Random tensors and the SYK model : Gaussian Universality of the Random Coupling

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The Sachdev-Ye-Kitaev (SYK) model

Quantum mechanical model ($1+0$ dimensions) with N Majorana fermions $\psi_i(t)$ with random degree q (even) interaction

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- Generalisation to a model with flavours (Gross, Rosenhaus)

$$H = \mathrm{i}^{q/2} J_{i_{1,1} \dots i_{1,q_1}, \dots, i_{f,1} \dots i_{1,q_f}} \prod_{a=1,\dots,f} \psi^a_{i_{a,1}} \cdots \psi^a_{i_{a,q_a}}$$

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Model in condensed matter (Sachdev, Ye, Georges, Parcollet) and AdS_2/CFT_1 at large N (Kitaev, Maldacena, Stanford, Polchinski, Rosenhaus, ...)

Lagrangian formulation and Feynman rules

Path integral formulation with Grassmann variables

$$L = \int dt \, \left\{ \psi_i \partial_t \psi_i - \mathrm{i}^{q/2} J_{i_1 \ldots i_q} \psi_{i_1} \cdots \psi_{i_q} \right\}$$

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- Free propagator

$$i \,\text{———}\, j \quad \rightarrow \quad G_0(t,t') = \langle T \psi_i(t) \psi_j(t') \rangle = \frac{\delta_{ij}}{2} \mathrm{sgn}(t - t')$$

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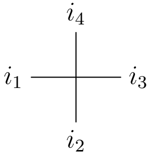
- Interaction vertex (for $q = 4$)



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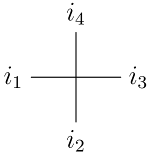

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- Sum over internal indices

Large N limit

Quenched disorder : evaluate connected graphs at fixed J and then average over J :
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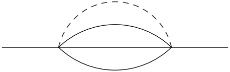
-  $\rightarrow \quad \frac{N^3}{N^3} = 1$

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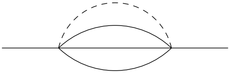
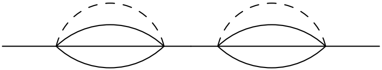
-  $\rightarrow \quad \frac{N^3}{N^3} = 1$
-  $\rightarrow \quad \frac{N^3 \times N^3}{N^3 \times N^3} = 1$

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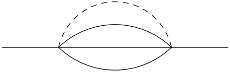
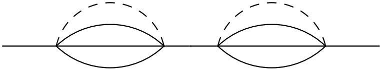
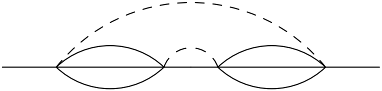
-  $\rightarrow \quad \frac{N^3}{N^3} = 1$
-  $\rightarrow \quad \frac{N^3 \times N^3}{N^3 \times N^3} = 1$
-  $\rightarrow \quad \frac{N \times N^3}{N^3 \times N^3} = \frac{1}{N^2}$

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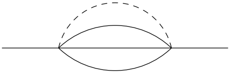
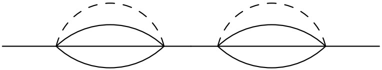
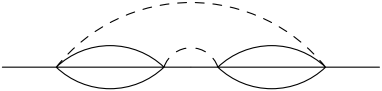
-  $\rightarrow \quad \frac{N^3}{N^3} = 1$
-  $\rightarrow \quad \frac{N^3 \times N^3}{N^3 \times N^3} = 1$
-  $\rightarrow \quad \frac{N \times N^3}{N^3 \times N^3} = \frac{1}{N^2}$

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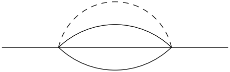
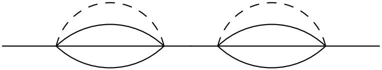
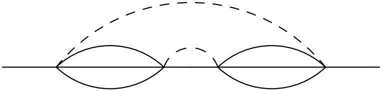
-  $\rightarrow \quad \frac{N^3}{N^3} = 1$
-  $\rightarrow \quad \frac{N^3 \times N^3}{N^3 \times N^3} = 1$
-  $\rightarrow \quad \frac{N \times N^3}{N^3 \times N^3} = \frac{1}{N^2}$

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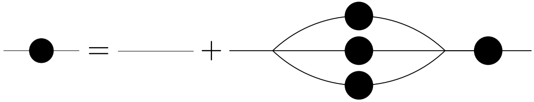
Similar results for the 4 point function summing ladder (geometric series) : expression in terms of hypergeometric function (nearly conformal)

Schwinger-Dyson equations

Graphical recursive construction at large N (only melonic graphs)



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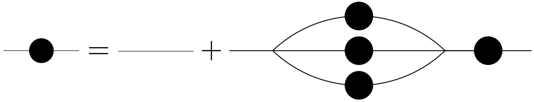

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Conformal solution at large N in the IR ($N \gg J|t-t'| \gg 1$), assuming $G \ll G_0$

$$\boxed{G_*(t,t') \propto \frac{\mathrm{sgn}(t-t')}{|t-t'|^{2\Delta}}}$$

with Δ anomalous dimension of ψ in the IR (trivial in the UV)

$$0 = 0 + 2 - (q-1) \times 2\Delta - 2\Delta \quad \Rightarrow \quad \Delta = \frac{1}{q}$$

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$$\boxed{\langle \log \mathcal{Z} \rangle_J = \int [DG][D\Sigma] \exp N S_{\text{eff}}[G,\Sigma]}$$

with O(N) invariant effective action for bilocals ($\star$ = convolution)

$$S_{\text{eff}}[G,\Sigma] = \frac{1}{2}\log\det\left(\partial - \Sigma\right)_\star + \frac{1}{2}\iint dt dt' \Sigma(t,t')G(t,t') + \sigma^2 G^q(t,t')$$

Large N limit and saddle point approximation

Path integral with effective action $\int [DG][D\Sigma] \exp N S_{\text{eff}}[G,\Sigma]$

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Saddle point $\rightarrow$ Schwinger-Dyson equation

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In the IR, we may simply drop $\delta(t-t')\partial_t$

$$\delta(t-t') = \int du\, G^{q-1}(t,u)G(u,t') \Leftrightarrow -1 = G^{q-1} \star G$$

Reparametrisation invariance

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In the IR (drop ∂_t), S_{eff} invariant under reparametrisation $t \to f(t)$ with $\Delta = \frac{1}{q}$ anomalous dimension

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$$G(t,t') \to \left|\frac{df}{dt}\right|^{\Delta}\left|\frac{df}{dt'}\right|^{\Delta} G(f(t),f(t'))$$

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$$\Sigma(t,t') \to \left|\frac{df}{dt}\right|^{1-\Delta} \left|\frac{df}{dt'}\right|^{1-\Delta} \Sigma(f(t),f(t'))$$

$G_*(t,t') \propto \frac{\text{sgn}(t-t')}{|t-t'|^{2\Delta}}$ only $\text{SL}_2(\mathbb{R})$ invariant $t \to f(t) = \frac{at+b}{ct+d}$

Spontaneous (and explicit by ∂_t) breaking of reparametrisation invariance $\to$ Schwarzian action for pseudo Goldstone modes (Kitaev, Witten, Stanford)

Non Gaussian averages and random tensors

Non Gaussian disorder ($V_N(J)$ perturbation) $\to J_{i_1\cdots q}$ random tensor

$$\langle \ldots \rangle_J = \frac{\int dJ \; \ldots \; \exp - \left\{ \frac{N^{q-1}}{2\sigma^2} J^2 + V_N(J) \right\}}{\int dJ \exp - \left\{ \frac{N^{q-1}}{2\sigma^2} J^2 + V_N(J) \right\}}$$

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Average of interaction term expressed as (after shift of integration $J \to J + K$)

$$\left\langle \exp \left\{ J_{i_1 \ldots i_q} \sum_r \int dt \psi^r_{i_1}(t) \cdots \psi^r_{i_q}(t) \right\} \right\rangle_J = \exp \left\{ \frac{N^{q-1}}{2\sigma^2} K^2 - V'_N(K) \right\}$$

with normalised background effective action $V'_N(K)$ for K

$$K_{i_1 \ldots i_q} = \frac{\sigma^2}{N^{q-1}} \sum_r \int dt \, \psi^r_{i_1}(t) \cdots \psi^r_{i_q}(t)$$

$$V'_N(K) = -\log \int dJ \exp - \left\{ \frac{N^{q-1}}{2\sigma^2} J^2 + V_N(K + J) \right\}$$

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- Generating function for higher dimensional random geometries

$$\int_{\text{rank } D \text{ tensors}} dT \exp -S_N(T) = \sum_{\substack{D\text{-valent Feynman graphs} \\ \Leftrightarrow D\text{-dimensional triangulations}}} W(\text{triangulation})$$

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- Reformulation of the SYK model without quenched disorder (Witten, Gurau, Klebanov, Tarnopolski, ...) : $\psi_{ijk\dots}(t)$ fundamental degrees of freedom instead of $\psi_i(t)$ (here the coupling is the random tensor instead)

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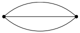
- Dipole 

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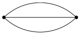
- Dipole 

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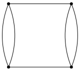
- Quartic melon 

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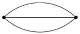
- Dipole 

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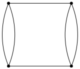
- Quartic melon 

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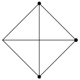
- Tetrahedron 

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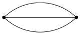
- Dipole 

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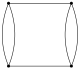
- Quartic melon 

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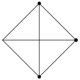
- Tetrahedron 

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For non symmetric complex models : black and white vertices $(T, \overline{T})$ and label $1, 2, .., D$ edges at each vertex (place of index)

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Find suitable exponents δ_Γ in such a way that

$$V'_N(T') = -\log \int dT \, \exp - \left\{ \frac{N^{q-1}}{2\sigma^2} T^2 + V_N(T' + T) \right\}$$

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- Other possibly relevant models : Matrix-Tensors, Necklaces, Series-Parallel,...

Large N limit from a Polchinski like equation

Effective action for a complex non symmetric rank q tensor

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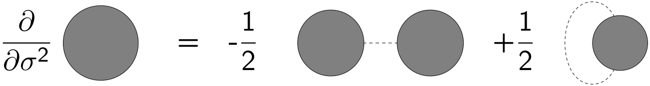

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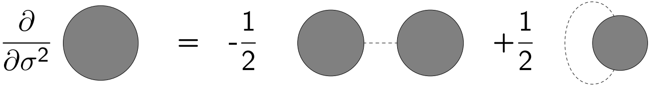

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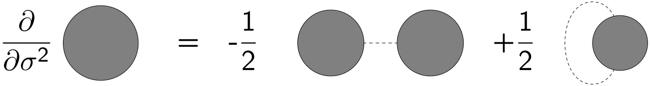

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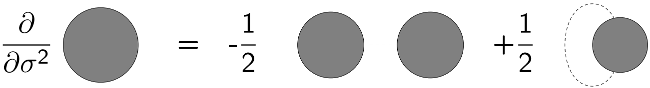

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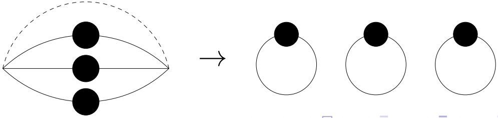
("melonic" graphs) 

Gaussian universality for bilocal fields

Inserting the bilocal field $G(t,t') = \frac{1}{N}\sum_i \psi_i^r(t)\psi_i(t')$, the scaling is $N^{\delta_\Gamma - 1 - v(q-1)+e} = N^{\delta_\Gamma - 1 - v(q/2-1)}$ ($2e = qv$ for q-valent graphs)

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- Similar to previous results on p-spin glasses (Bonzom, Gurau, Smerlak)

Details for a complex coloured model

Model with q complex field $\to$ random coupling = complex non symmetric tensor (simplest combinatorics)

$$H = \mathrm{i}^{\frac{q}{2}} \sum_{i_1,\ldots,i_q} \bar{J}_{i_1,\ldots,i_q} \psi^1_{i_1} \cdots \psi^q_{i_q} + \mathrm{i}^{\frac{q}{2}} \sum_{i_1,\ldots,i_q} J_{i_1,\ldots,i_q} \bar{\psi}^1_{i_1} \cdots \bar{\psi}^q_{i_q}$$

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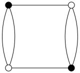

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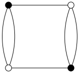

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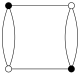

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Leading order non Gaussian correction to the effective action

$$\propto \frac{1}{N^2} \int dt_1 dt_2 dt_3 dt_4 G^3(t_1, t_2) G(t_1, t_3) G(t_2, t_4) G^3(t_3, t_4)$$

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Gaussian universality also holds for i.i.d. couplings with mean 0 and moments $\langle \mathcal{J}^k \rangle \ll N^{-\frac{q-1}{k}}$ ($\sim N^{-\frac{q-1}{2}}$ in the quadratic case) :

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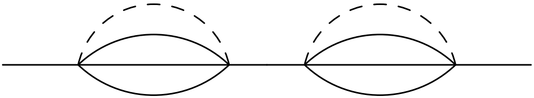

Gaussian universality for independent couplings

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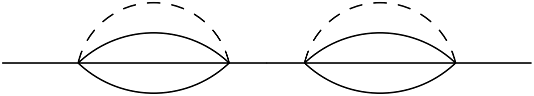

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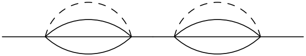

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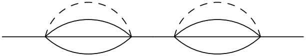

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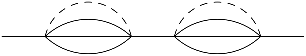

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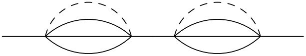

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- Gaussian universality : average over non Gaussian disorder equivalent to a Gaussian one