Mixed Global **Anomaly** and **Boundary** Conformal Field Theories

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Based on a work with **Tokiro Numasawa** (Osaka U., McGill U.)

arXiv:1712.09361 [hep-th]

3d SPT phase

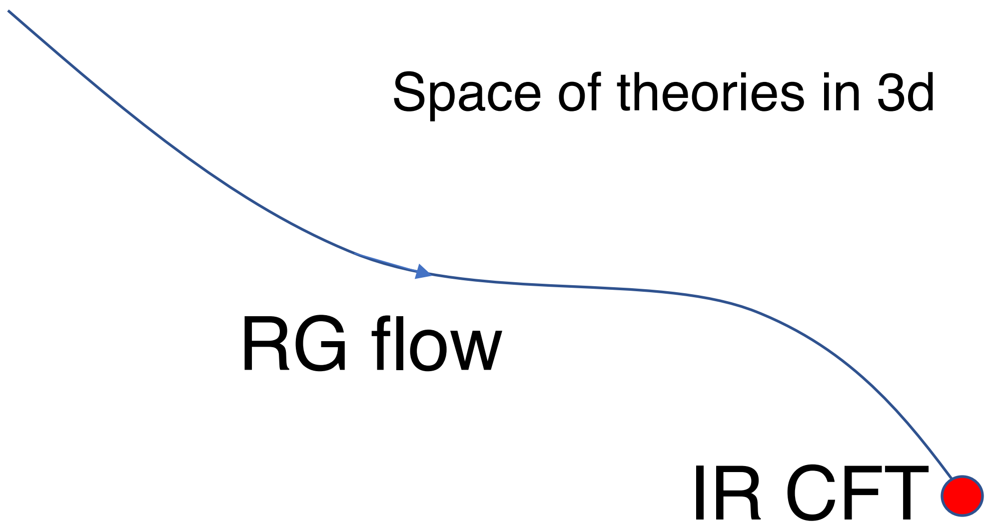
↓ Motivation

't Hooft anomaly in 2d CFT

↕ **New interesting observation!**

2d boundary CFT

RG flow



(maybe empty)

Let's look more closely at this

→ SPT phase

SPT phases

(symmetry protected topological)

(In my impression)

Asking

Is this “empty theory” really empty?

No particle, no massless excitation, no spontaneous symmetry breaking, but there is still some non-trivial phase structure.

Example: 3d massive free Dirac fermion

$$S = \int d^3x \left(i\bar{\psi}\gamma^\mu \partial_\mu \psi + m\bar{\psi}\psi - \frac{1}{\Lambda}\bar{\psi}\partial^2\psi \right)$$

Different

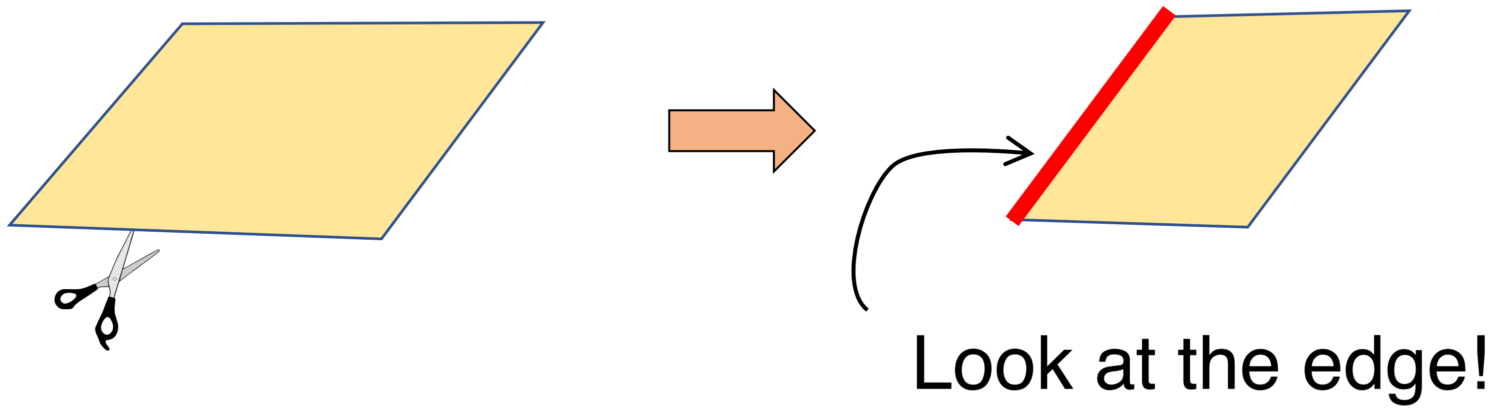
$$S = \int d^3x \left(i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - \frac{1}{\Lambda}\bar{\psi}\partial^2\psi \right)$$

$m > 0$

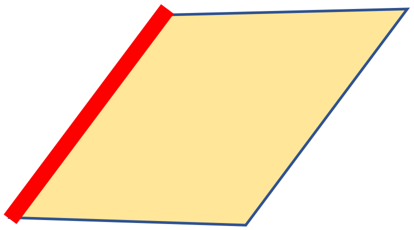
Remnant of the regularization
Eg. Pauli-Villars
Wilson term in lattice

How to distinguish?

How to distinguish?



Example: 3d massive Dirac fermion



$$S = \int d^3x \left(i\bar{\psi}\gamma^\mu \partial_\mu \psi + m\bar{\psi}\psi - \frac{1}{\Lambda}\bar{\psi}\partial^2\psi \right)$$

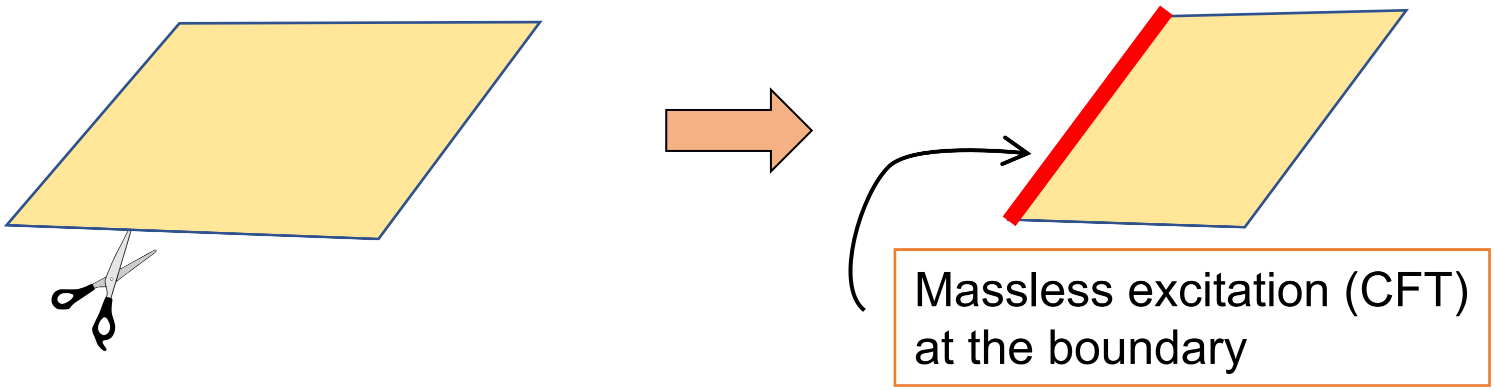
➡ Edge Weyl fermion

(non-empty 2d CFT)

$$S = \int d^3x \left(i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi - \frac{1}{\Lambda}\bar{\psi}\partial^2\psi \right)$$

➡ Empty

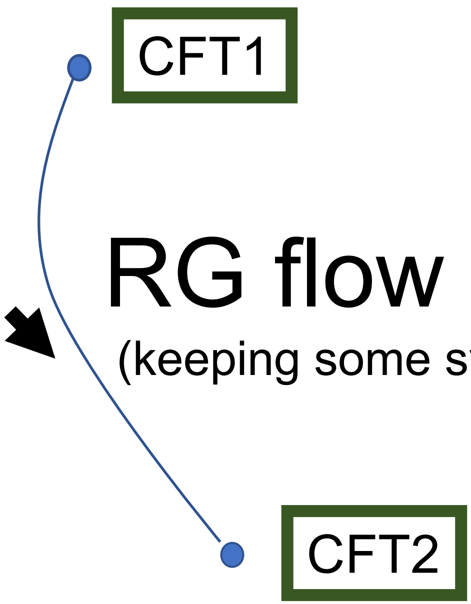
A way to find SPT phases



SPT phase in 3 dim $\sim$ **“classification” of 2-dim CFT**

“Classification”

(should not depends on small perturbation)



RG flow

(keeping some symmetry)

⇔

CFT1 ~ CFT2

same class

Need “RG invariants”

’t Hooft anomaly is an RG invariant

Obstruction to gauge a global
symmetry of the theory

(Gauge non-invariance in the presence of
backgound gauge field)

Example: 2d Weyl fermion (appear at the edge of SPT phase)

$$S = \int d^2x i\bar{\psi}_+(\partial_0 - \partial_1)\psi_+$$

cannot go to empty by perturbation
since U(1) symmetry

$$\psi_+ \longrightarrow e^{i\alpha}\psi_+$$

is anomalous (when you introduce background gauge field).

SPT phase in 3 dim

$\sim$ “classification” of the **anomaly** of 2-dim **CFT**

In particular

If a CFT $\xrightarrow{\text{Perturbation and RG}}$ Empty

this CFT is called "gappable"

(no anomaly)

3d SPT phase

Motivation

Done

't Hooft anomaly in 2d CFT

New interesting observation!

2d boundary CFT

2d boundary CFT

[Ishibashi], [Ishibashi-Onogi], [Cardy]

$$|B\rangle_C$$

Described by a boundary state

If one can introduce a boundary without breaking the symmetry

$\exists |B\rangle_C$ symmetry invariant

$\Leftrightarrow$ CFT is called **"Edgeable"**

Gappable
(no anomaly)

↔

Edgeable
(symmetry invariant boundary state exsists)

closely related

[Han, Tiwari, Hsieh, Ryu 17]

Examples

2d Weyl fermion

- Ungappable(anomaly)
- Unedgeable(boundary cannot exist)

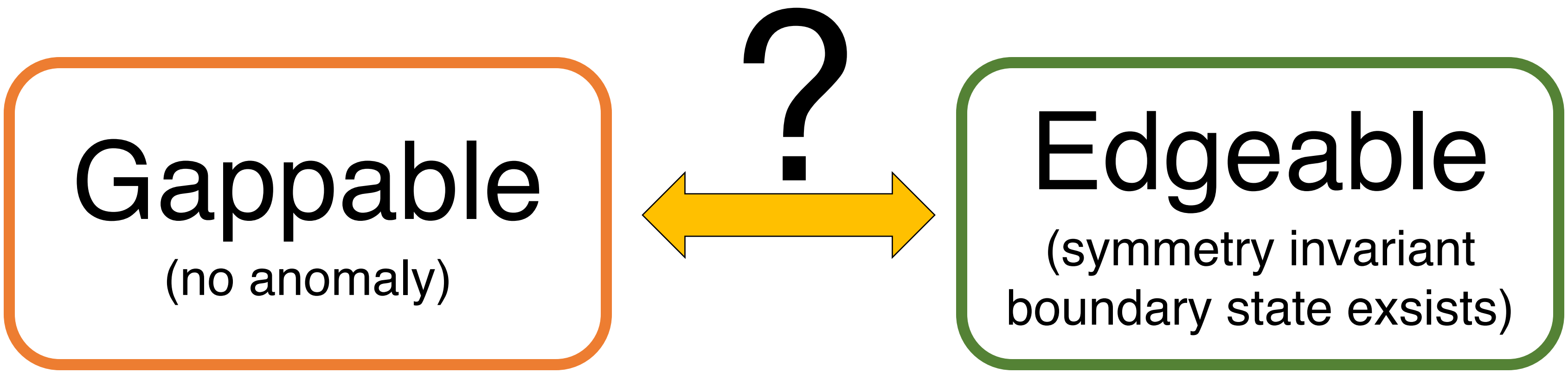
cannot be reflected

2d Dirac fermion

- Gappable(no anomaly and mass term is possible)
- Edgeable(Boundary can exist)

Gappable = edgeable work nicely.

We want to look at



in WZW model keeping center symmetry
and diffeomorphism.

We find

Gappable
(no anomaly)

⟺

Edgeable
(“**symmetry**” invariant boundary state exsists)

holds in a rather nontrivial way.

(involving the charge conjugation)

WZW model

(non-chiral) Wess-Zumino-Witten model

Field: $g(x) \in SU(N)$

Action: $S = \frac{k}{8\pi}\int_{\Sigma_2} d^2x \mathrm{tr}(\partial_\mu g \partial^\mu g^{-1}) + \frac{k}{12\pi}\int_{M_3} \mathrm{tr}(g^{-1}dg)^3$

$\partial M_3 = \Sigma_2$

Parameter: $k \in \mathbb{Z}_{\geq 0}$ "Level" $k \sim \frac{1}{\hbar}$

$$SU(N)_k$$

※Different from chiral WZW model that appear at the boundary of Chern-Simons theory

WZW model has affine Lie algebra symmetry

↓

Solved

highest weight states $|\hat{\lambda}, \hat{\lambda}\rangle$ (for diagonal theory)

$$\hat{\lambda} = [\lambda_0, \lambda_1, \cdots, \lambda_{N-1}] \qquad \lambda_j \in \mathbb{Z}_{\geq 0}$$

Affine Dynkin label

Level: $k = \lambda_0 + \lambda_1 + \cdots + \lambda_{N-1}$

(non-chiral) WZW model

$$S = \frac{k}{8\pi}\int_{\Sigma_2} d^2x \mathrm{tr}(\partial_\mu g \partial^\mu g^{-1}) + \frac{k}{12\pi}\int_{M_3} \mathrm{tr}(g^{-1}dg)^3$$

$$SU(N)_k$$

$$g(x) \in SU(N) \quad k \in \mathbb{Z}_{\geq 0}$$

We focus on

- center $\quad g(x) \to hg(x), \quad h \in \mathbb{Z}_N \subset SU(N)$
- diffeo

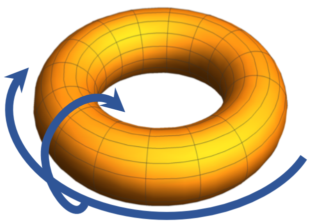
No perturbative anomaly

Global anomaly (anomaly for a large gauge transf.)

[Gepner, Witten 86]

WZW model on

- torus
- gauge field for $\mathbb{Z}_N$



Holonomy (Wilson line)

Large diffeo(modular transformation) invariant?

Metric

Coordinates (x, y) $\quad x \sim x + 2\pi, \quad y \sim y + 2\pi$

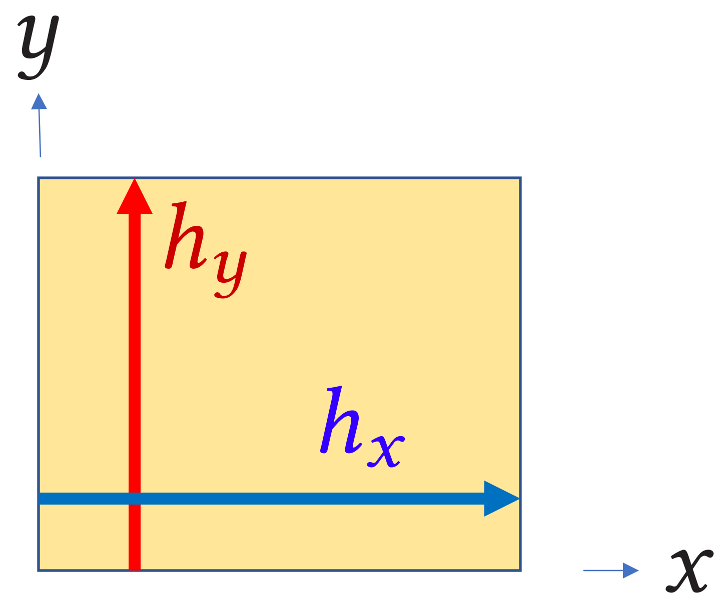
$$ds^2 = |dx + \tau dy|^2$$

$\tau = \tau_1 + i\tau_2$ modular parameter

Gauge field

$$h_x, h_y \in \mathbb{Z}_N$$

(Wilson line)

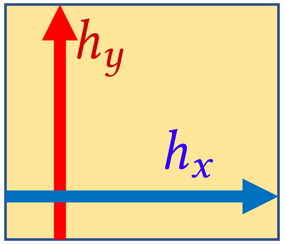


Partition function

$$Z(\tau, h_x, h_y)$$

Large diffeo (modular transformation)

$$x \sim x + 2\pi, \quad y \sim y + 2\pi \qquad ds^2 = |dx + \tau dy|^2$$



$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix} \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

* This is not continuously connected to the identity

Background fields

$$(\tau, h_x, h_y) \to (\tau' = \frac{a\tau + b}{c\tau + d}, h'_x = h_x^d h_y^c, h'_y = h_x^b h_y^a)$$

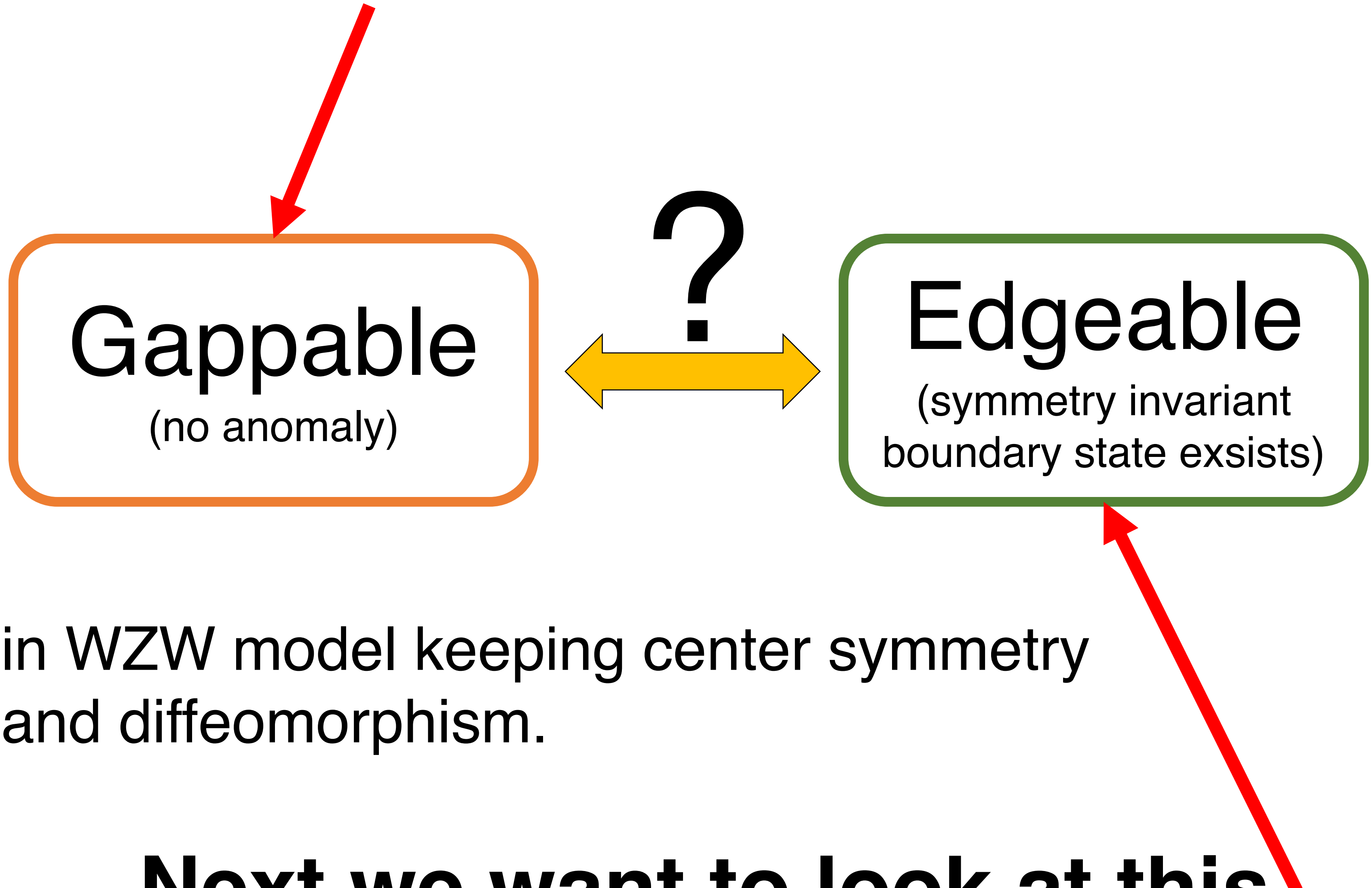
No anomaly?

$$Z(\tau, h_x, h_y) \overset{?}{=} Z(\tau', h'_x, h'_y)$$

Fact: [Gepner, Witten 86], [Freed, Vafa 87],…
[Sule, Chen, Ryu 13], [Furuya and Oshikawa 15],…
[Numasawa, SY 17],
[Di Francesco; Mathieu, Sénéchal "CFT" book]

N: odd		no anomaly (gappable)
N: even	k:even	no anomaly (gappable)
	k:odd	anomaly(ungappable)

We have seen this



in WZW model keeping center symmetry and diffeomorphism.

Next we want to look at this

Boundary WZW model

Boundary state in WZW model

[Ishibashi], [Ishibashi-Onogi], [Cardy]

$|\hat{\lambda}\rangle_C$

$$\hat{\lambda} = [\lambda_0, \lambda_1, \ldots, \lambda_{N-1}]$$

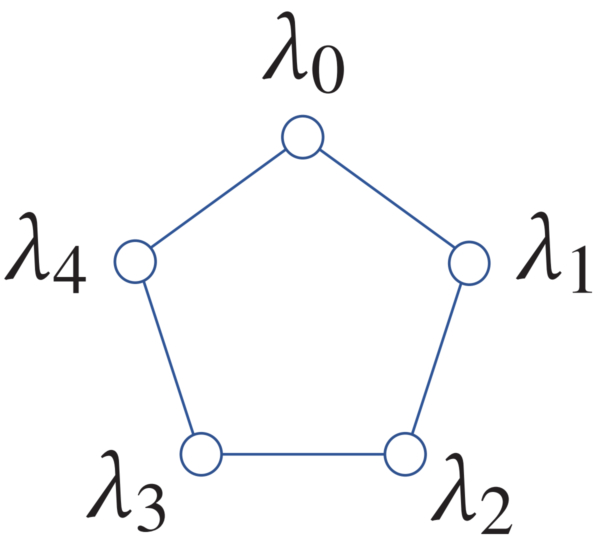
Affine Dynkin label

(same label as the highest weight state)

$$\lambda_j \in \mathbb{Z}_{\geq 0}$$

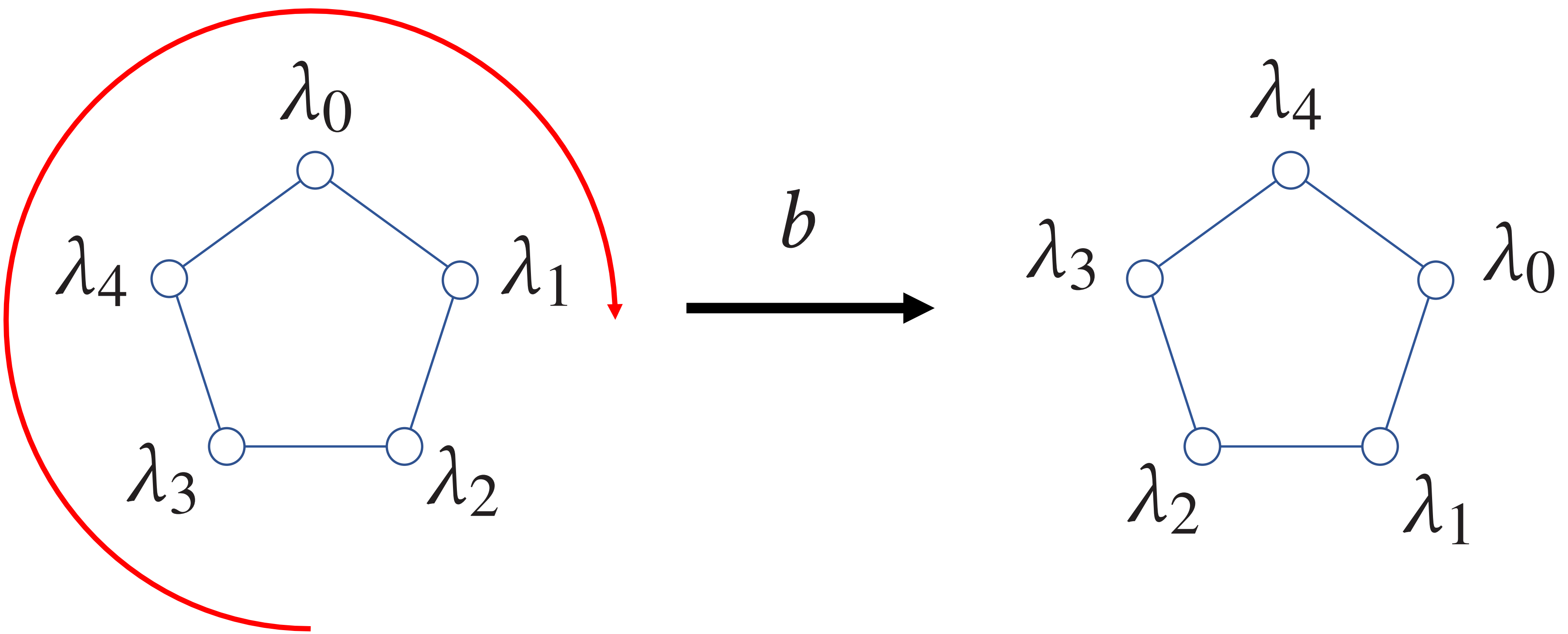
level $k = \lambda_0 + \lambda_1 + \cdots + \lambda_{N-1}$

Extended Dynkin diagram



Action of the center $\mathbb{Z}_N$ to the boundary state

$b \in \mathbb{Z}_N$ the generator



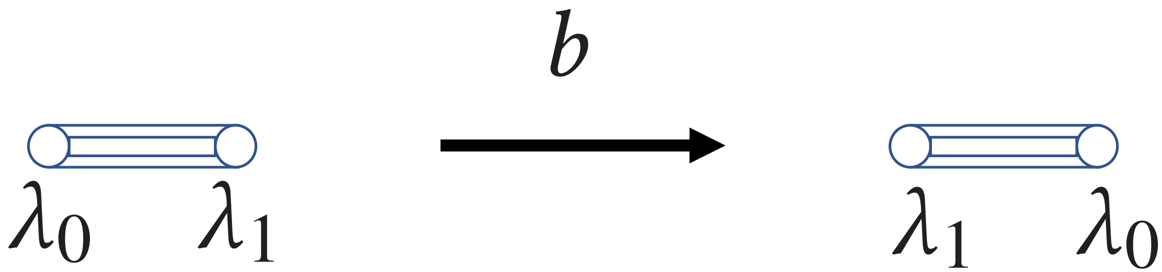
Action of modular transformation to boundary states

I have no idea

(Does modular transformation act on elements of the Hilbert space?)

Let's consider $\mathbb{Z}_N$ invariant boundary state

Example: SU(2)



$\mathbb{Z}_2$ invariance ⟷ $\lambda_0 = \lambda_1$

⟶ $k = \lambda_0 + \lambda_1 = 2\lambda_0$

$\mathbb{Z}_2$ invariant boundary state exist if and only if k is an even integer.

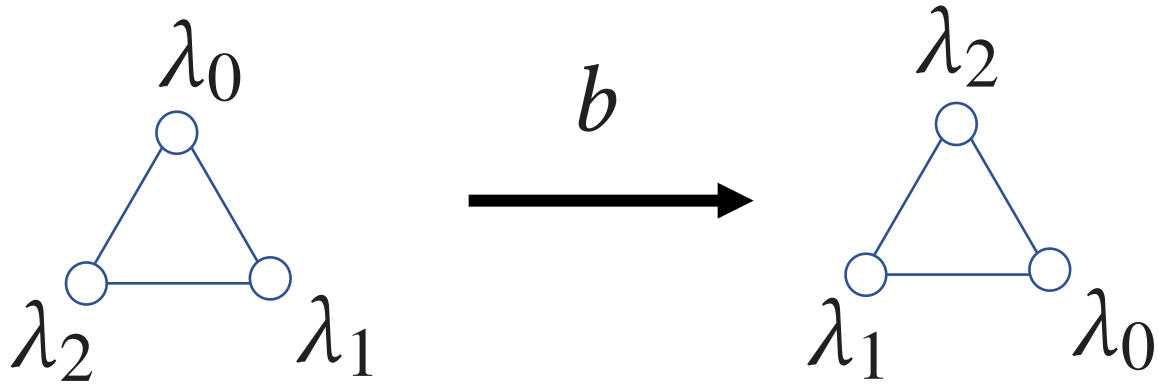
Gappable
(no anomaly)

⟺

Edgeable
($\mathbb{Z}_2$ invariant boundary state exsists)

holds!

Example: SU(3)



$\mathbb{Z}_3$ invariance ⟷ $\lambda_0 = \lambda_1 = \lambda_2$

⟶ $k = \lambda_0 + \lambda_1 + \lambda_2 = 3\lambda_0$

$\mathbb{Z}_3$ invariant boundary state exist if and only if k is a multiple of 3.

Gappable
(no anomaly)

⟺

Edgeable
($\mathbb{Z}_3$ invariant boundary state exsists)

fails!

Gappable
(no anomaly)

⇔

Edgeable
($\mathbb{Z}_N$ invariant boundary state exsists)

fails!

for N>2

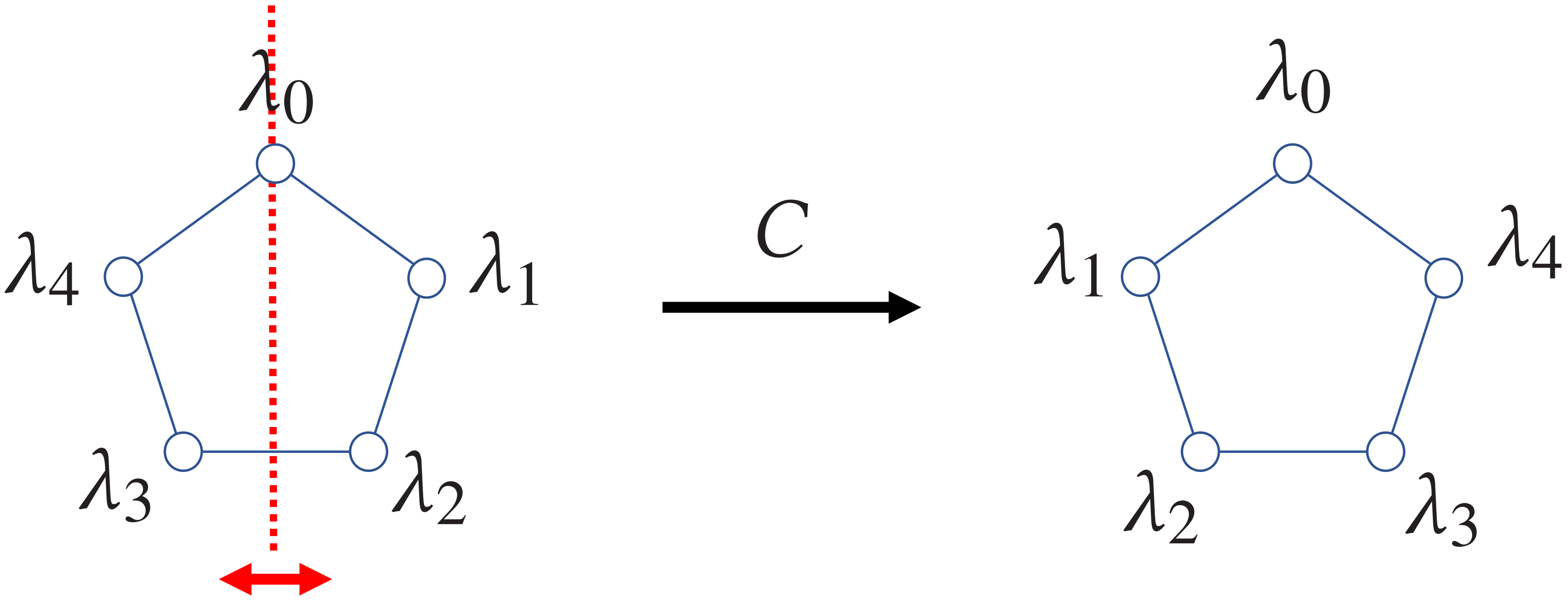
Gappable theory is not always edgeable.

Question:

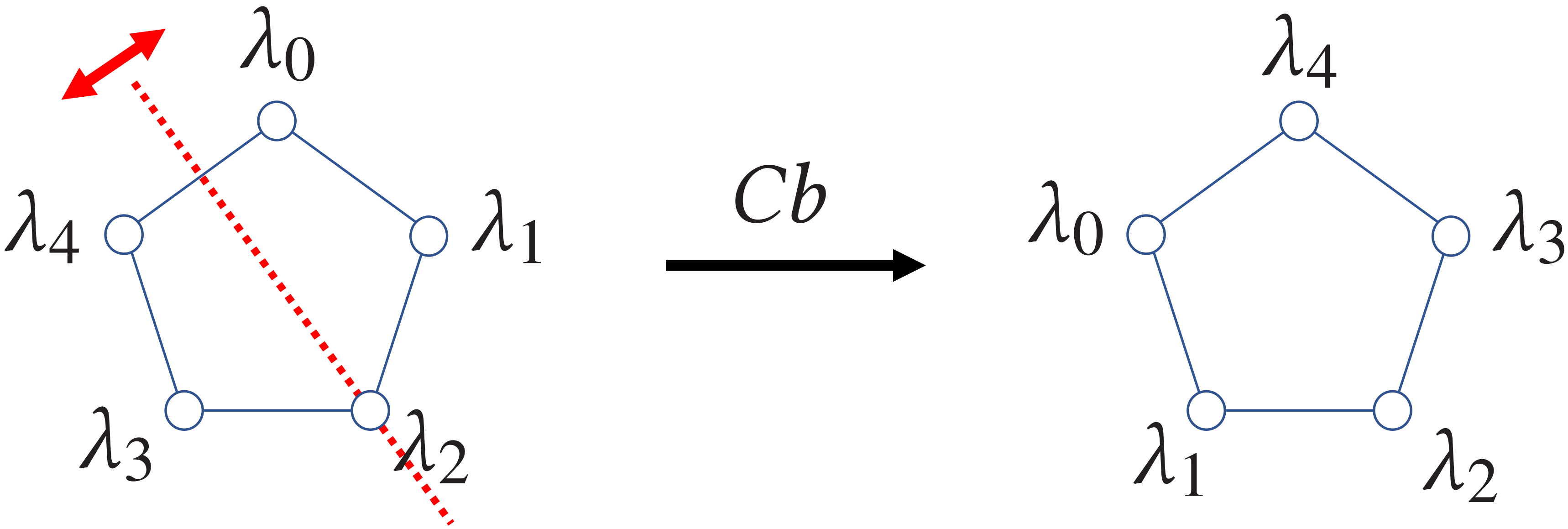
Can we modify the “edgeability” such that the gappability⇔edgeability relation holds?

YES

Charge conjugation



Action of Cb

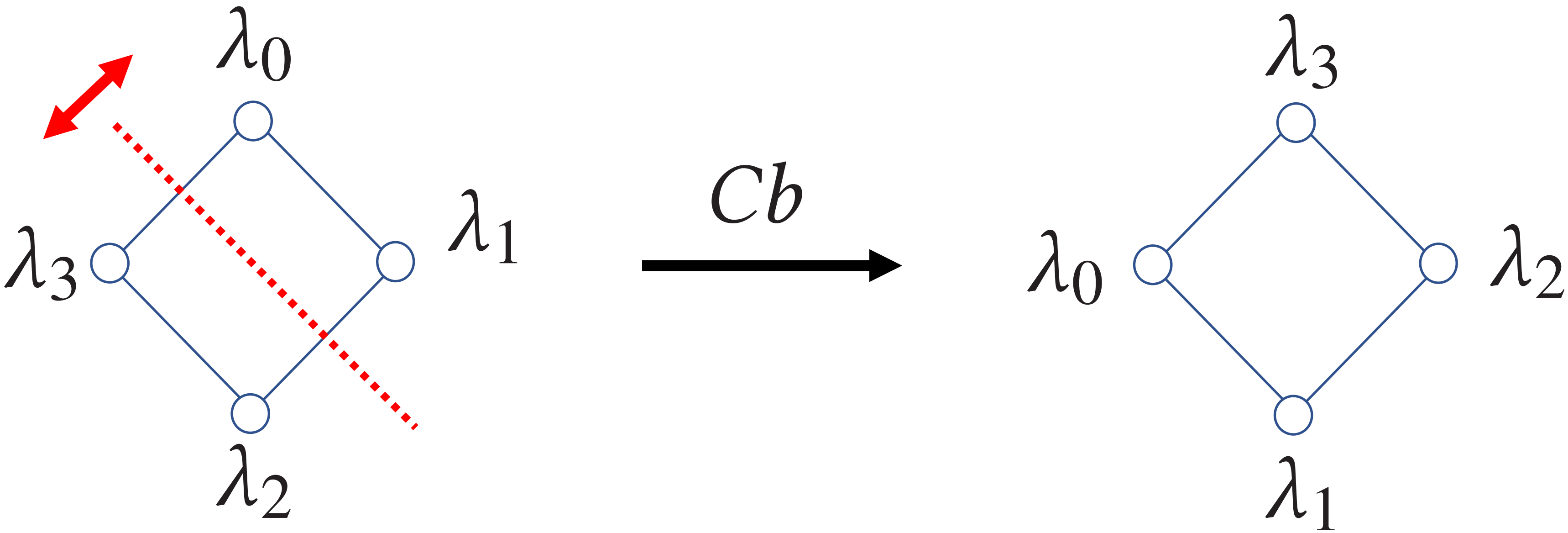


For N:odd $\hat{\lambda} = [0; 0, \ldots, k, \ldots, 0]$

$\frac{N-1}{2}$

is a Cb invariant boundary state for any k.

Action of Cb



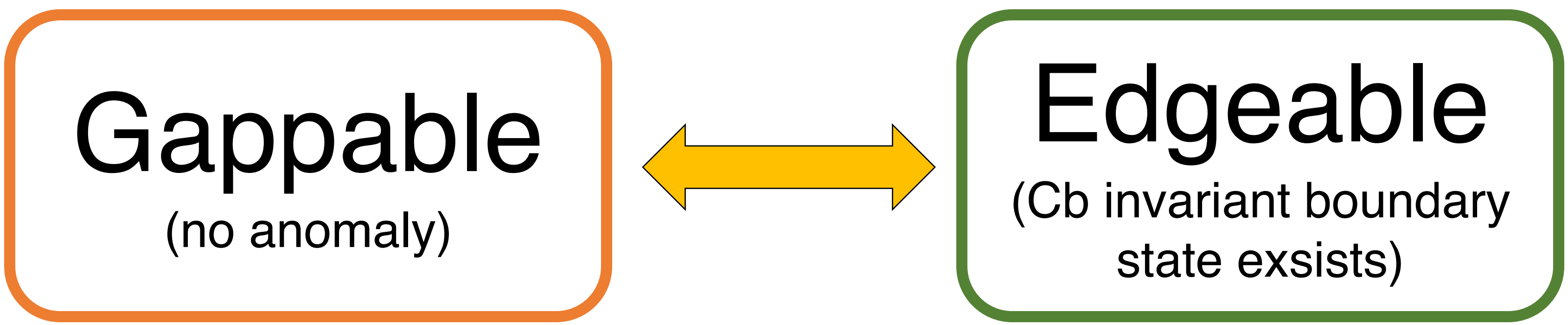
For N:even Cb invariance

$$\rightarrow \lambda_0 = \lambda_{N-1}, \lambda_1 = \lambda_{N-2}, \ldots, \lambda_{N/2-1} = \lambda_{N/2}$$

$k = \lambda_0 + \cdots + \lambda_{N-1} = 2(\lambda_0 + \cdots + \lambda_{N/2-1})$ is even.

Does a Cb invariant boundary state exists?

N: odd		Yes
N: even	k:even	Yes
	k:odd	No


Gappable
(no anomaly)
Edgeable
(Cb invariant boundary state exsists)

holds.

Summary

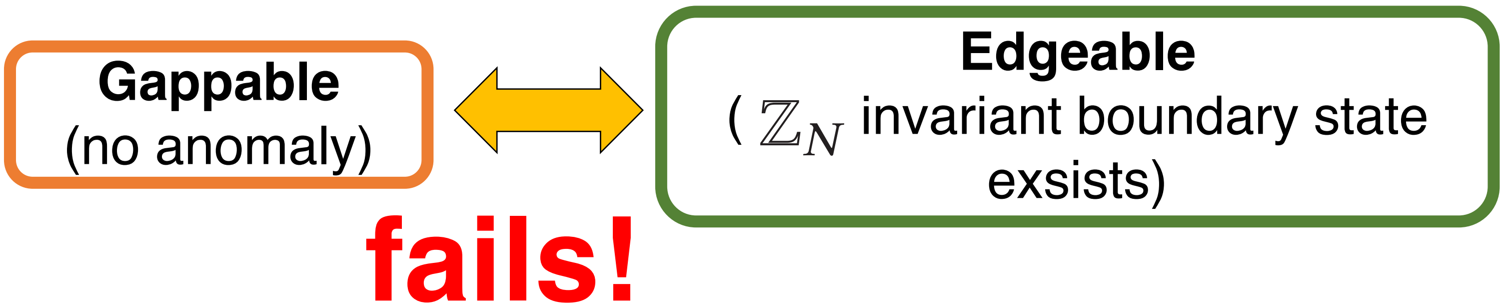
SU(N) WZW model

center and large diffeo

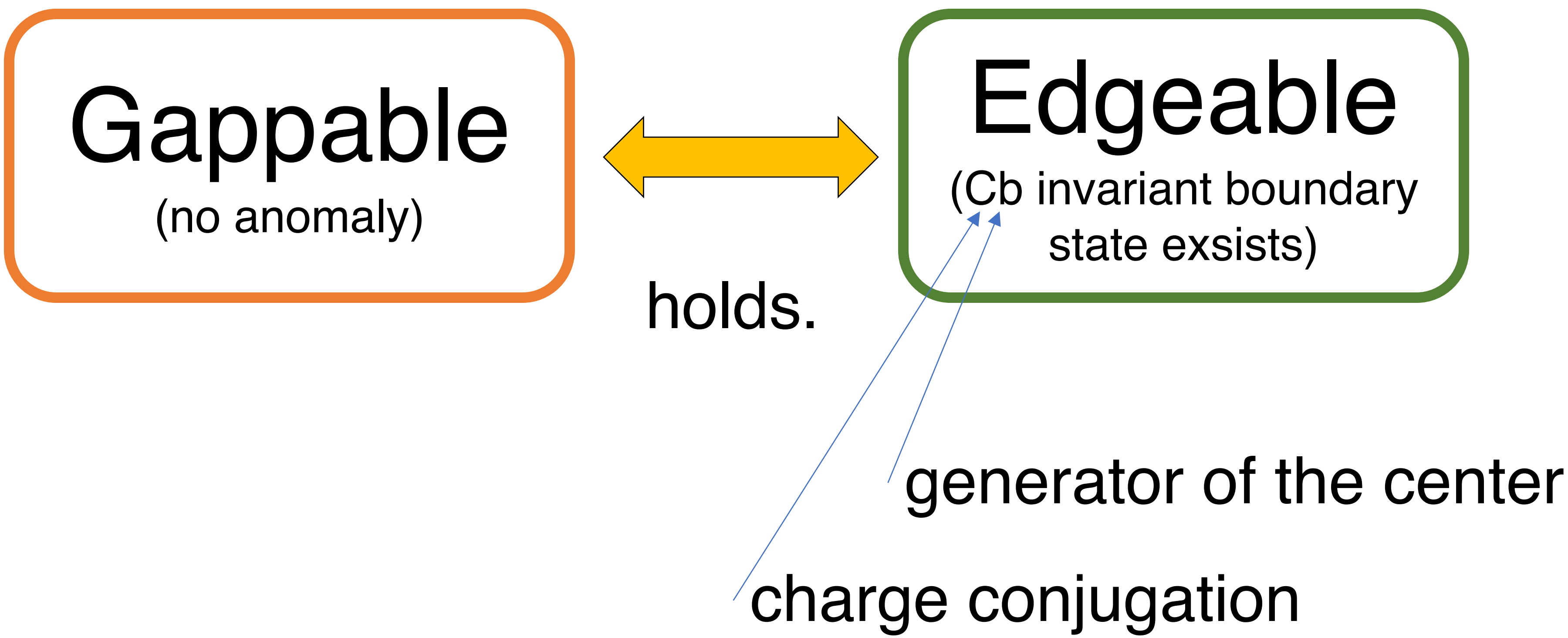
’t Hooft anomaly in 2d CFT

New interesting observation!

2d boundary CFT


Gappable
(no anomaly)
Edgeable
($\mathbb{Z}_N$ invariant boundary state exsists)
fails!

for N>2


Gappable
(no anomaly)
Edgeable
(Cb invariant boundary
state exsists)
holds.
generator of the center
charge conjugation

Comments

- This relation also holds for simple and simply connected compact group with center $\mathbb{Z}_N$

$$A_n, B_n, C_n, D_{2m+1}, E_6, E_7$$

* centers of E_8, F_4, G_2 are trivial

- This relation also holds for subgroup of the center.

- This relation fails for product groups.

Special case of more general contition?

Discussion

Why involving the charge conjugation?

In the anomaly analysis, we consider mixed modular/center anomaly. However in the boundary state analysis, we did not consider modular invariance.

$$C = S^2$$

A part of modular transformation is included

Modular S transformation

Original edgeability condition

Gappable
(no anomaly)

Edgeable
($\mathbb{Z}_N$ invariant boundary state exsists)

An edgeable theory is always gappable, though an gappable theory is not always edgeable.

2d SPT phase ?