

Gravity EASY

Gauge HARD

Gravity HARD

Gauge EASY

[MCSMC 2019, in preparation]

Preliminary new results

$$\frac{E}{N^2} = a_0 T^{p_0} + a_1 T^{p_0+1.8} + a_2 T^{p_0+3}$$

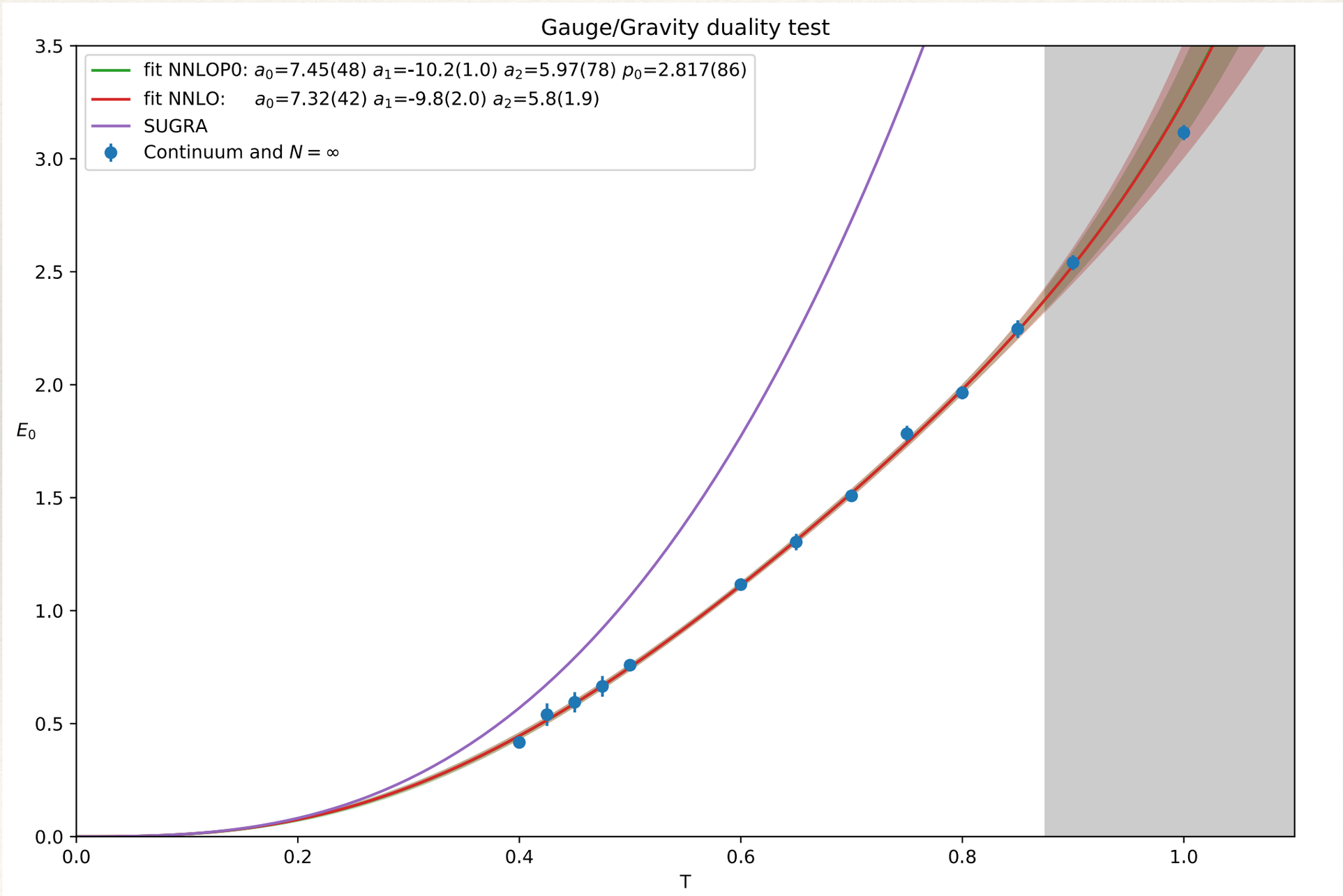
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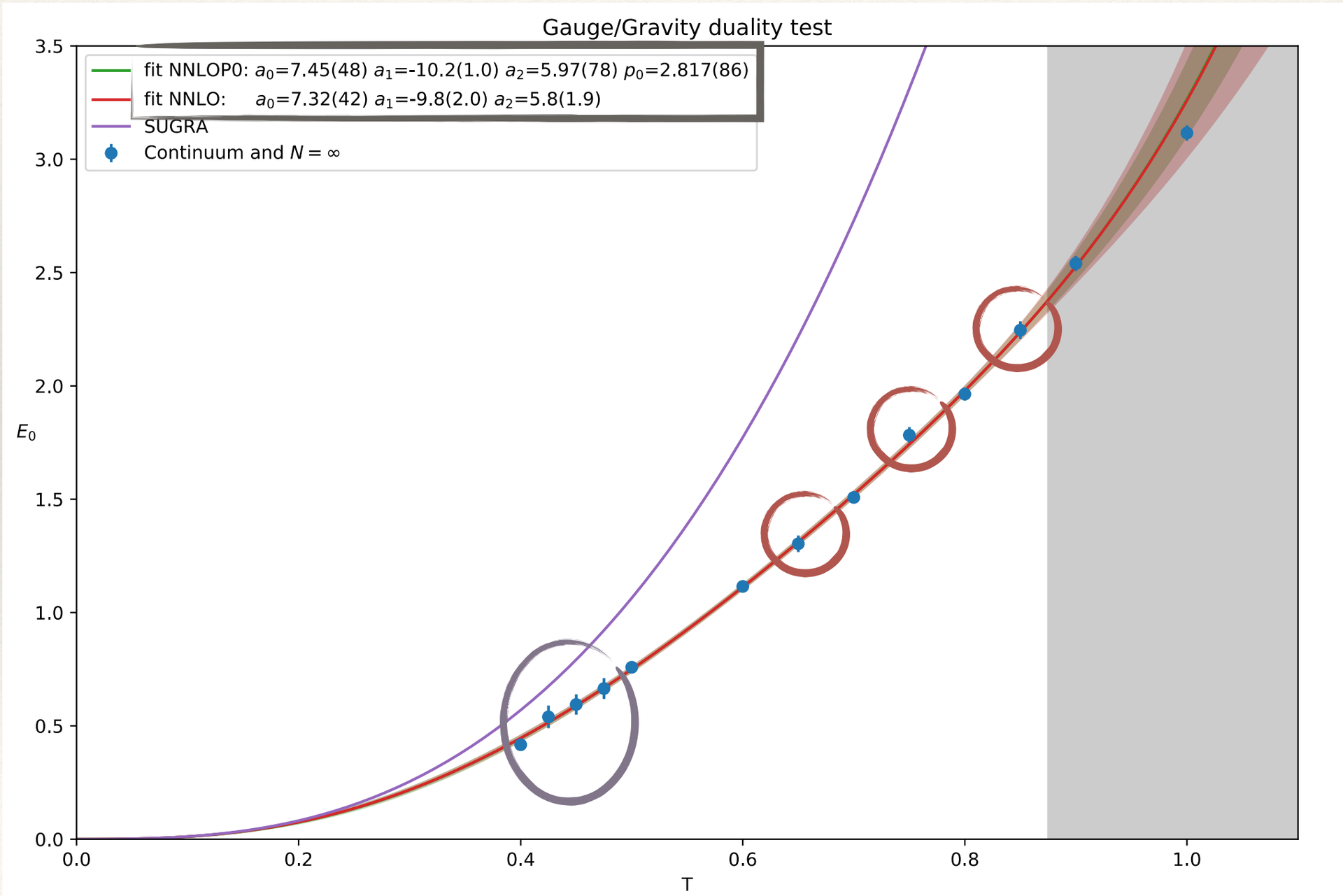


[MCSMC 2019, in preparation]

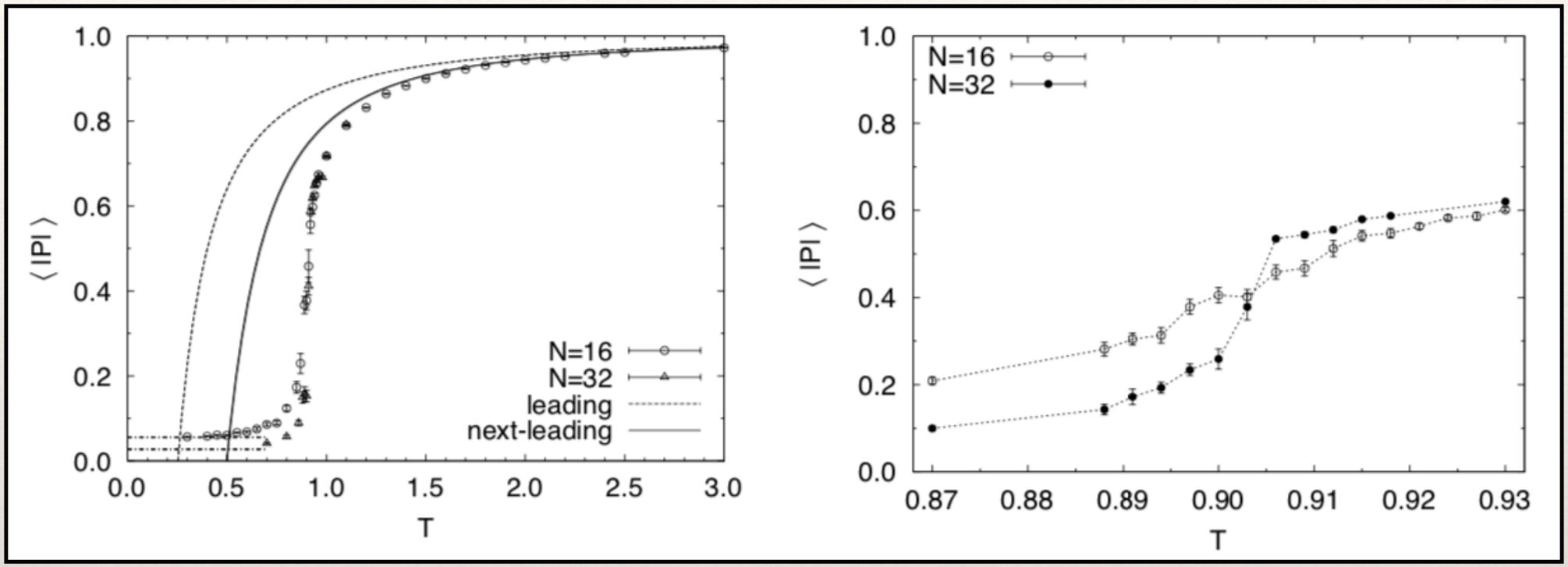
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Phase transitions



[Kawahara, Nishimura, Takeuchi 2007]

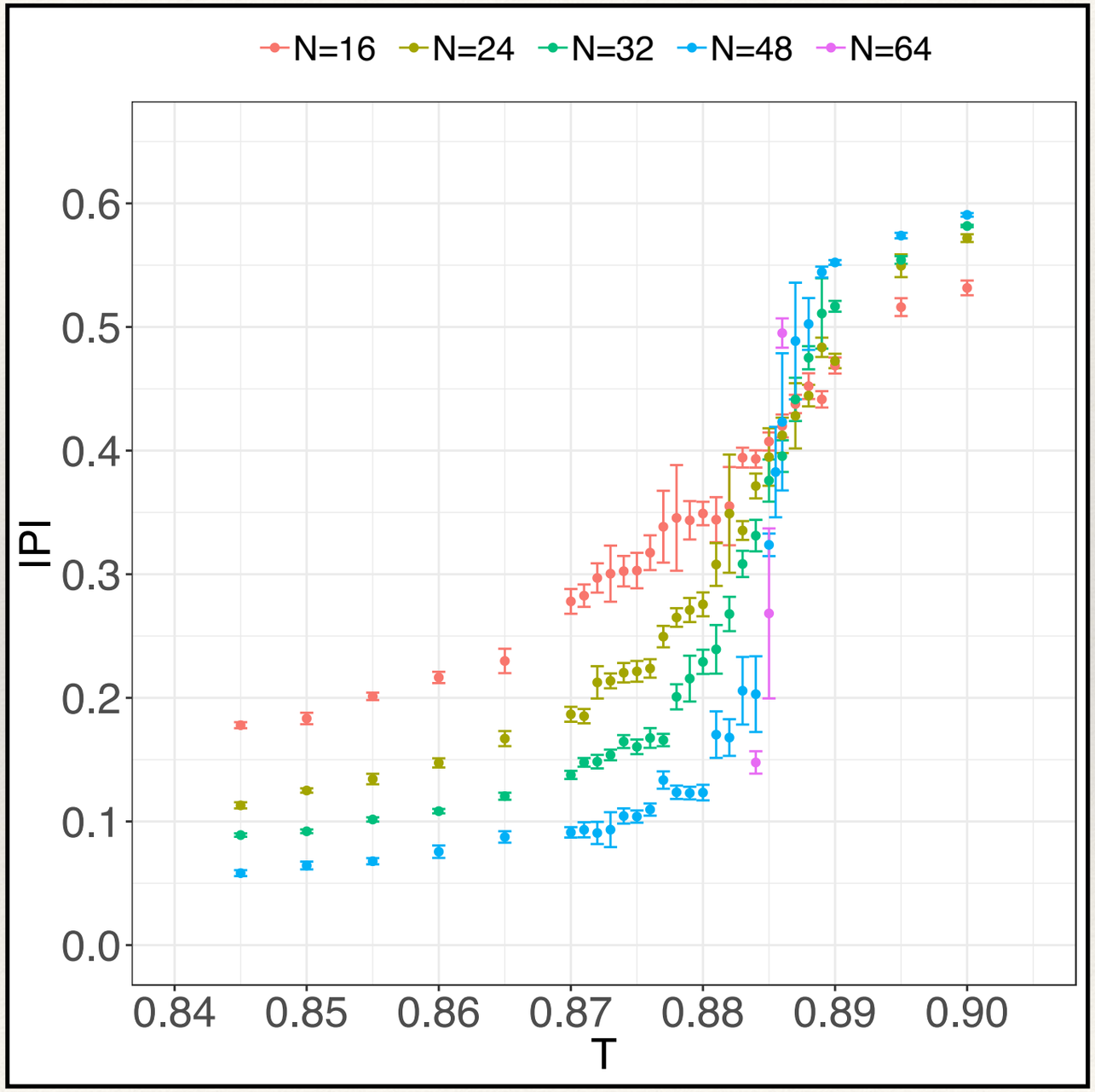
- Temperature corresponds to spatial circle of 2d SYM
- Phase transition corresponds to ***black hole/black string*** topology change in the dual gravity picture

- Previous results at finite N
- and finite cutoff
- Qualitative agreement with transition of 2nd order

[MCSMC 2019, in preparation]

Bosonic BFSS to large N

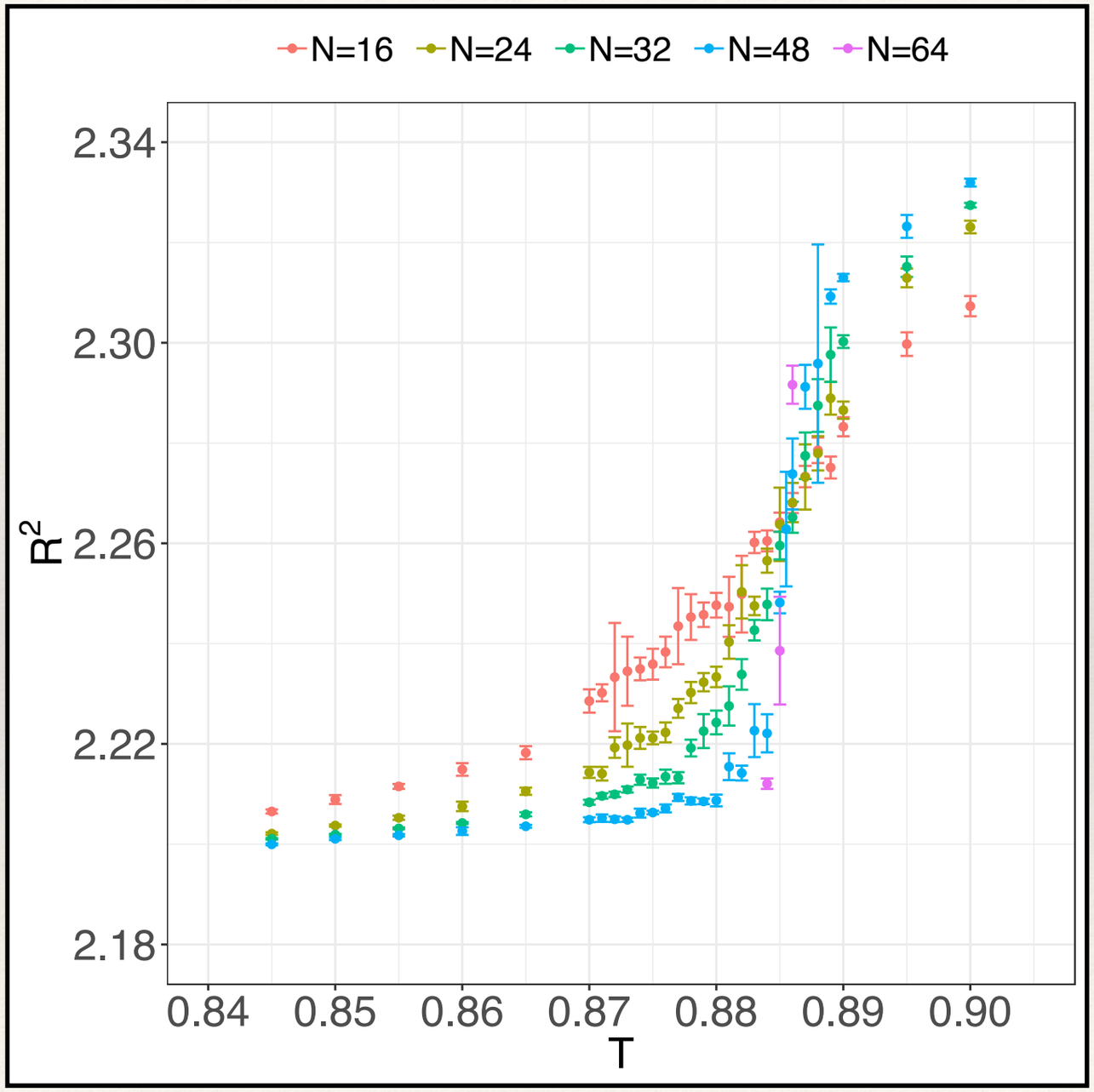
- Increase to N =64
- Order parameter becomes more discontinuous
- Larger N indicates transition of 1st order
- Remove the cutoff
- Other observables have the same qualitative behavior



[MCSMC 2019, in preparation]

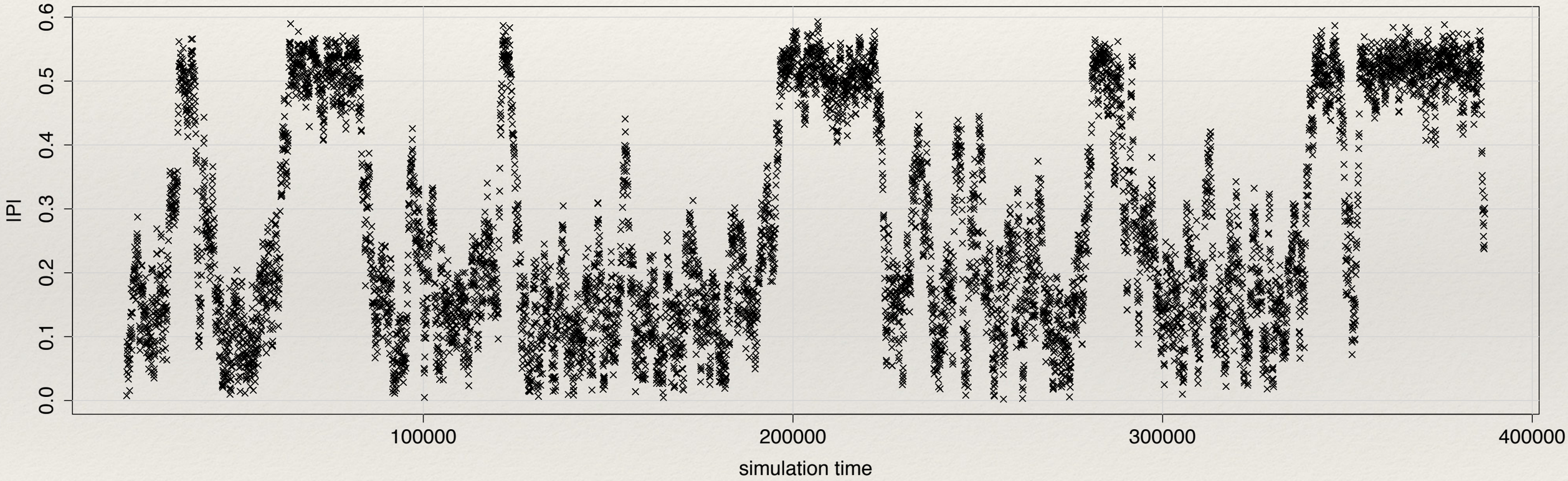
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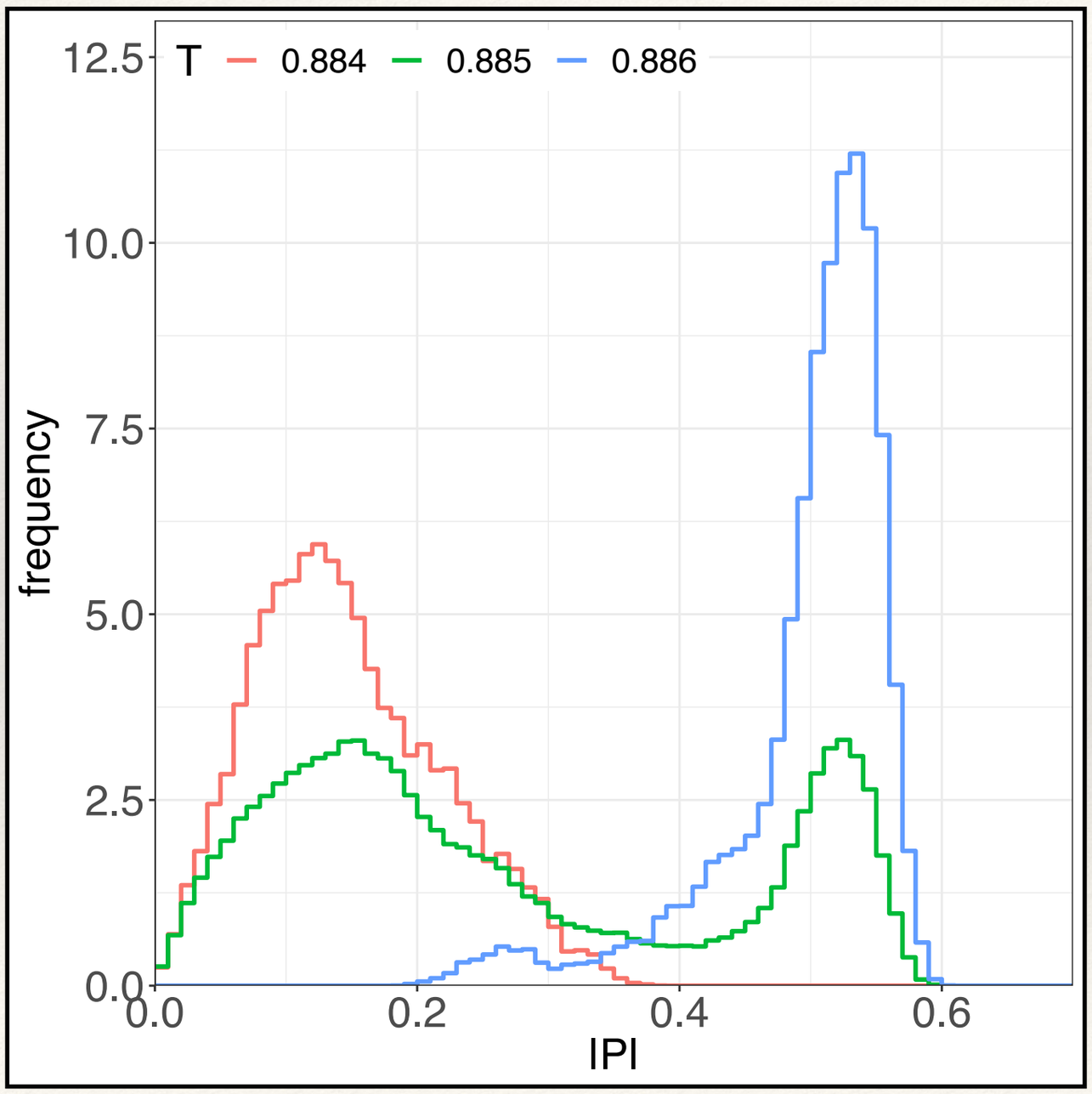
[MCSMC 2019, in preparation]

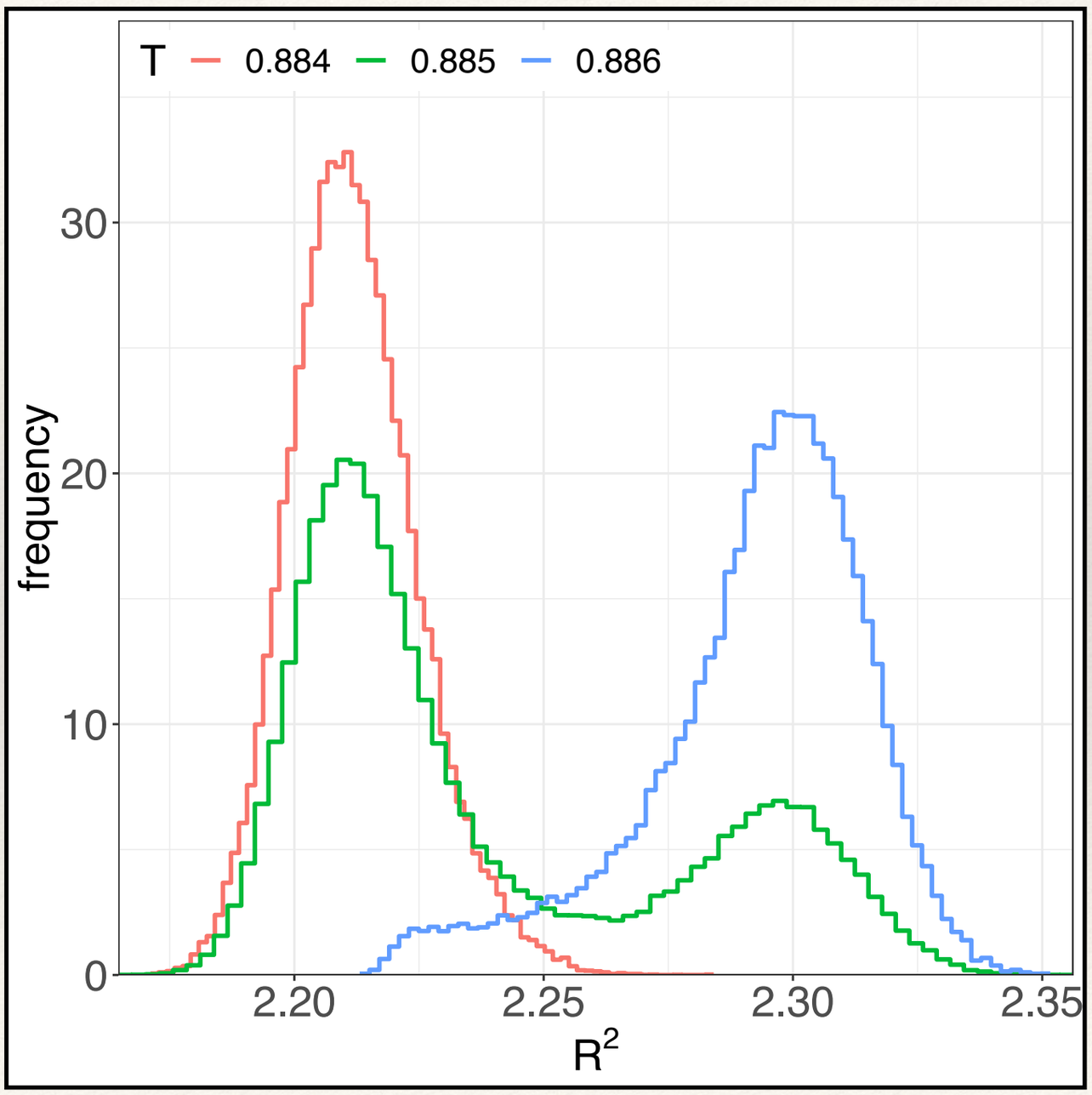
Order of transition



[MCSMC 2019, in preparation]

Order of transition

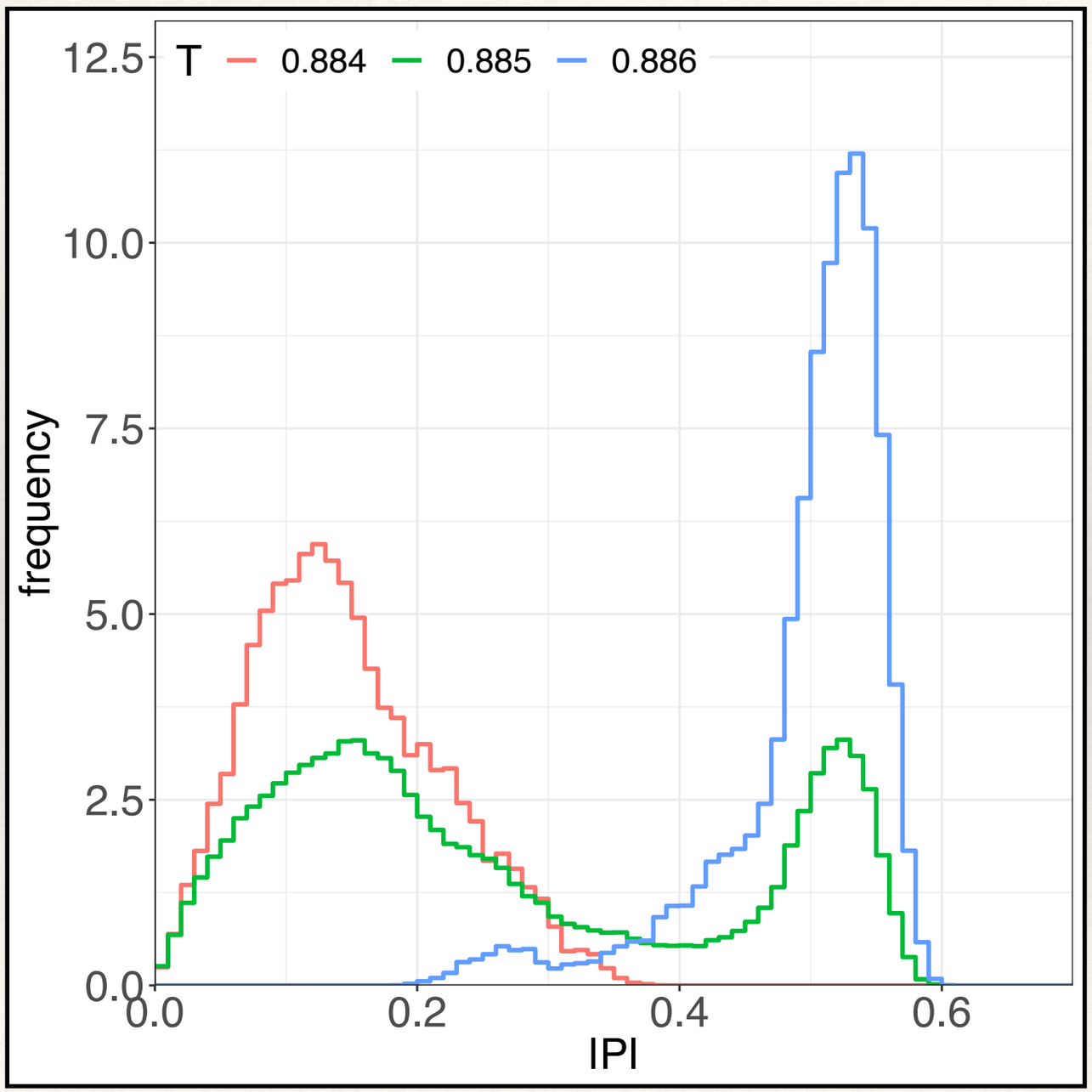


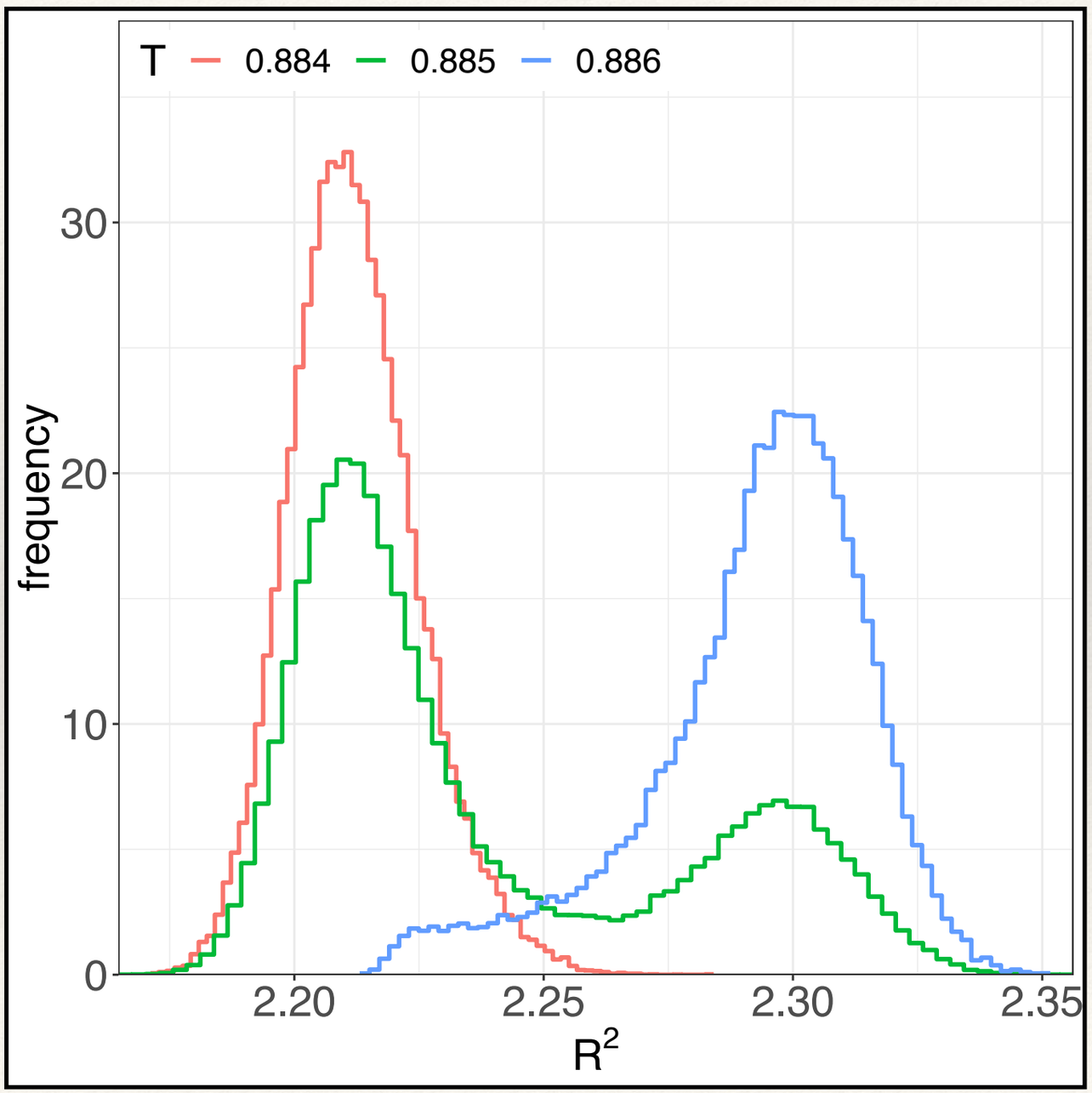


[MCSMC 2019, in preparation]

Order of transition

Transition is 1st order: consistent with gauge/gravity prediction in 2d SYM





Summary and future work

- Testing the gauge/gravity duality is the first step towards using it to define a quantum theory of gravity
- Numerical tests are now mature for low-dimensional systems: control over continuum and planar limit
- Lower temperatures can probe the M-theory regime of the model: new results are already coming…
- Higher-dimensional SYM can be studied on the lattice: first step using effective lower-dimensional models
- Phase diagram elucidated with numerical simulations.

Extra slides

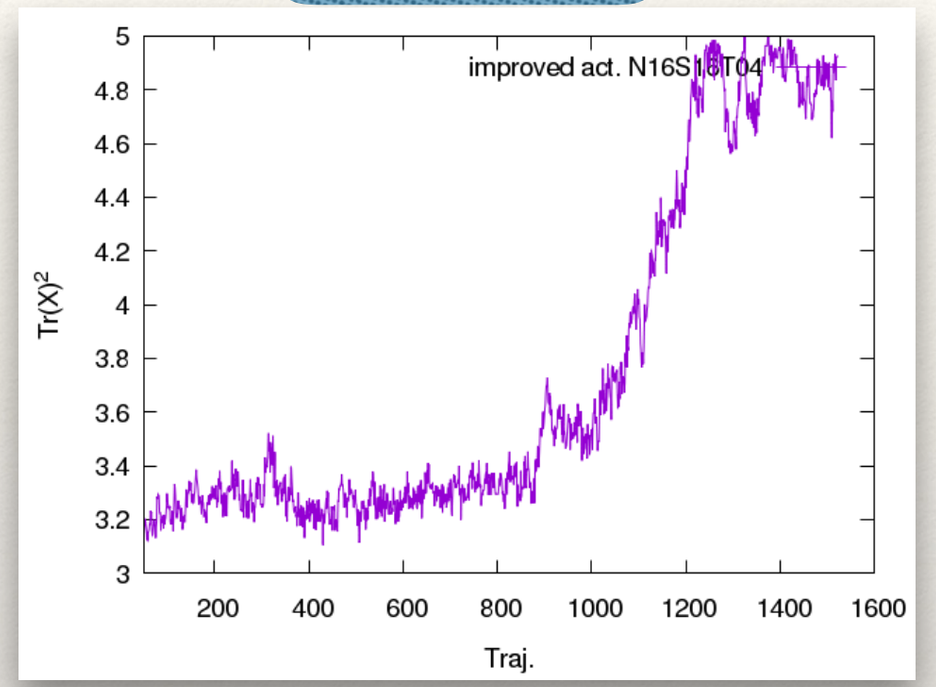
[MCSMC, Phys. Rev. D94 (2016) 094501]

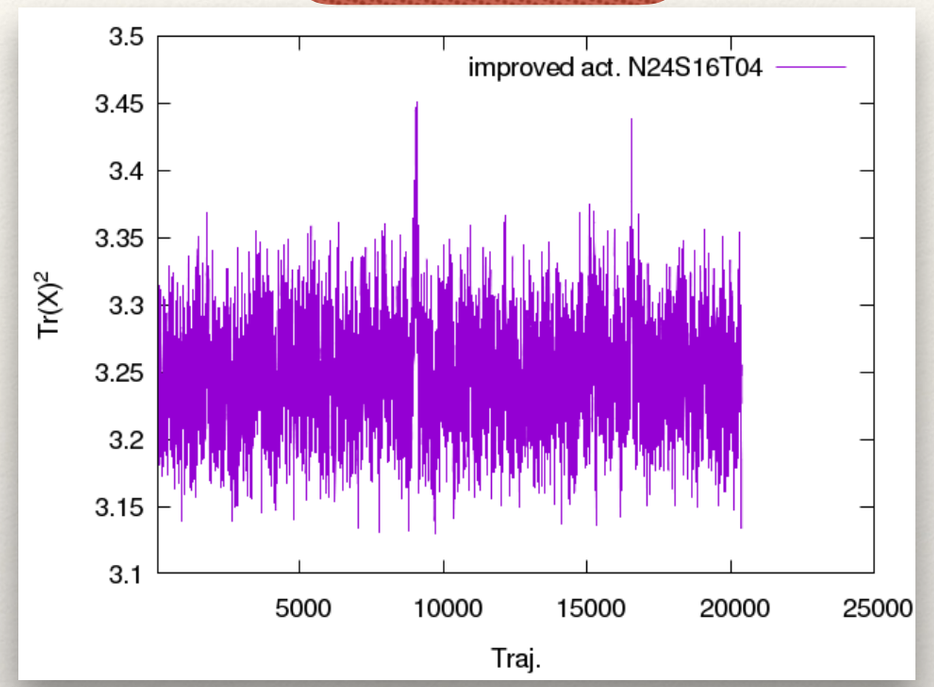
Challenge: flat direction

N=16

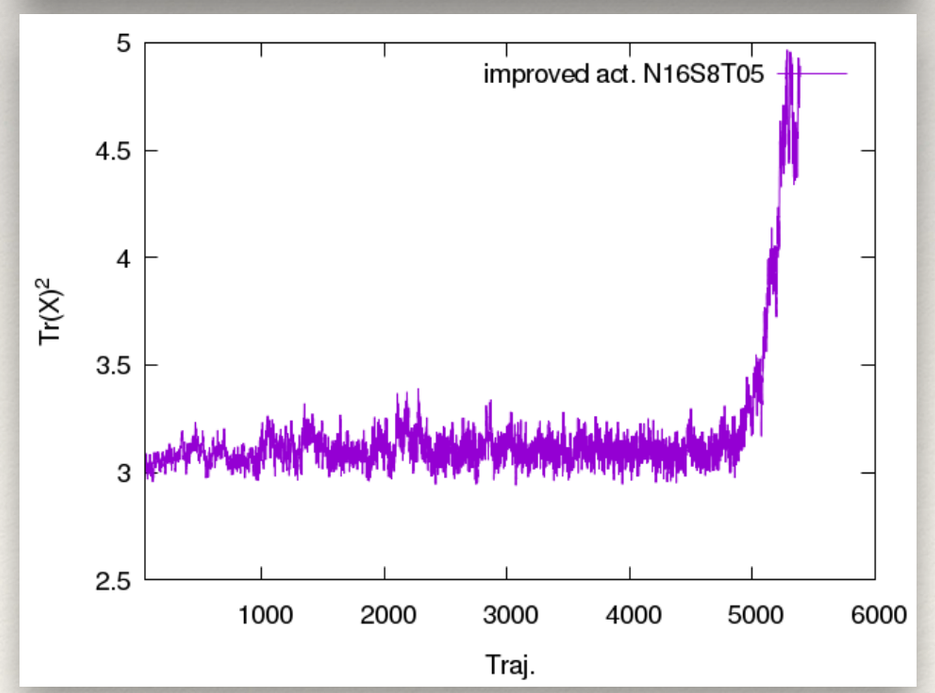
N=24

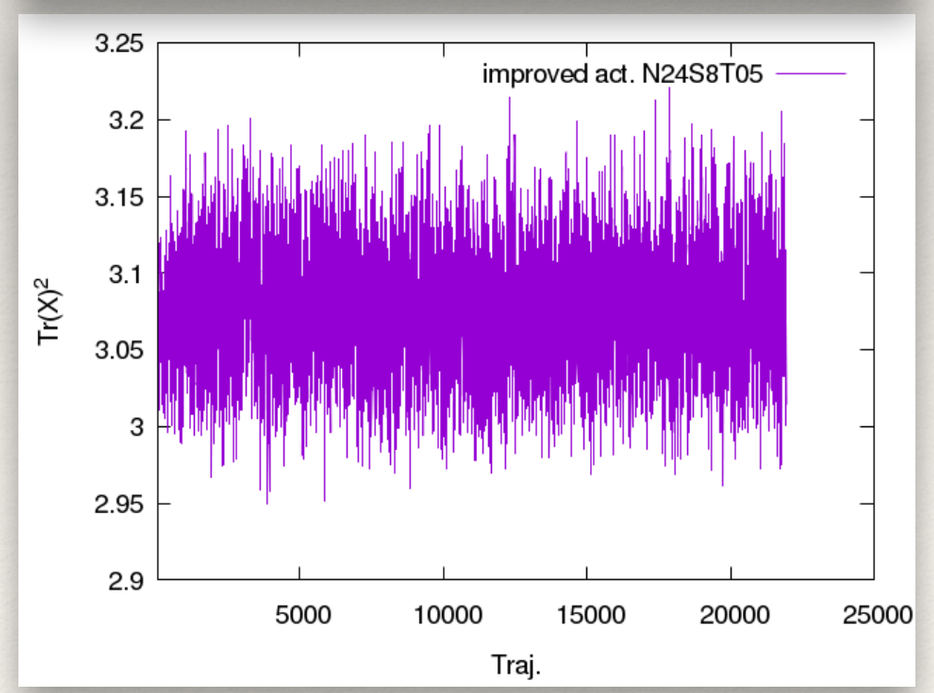
T=0.4





T=0.5





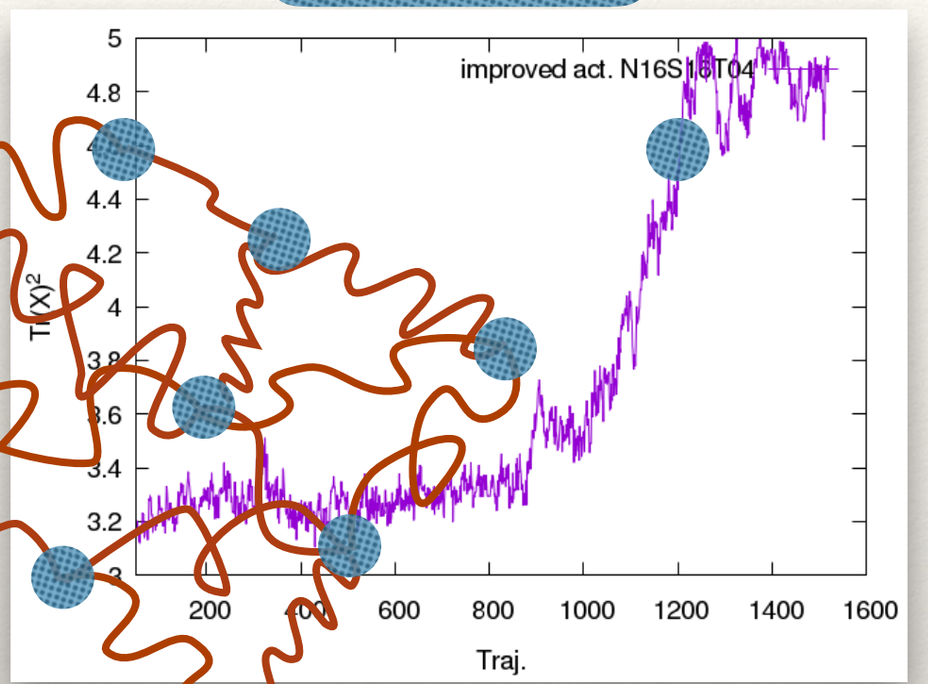
[MCSMC, Phys. Rev. D94 (2016) 094501]

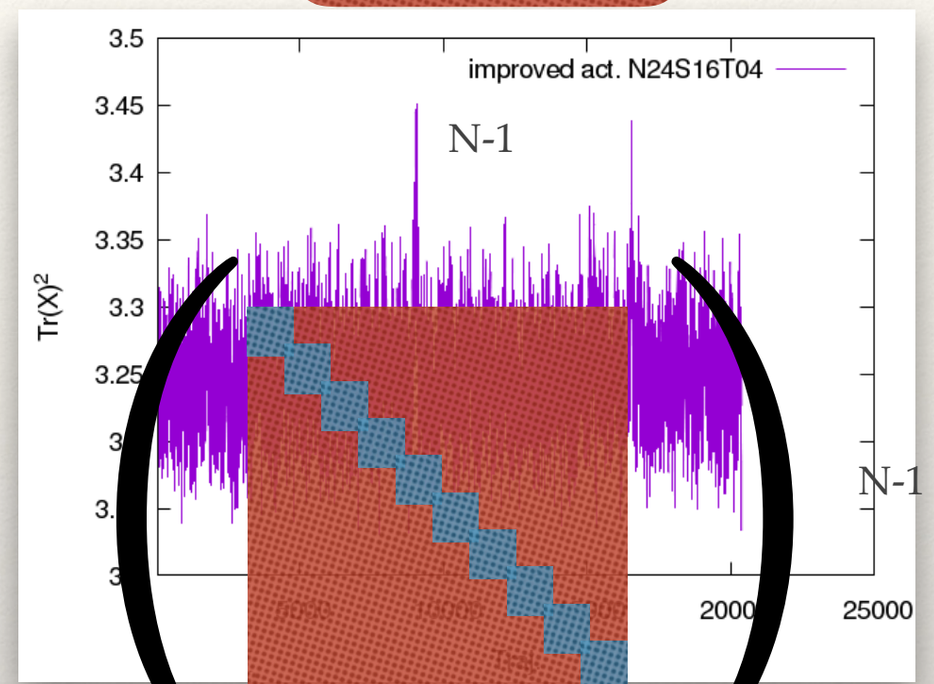
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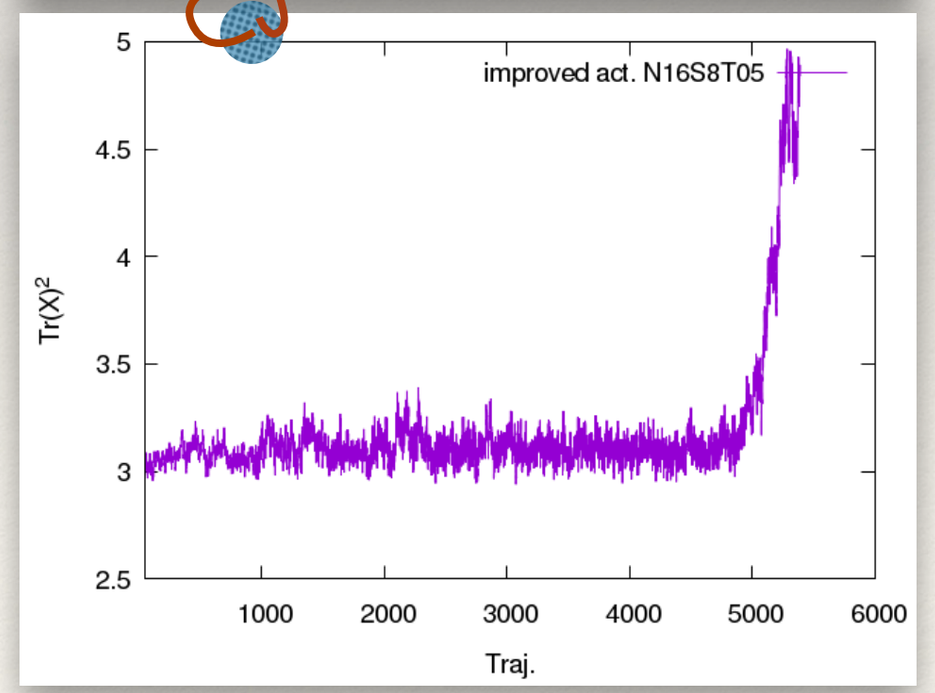
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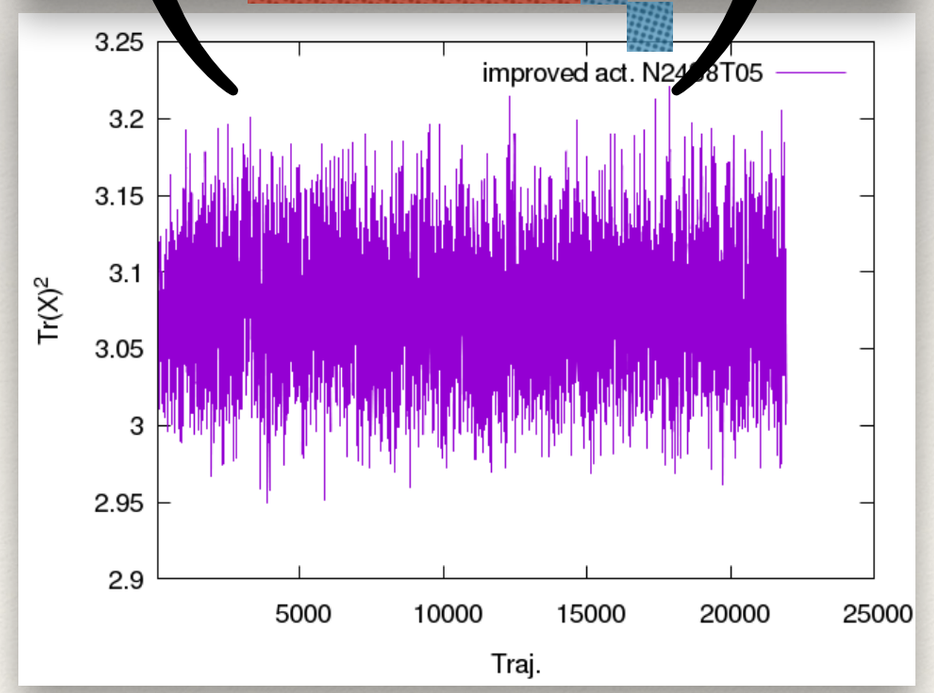
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T=0.5



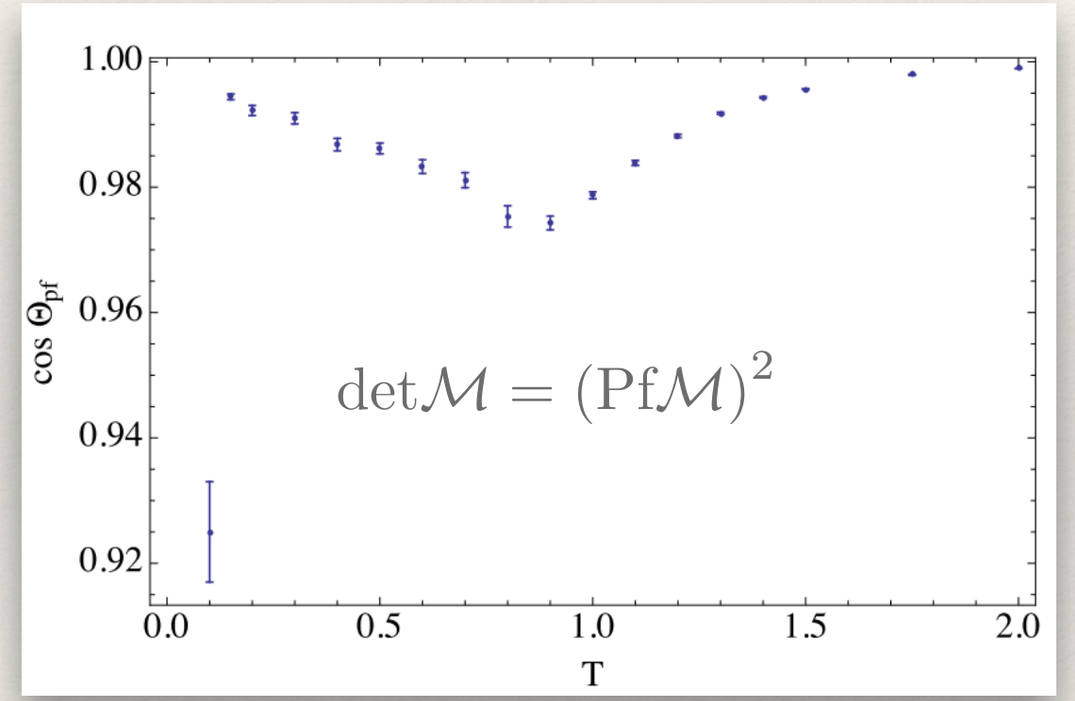


[Filev, O'Connor arxiv:1506.01366]
[Hanada, Nishimura, Sekino, Yoneya, arxiv:1108.5153]

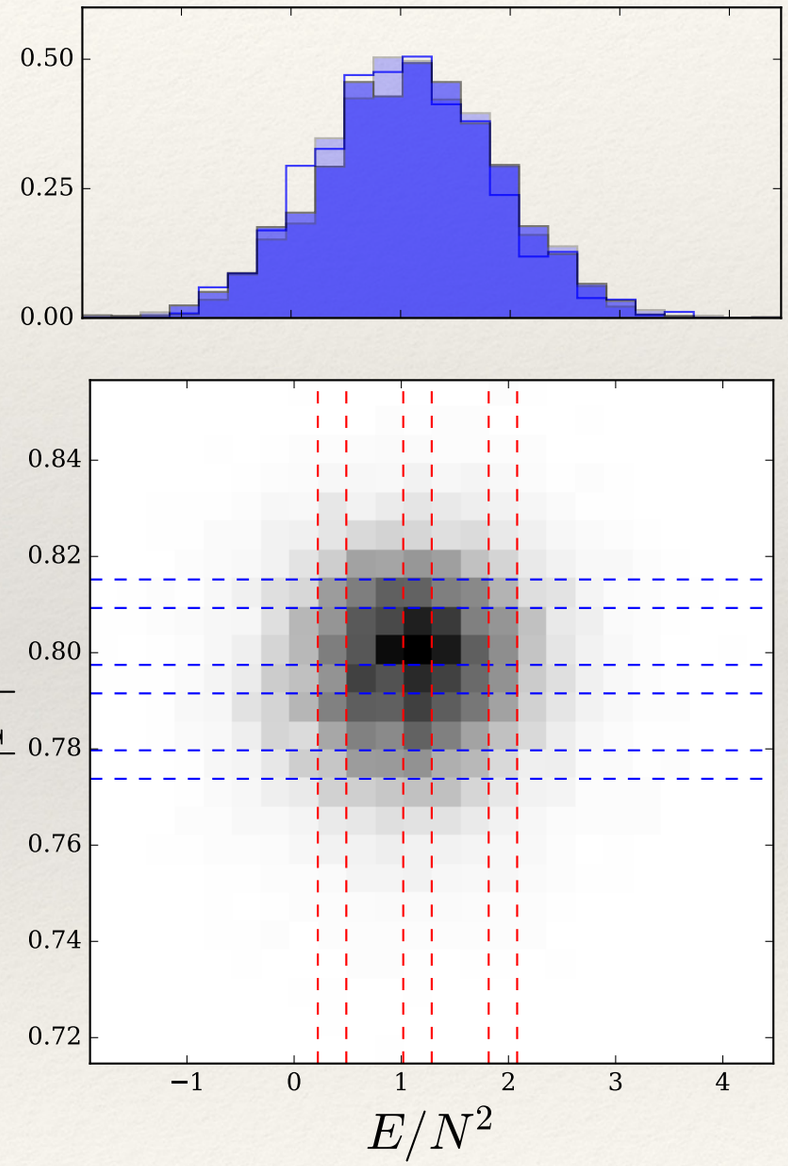
[MCSMC, Phys. Rev. D94 (2016) 094501]

Challenge: Pfaffian

N=3 L=4 (+ small regulator mass)



T=0.5 N=16 L=16 O(a) improved action



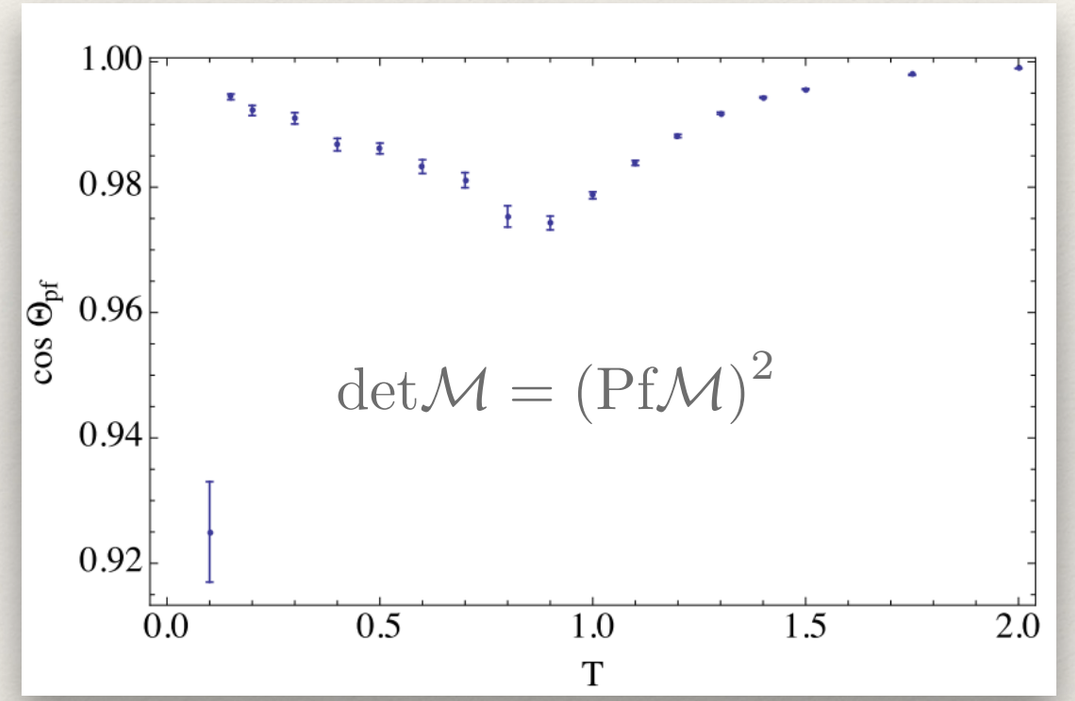
[Filev, O'Connor arxiv:1506.01366]
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[MCSMC, Phys. Rev. D94 (2016) 094501]

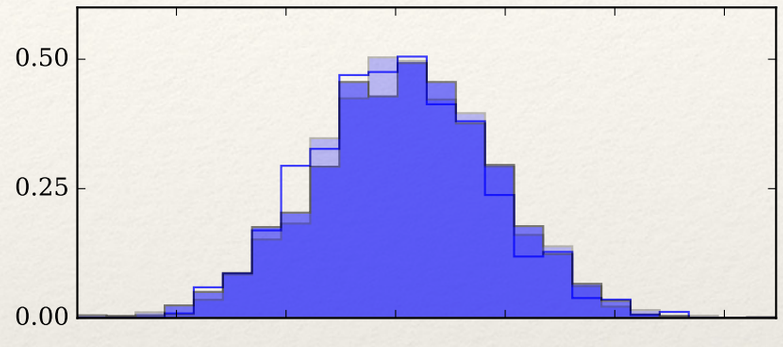
Challenge: Pfaffian

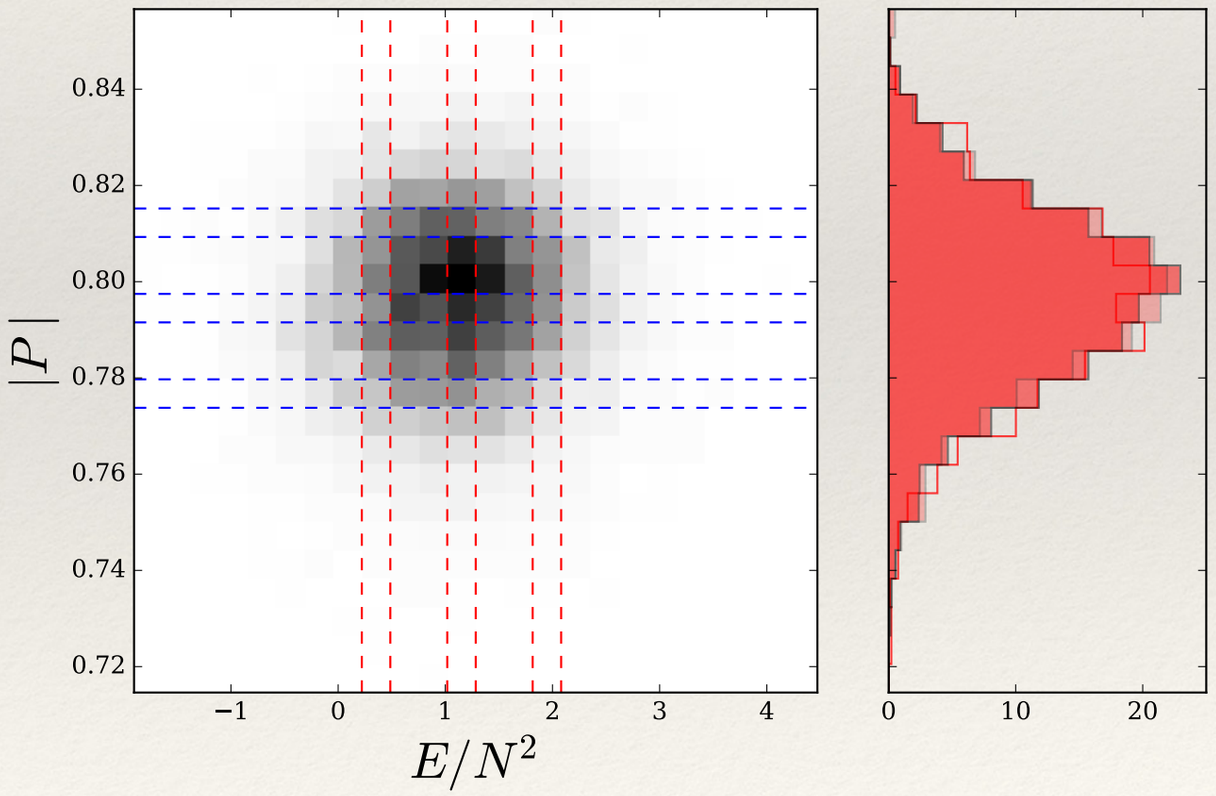
phase reweighting: $\langle \mathcal{O} \rangle_{\mathrm{F}} = \dfrac{\langle \mathcal{O} \cdot e^{i\theta} \rangle_{\mathrm{PQ}}}{\langle e^{i\theta} \rangle_{\mathrm{PQ}}}$

N=3 L=4 (+ small regulator mass)



T=0.5 N=16 L=16 O(a) improved action





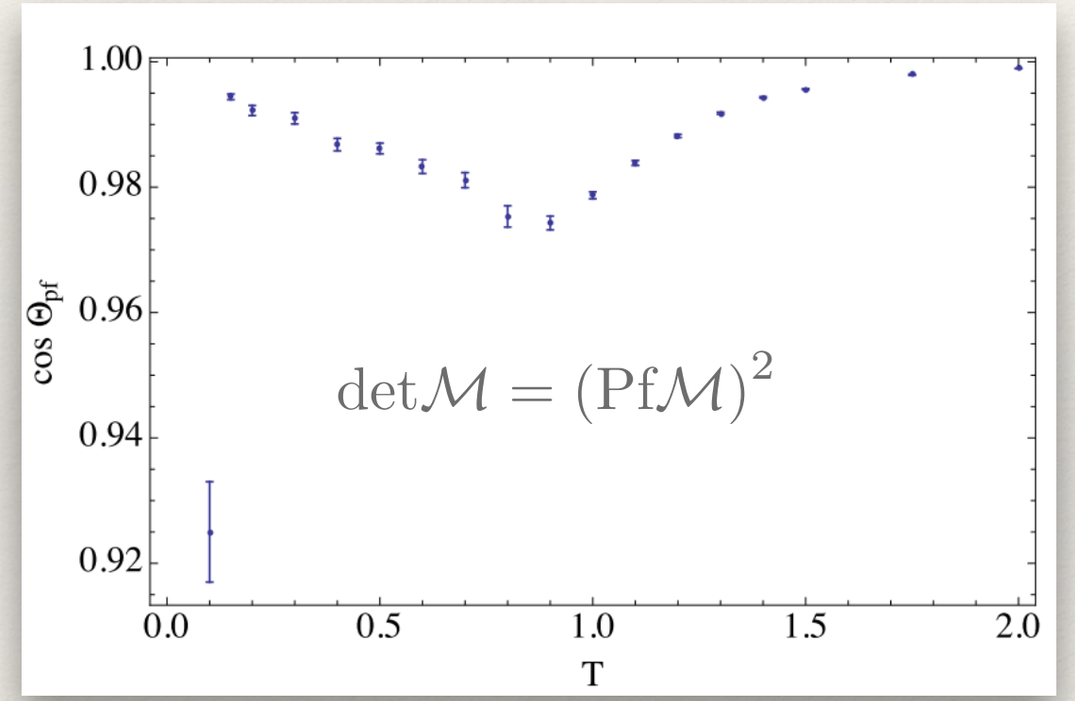
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[MCSMC, Phys. Rev. D94 (2016) 094501]

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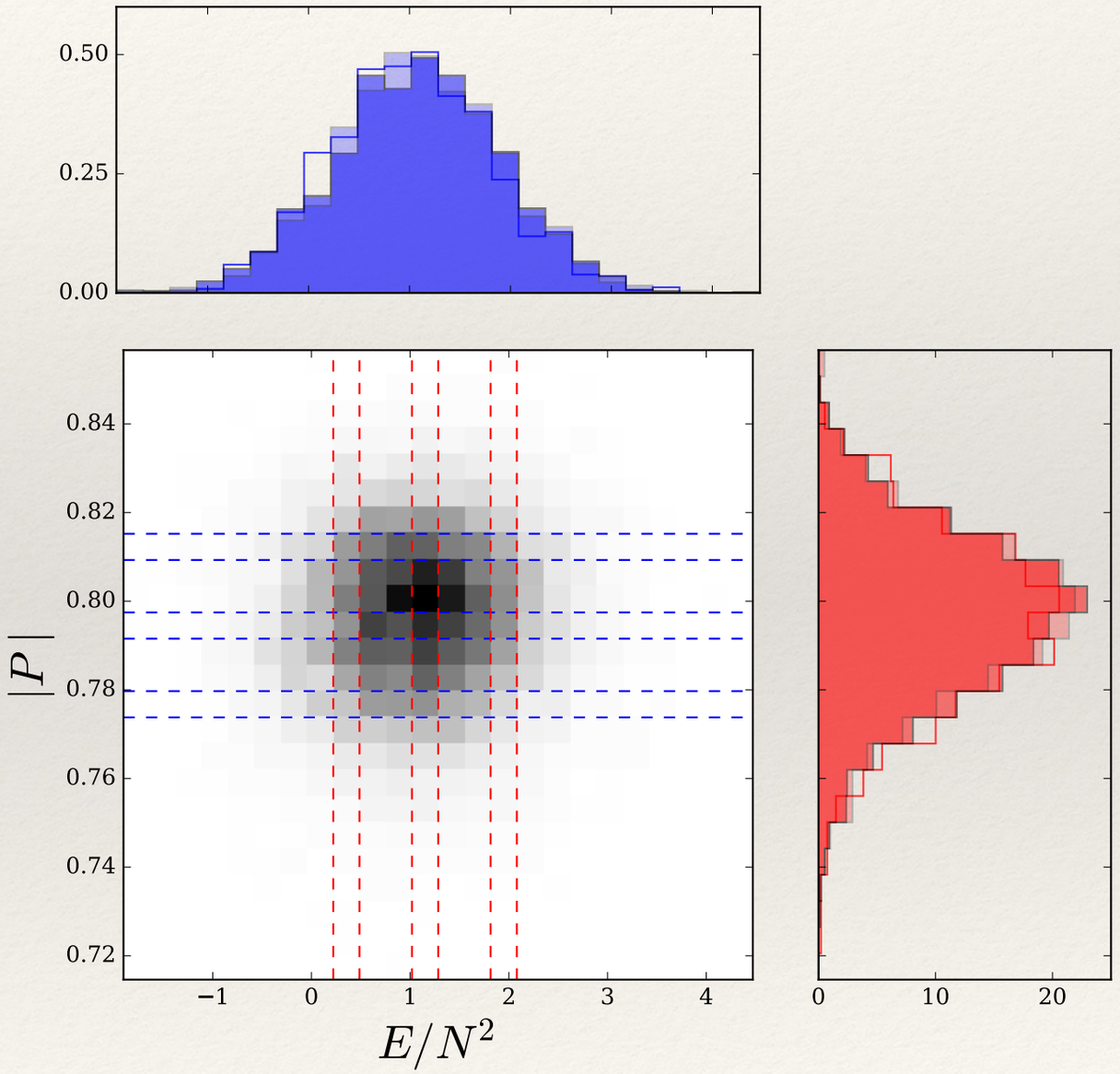


$$\langle \mathcal{O} \rangle_{\mathrm{PQ}} = \int dx\ x\rho(x)$$

$$\langle \mathcal{O} \cdot e^{i\theta} \rangle_{\mathrm{PQ}} = \int dx\ x\rho(x) w_x$$

$$\langle e^{i\theta} \rangle_{\mathrm{PQ}} = \int dx\ \rho(x) w_x$$

T=0.5 N=16 L=16 O(a) improved action



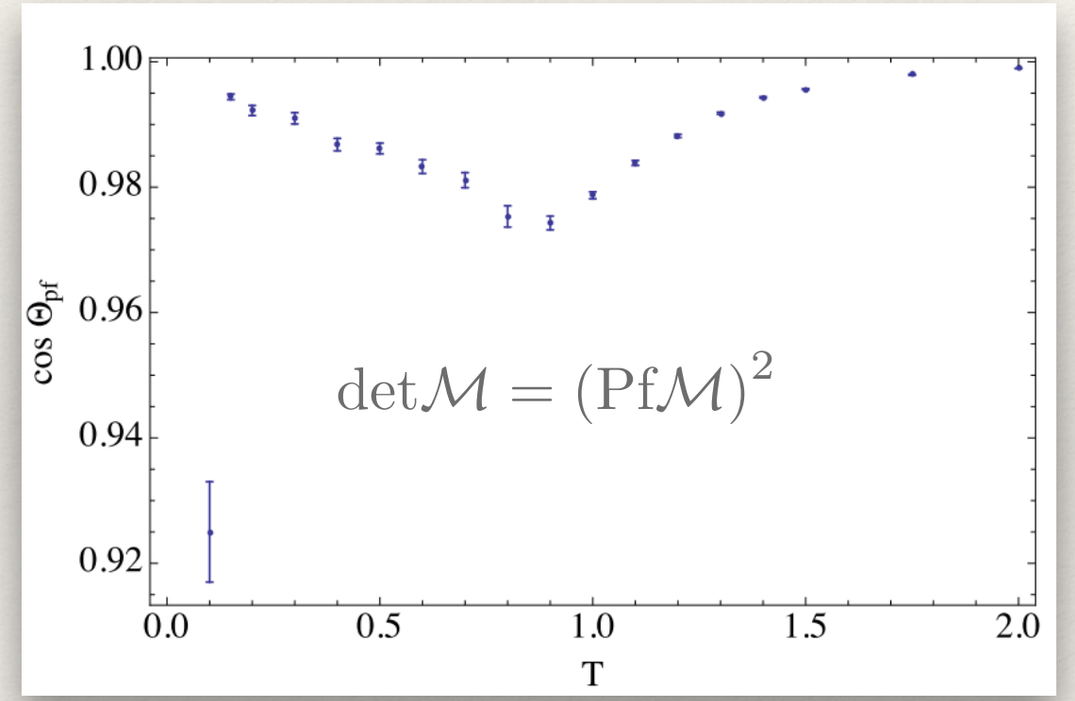
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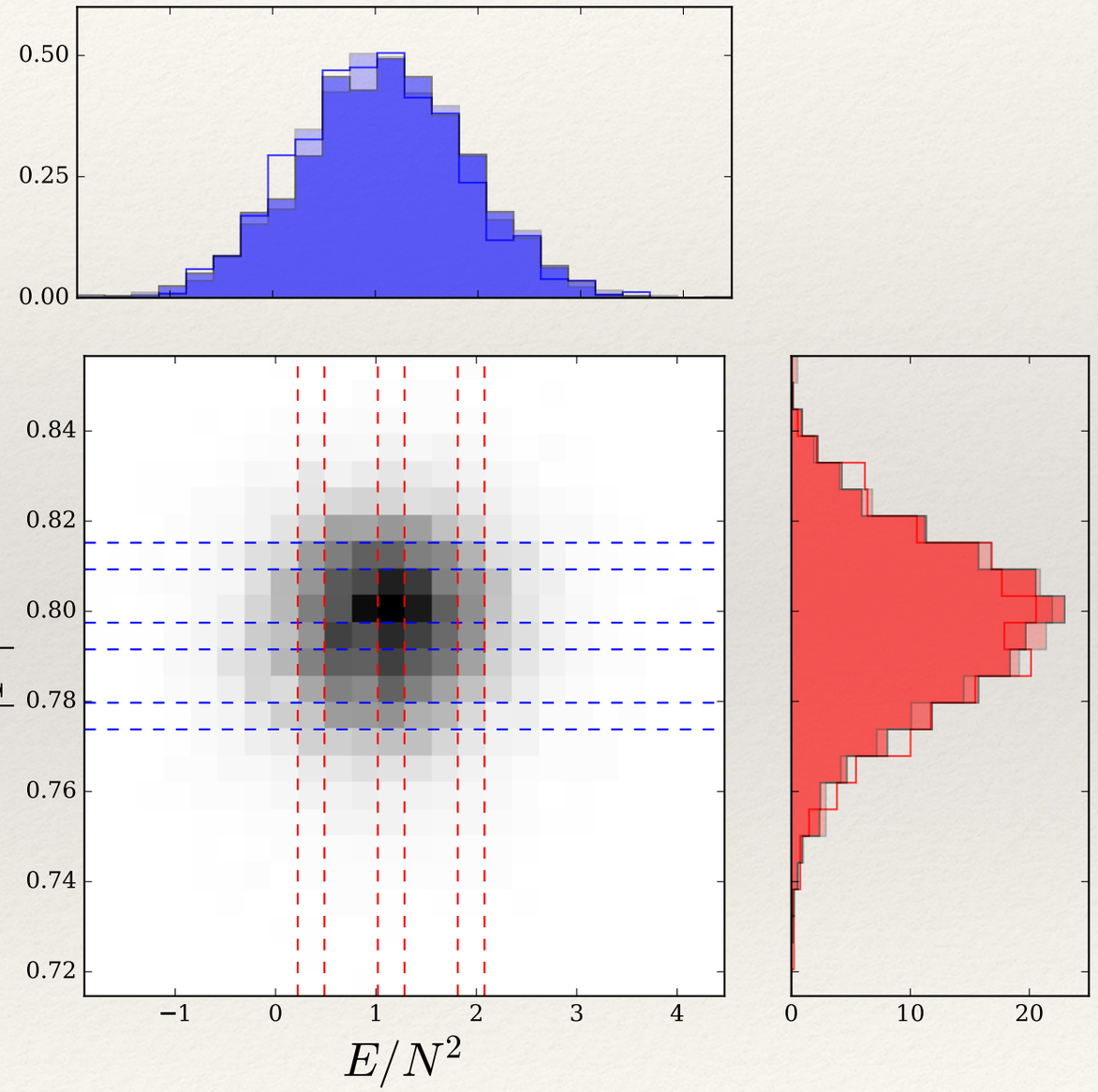


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T=0.5 N=16 L=16 O(a) improved action



No correlations between |P| and E

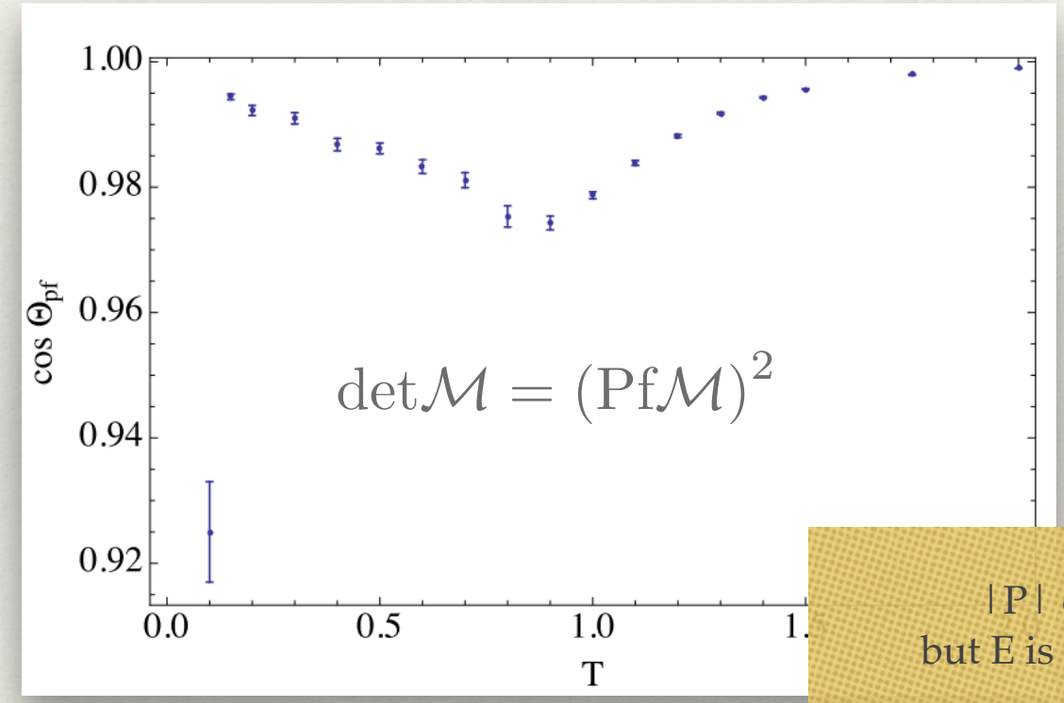
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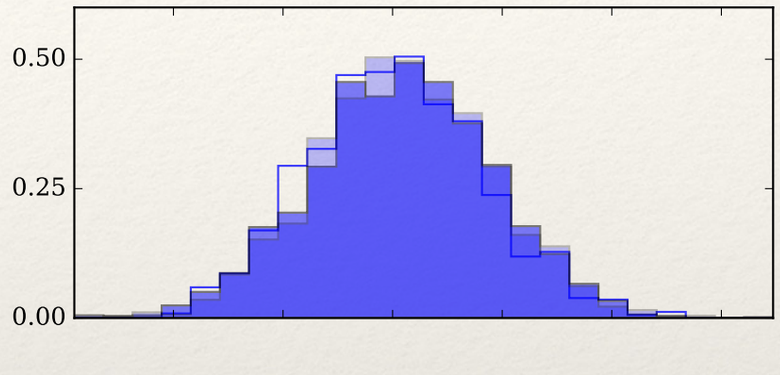


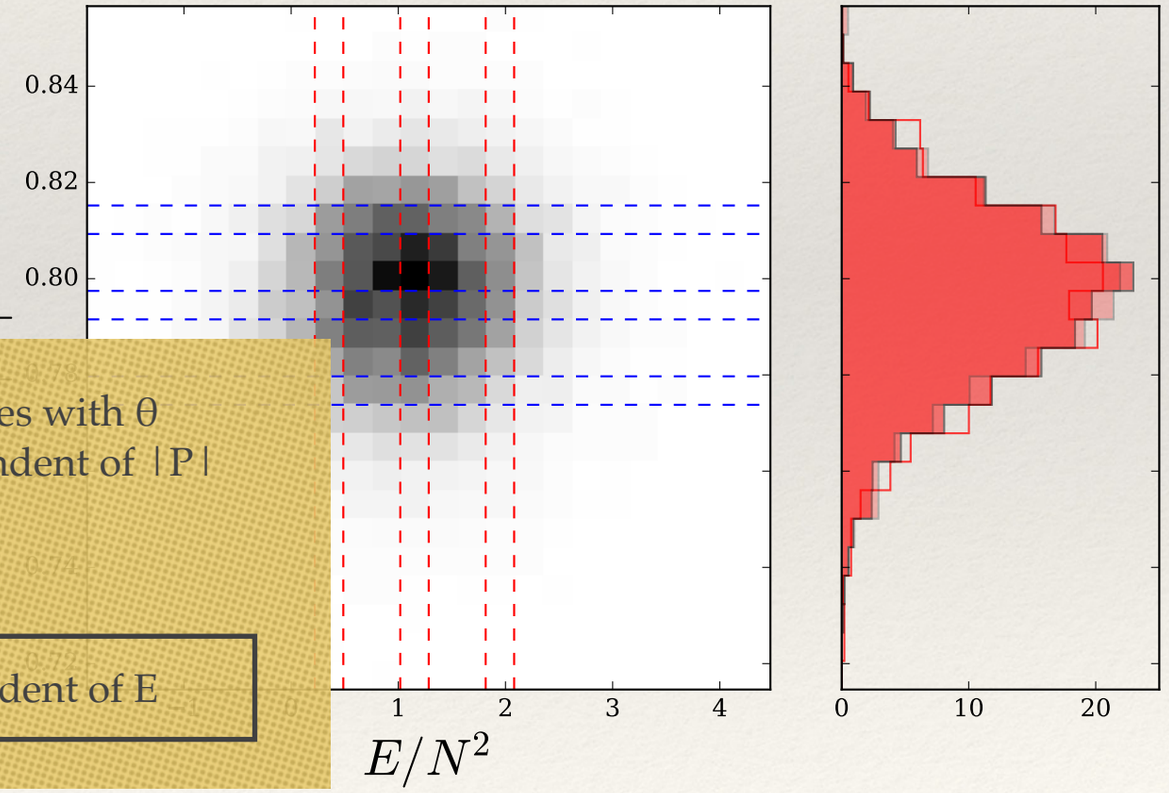
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T=0.5 N=16 L=16 O(a) improved action





|P| correlates with θ
but E is independent of |P|

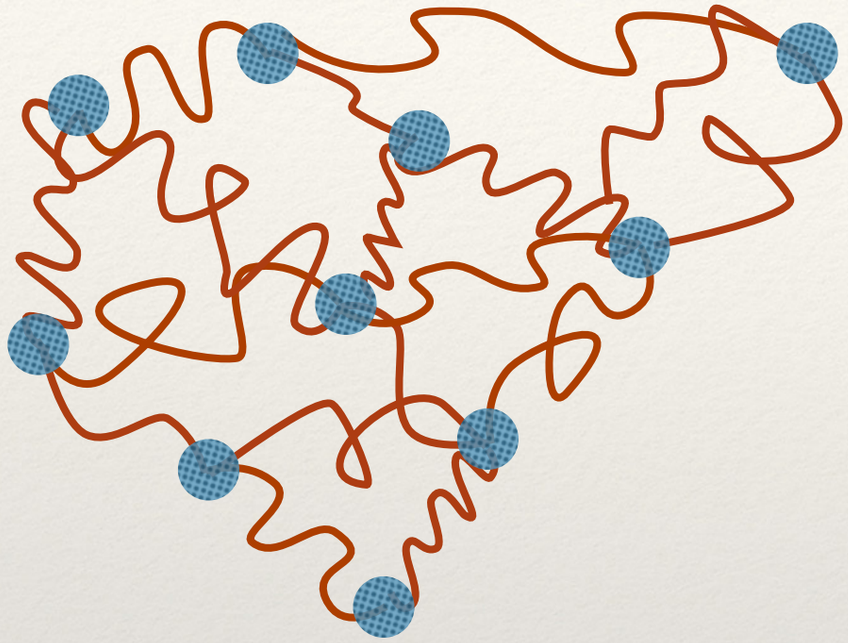
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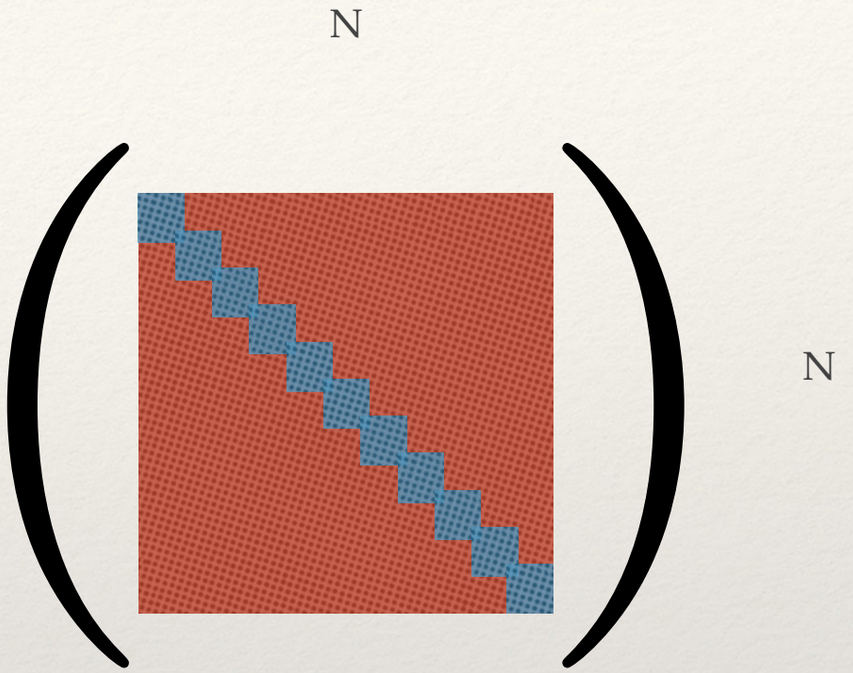
θ is independent of E

No correlations between |P| and E

[Berkowitz, Hanada, Maltz, Phys. Rev. D94 (2016) 126009]

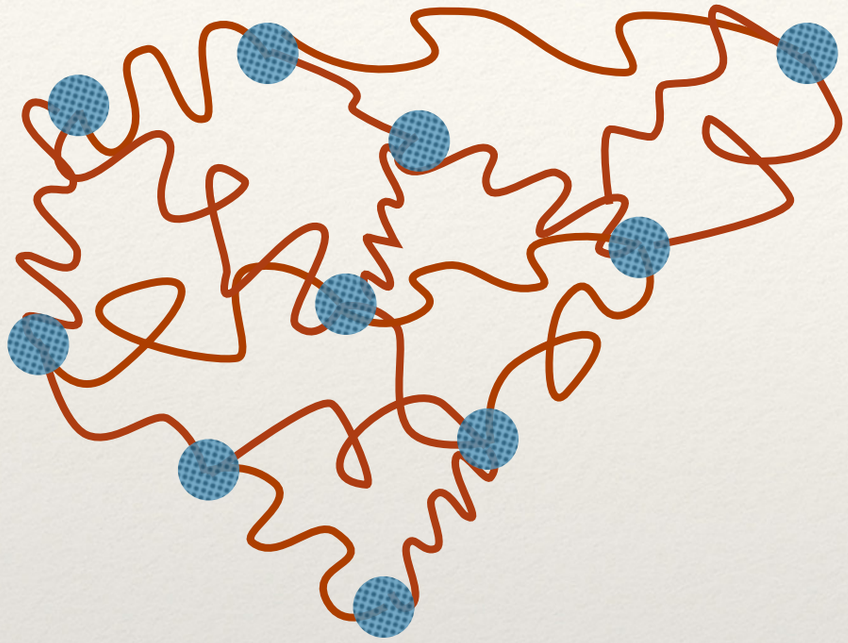
Flat direction: evaporation



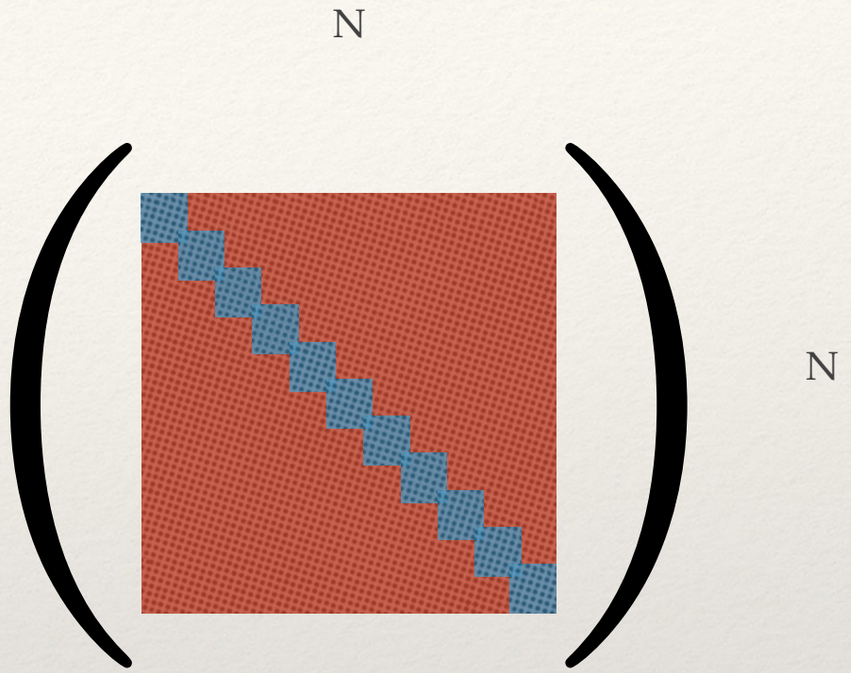


[Berkowitz, Hanada, Maltz, Phys. Rev. D94 (2016) 126009]

Flat direction: evaporation

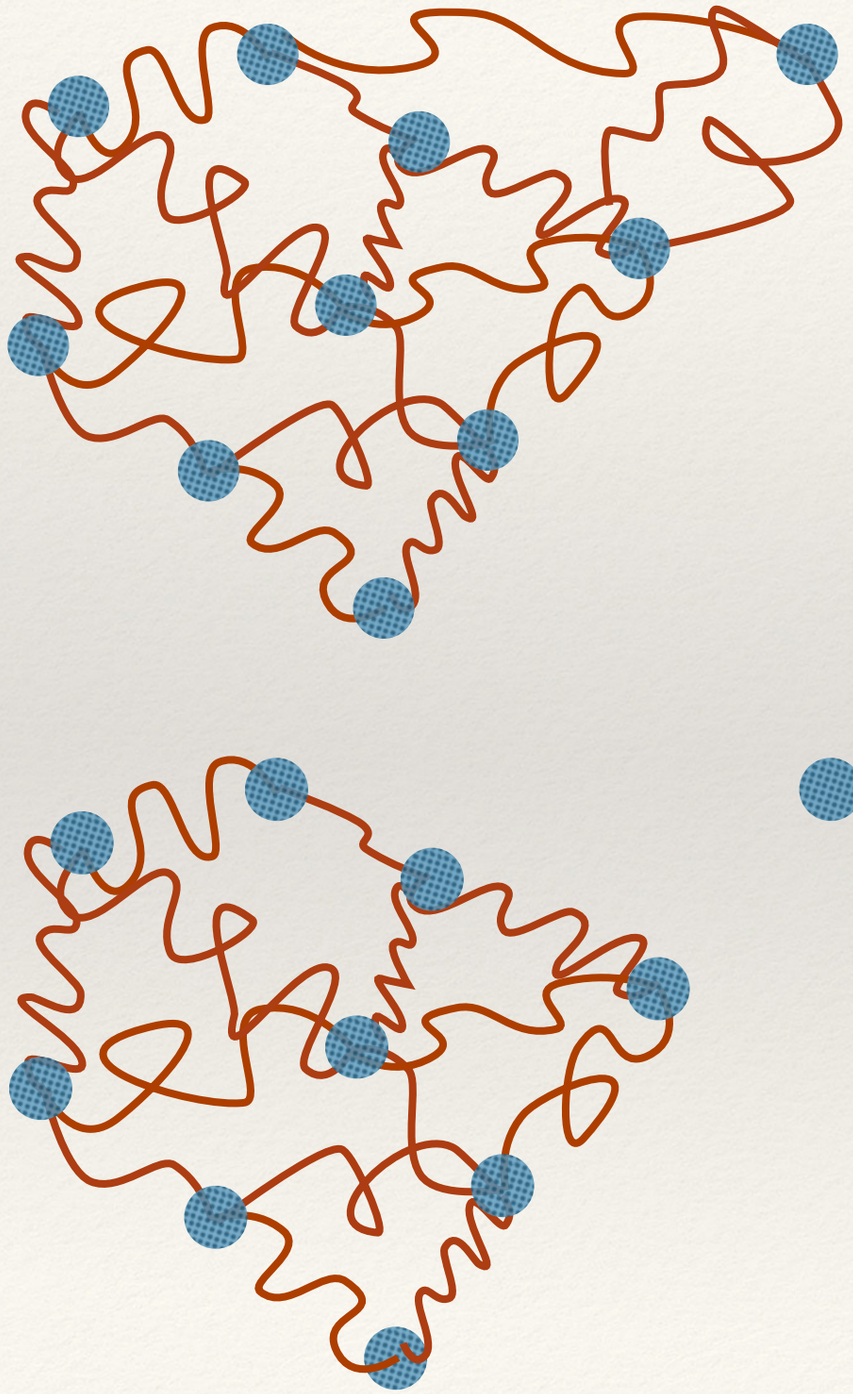


$\#\,\mathrm{dof}=N^2$

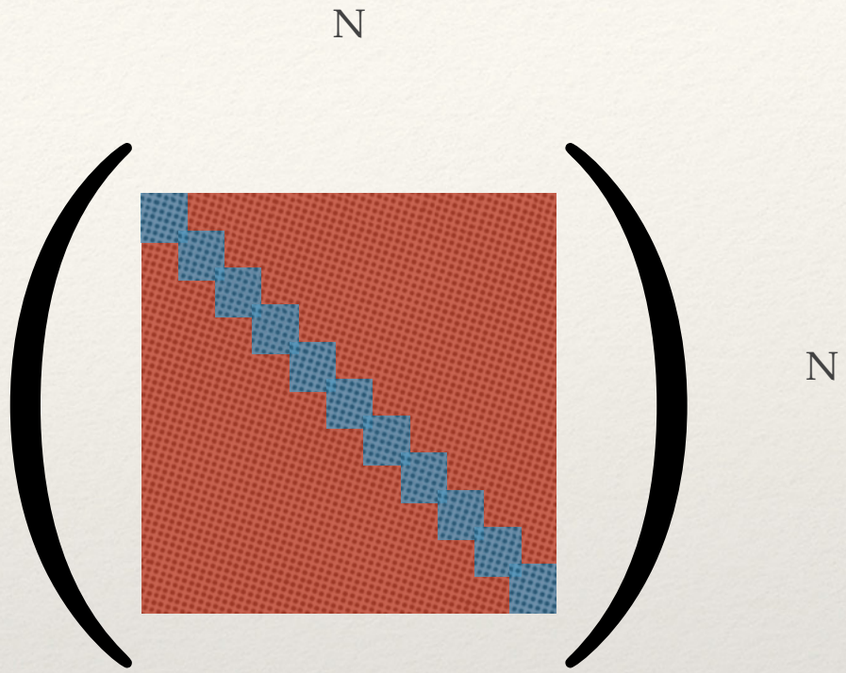


[Berkowitz, Hanada, Maltz, Phys. Rev. D94 (2016) 126009]

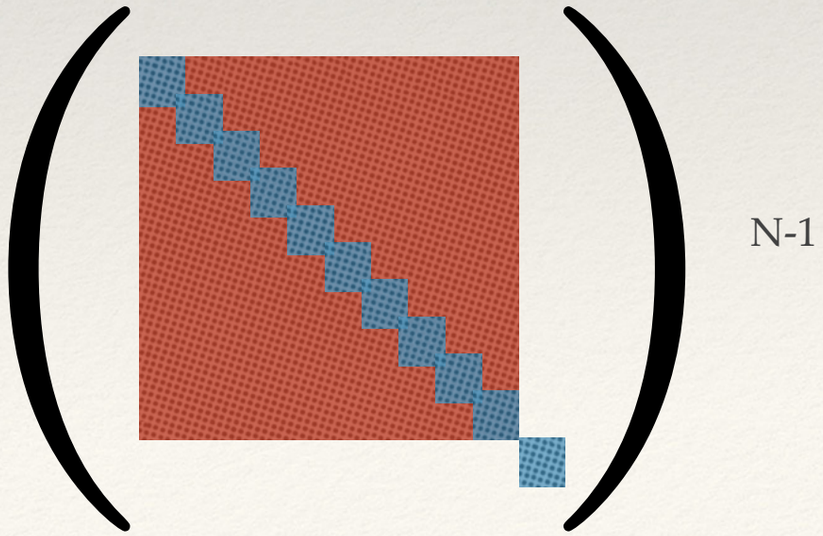
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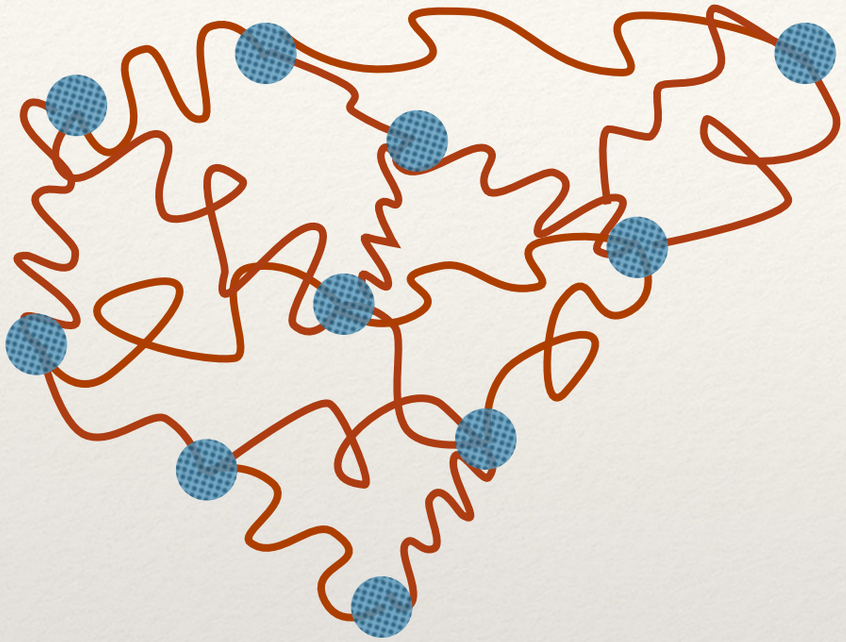


N-1

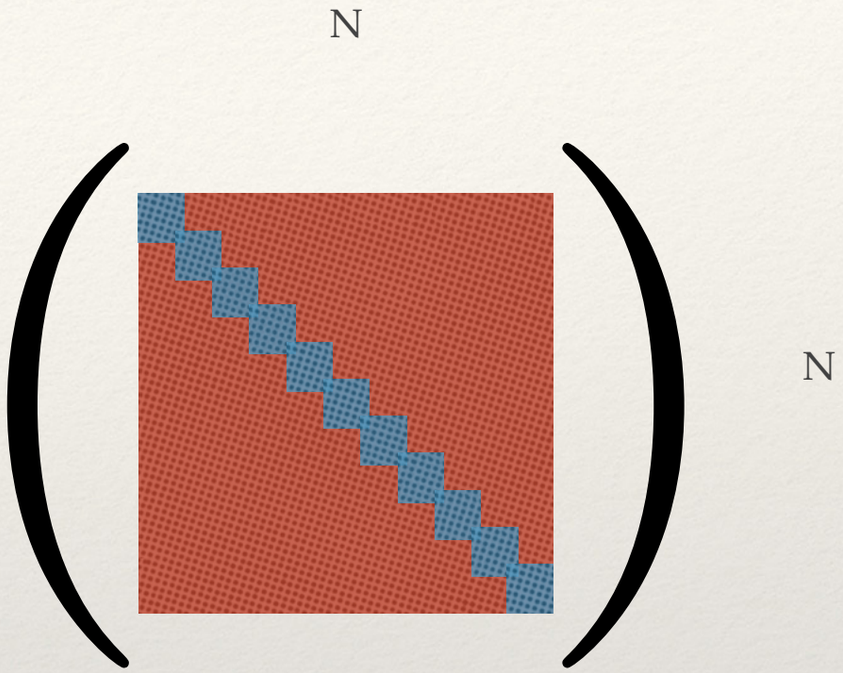


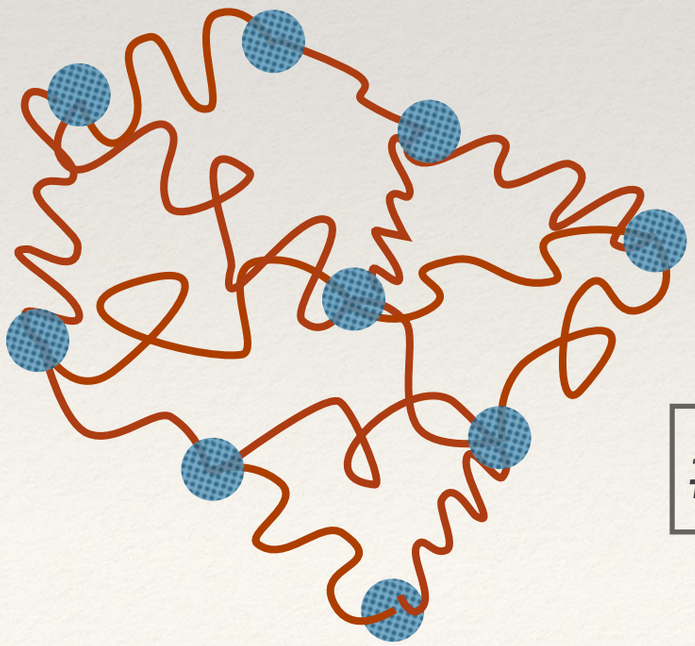
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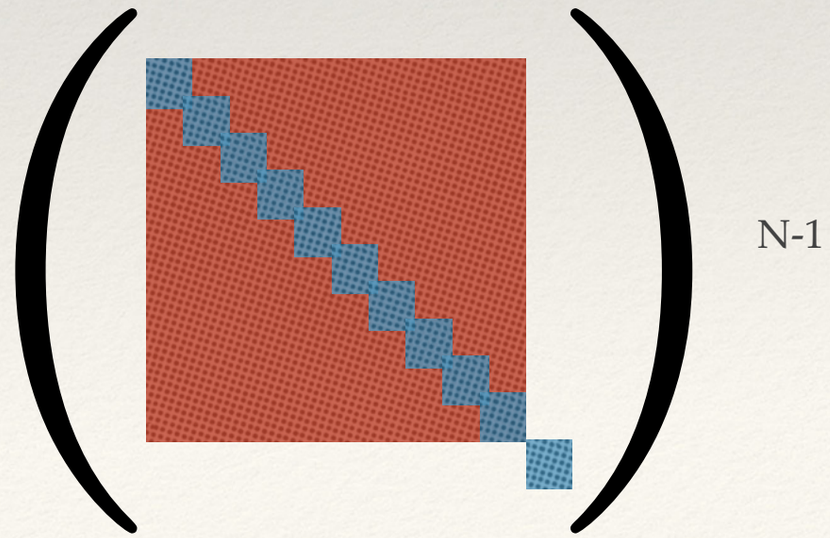


$\#\,\mathrm{dof}=N^2$





$\#\,\mathrm{dof}=1+(N-1)^2$

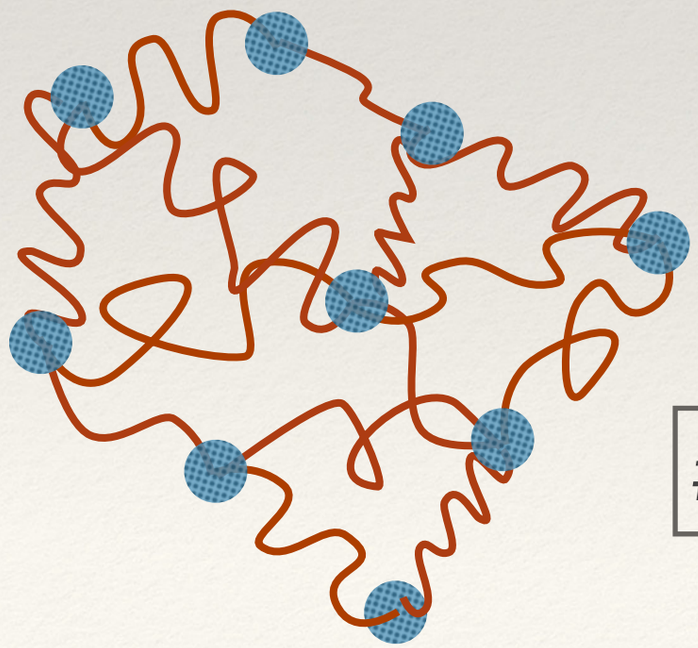


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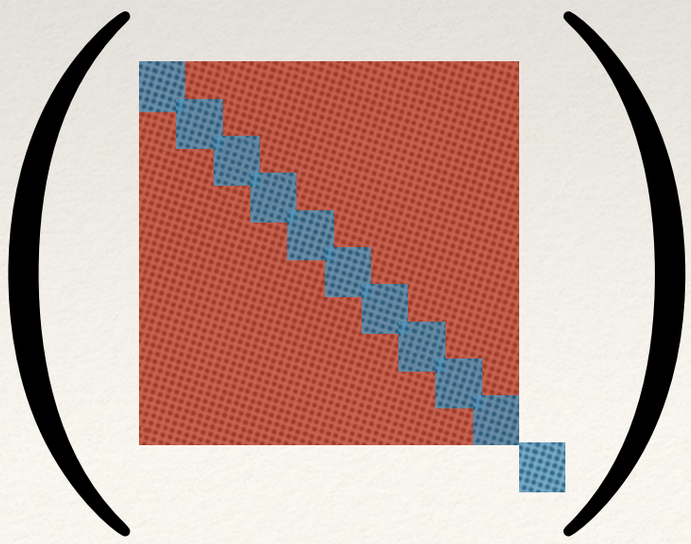


emission is entropically disfavored at large-N



N-1

#dof=1+(N-1)²

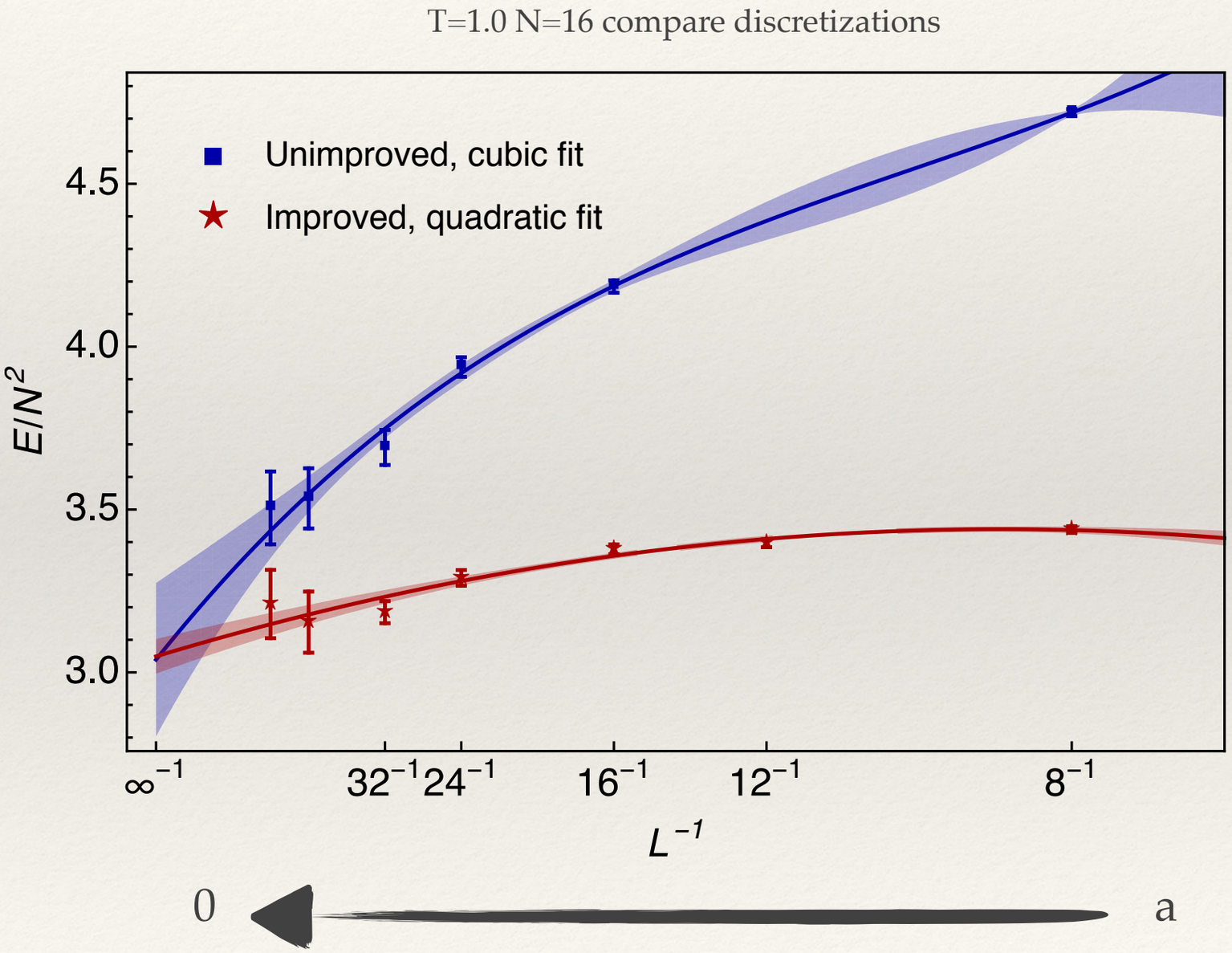


N-1

[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

Remove regularization effects

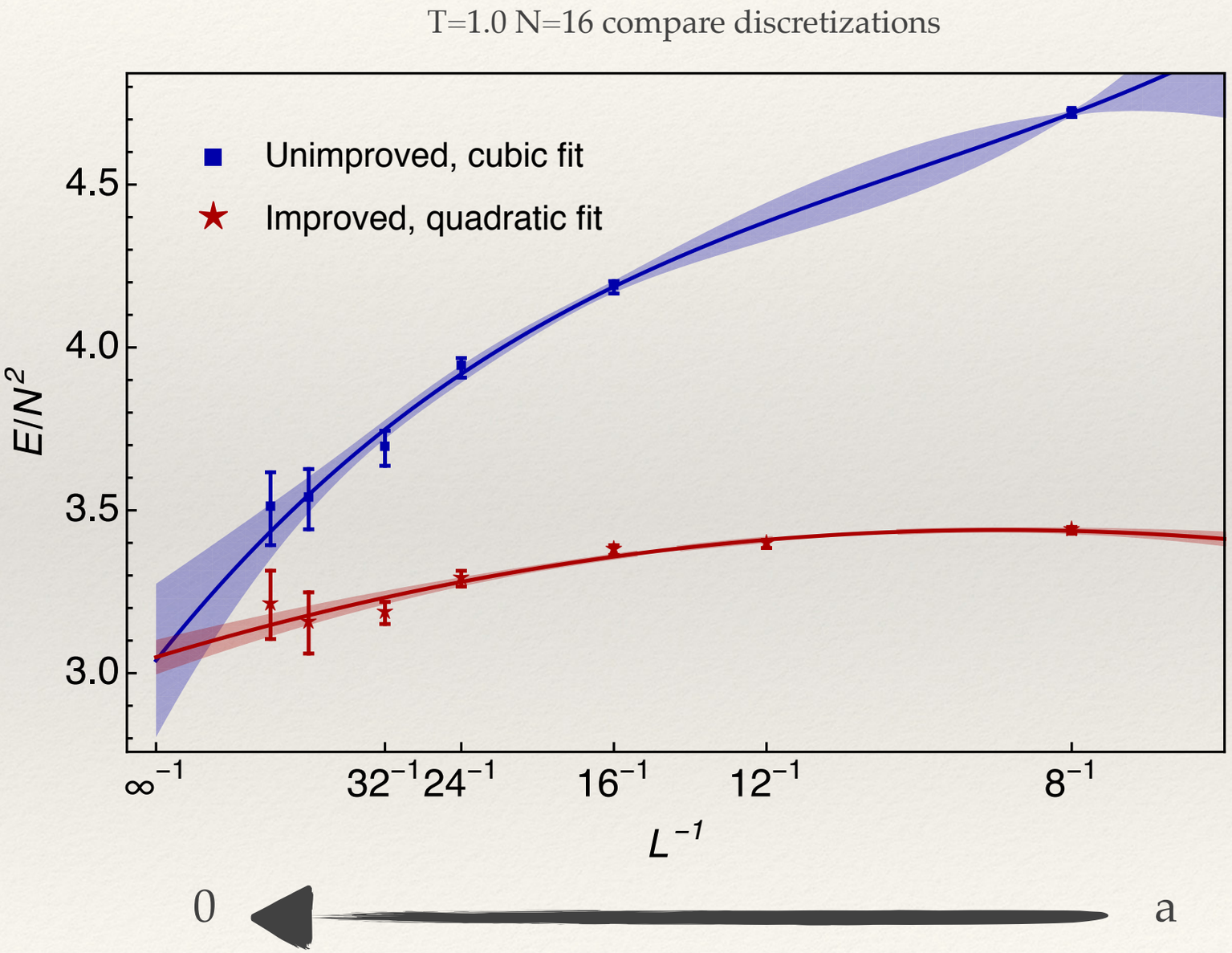
- Numerical simulations are done at fixed cutoff e.g. a



[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

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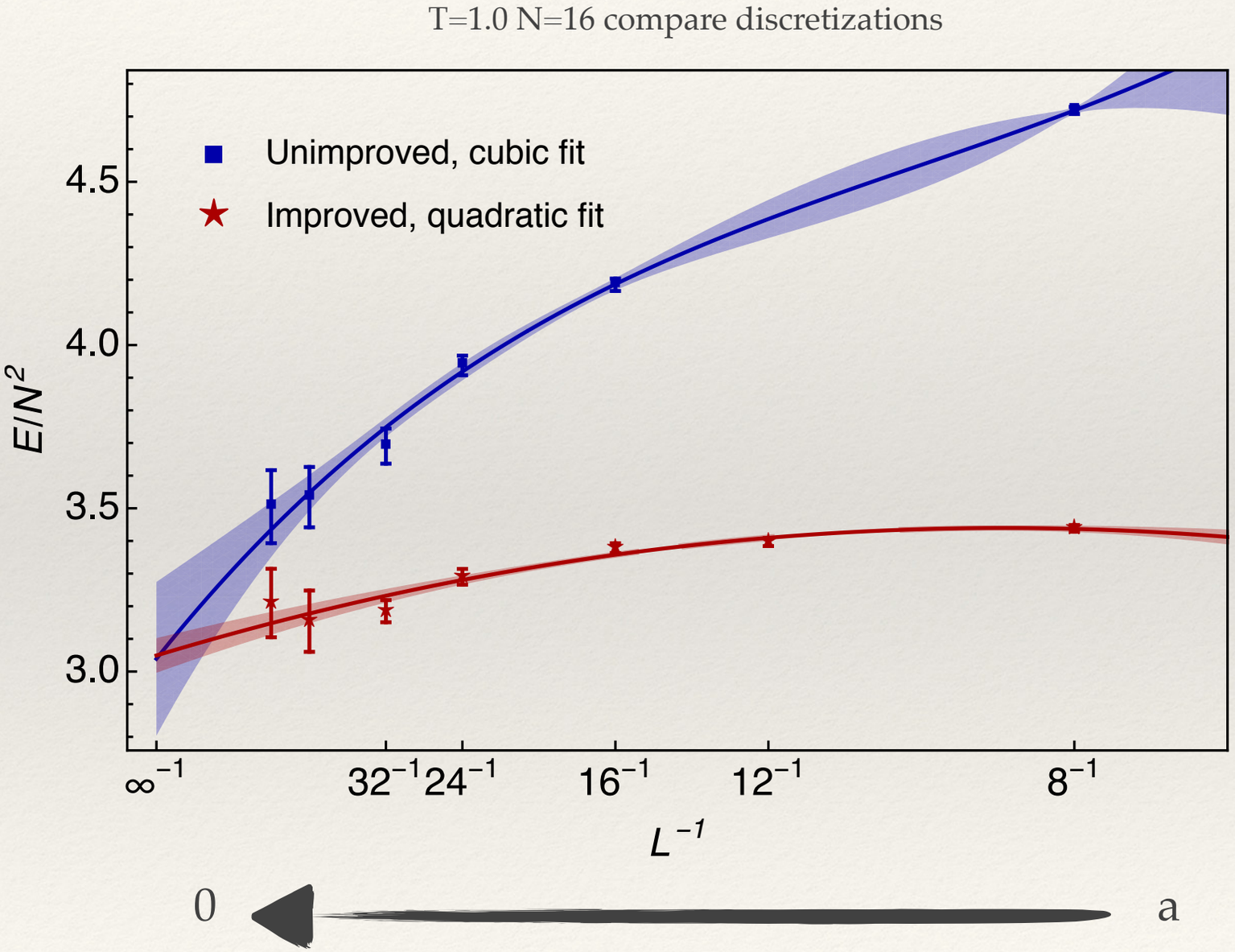
- Numerical simulations are done at fixed cutoff e.g. a
- Need to remove regulator effects



[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

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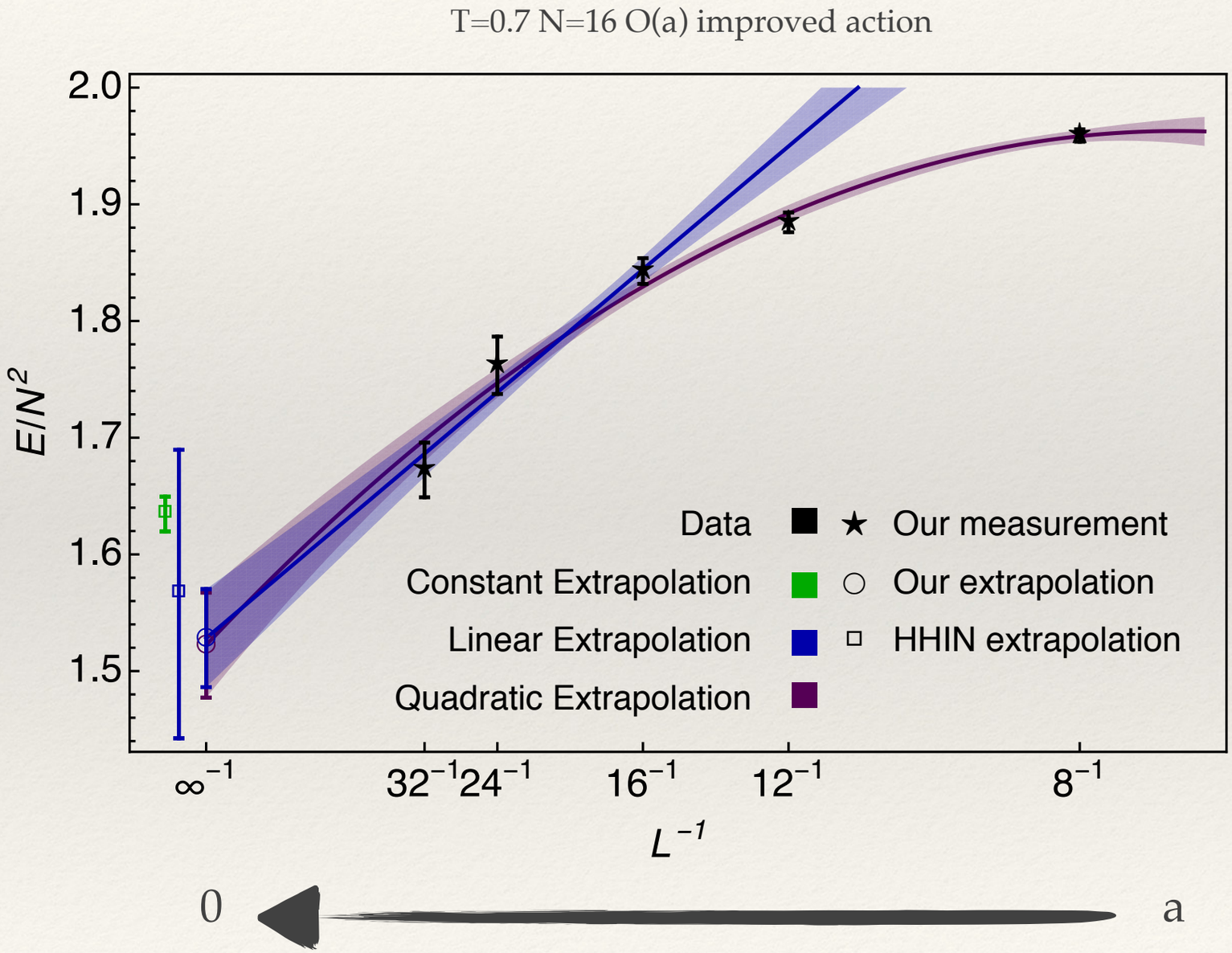
- Numerical simulations are done at fixed cutoff e.g. a
- Need to remove regulator effects
- Different discretization forms give consistent results



[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

Remove regularization effects

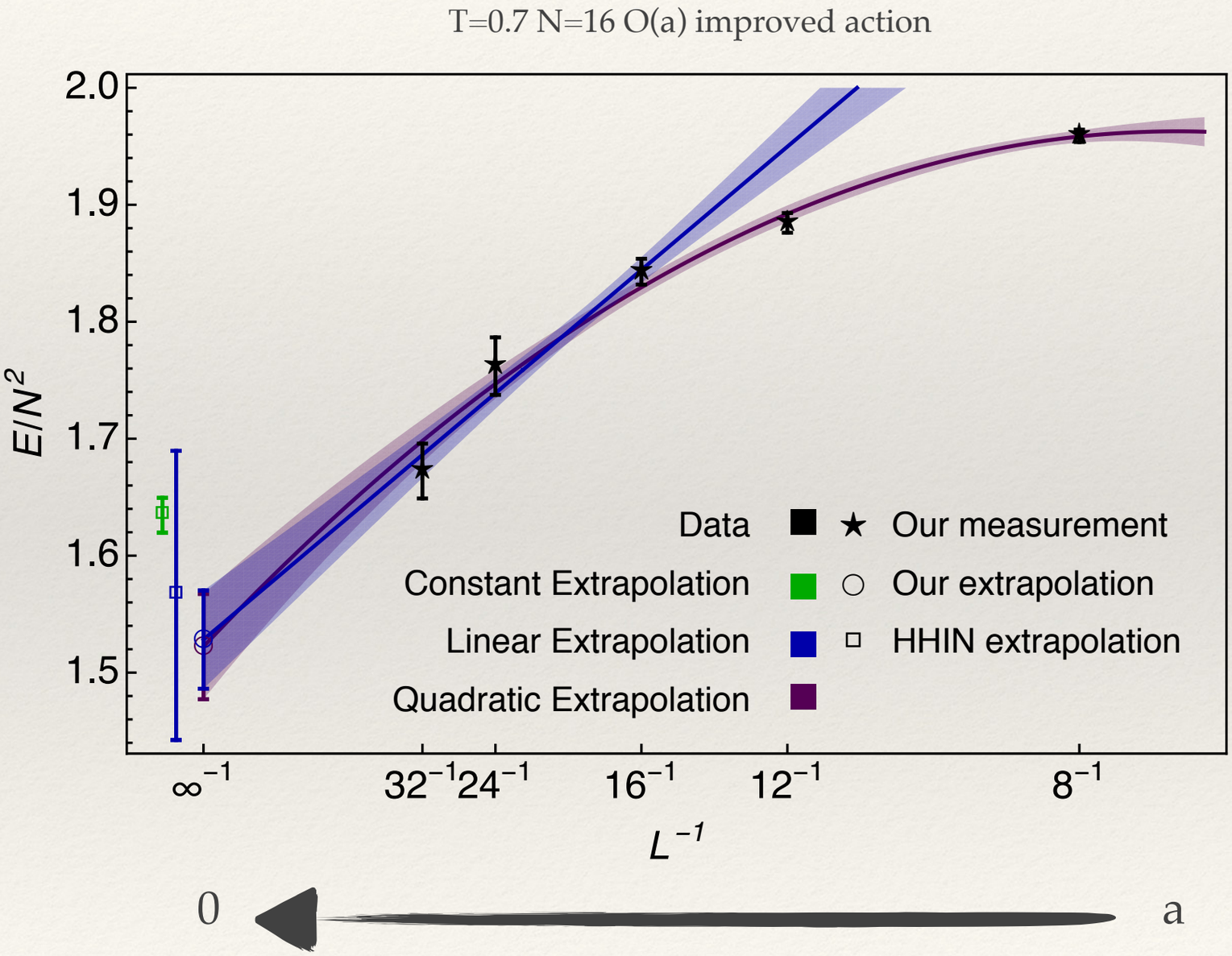
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[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

Remove regularization effects

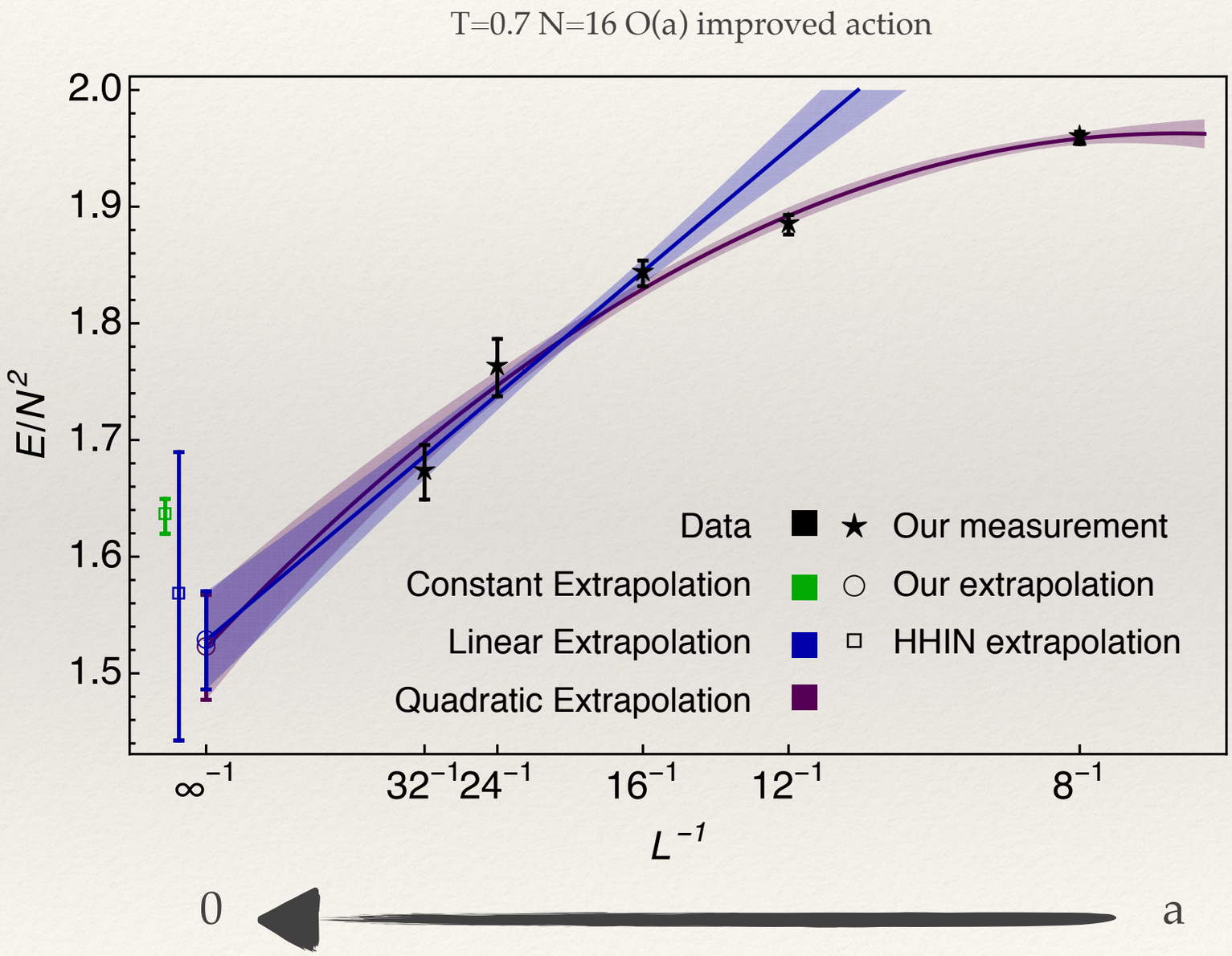
- Numerical simulations are done at fixed cutoff e.g. a
- Need to remove regulator effects
- Different discretization forms give consistent results
- Compare with previous results in the literature



[Hanada, Hyakutake, Ishiki, Nishimura, arXiv:1603.00538] [MCSMC, Phys. Rev. D94 (2016) 094501]

Remove regularization effects

- Numerical simulations are done at fixed cutoff e.g. a
- Need to remove regulator effects
- Different discretization forms give consistent results
- Compare with previous results in the literature
- Physics is independent of the regulator once cutoff effects are accurately removed

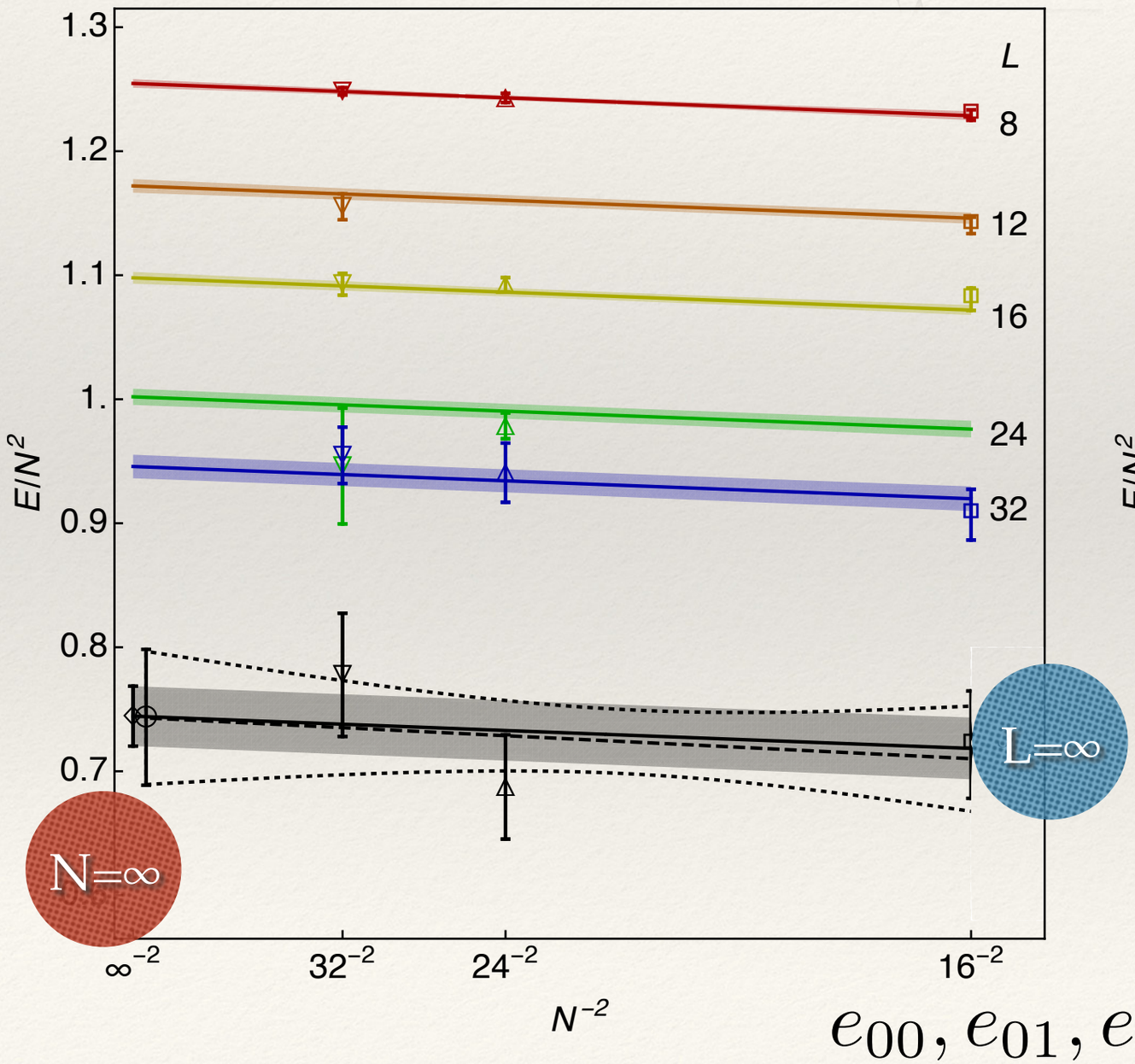


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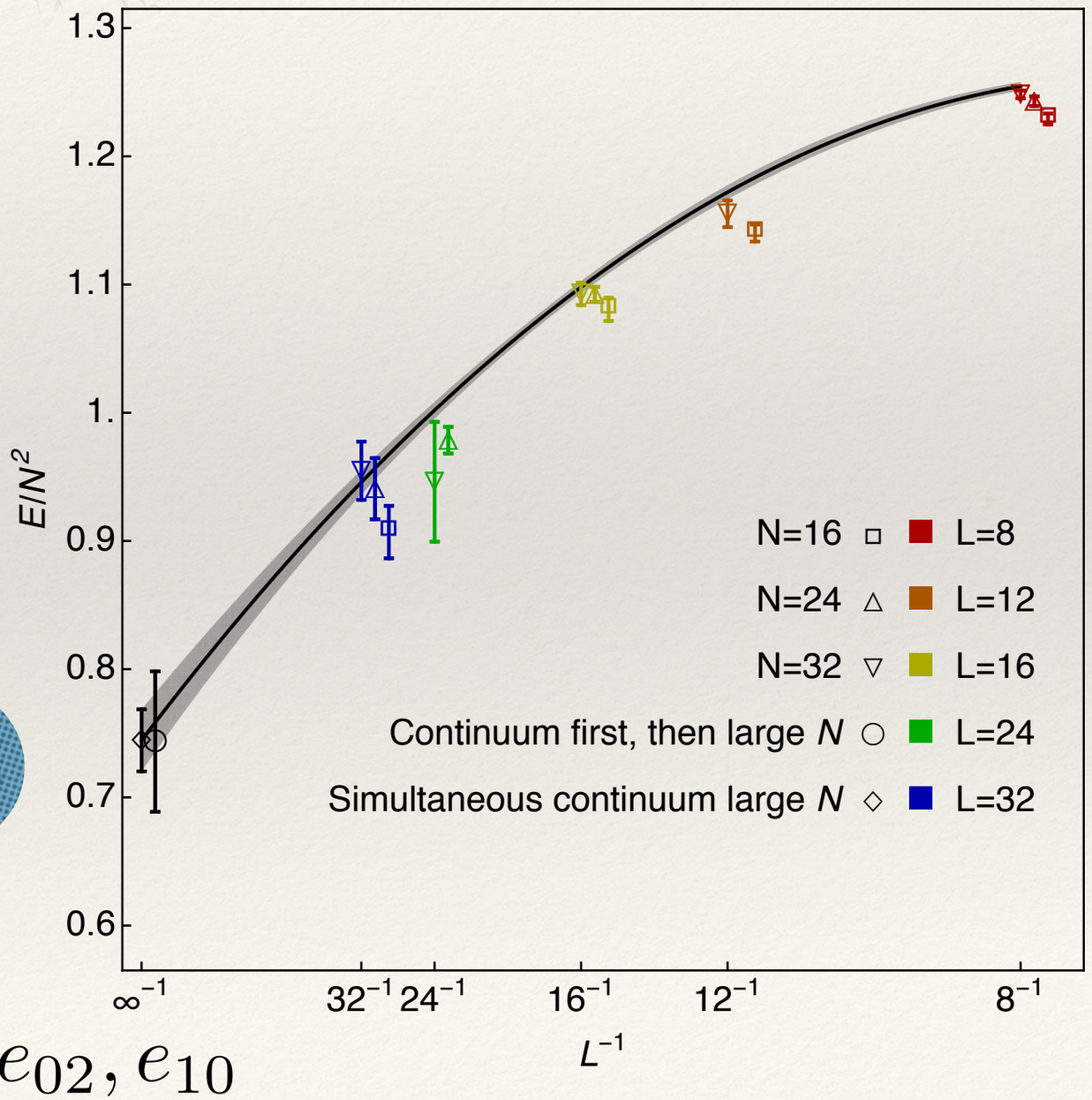
Continuum & large-N limit

$$\frac{E}{N^2} = \sum_{ij \geq 0} \frac{e_{ij}}{N^{2i} L^j}$$

T=0.5 O(a) improved action



T=0.5 O(a) improved action



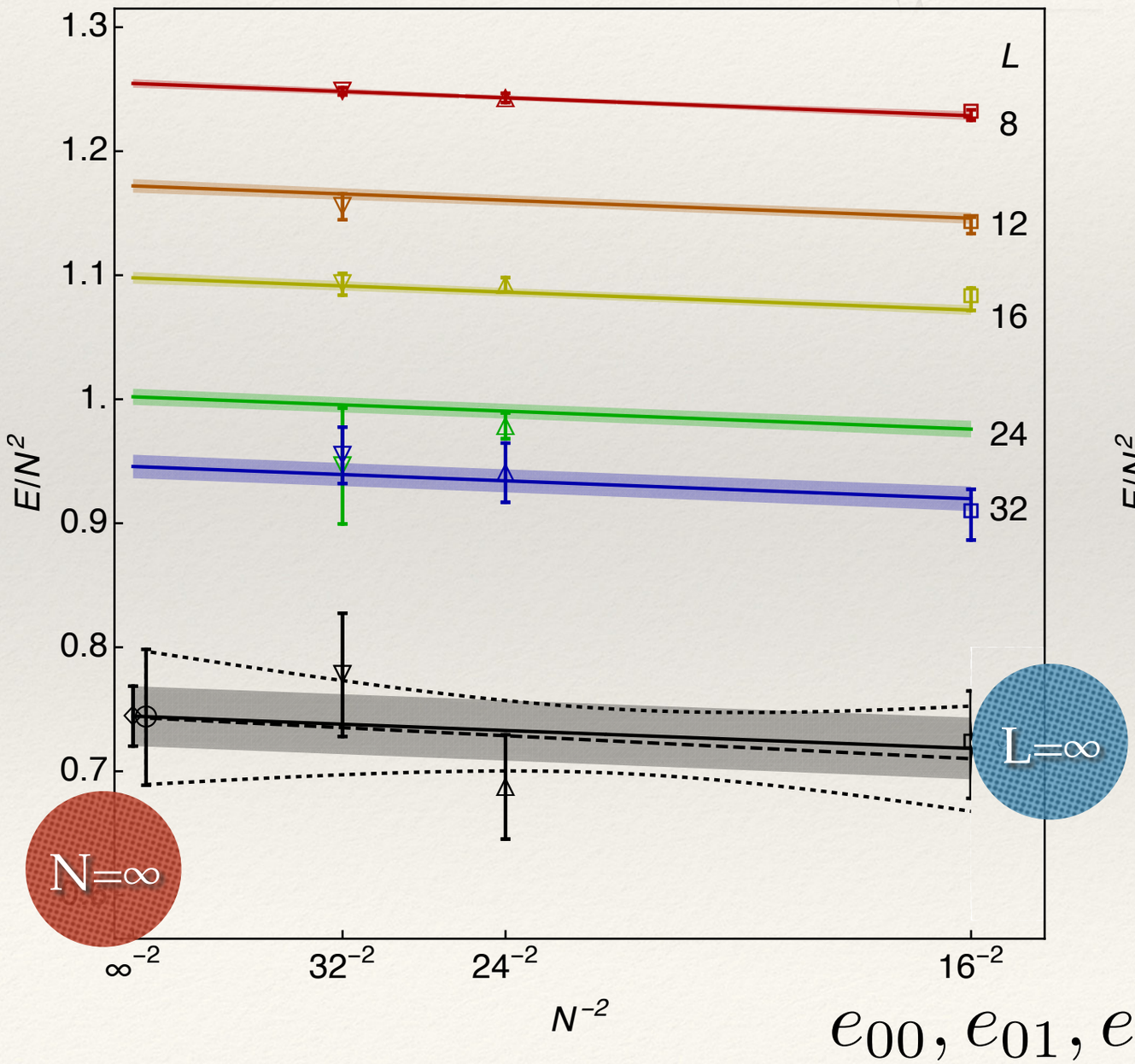
$$e_{00}, e_{01}, e_{02}, e_{10}$$

[MCSMC, Phys. Rev. D94 (2016) 094501]

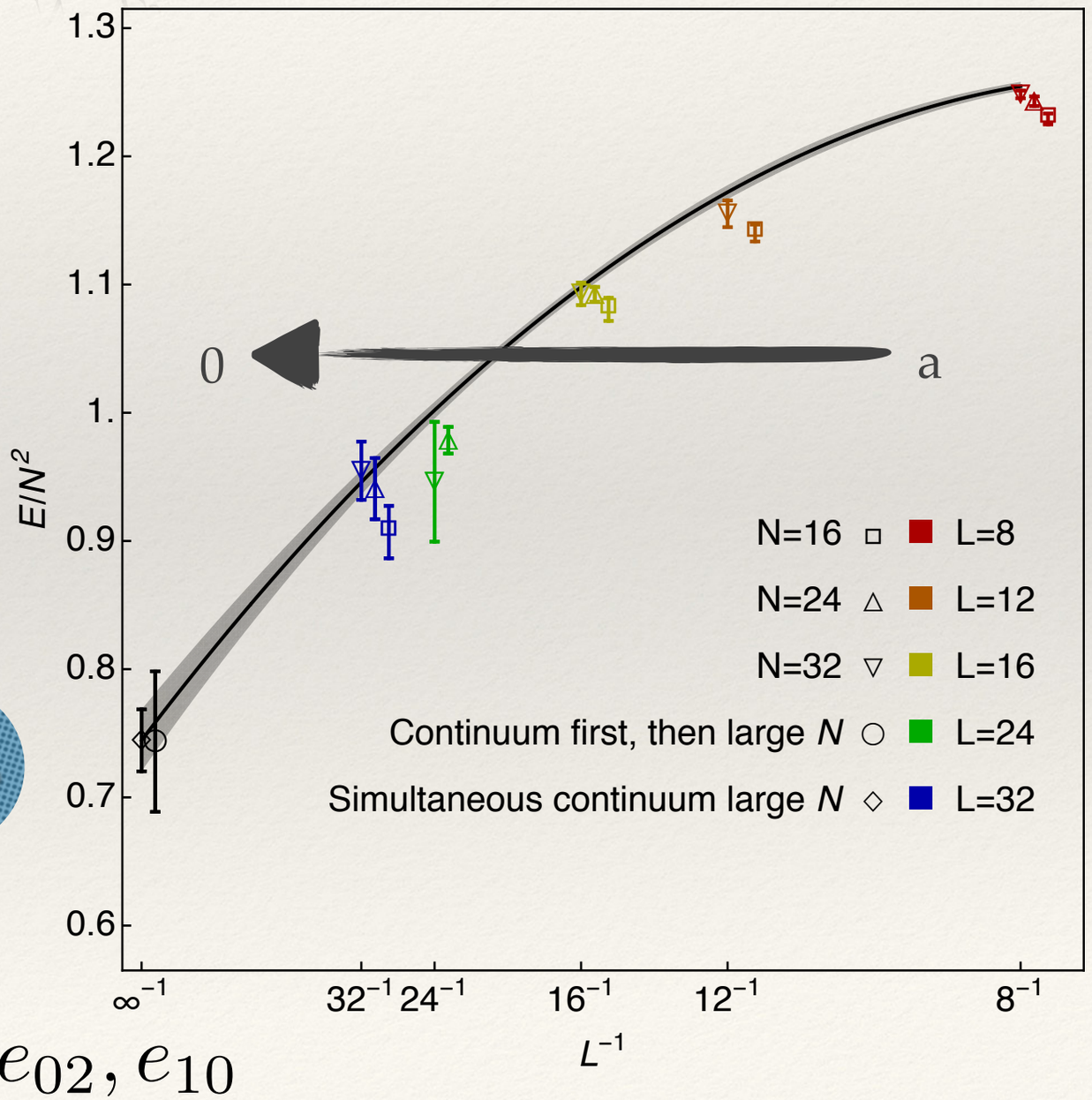
Continuum & large-N limit

$$\frac{E}{N^2} = \sum_{ij \geq 0} \frac{e_{ij}}{N^{2i} L^j}$$

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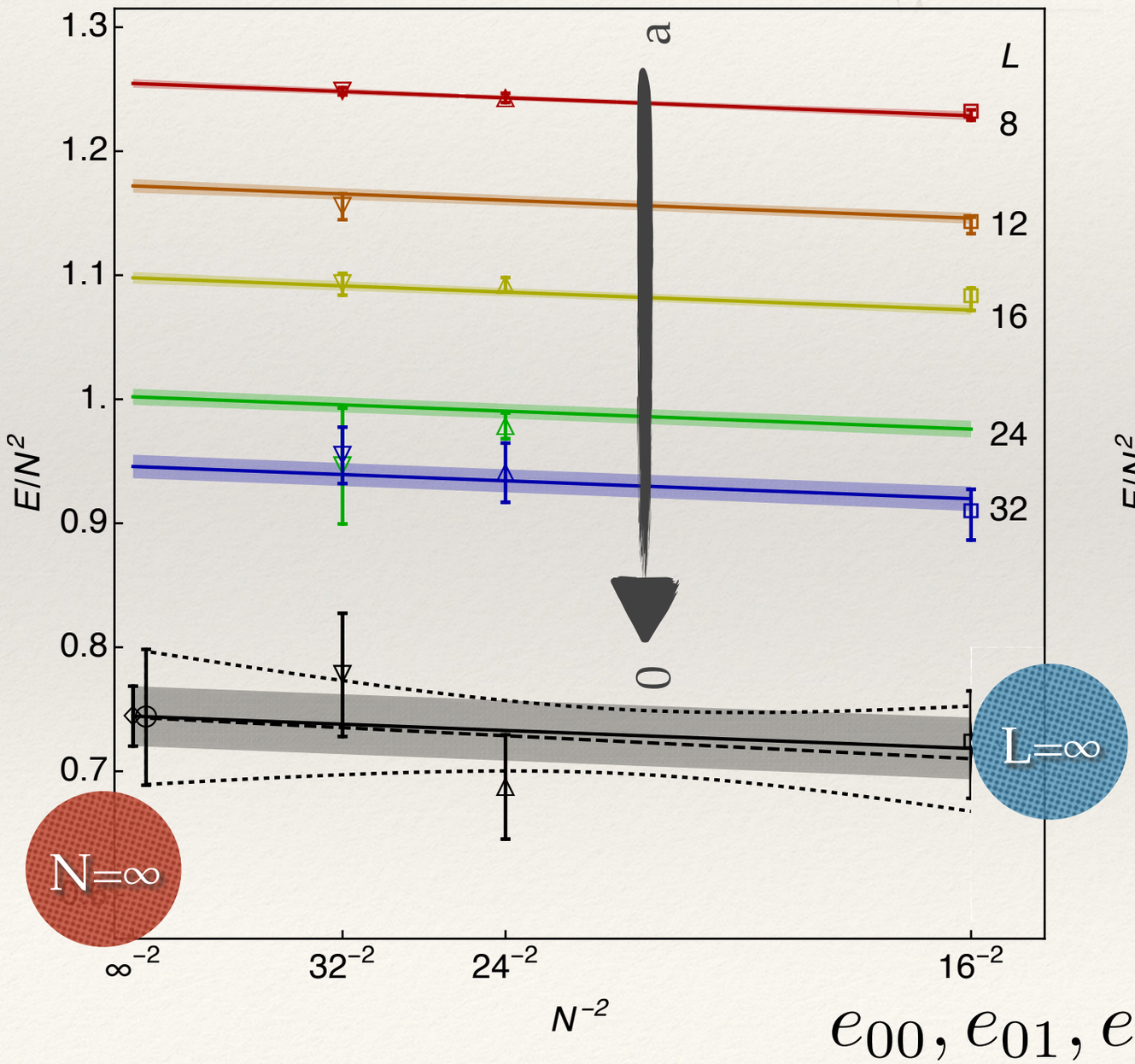
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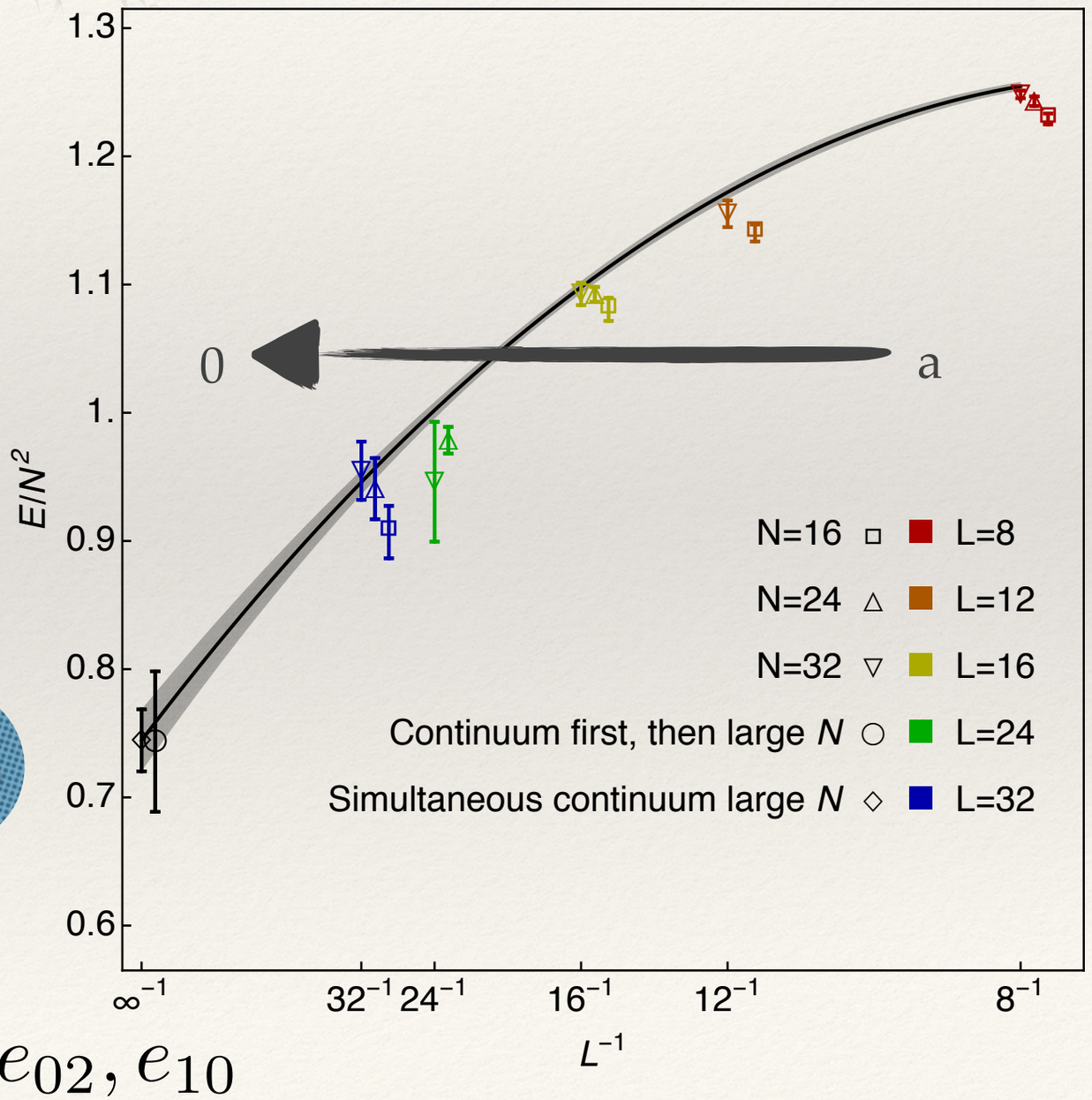
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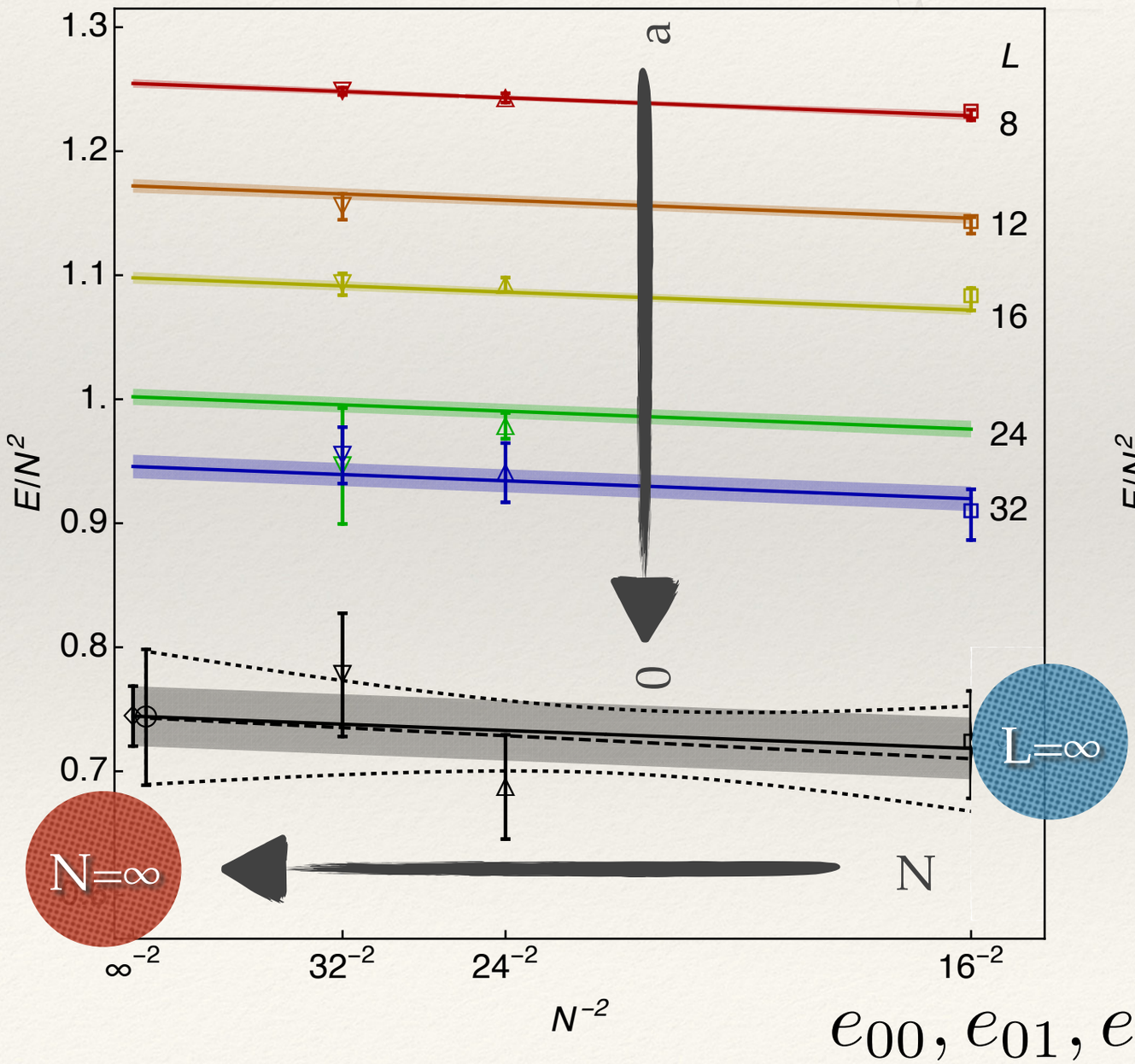
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[MCSMC, Phys. Rev. D94 (2016) 094501]

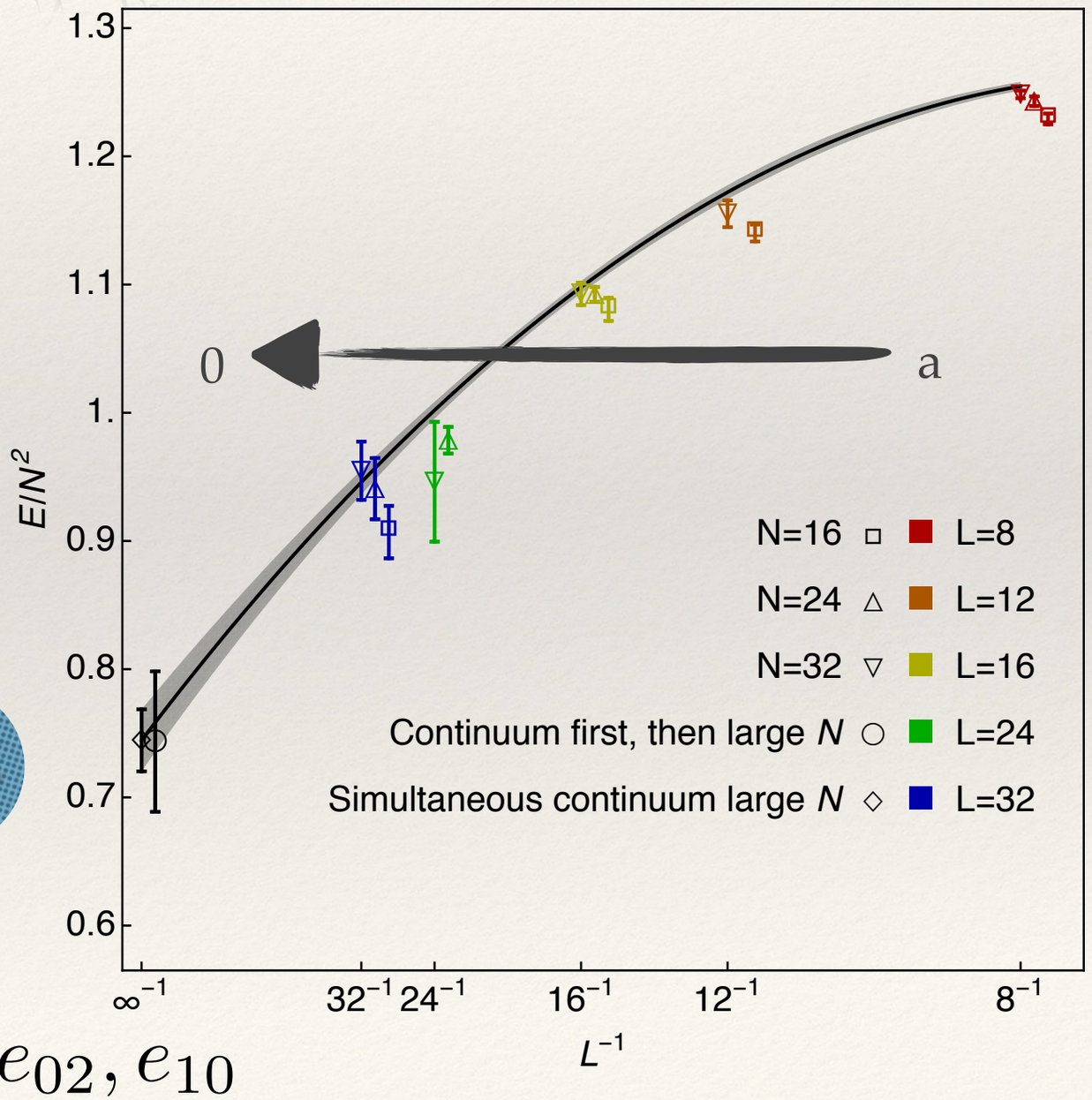
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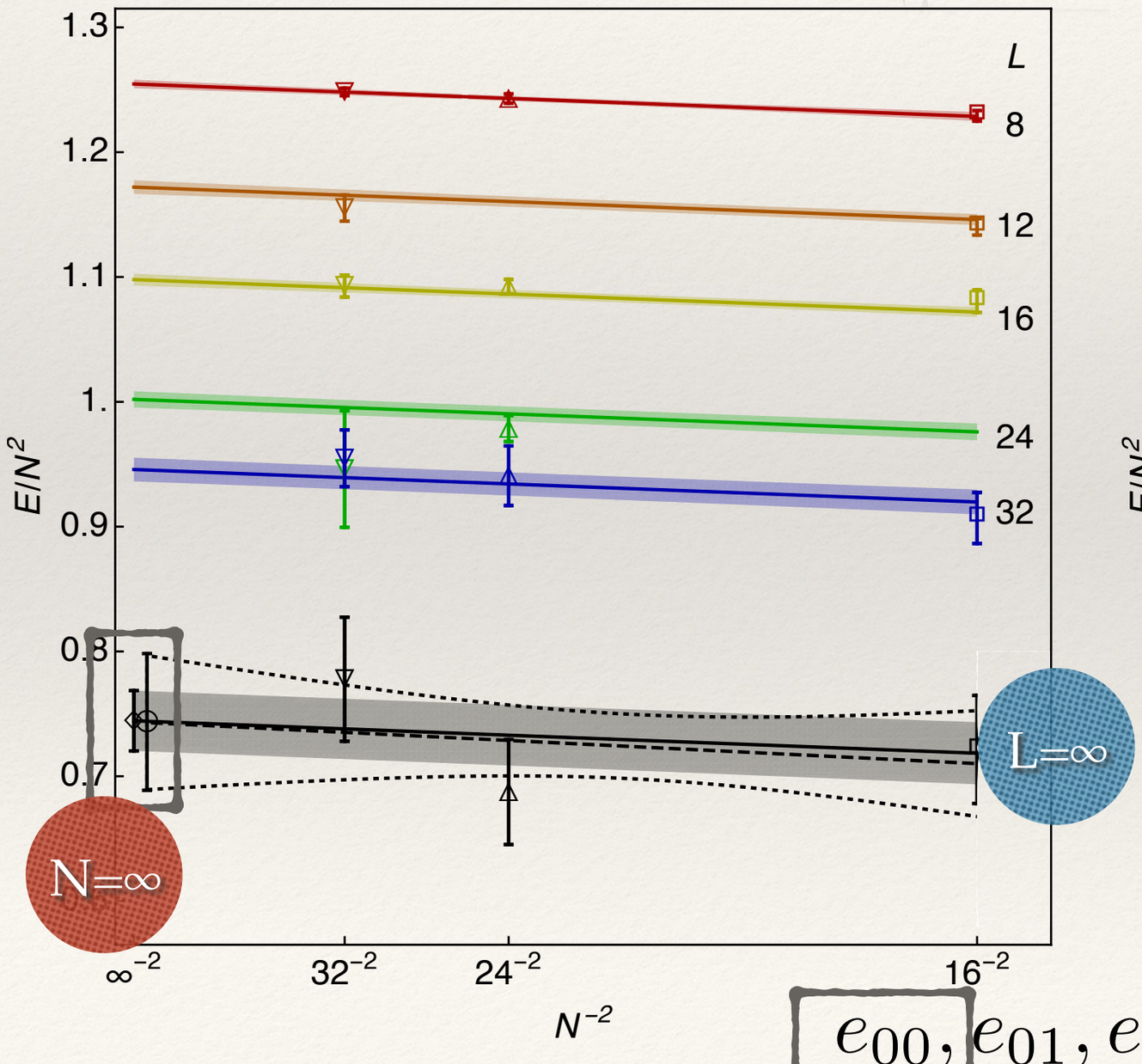
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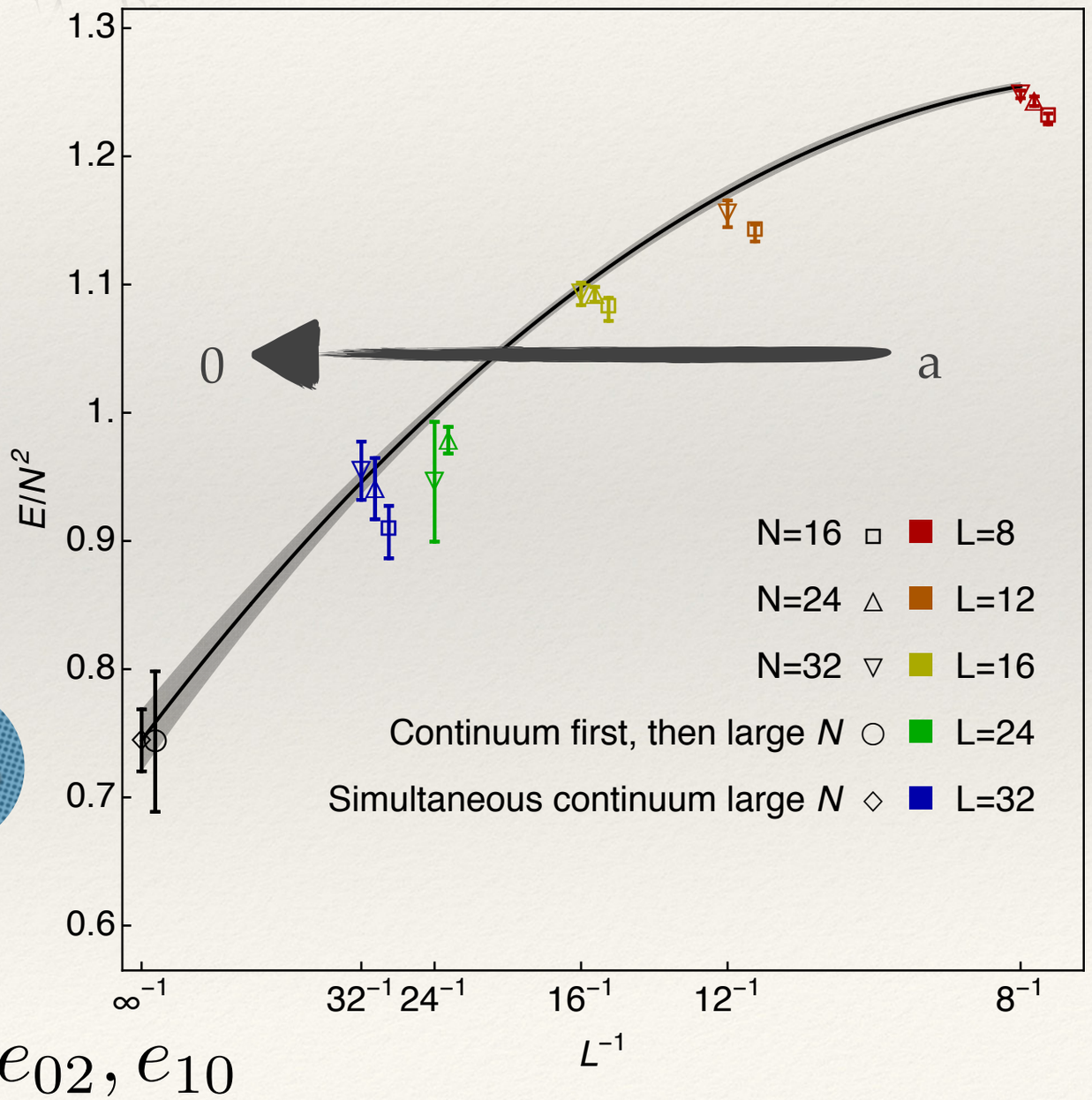
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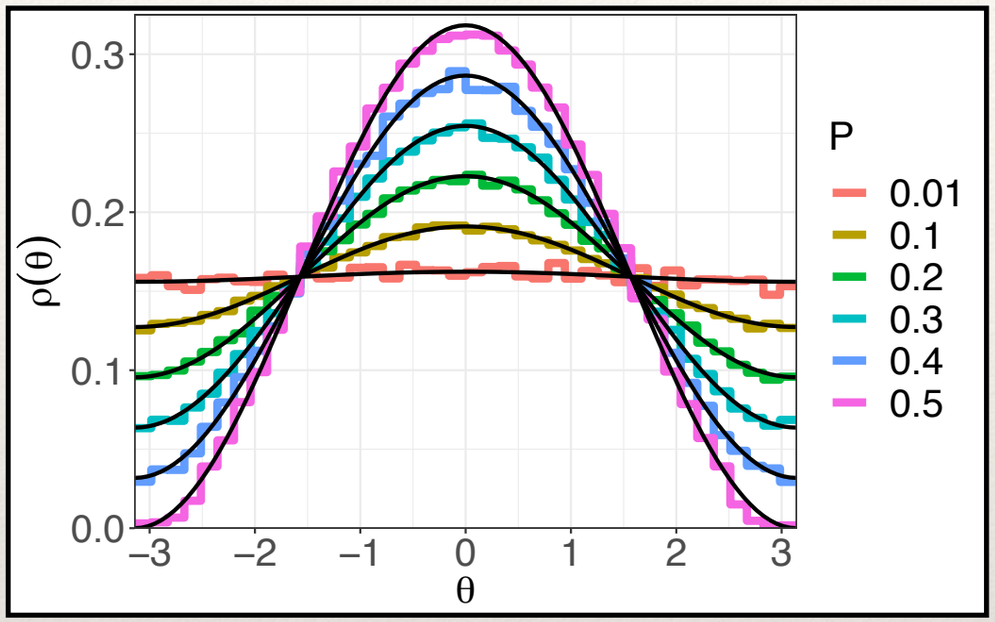
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$$e_{00}, e_{01}, e_{02}, e_{10}$$

[Hanada, Maltz '16, Hanada, Ishiki, Watanabe '18]

Partial Deconfinement



$$\rho_P(\theta) = \frac{M}{N}\rho_{\frac{1}{2}}(\theta) + \left(1 - \frac{M}{N}\right)\rho_0(\theta)$$

$$\downarrow$$

$$E(|P|, T) = \varepsilon_0 + f(T) \cdot |P|^2$$

$$R^2(|P|, T) = R_0 + g(T) \cdot |P|^2$$

