A renormalizable topological quantum field theory for gravity

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Outline

Introduction

The problem of $d = 4$ perturbative quantum gravity

Conceptually relativistic gravity is necessarily *background independent* while QFT is not.

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Conceptually relativistic gravity is necessarily *background independent* while QFT is not.

Technically the Einstein-Hilbert action

$$S_{\mathrm{EH}}\left[g_{\mu\nu}\right] = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R\ ,$$

is not polynomial: no free theory to perturbate around.

Introduction

The problem of $d = 4$ perturbative quantum gravity

Conceptually relativistic gravity is necessarily *background independent* while QFT is not.

Technically However, if we jeopardize background independence

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \ , \tag{1}$$

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Technically we get an action for a propagating spin-2 field, schematically,

$$S[\phi_{\mu\nu}] = \int d^4x \left[-\frac{1}{2} (\partial\phi)^2 + 8\pi G\phi (\partial\phi)^2 + (8\pi G)^2 \phi^2 (\partial\phi)^2 + \cdots \right] \tag{1}$$

where $\phi_{\mu\nu} \equiv \frac{1}{8\pi G} h_{\mu\nu}$.

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where $\phi_{\mu\nu} \equiv \frac{1}{8\pi G}h_{\mu\nu}$. Nonetheless, it is power-counting nonrenormalizable.

Remark 1: It does not mean this framework is completely useless though

$$U(r) = -\frac{Gm_1m_2}{r}\left(1 + \frac{43}{30\pi}\frac{G}{r^2} + \cdots\right) . \tag{2}$$

Introduction

Quantum gravity approaches

There are usually two branches:

1. Background independence is **ex**plicitly built-in: LQG, CDT, Tensor Models, Causal Sets, etc;
2. Background independence is **im**plicitly built-in (at best): ASQG, HDQG, Supergravities, Strings, and induced gravities in general.

The scenario explored here interpolates between branch 1. and 2..

Topological quantum field theories

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If $\mathcal{O}$ is an observable then

$$\frac{\delta}{\delta g_{\mu\nu}}\langle\mathcal{O}\rangle = 0\;, \tag{3}$$

where

$$\langle\mathcal{O}\rangle = \frac{\int_{\mathcal{A}}\mathcal{D}\Phi e^{-S_{\text{TQFT}}[\Phi]}\mathcal{O}}{\int_{\mathcal{A}}\mathcal{D}\Phi e^{-S_{\text{TQFT}}[\Phi]}}\;. \tag{4}$$

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Indeed $\langle \mathcal{O} \rangle$ carries global information about spacetime. For instance, its topology or smooth structure. In this sense, TQFTs are examples of explicitly background independent QFTs.

Topological quantum field theories

Examples

$d = 2$ topological σ-models: related to Gromov-Witten invariants [1];

$d = 2$ gravity: related to Mumford-Morita-Miller invariants [2];

$d = 3$ Chern-Simons theories: related to link and knots invariants [3];

$d = 3$ gravity: tree-level related to $d = 3$ Chern-Simons [4].

$d = 4$ topological Yang-Mills theories: related to Donaldson invariants [5];

$d = 4$ gravity: ???

Topological quantum field theories

$d = 4$ topological Yang-Mills theory

There are multiple ways to define such theory:

Witten's original formulation: twisted version of $\mathcal{N} = 2$ Super-Yang-Mills theory [6];

Mathai-Quillen formulation: (generalized) Euler invariant of $\mathcal{M}_{\text{ins}}$ [7];

Baulieu-Singer formulation: BRST quantization of the 2nd Chern class [8].

Topological quantum field theories

$d = 4$ topological Yang-Mills theory (Baulieu-Singer formulation)

Action functional:

$$S_{\mathrm{TYM}} = \int \mathrm{Tr}\,(FF) \qquad \text{(Pontryagin action)} \tag{5}$$

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$$\delta A = -D\alpha + \beta\ . \tag{6}$$

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BRST technology:

$$\delta \longrightarrow s \quad ; \quad s^2 = 0\ , \tag{7}$$

$$\alpha \longrightarrow c \quad ; \quad [c, c] = cc + cc\ , \tag{8}$$

$$\beta \longrightarrow \psi \quad ; \quad [\psi, \psi] = \psi\psi - \psi\psi\ . \tag{9}$$

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Gauge fixing:

$$\left.\begin{aligned} d \star A &= 0 \,, \\ d \star \psi &= 0 \,, \\ F \pm \star F &= 0 \,. \end{aligned}\right\} \quad \text{(Anti-)Self-dual Landau gauge}$$

Topological quantum field theories

$d = 4$ topological Yang-Mills theory (Baulieu-Singer formulation)

Our first work [9]:

- ◇ We showed that TYM in such gauge renormalizes with only one (unphysical) parameter;
- ◇ We showed that $\langle A(x)A(y)\rangle = 0$ exactly.

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Our second [10]:

- ⋄ We showed that TYM is actually tree-level exact.

TQFT for $d = 4$ gravity

Motivation

- Failure of a traditional QFT description of gravity;
- TQFTs are the only examples of renormalizable perturbative QFTs that are also background independent;
- TQFTs and gravity are proven to be very closely related in lower spacetime dimensions.

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These remarkable features led Edward Witten to hypothesize if TQFTs are also related to gravity in $d = 4$. In particular, if TYM theory could be physically understood as an unbroken phase of gravity.

If yes, all local physics in the smallest scales ($< 10^{-35}$ m) would be trivial. Quantum gravity would be just about global features of spacetime: its smooth structure, etc.

Unbroken phase

A renormalizable TYM theory

Let us consider the most general metric independent action functional invariant under $SO(1,3)$

$$S_0 = \int \mathrm{Tr}\left(g_1 FF + g_2 FF^\star\right) \ , \tag{5}$$

where * stands for the (metric independent) color dual.

Remark 2: S_0 has nothing to do with usual gravity *à priori*.

Unbroken phase

A renormalizable TYM theory

Now, let us consider a pair of BRST doublets

$$sY = X \;, \tag{5}$$

$$sX = 0 \;. \tag{6}$$

Remark 3: Observables are not altered by the introduction of a pair of fields with BRST doublet structure (Doublet Theorem).

Unbroken phase

A renormalizable TYM theory

The most general action functional of A, X and Y which is s-exact is then

$$\begin{aligned} S_{\rm triv} &= s\int\left[Y\left(g_3F+g_4\star F+g_5F^\star+g_6\star F^\star+g_7X+g_8\star X+g_9X^\star+g_{10}\star X^\star\right)\right] , \\ &= \int\{\left(g_3F+g_4F\star+g_5F^\star+g_6F^\star\star+g_7X+g_8X\star+g_9X^\star+g_{10}X^\star\star\right)X+ \\ &+Y\left[g_3\left(D\psi+[c,F]\right)+g_4\star\left(D\psi+[c,F]\right)+g_5\left(D\psi+[c,F]\right)^\star+ \right. \\ &+\left. g_6\star\left(D\psi+[c,F]\right)^\star\right]\} . \end{aligned} \tag{5}$$

Remark 4: The presence of the metric dependent $\star$ operator does not spoil the topological nature of this theory.

Unbroken phase

A renormalizable TYM theory

Finally, the action we are considering is

$$S = S_0 + S_{\text{triv}} \tag{5}$$

Remark 5: Action S and S_0 are physically indistinguishable;

Remark 6: After gauge-fixing, we showed that S is renormalizable to all orders.

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But why S???

Broken phase

Physical values of sources

Let us consider that

$$Y\big|_{\text{phys.}} = 0 \,, \tag{6}$$

$$X\big|_{\text{phys.}} = \mu^2 ee \,. \tag{7}$$

where μ is a mass parameter and e is the vierbein field.

Broken phase

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where μ is a mass parameter and e is the vierbein field.
Consequently

$$S\big|_{\text{phys.}} = S_0 + \mu^2 \int \text{Tr}\left[(g_4 + g_5)\, F \star (ee) + \mu^2\,(g_8 + g_9)\, ee \star (ee) + (g_3 + g_6)\, Ree\right]\ , \quad (8)$$

which is exactly is the $d = 4$ Lovelock-Cartan action of gravity.

TQFT for $d = 4$ gravity

Symmetry breaking

The topological BRST operator can be decomposed as

$$s = s_{\mathrm{YM}} + s_{\mathrm{T}} \ , \tag{9}$$

then

$$sS = s_{\mathrm{YM}}S + s_{\mathrm{T}}S \ . \tag{10}$$

The physical values of sources Y and X induces a break

$$sS = s_{\mathrm{YM}}S + \cancel{s_{\mathrm{T}}S} \ , \tag{11}$$

freeing the gravitational degrees of freedom.

Conclusions

- Indeed there is a way for a TYM theory to generate the whole Lovelock-Cartan family of gravities via an explicit breaking of its topological symmetry;
- Such a TYM theory for $d = 4$ gravity is renormalizable to all orders.

Perspectives

In this specific approach:

- What is the agent responsible for the breaking mechanism? Corrections coming from a "topological matter" couplings? Higgs-like mechanism?

More generally:

- The role topology plays in QG. How to evaluate topology-changing processes?

Thank you

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