Goldstone counting and Inhomogeneous Ground States at Large Global Charge

Masataka Watanabe (Kavli IPMU, U. Tokyo)

23. Mar, 2018

Bootstrap Approach to Conformal Field Theories and Applications

Based on

[1505.01537] with

Simeon Hellerman, Domenico Orlando, Susanne Reffert

[1705.05825] and work *almost done* with

Simeon Hellerman, Nozomu Kobayashi, Shunsuke Maeda

Recouping Theory of Big-J

$O(2)$ model at Big-J

$O(4)$ model at Big-J

Goldstone counting and inhomogeneity

Recouping Theory of Big-J

$O(2)$ model at Big-J

$O(4)$ model at Big-J

Goldstone counting and inhomogeneity

Theory of Big-J

- Strongly-coupled quantum field theories are interesting, but we do not have many tools for them.
- Semiclassical analysis can sometimes give us interesting tools to study them.
- Giving system large charges, J, we can sometimes analyse strongly-coupled theory in the semi-classical regime, where the full Lagrangian is then weakly coupled in units of $1/J$.
- We consider strongly-coupled QFT on the spatial slice S^2 with radius R in this talk. We give charge density ρ to the state and mostly set $R = 1$ by rescaling.

Theory of Big-J

- Let me tell you how Big-J works. You give large dimensionful VEV to the fields associated with the symmetry. Then there is a large hierarchy between UV and IR energies.
- In this case, we can expect $\Lambda_{\mathrm{UV}} = \sqrt{\rho}$ and $\Lambda_{\mathrm{IR}} = 1/R$ by dimensional analysis. This is equivalent to expecting the homogeneity of the ground state of the IR Lagrangian.
- Incidentally, this is only an assumption right at this moment, but I will show that this is all consistent later.
- Now then, when we take the limit of $\rho/R^2 \to \infty$, small ratio of $\Lambda_{\mathrm{IR}}/\Lambda_{\mathrm{UV}}$ should render the theory weakly-coupled!

Recouping Theory of Big-J

$O(2)$ model at Big-J

$O(4)$ model at Big-J

Goldstone counting and inhomogeneity

RG flow of the $O(2)$ model at large charge

- Let me start with the simplest model of all to be analysed at large charge, J.
- We start with the following UV action using a complex field $\phi \equiv a \times e^{i\chi}$,

$$\mathcal{L}_{\mathrm{UV}} = -\partial\phi\partial\bar{\phi} - m^2|\phi|^2 - g^2|\phi|^4$$

  This system has the $O(2)$ symmetry.
- By fine-tuning the value of m^2, we get the conformal Wilson-Fischer fixed point in the IR.
- We now break the $O(2)$ symmetry spontaneously by giving $a = |\phi|$ a large dimensionful VEV. Then the RG flow becomes classical at $\Lambda_0 = |\phi| \propto \sqrt{\rho}$. Here χ becomes the GB of the theory from which we construct the effective Lagrangian.

RG flow of the $O(2)$ model at large charge

- Now, scale everything far below Λ_0.
- Then the IR Lagrangian at large dimensionful VEV, a, has to be classically conformally invariant, at leading order in large ρ. This is because quantum fluctuations are suppressed in powers of Λ/Λ_0.
- If you need to write the IR quantum Lagrangian, you can do so after writing down the classical Lagrangian, by setting the RG evolution to be vanishing at the fixed point.
- All our tasks, therefore, has reduced to writing down the classically conformally invariant operator according to J-scaling.

RG flow of the $O(2)$ model at large charge

- I will now give you the leading order $O(2)$ invariant IR Lagrangian, which is also classically conformally invariant,

$$\mathcal{L}_{\mathrm{IR}} = -\frac{1}{2}(\partial a)^2 - \frac{1}{2}\kappa a^2(\partial\chi)^2 - \frac{h^2}{12}a^6 + \circ\circ\circ$$

- You can also integrate out the a field, whose mass is of order $\sqrt{\rho} \gg 1$, resulting in

$$\mathcal{L}_{\mathrm{IR}} = b_\chi |\partial\chi|^3 + \circ\circ\circ$$

  where $b_\chi = \frac{\sqrt{2}\kappa^{3/2}}{3h}$.
- By virtue of Noether theorem, we have $\rho = 3b_\chi|\partial\chi|^2$ and $J = 4\pi R^2\rho$

Sorting operators at Big-J

- Now it's time to list operators in the effective action.
- After integrating out the a field, you are free to put its mass to the denominator of effective operators, which is, in this case, $|\partial\chi|$.
- In order to do this, we have to know the EOM and its classical solution for χ. Because the lowest energy solution of the EOM of χ is homogeneous (either by direct computation or by an argument given later), we can solve it for $\chi = \omega t$.
- Here, we have $\omega \propto \sqrt{J}$.

Sorting operators at Big-J

- We now have all the necessary tools. The scaling of $|\partial\chi|$ is as follows,

$$|\partial\chi| \propto \sqrt{J}$$

- You are also allowed to put operators like $\partial^k|\partial\chi|$. This does not scale like $\sqrt{J}$, because using the solution to the leading EOM, $\partial^k|\partial\chi|$ is vanishing, meaning it should only come from the fluctuation part of χ, so that

$$\partial^k|\partial\chi| \propto J^{-1/4}$$

- One more rule to remember is that you can use the EOM for the leading action. That is, $\partial_\mu\left(|\partial\chi|\partial^\mu\chi\right) = 0$ can be used to eliminate operators (they can be traded for something of the lower J scaling).

Sorting operators at Big-J

- I am only giving you the result of the listing here.
- Order $J^{3/2}$

$$|\partial\chi|^3$$

- Order $J^{1/2}$

$$\mathtt{Ric}_3|\partial\chi| + \frac{2(\partial|\partial\chi|)^2}{|\partial\chi|} = O(J^{1/2}) + O(J^{-1})$$

- Note that the Weyl completing term is lower in scaling than the first term.
- These are actually the only operators that appear at or above $O(J^0)$!

Universal IR Lagrangian at Big-J

- Now we know everything to describe the IR Lagrangian at large charge.
- This Lagrangian is universal when the theory is in the WF fixed point,

$$\mathcal{L}_{\mathrm{IR}} = c_{3/2}|\partial\chi|^3 + c_{1/2}\mathtt{Ric}_3|\partial\chi| + O(J^{-1/4})$$

- We are now in a position to calculate the operator dimension at large charge.

Operator dimension at large charge

- Let us calculate the operator dimension at large charge.
- The classical piece is given by just

$$k_{3/2}J^{3/2} + k_{1/2}J^{1/2}$$

- Notice this leading J-scaling, $3/2$. This is consistent with the analysis of Nakayama that the upper bound for the scaling of the lowest operator dimension, which was $J^{1.6}$.

Operator dimension at large charge

- The leading quantum piece of the operator dimension is given by the one-loop vacuum contribution from the $O(J^{3/2})$ piece.
- This is calculated by separating $\chi = \chi_0 + |\partial\chi|^{-1/2}\hat{\chi}$ into VEV and (normalised) fluctuations,

$$\mathcal{L}_{\text{leading}}/b_\chi = |\partial\chi_0^3| + \frac{3}{2}\hat{\chi}\left(\partial_t^2 + \frac{1}{2}\triangle_{S^2}\right)\hat{\chi} + \circ\circ\circ$$

- Now, the contribution is the standard Coleman-Weinberg formula on the sphere.

$$E_0 = \sum_{\ell=0}^{\ell=\infty}(2\ell+1)\sqrt{\ell(\ell+1)}$$

but now you have to regulate and renormalise the sum, say using zeta-function regularization.

Operator dimension at large charge

- It is actually a bit tricky because you can sometimes naively use zeta-function regularization to get a wrong result.
- But there is certainly a way to correctly compute this using zeta-functions, which gives

$$E_0 = -0.094$$

- There were no operators available at $O(J^0)$, so this is the only universal contribution to the dimension at this order. We therefore get

$$k_{3/2} J^{3/2} + k_{1/2} J^{1/2} - 0.094$$

Comments

- Note that the speed of the GB is $1/\sqrt{2}$ times the speed of light. This follows directly from conformal symmetry.
- All the states whose dimensions are $O(1)$ above Big-J ground state can also be written down – at spin ℓ, the energy of the excited state increases by $\Delta E(\ell) = \sqrt{\ell(\ell+1)/2}$
- Therefore the spin $\ell = 1$ state is just the descendent of the ground state. Others are just new primaries.

Comments

- $\mathcal{N} = 2$, $D = 3$ SUSY theory with superpotential $W = \Phi^3$ with large R-charge i s in the same universality class too. Notice the large discrepancy from the BPS bound.
- In this case, there is no moduli space of vacua, and you get an EFT with a fermion, and the GB encountered also in the non-SUSY case.
- Now, superconformal symmetry does *not* fix the mass of the fermion to zero – rather, it fixes the mass at $O(\sqrt{J})$.
- Then you can integrate the fermions out, and you get the same EFT as in the non-SUSY case.
- When there is a moduli space of vacua, the story is totally different. Wait for Simeon's talk, it's going to be fun....

Monte Carlo numerics

- Now you get a nontrivial universal number, you should check this numerically too.
- You can use Monte Carlo simulations to verify the sum rule for the operator dimension.
- The result, done by Banerjee, Chandrasekharan, and Orlando (one of the authors of the original $O(2)$ paper!), suggests the remarkable fit even up to $J \sim 1$!

Monte Carlo numerics

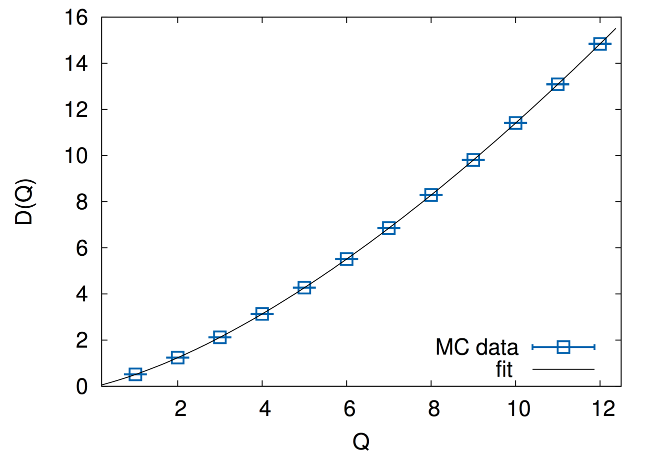

Figure 1: Domenico said on the unit sphere the fitted number is $k_{3/2} = 1.195/\sqrt{4\pi}$ and $k_{1/2} = 0.075\sqrt{4\pi}$. $E_0 \approx -0.094$ comes out right.

Bootstrap at large charge?

- It is clear from the construction that the straightforward numerical bootstrap program slows down at larger charges.
- Analytic bootstrap should be interesting, but not exactly parallel with analytic bootstrap at large spin. The theory is not going to be (generalised) free at leading order (the dimension scales as $J^{3/2}$ instead of J).
- Recently Jafferis, Mukhametzhanov and Zhiboedov have put out a nice paper studying Big-J bootstrap.
- It shows that the EFT we derived is the only possible EFT when there is only one Regge trajectory ($O(1)$ excitations with spin more than 2, c.f., [Caron-Huot]).

Recouping Theory of Big-J

$O(2)$ model at Big-J

$O(4)$ model at Big-J

Goldstone counting and inhomogeneity

Inhomogeneity of the $O(4)$ large charge grouns states

- Now finally we have all the tools to study the cases where the symmetry algebra has more than one Cartans.
- Why is this important? This is because we used homogeneity to say that Λ_{IR} is small compared to Λ_{UV}. This was a key fact establishing the classical scale invariant EFT.
- In the first part of the talk I just set this ansatz and later proved homogeneity by explicitly computing the classical ground state configuration.

Inhomogeneity of the $O(4)$ large charge ground states

- In 2017, Alvarez-Gaume, Loukas, Orlando and Reffert proved that if we persist in having the homogeneous ground state configuration at large charge in the $O(N)$ model, you can only consider cases where you excite only one Cartan of the symmetry group.
- You can intuitively observe this fact even when you consider free $SU(2)$ bosonic Lagrangian – In the language of the $O(4)$ model, when you excite two Cartans by the same amount
- You can then see what operator describes the large charge ground state with spin 0 – the option is just $\left|\epsilon_{ab}q^a\partial_\mu q^b\right|^n$, where q transform as a doublet in $SU(2)$. Classical configuration cannot be homogeneous on the spatial slices then.

Inhomogeneity of the $O(4)$ large charge ground states

- So inhomogeneity really is a problem when you want to study the large charge expansion of the $O(4)$ theory with generic ρ_1 and ρ_2, eigenvalues of the charge density matrix.
- Why? If the configuration has the instability towards inhomogeneity in the scale of the charge density itself, $1/\sqrt{\rho} = 1/\Lambda_{\mathrm{UV}}$, the EFT is definitely going to break down.
- On the other hand, if it's leaned towards the IR side, the EFT is still applicable even for generic charge density ratios, because you still have the large hierarchy between UV and IR.

Looking for the $O(4)$ large charge ground states

- Let us now look for the large charge ground state configuration of the theory.
- It just the same as in the $O(2)$ case. You decompose the doublet Q appearing in the UV Lagrangian as $Q = A \times q$ and give VEV to A.
- Here we require $q^\dagger q = 1$, so along with the helical ansatz, let's parametrise the helical solution as

$$q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} e^{i\omega_1 t} \sin(p(x)) \\ e^{i\omega_2 t} \cos(p(x)) \end{pmatrix}$$

- The leading IR Lagrangian is

$$\mathcal{L}_{\mathrm{IR}} = b_q \left(\partial q^\dagger \partial q \right)^{3/2}$$

as you might have already rightly guessed from the $O(2)$ case.

Looking for the $O(4)$ large charge ground states

- This time, for simplicity, let us put the system on $T^2 \times \mathbb{R}$, where the volume of this torus is $\mathcal{V}$.
- I am still working on the $S^2 \times \mathbb{R}$ case to compute the operator dimensions....
- The charge densities and the energy density associated with a specific configuration of q is

$$
\begin{aligned}
\rho_1 &= \frac{8b_q}{3\mathcal{V}} \int dx^i \, \omega_1 \sqrt{-p'(x)^2 + V(p(x))} \sin^2(p(x)) \\
\rho_2 &= \frac{8b_q}{3\mathcal{V}} \int dx^i \, \omega_2 \sqrt{-p'(x)^2 + V(p(x))} \cos^2(p(x)), \\
\mathcal{E} &= \frac{b_q}{\mathcal{V}} \int d^2x \sqrt{-p'(x)^2 + V(p(x))} (p'(x)^2 + 2V(p(x)))
\end{aligned}
$$

where $V(p) = \omega_2^2 + \left(\omega_1^2 - \omega_2^2\right) \sin^2(p)$.

Looking for the $O(4)$ large charge ground states

- Let us solve the EOM to actually compute the lowest classical configuration. First assume that the configuration is only inhomogeneous in one direction, say, x-axis.
- I will come back to the consistency of this assumption later, so let's just assume this.
- Now the EOM is just $\partial_\mu T_{xx} = 0$, where this stress tensor component is given by

$$T_{xx} = b_q \sqrt{-p'(x)^2 + V(p(x))} \left(2p'(x)^2 + V(p(x))\right)$$

- Let us therefore set $\kappa = {b_q}^{-1/3} {T_{xx}}^{1/3}$, where κ is some constant with mass dimension 1.

Looking for the $O(4)$ large charge ground states

- Now the EOM just reduces to the following equation,

$$-\frac{\kappa^6}{4} = \left(p'(x)^2 - V(p(x))\right)\left(p'(x)^2 + \frac{V(p(x))}{2}\right)^2$$

- Let us use the assumption about the almost homogeneity of the ground state configuration, (which is shown to be consistent later).
- One observation is important here in disentangling the complication of this EOM. That is, there are two arbitrary scales in this story. One is $\omega_2 \sim \omega_1$, which controls the charge density itself. The other is the difference of them, $\omega_1 - \omega_2 (> 0)$. This controls the inhomogeneity, or the amplitude of the solution $p(x)$ of the EOM.

Looking for the $O(4)$ large charge ground states

- The alternative view is that when $\omega_1 = \omega_2$, the solution just should come down to a homogeneous one, for any values of ω. So $\omega_1 - \omega_2$ and $\omega_2 \sim \omega_1$ generically must scale completely differently.
- So let's take $\omega_{1,2} \sim O(\sqrt{J})$ and $p'(x) \sim \omega_1 - \omega_2 \sim O(1)$, using which the EOM above can be simplified dramatically,

$$(p'(x))^2 = 2\omega_2(\omega_1 - \omega_2)\left(\sin^2(p_0) - \sin^2(p(x))\right)$$

where p_0 is the maximal value of $p(x)$.

$O(4)$ large charge ground states

- This differential equation is actually the same as the EOM for a classical pendulum problem in a uniform gravitational field. And, we know the answer to this type of differential equation very well.
- The solution to the EOM writes

$$\frac{\sin(p(x))}{\sin(p_0)} = \mathrm{sn}\left(\frac{x}{\ell}; \sin(p_0)\right)$$

where the period of the solution, $4L$, is related with ℓ by $L = \ell F\left(\frac{\pi}{2}; \sin(p_0)\right)$, and also $\Delta\left(\frac{\pi}{2}; \sin(p_0)\right) = \frac{\rho_1}{\rho_1+\rho_2}$

- The definitions follow in a moment.

$O(4)$ large charge ground states

- From the previous slide;

$$\frac{\sin(p(x))}{\sin(p_0)} = \text{sn}\left(\frac{x}{\ell}; \sin(p_0)\right)$$

$$L = \ell F\left(\frac{\pi}{2}; \sin(p_0)\right), \quad \Delta\left(\frac{\pi}{2}; \sin(p_0)\right) = \frac{\rho_1}{\rho_1 + \rho_2}$$

- Definitions;

$$\Delta(p; k) \equiv \frac{F(p; k) - E(p; k)}{F(p; k)}$$

$$F(x; k) \equiv \int_0^p \frac{d\hat{p}}{\sqrt{1 - k^2 \sin^2(\hat{p})}}, \; E(p; k) \equiv \int_0^p d\hat{p}\sqrt{1 - k^2 \sin^2(\hat{p})}$$

Behaviour for Δ

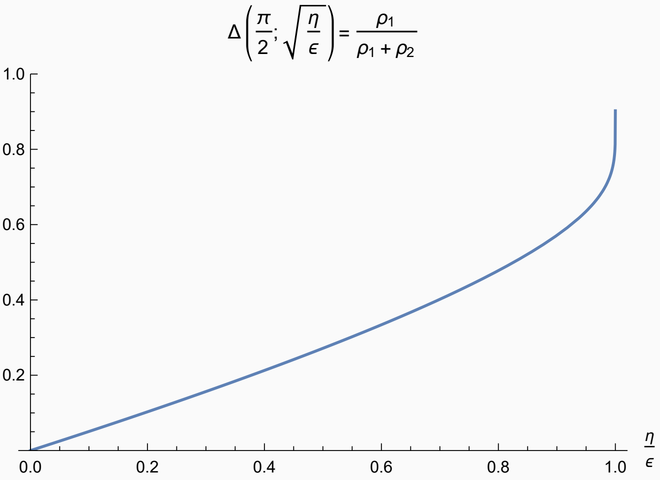

Figure 2: At $\rho_1 = \rho_2$, we have $\sin^2(p_0) = 0.826\ldots$.

Ground state has preference for homogeneity

- You can now use this solution to the EOM to compute the energy associated with each solution with different periods, or equivalently, different ℓ.
- The explicit result is complicated, but like this,

$$\mathcal{E} = \frac{3\sqrt{3}}{8\sqrt{2b_q}}(\rho_1 + \rho_2)^{3/2} \times \left(1 + \frac{A}{\ell^2}\right)$$

  where

$$A \equiv \frac{2b_q}{3(\rho_1 + \rho_2)}\left(\sin^2(p_0) + \frac{\rho_1}{\rho_1 + \rho_2}\right) > 0$$

- So making the period larger is energetically favourable! The period for the ground state configuration becomes the (longer) period of the torus!

Comments

- This is all about in the regime where $\rho_1 < \rho_2$. After ρ_2 has become bigger than ρ_1, just replace them with each other. This means there is a first order phase transition at $\rho_1 = \rho_2$.
- There are "winded" solutions that I didn't mention. The quotations are because there are no topological windings on the torus, but nevertheless they appear as soft modes on top of the ground state solution.

Observables

- As in the case of the $O(2)$ model, you can compute the leading energy of the large charge ground state.
- We put the theory on a torus with periods $\ell_2 < \ell_1$. When $\rho_1 = \rho_2 \equiv \rho/2 = J/(\ell_1 \ell_2)$, the energy of the ground state at charge J is calculated classically to be

$$E = \frac{3\sqrt{3} J^{3/2}}{8\sqrt{2 b_q \ell_1 \ell_2}} \times \left(1 + \frac{114.2 \times b_q \ell_2}{3 J \ell_1}\right)$$

- The result is correct up to $J^{1/2}$, because $\mathtt{Ric}_3 |\partial q|$, the only term of order $O(J^{1/2})$ is absent on the torus, which is flat.

Observables

- Two-point functions of the field $q = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$ can also be calculated.
- At leading order there should only be classical contributions, so you integrate on the torus over the homogeneous direction.
- This procedure gives (I will denote $\sigma \equiv \sin(p_0)$ for simplicity)

$$\langle q_1^*(0) q_1(x) \rangle \propto \int dy_1 dy_2 \, q_1^*(y_1, y_2) q_1(x_1 + y_1, x_2 + y_2)$$

$$\propto \sigma^2 \int_0^{4L} dy_1 \, \mathrm{sn}\left(\frac{y_1}{\ell}; \sigma\right) \mathrm{sn}\left(\frac{x_1 + y_1}{\ell}; \sigma\right)$$

- $\langle q_2^*(0) q_2(x) \rangle$ can be calculated too. Replace sn with dn to get the result.

Behaviour of the two-point functions

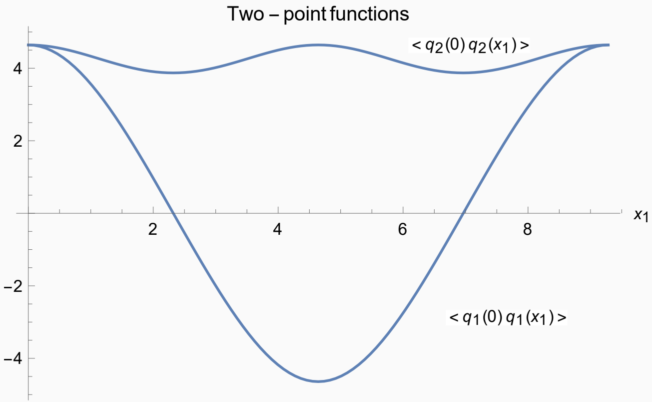

Figure 3: The graph is when $\rho_1 = \rho_2$. Negative value at antipodal point seems interesting as a direct consequence on observables of inhomogeneity.

Monte-Carlo numerics of the $O(4)$ model

- We want somebody to check these statements about observables using Monte-Carlo simulations.
- Anyone ○ ○ ○ ○ ○ ○ ○ ○ ○ ???
- I think however we should caution anyone who are interested. Because of the inevitable soft modes present on the torus, the temperature might have to be really low to see even the leading order scaling....
- Again, anyone ○ ○ ○ ○ ○ ○ ○ ○ ○ ???

Recouping Theory of Big-J

$O(2)$ model at Big-J

$O(4)$ model at Big-J

Goldstone counting and inhomogeneity

Inhomogeneity of the ground states and Goldstone counting

- We can actually prove several cute facts about inhomogeneity using Goldstone counting.
- The symmetry is first explicitly broken by adding chemical potential, which then is spontaneously broken by the solution to the EOM itself.
- For example, we can actually prove that the ground state for the $O(2)$ model at large charge must be homogeneous.

Homogeneity: $O(2)$ at large charge

- We can prove the homogeneity of the ground state at large charge of the $O(2)$ model even without resorting to a complicated argument like explicit breaking or anything.
- Assume otherwise; then in the EFT there are two or more GBs, namely, the axion and the GB(s) from the translational symmetry breaking.
- But you started from a theory of a complex scaler, whose dof is two.
- Then the EFT should contain less than two dof, which would contradict with the presence of the translational GB(s).

Inhomogeneity: $O(4)$ at large charge

- We prove that the ground state configuration is still homogeneous if you only excite one of the two Cartans, i.e., $\rho_1 = \rho$ and $\rho_2 = 0$.
- One easy way to see the breaking pattern caused by this constraint is to think of what transformations preserve the condition $\rho_2 = 0$, using

$$
\begin{aligned}
-\frac{2ib_q}{3}\int dx^i \sqrt{\mathcal{L}_0}\left[q^\dagger \partial_t q - \mathrm{c.c.}\right]\Big/\mathcal{V} &= \rho_1 + \rho_2 \quad (1) \\
-\frac{2ib_q}{3}\int dx^i \sqrt{\mathcal{L}_0}\left[q^\dagger \sigma^3 \partial_t q - \mathrm{c.c.}\right]\Big/\mathcal{V} &= \rho_1 - \rho_2, \quad (2)
\end{aligned}
$$

- The condition, then, becomes $q_1 = q_2$. The symmetry actions that preserve this condition is just either overall phase rotation or the elements of diagonal SU(2).

Inhomogeneity: $O(4)$ at large charge

- So the explicit breaking from the chemical potential becomes

$$\mathrm{SU}(2) \times \mathrm{SU}(2) \xrightarrow{\text{explicit breaking}} \mathrm{U}(1) \times \mathrm{SU}(2)$$

- Then we solve the EOM to find $\omega_1 = \omega_2$. Then one of the vacuum configuration becomes $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ so the spontaneous breaking pattern is

$$\mathrm{U}(1) \times \mathrm{SU}(2) \xrightarrow{\text{spontaneous breaking}} \mathrm{U}(1)$$

assuming the homogeneity of the configuration.

Inhomogeneity: $O(4)$ at large charge

- Because $\dim(\mathrm{U}(1) \times \mathrm{SU}(2)/\mathrm{U}(1)) = 3$, if the translational symmetry is further broken, there are four or more Goldstone modes in the system.
- But there are only three light real fields in the spectrum, so this cannot happen.
- We now have proven that the large charge ground state configuration where only one Cartan is excited is homogeneous!

Inhomogeneity: $O(4)$ at large charge

- We now prove that the ground state configuration becomes inhomogeneous only in one direction even if you generically excite two Cartans, i.e., $\rho_1,\ \rho_2 \neq 0$
- The explicit breaking from the chemical potential is

$$\mathrm{SU}(2) \times \mathrm{SU}(2) \xrightarrow{\text{explicit breaking}} \mathrm{U}(1) \times \mathrm{U}(1)$$

- Then we solve the EOM, but you already know from Gaume et.al., that the configuration cannot be homogeneous.
- Assume the inhomogeneity in only one direction. The spontaneous breaking pattern is then

$$\mathrm{U}(1) \times \mathrm{U}(1) \times \{\text{translation}\} \xrightarrow{\text{spontaneous breaking}} \{\text{trivial}\}$$

Inhomogeneity: $O(4)$ at large charge

- The dimension of the coset is, again, 3. So now if the translational symmetry is further broken, there are four or more Goldstone modes in the system
- But there are only three light real fields in the spectrum, so this cannot happen.
- So we now have established that the large charge ground state configuration where two Cartans are excited is only homogeneous in one direction!

Take-home messages

- Large charge expansion is interesting, and makes it possible to analyse a strongly-coupled theory like a weakly-coupled one.
- This analysis is largely dependent on the large separation of UV and IR energies.
- Sometimes the ground state configuration at large charge is inhomogeneous, but in our examples the inhomogeneity is at the scale of the underlying geometry itself, and the EFT is still applicable.
- You can directly extract interesting information from the inhomogeneity, such as two-point functions.
- The inhomogeneity is the spontaneous breaking of the translation symmetry, and it is possible to analyse the breaking pattern by matching the number of Goldstones with the available light modes in EFT.