Asymptotically Safe Gravity with Riemann and Ricci Tensors[1]

Yannick Kluth

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UNIVERSITY OF SUSSEX

[1] based on work with Daniel Litim

This Talk ...

Introduction

Pure Quantum Gravity

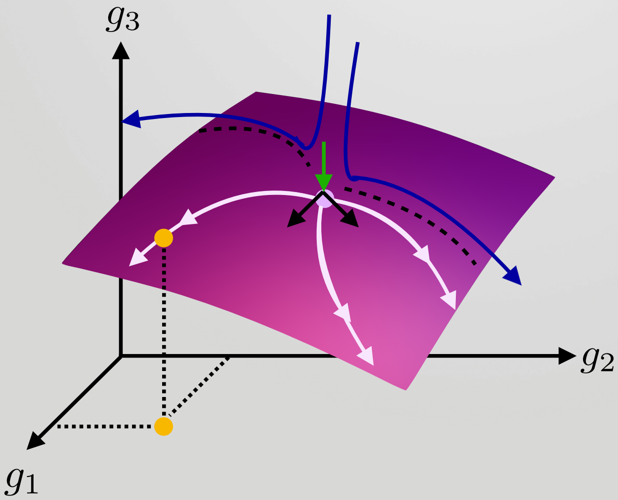
- General Relativity is tested to very high precision but can't be end of story: Black holes, consistency with QM, ...
- Quantum Field Theory describes all forces except for gravitation to very high precision
- What do we know about gravity in quantum field theory?
  - Gravity is perturbatively non-renormalisable
  - Infinite number of counterterms
  - Only predictive as low energy effective field theory

$$\mathcal{S} = \int \mathrm{d}^d x \sqrt{g} \left[\lambda_0 + \lambda_1 R + \lambda_{2,1} R^2 + \lambda_{2,2} R_{\mu\nu} R^{\mu\nu} + \lambda_{2,3} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \lambda_{2,4} \Box R + \dots \right]$$

- How to determine open coupling constants?

Asymptotic Safety

RG Flow



- Trajectories flow towards IR
- All trajectories which flow into fixed point are on UV critical surface (purple)
- Dimensionality of UV critical surface is given by number of negative eigenvalues of stability matrix

Graphic taken from [Eichhorn, 2018]

Stability Matrix

Expand beta functions around fixed point $\tilde{g}^*$:

$$\tilde{\beta}_i = \left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* (\tilde{g}_j - \tilde{g}_j^*) + \mathcal{O}(\tilde{g}_j - \tilde{g}_j^*)^2$$

This is solved by

$$\tilde{g}_i = \tilde{g}_i^* + \sum_n C_n k^{\theta_n} v_i^{(n)}$$

with

$$\left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* v_j^{(n)} = \theta_n v_i^{(n)}$$

The C_n are now the open parameters of the theory

- $\Re(\theta_i) > 0$: Eigenmode diverges in the UV $\rightarrow$ $C_i = 0$ (Irrelevant direction)
- $\Re(\theta_i) < 0$: Eigenmode vanishes in the UV $\rightarrow$ $C_i =$ unknown (Relevant direction)

➤ **Number of relevant directions is given by number of negative eigenvalues**

Gaussian Scaling

Relation between massive and massless couplings:

$$\tilde{g}_i = k^{-d_i} g_i$$
$$\Rightarrow \tilde{\beta}_i = -d_i \tilde{g}_i + k^{-d_i} \beta_i$$

Stability matrix:

$$\left.\frac{\partial \tilde{\beta}_i}{\partial \tilde{g}_j}\right|_* = -d_i \delta_{ij} + \text{quantum corrections}$$

At Gaussian fixed point ($\tilde{g}_i = 0$):

$$\theta_i = -d_i$$

At Gaussian fixed point the number of relevant directions is given by number of couplings with positive mass dimension. What about interacting fixed points?

Bootstrap Approach

- Quantum Corrections change eigenvalues at NGFP
- Can any operator become relevant?

Bootstrap Approach to asymptotic safety: [Falls, Litim, Nikolakopoulos, Rahmede, 2013]

- Hypothesis: Canonical mass dimension remains to be a good ordering principle
- Newly included operators with higher mass dimension should turn out to be more irrelevant

Functional Renormalisation Group & Truncations

Functional Renormalisation Group

- Use non-perturbative Functional Renormalisation Group (FRG) [Wetterich, 1992]

$$\partial_t \Gamma_k = \frac{1}{2} \operatorname{Tr} \left[\left(\partial_t \mathcal{R}_k \right) \left(\Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \right]$$

- $\mathcal{R}_k$ is a k-dependent mass term (IR regulator)
- Γ_k is effective average action. (Scale dependent effective action)
- Solve FRG approximately by making an ansatz for the form of Γ_k
    - ➤ Extract beta functions
- Avoid non-minimal operators by using spherical background
    - ➤ Then, only operators with different mass dimensions can be distinguished

State-of-the-Art

Truncations that have been studied so far:

- $F(R)$
- $U\left(\mathrm{Ric}^2\right) + R\,V\left(\mathrm{Ric}^2\right)$
- $\lambda_0 + \lambda_1 R + \lambda_{2,1} R^2 + \lambda_{2,2}\mathrm{Ric}^2 + \lambda_{2,3}\mathrm{Riem}^2$
- $\lambda_0 + \lambda_1 R + \lambda_C C^{\mu\nu}{}_{\rho\sigma} C^{\rho\sigma}{}_{\alpha\beta} C^{\alpha\beta}{}_{\mu\nu}$

$$\vdots$$

Usually, the UV critical surface is three dimensional

Which operators do we need?

	Curvature Invariants
dim-2	R
dim-4	$R^2, R_{\mu\nu}R^{\mu\nu}, R_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, \Box R$
dim-6	$\nabla_\mu R \nabla^\mu R, \nabla_\rho R_{\mu\nu}\nabla^\rho R^{\mu\nu}, R^3, RR_{\mu\nu}R^{\mu\nu}, RR_{\rho\sigma\mu\nu}R^{\rho\sigma\mu\nu}, R_{\mu\nu}R^{\nu\rho}R^\mu_\rho,$ $R_{\mu\nu}R_{\rho\sigma}R^{\mu\nu\rho\sigma}, R^\mu_\nu R^{\nu\alpha\beta\gamma}R_{\mu\alpha\beta\gamma}, R^{\mu\nu}{}_{\rho\sigma}R^{\rho\sigma}{}_{\alpha\beta}R^{\alpha\beta}{}_{\mu\nu}, R_{\mu\nu\rho\sigma}R^{\mu}{}_{\alpha}{}^{\rho}{}_{\beta}R^{\nu\alpha\sigma\beta}$
⋮	

Gravitational Truncations

Consider the following truncations

$$\Gamma_k = \int \mathrm{d}^d x \sqrt{g}\, F(\mathrm{Ric}) = \int \mathrm{d}^d x \sqrt{g}\, \left[\lambda_0 + \lambda_1 R^\mu{}_\mu + \lambda_2 R^\mu{}_\nu R^\nu{}_\mu + \lambda_3 R^\mu{}_\nu R^\nu{}_\rho R^\rho{}_\mu + \ldots\right]$$

and

$$\begin{aligned}\Gamma_k &= \int \mathrm{d}^d x \sqrt{g}\, \left[U\left(\mathrm{Riem}^2\right) + R\, V\left(\mathrm{Riem}^2\right)\right] \\ &= \int \mathrm{d}^d x \sqrt{g}\, \left[\lambda_0 + \lambda_1 R + \lambda_2 \mathrm{Riem}^2 + \lambda_3 R\, \mathrm{Riem}^2 + \ldots\right]\end{aligned}$$

truncated to include N different operators.

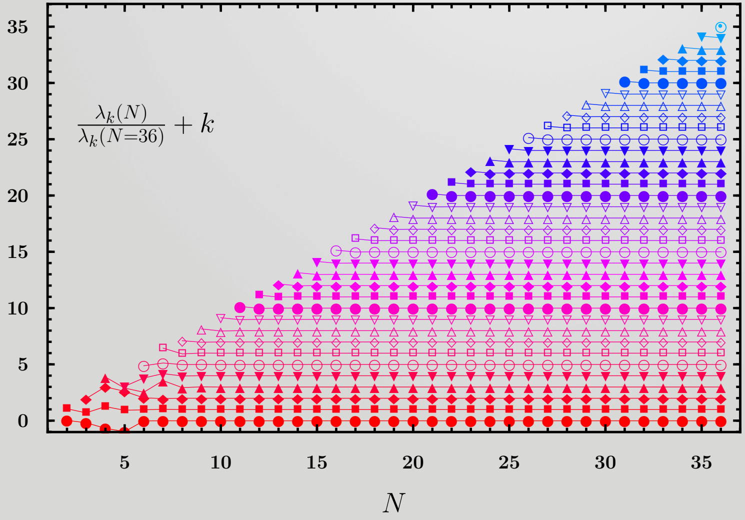
Results

Results

- In both truncations two (stable) fixed points can be found at different orders N. This includes
  - Martin Reuter Fixed Point ($\lambda_0 \approx 0.25$, $\lambda_1 \approx -0.9$)
  - Fixed Point with smaller couplings ($\lambda_0 \approx 0.06$, $\lambda_1 \approx -0.3$)
    - Both of them have **4 relevant directions** in both truncations!
  - Additionally, spurious solutions and much less stable fixed points can be found

- The two stable fixed points are very similar in both truncations

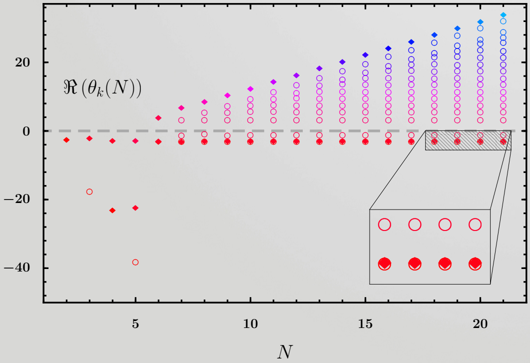
Couplings for MR in $F(\mathrm{Ric})$



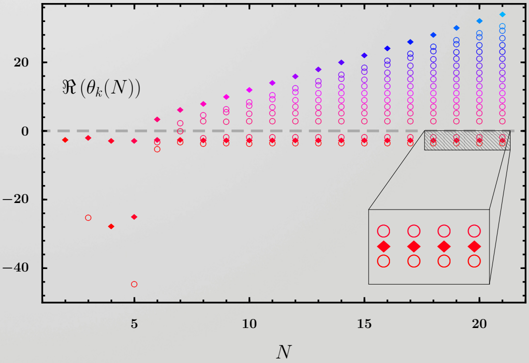
- Convergence of fixed point couplings $\lambda_k(N)$ with N being the number of included operators
- High orders show strong convergence
- Unstable below $N = 6$

Eigenvalues for MR FP_4's

$U\left(\mathrm{Riem}^2\right) + RV\left(\mathrm{Riem}^2\right)$

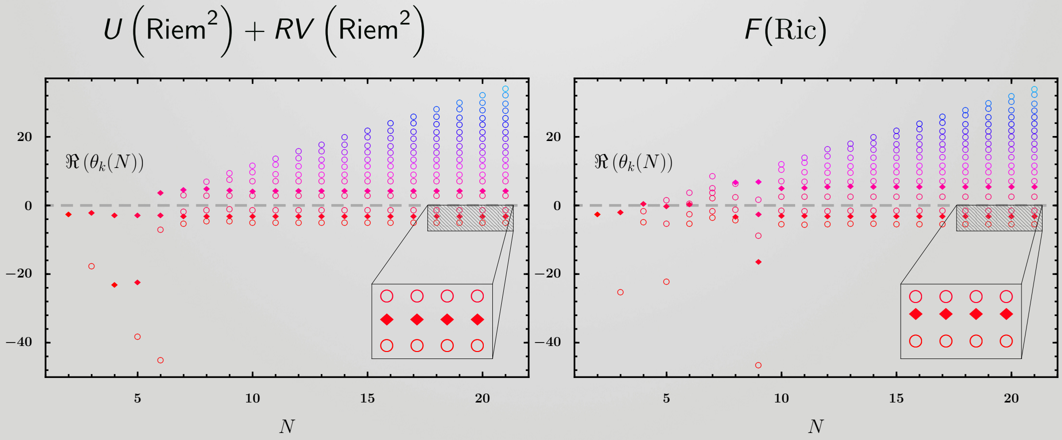


$F(\mathrm{Ric})$



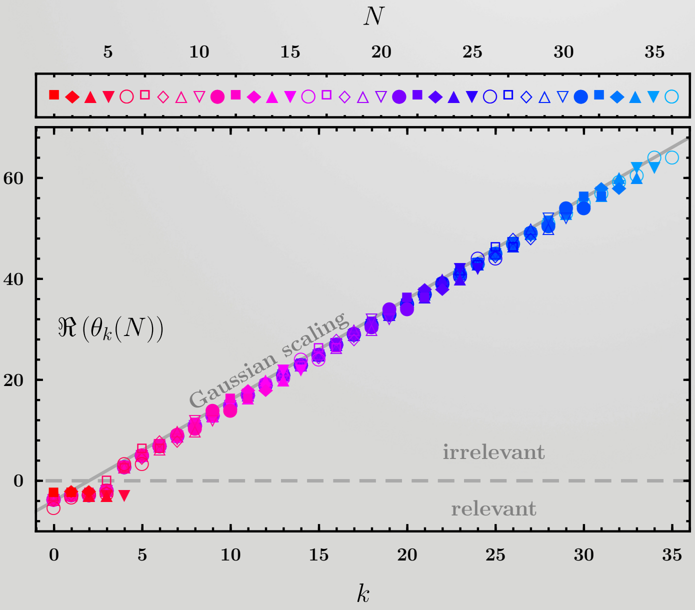
Empty Circles ⇔ Real eigenvalues // Filled Diamonds ⇔ Complex conjugate eigenvalues

Eigenvalues for small FP_4's



Empty Circles ⇔ Real eigenvalues // Filled Diamonds ⇔ Complex conjugate eigenvalues

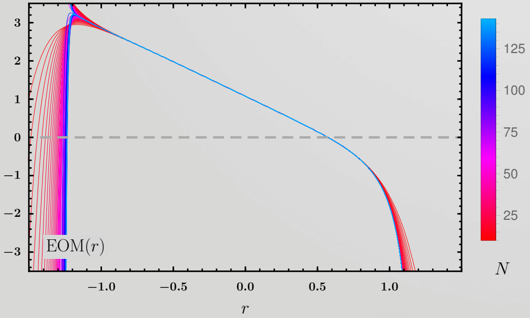
Gaussian Scaling of MR FP_4 in f(Ric)



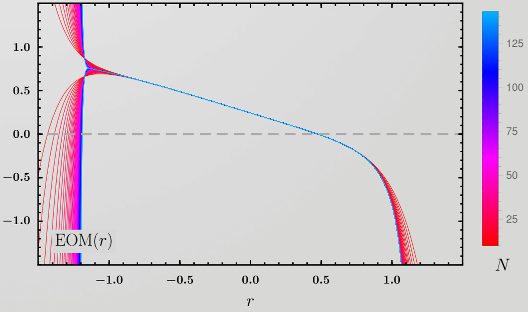
- All eigenvalues up to $N = 36$ are shown (different symbols for different orders)
- Eigenvalues close to Gaussian scaling
- Other FP_4's show the same Gaussian scaling in both truncations

Equations of Motion for MR FP_4's

$$U\left(\text{Riem}^2\right) + RV\left(\text{Riem}^2\right)$$



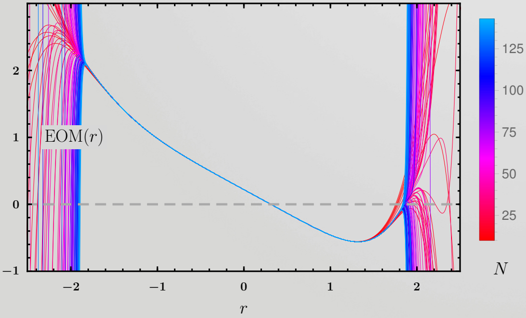
$$F(\text{Ric})$$



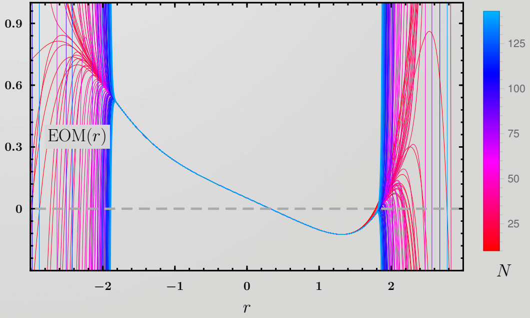
Notice pole-like behaviour on positive real axis

Equations of Motion for small FP_4's

$$U\left(\mathrm{Riem}^2\right) + RV\left(\mathrm{Riem}^2\right)$$



$$F(\mathrm{Ric})$$



Radius of convergence can be extended by Pade approximation

RG Trajectories

- The theory should be on an RG trajectory from the UV fixed point to the Gaussian fixed point $\Lambda = 0$ and $G_N = 0$
- IR connecting RG trajectories for MR FP's can be found (in both truncations)
- For small FP's no such RG trajectory can be found
  - Coupling space is divided in two separate part by hypersurface where denominator of beta functions vanish
  - Small FP's and Gaussian FP's are in different parts
  - No RG trajectories possible

Summary

Summary

- First studies of $F(\mathrm{Ric})$ and $F(R, \mathrm{Riem}^2)$
- Two truncations with four relevant directions
  - First indications that dim-6 operators can become relevant
- Results for stable fixed points are similar in both truncations
  - Eigenvalue pattern
  - Equations of Motion
  - RG trajectories
- What happens when matter is included?

Thank you!