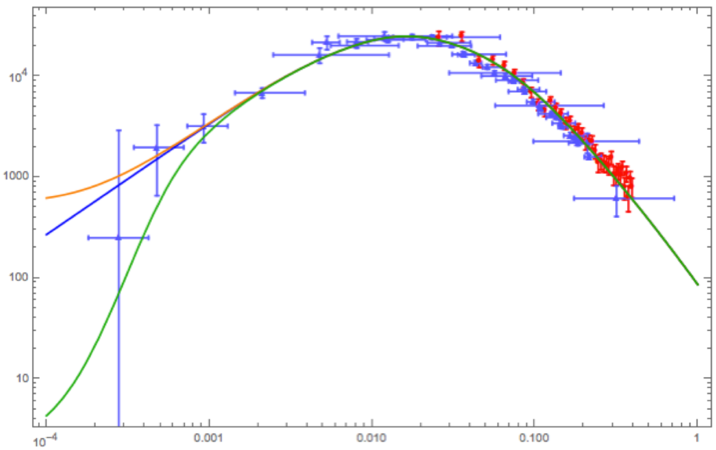
Gravitational Fluctuations as an alternative to Inflation

– Testing QG in Cosmology –



Sunny Yu

University of California, Irvine

lhyu1@uci.edu

[with H.W. Hamber]

QUANTUM AND GRAVITY IN OKINAWA

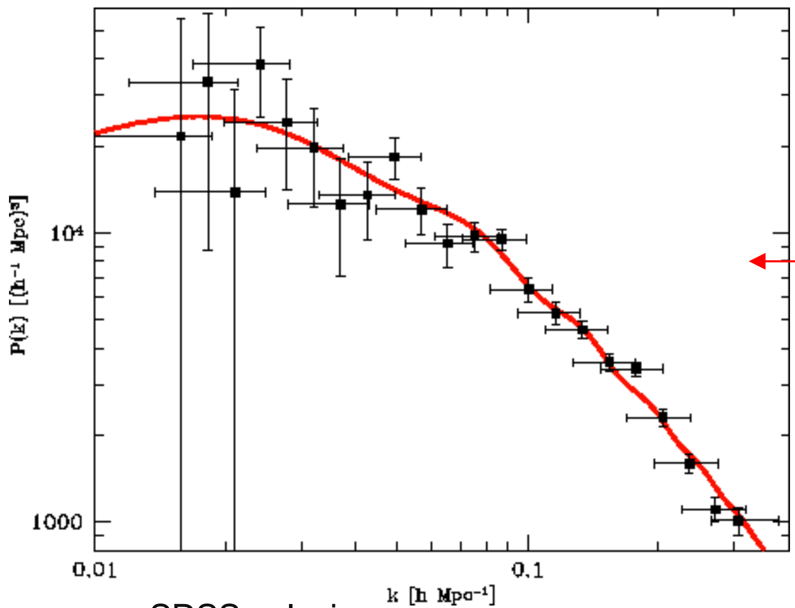
OIST

OIST / 2019-07-23

General reference:

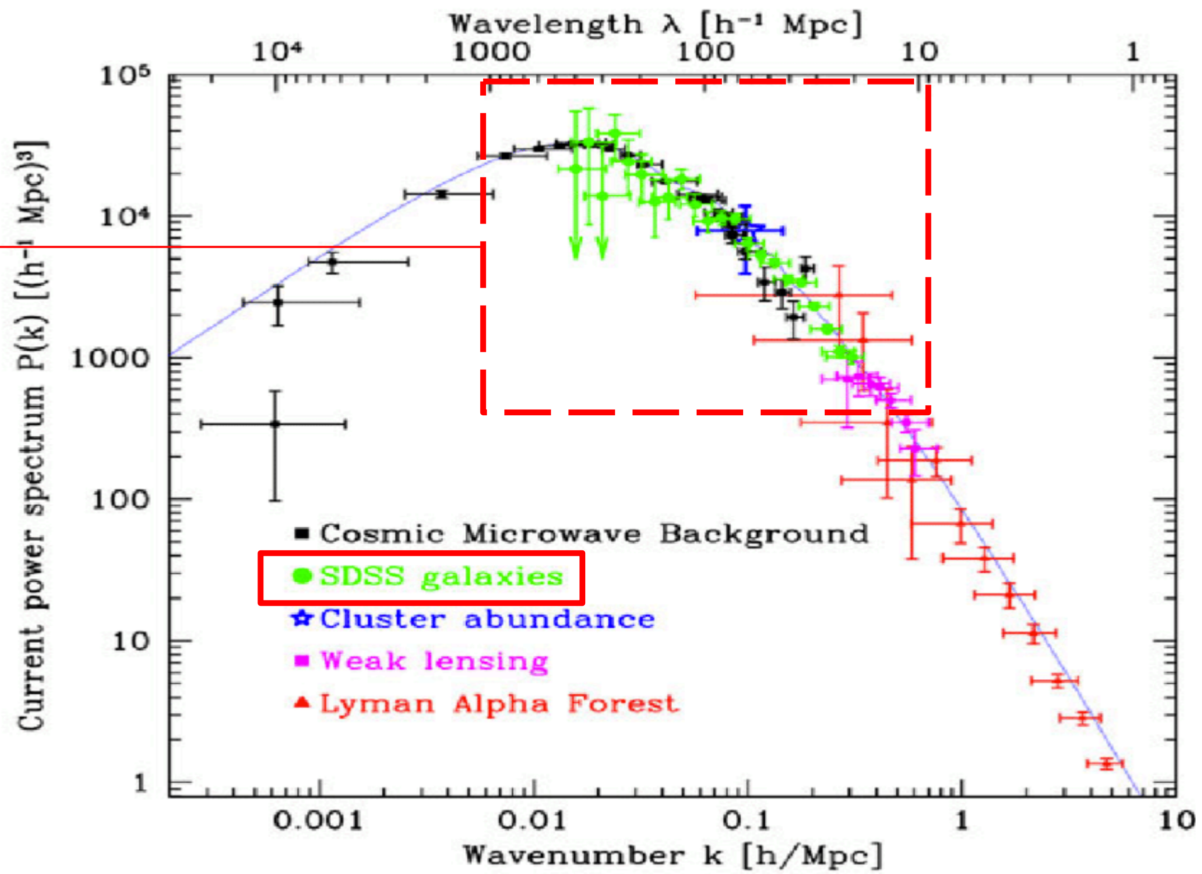
"Gravitational Fluctuations as an alternative to Inflation" (arxiv.org/abs/1807.10704; MDPI Universe 2019, 5(1), 31)

The Matter Power Spectrum $P(k)$



- SDSS galaxies

Tegmark et al. (2004)



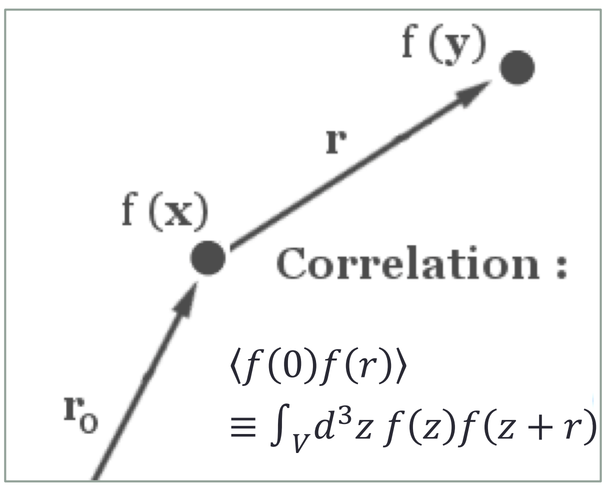
"Triumph" of Inflation?

The Matter Power Spectrum $P(k)$
What is it?

$P(k) \Rightarrow$ **Correlation function** of how matter is distributed (*in k-space*)

$$G_\rho(x-y) = \langle \delta\rho(x)\delta\rho(y) \rangle \;\; \Leftrightarrow \;\; P(k) = \langle |\delta(k)|^2 \rangle$$

(real space) (momentum(/k)-space)



$\delta\rho \equiv \rho(x) - \bar{\rho}$ overdensity from average

$\delta \equiv \frac{\delta\rho}{\bar{\rho}}$ "density-contrast" (fractional overdensity)

Standard theory of Inflation:

$$\langle \phi\phi \rangle \Rightarrow \langle \delta\rho\delta\rho \rangle$$

Q: Is Inflation the only explanation?
– Any other explanations?

QFT & Correlation Functions

QFT is great at calculating correlation functions!

before applying the functional derivative:

$$\frac{\delta}{\delta J(x)}\int d^4y\,\partial_\mu J(y)V^\mu(y) = -\partial_\mu V^\mu(x). \qquad (9.33)$$

The basic object of this formalism is the *generating functional* of correlation functions, $Z[J]$. (Some authors call it $W[J]$.) In a scalar field theory, $Z[J]$ is defined as

$$Z[J] \equiv \int \mathcal{D}\phi \, \exp\Big[i\int d^4x\,[\mathcal{L} + J(x)\phi(x)]\Big]. \qquad (9.34)$$

This is a functional integral over ϕ in which we have added to $\mathcal{L}$ in the exponent a *source term*, $J(x)\phi(x)$.

Correlation functions of the Klein-Gordon field theory can be
puted by taking functional derivatives of the generating functiona
ple, the two-point function is

$$\langle 0|T\phi(x_1)\phi(x_2)|0\rangle = \frac{1}{Z_0}\left(-i\frac{\delta}{\delta J(x_1)}\right)\left(-i\frac{\delta}{\delta J(x_2)}\right)Z[J]\Big|_{J=0}$$

where $Z_0 = Z[J=0]$. Each functional derivative brings down a f
the numerator of $Z[J]$; setting $J=0$, we recover expression (9.18).
higher correlation functions we simply take more functional deri

Formula (9.35) is useful because, in a free field theory, $Z[J]$ can be rewritten in a very explicit form. Consider the exponent of (9.34) in the free Klein-Gordon theory. Integrating by parts, we obtain

$$\int d^4x\,[\mathcal{L}_0(\phi) + J\phi] = \int d^4x\,[\tfrac{1}{2}\phi(-\partial^2 - m^2 + i\epsilon)\phi + J\phi]. \qquad (9.36)$$

Q: Can we apply QFT to Gravity?

Q: Are such predictions reliable?

⇒ *Yes!*

Problems of Quantum Gravity

(1) Theory is not perturbatively renormalizable.

(2) Short distance theory is uncertain.

What is Quantum Gravity ?

The combination of:

(1) Einstein's 1916 classical General Relativity

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

$$\Leftrightarrow \boxed{I_{EH}[g_{\mu\nu}] = \frac{1}{16\pi G} \int d^d x \sqrt{g(x)}\, R(x) - 2\lambda}$$

(2) Quantum mechanics, in covariant Path Integral

$$\boxed{Z = \int [d g_{\mu\nu}] \exp\left\{ \frac{i}{\hbar} I_{EH}[g_{\mu\nu}] \right\}}$$

$[d g_{\mu\nu}] \equiv$ *DeWitt functional measure*

Uniqueness

$$Z_{cont} = \int [d\,g_{\mu\nu}] \exp\left\{-\lambda_0 \int dx\,\sqrt{g} + \frac{1}{16\pi G}\int dx\sqrt{g}\,R\right\}$$

While:

Higher order operators ⇒ Short distance physics,

Long distance theory is *unique**!

*Feynman 1963 ⇒ $I_{EH+\lambda}$ = unique theory of *m=0, s=2 field.*

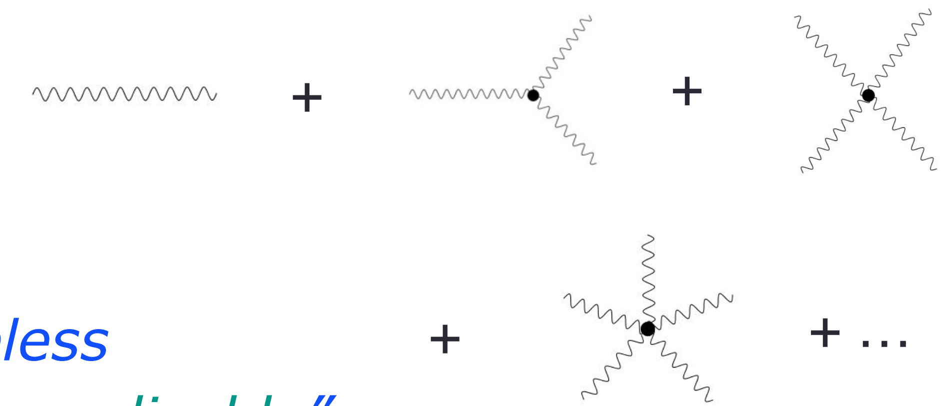
… just like QED and QCD.

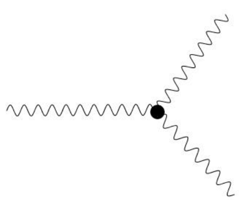
Long Distance Difficulties

$$Z_{cont} = \int [d\, g_{\mu\nu}] \exp\left\{ -\lambda_0 \int dx\, \sqrt{g} + \frac{1}{16\pi G} \int dx \sqrt{g}\, R \right\}$$

- *Theory is Highly Non-Linear*
- *"Gravity gravitates"*
- *Perturbation Theory in G is useless*

  "Not perturbatively renormalizable"





$$\longrightarrow \quad V(k) \sim h \cdot \partial h \cdot \partial h \sim k^2$$

⇒ Nonperturbative methods?

Nonperturbative Methods, Predictions

Methodology:

- Analytical methods:
- $2+\varepsilon$ expansion, Modern RG analysis, Large-*N* expansion…
- Numerical methods: Lattice calculations

Predictions?

- Non trivial RG fixed point, Phase Transitions!
- Nonperturbative scale ξ
- Nontrivial Scaling exponents of correlation functions ν
- RG running of coupling constants ($g \rightarrow g(k^2)$)
- and so on…

All = common features of perturbatively-nonrenormalizable theories!

Predictability of not pert. renormalizable theories?

Q: Are there other QFTs where perturbation theory fails, yet makes useful/physical predictions?

Yes!

- QCD
  Quarks and gluons are confined. *Main theoretical evidence for confinement and chiral symmetry breaking is possibly from the lattice.*

$$V(r) \underset{r\to\infty}{\sim} \kappa\, r \qquad \kappa \simeq e^{-\frac{1}{2\beta_0\, g^2}}$$

- Superconductor
  BCS Theory: *Fermions close to the Fermi surface condense into Cooper pairs.*

$$\Delta = 2\,\hbar\,\omega_D\, e^{-\frac{1}{N(0)\,g}}$$

- Superfluid
  Described by condensate density.

$$E_k = \hbar\sqrt{n_0(T)\, V_{\mathbf{k}=0}/m}\; k$$

- Degenerate Electron Gas
  Screening due to *Thomas-Fermi mechanism.*

$$V(r) \sim \frac{e^2}{r}\, e^{-q_{TF}\, r} \qquad q_{TF} = e\sqrt{m\Lambda}$$

- Homogeneous Turbulence
  Observables $\sim (R - R_c)^\gamma$ R_c = *critical Reynolds no. (Kolmogorov)*
- Ferromagnets
  Spontaneous Symmetry Breaking & dimensional transmutation

$$\langle \frac{1}{V}\sum_i S_i \rangle \underset{T\to T_c}{\sim} (T_c - T)^\beta$$

Predictability of not pert. renormalizable theories?

Yes!

**The Nonlinear Sigma-model* (NLSM):

- ⇒ a not perturbative renormalizable theory in 3D,
- ⇒ *second most accurate* prediction *of QFT!*

(on critical exponents $\alpha, \beta, \ldots$)   (after $g-2$ measurements of QED)

Q: critical exponents for gravity?

Critical Exponent for Gravity ν

- Nontrivial Scaling exponents of correlation functions ν

$$\langle \delta R(x) \delta R(y) \rangle \sim \frac{1}{r^{\,2(d-1/\nu\,)}} \qquad \sim \frac{1}{r^{2}}$$

- Various methods:

Method used to compute ν in d=4	Universal Exponent ν
Euclidean Lattice Quantum Gravity	$\nu^{-1} = 2.997(9)$
Perturbative $2+\epsilon$ expansion to one loop [22]	$\nu^{-1} = 2$
Perturbative $2+\epsilon$ expansion to two loops [23]	$\nu^{-1} = 22/5 = 4.40$
Einstein-Hilbert RG truncation [56]	$\nu^{-1} \approx 2.80$
Recent improved Einstein-Hilbert RG truncation [57]	$\nu^{-1} \approx 3.0$
Geometric argument [33] $\rho_{vac\ pol}(r) \sim r^{d-1}$	$\nu^{-1} = d - 1 = 3$
Lowest order strong coupling (large G) expansion [29]	$\nu^{-1} = 2$
Nonlocal field equations with $G(\Box)$ for the static metric [46]	$\nu^{-1} = d - 1$ for $d \geq 4$

- From lattice (numerical): $\nu = 0.334(4)$ $\quad \nu^{-1} \approx 3$

Q: How to relate this to testable objects?

Relating $G_R \to G_\rho$ (& $P(k)$)

- Using $\nu = 3$; Recall:

$$G_R(x-y) = \langle \delta R(x) \delta R(y) \rangle \sim \frac{1}{r^{2(d-1/\nu)}} = \frac{1}{r^2}$$

- Einstein Field Equations: $(R \to \rho)$

$$R(x) = 8\pi G\, \rho(x)$$

$$\underbrace{\langle\, R\, R\, \rangle}_{G_R} = (8\pi G)^2 \underbrace{\langle\, \delta\rho\, \delta\rho\, \rangle}_{G_\rho}$$

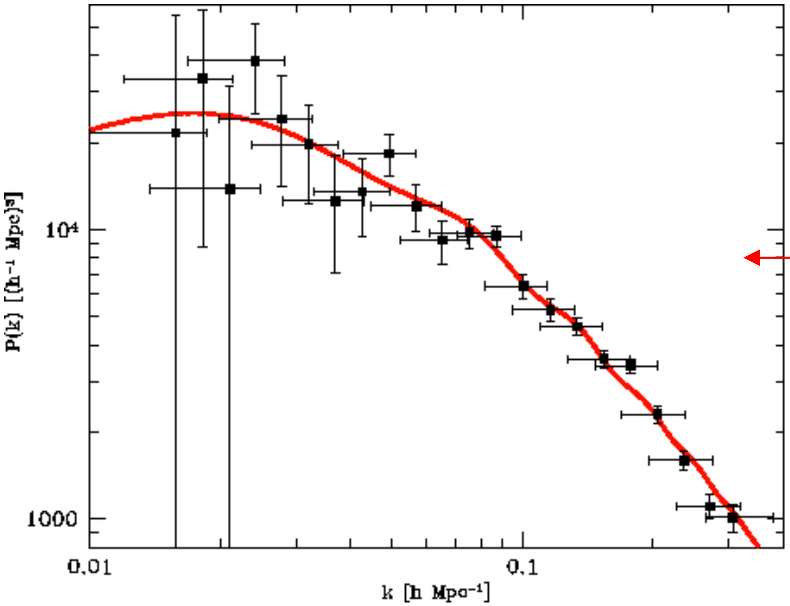
$$\therefore\ G_R \sim \frac{1}{r^2} \ \Rightarrow\ G_\rho \equiv \langle\, \delta\rho\, \delta\rho\, \rangle \sim \frac{1}{r^2}$$

- Fourier transform:

$$G_\rho \equiv \langle\, \delta\rho\, \delta\rho\, \rangle \sim \frac{1}{r^2} \ \Rightarrow\ \boxed{P_{matter}(k) \sim \frac{1}{k}}$$

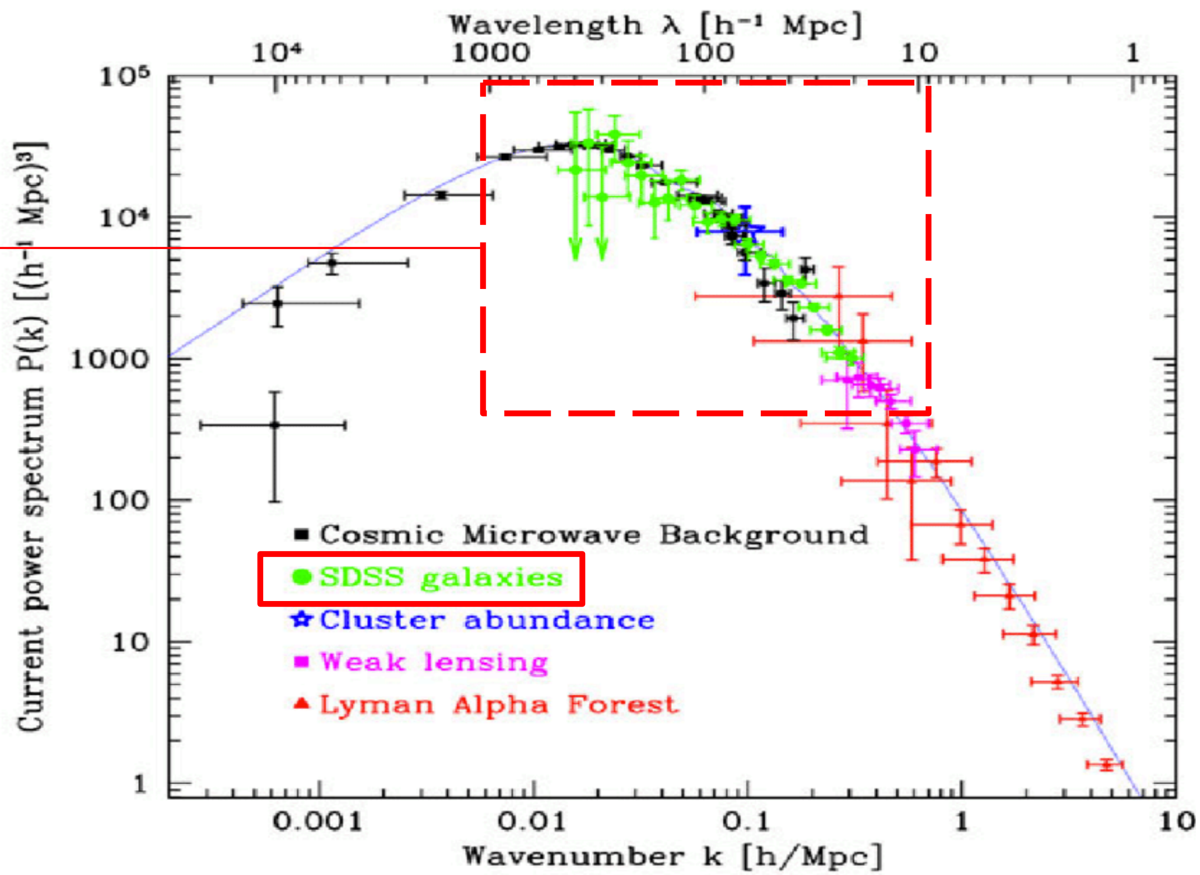
How does this prediction compare with observations?

Matter Power Spectrum



- SDSS galaxies

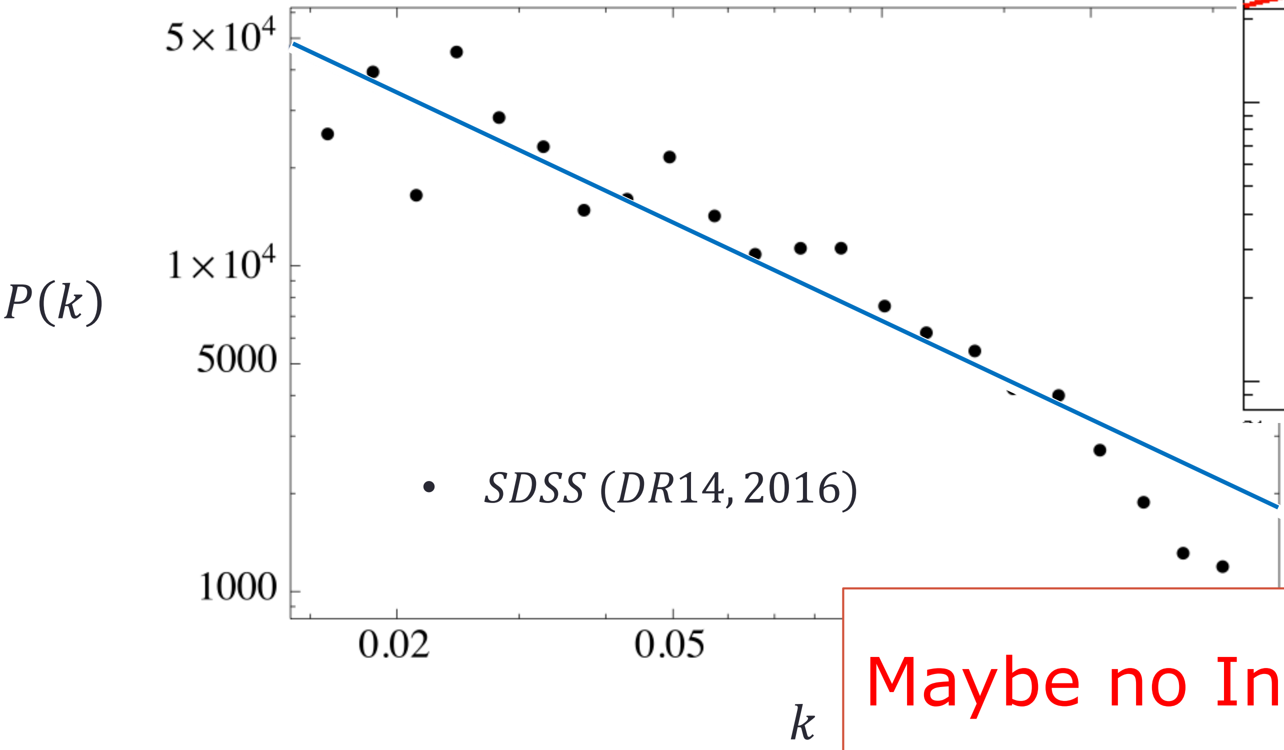
Tegmark et al. (2004)

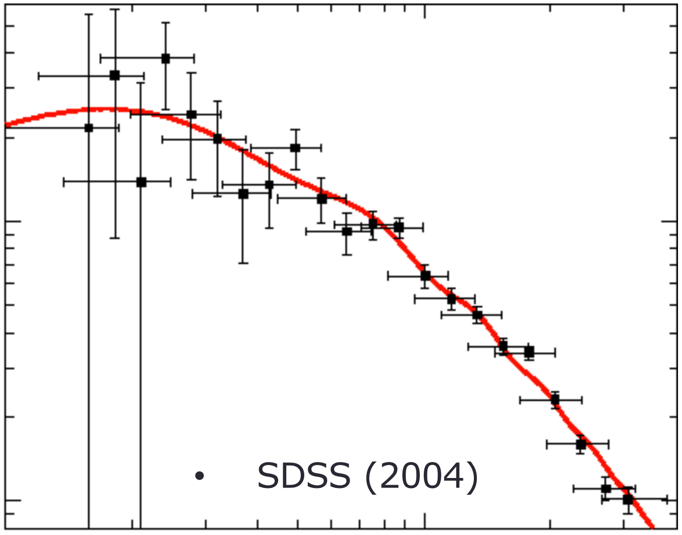


Predictions of Quantum Gravity

$$P_{matter}(k) \sim \frac{1}{k}$$

- Matter Power Spectrum from SDSS Galaxy Survey:





Maybe no Inflation Needed…?

Q: How to reproduce CMB data in small-k regime?

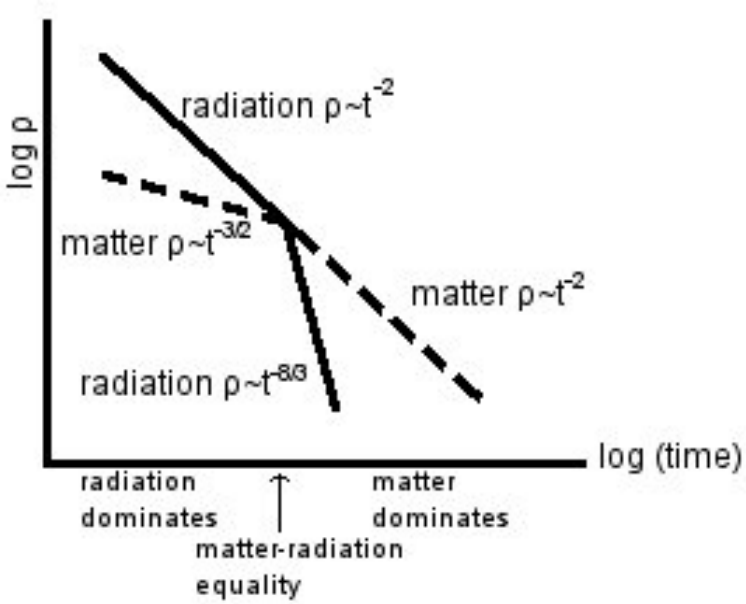
Extending the prediction to larger distance (small k)?

Larger distance → earlier in time

→ switches from **matter dominated** → **radiation dominated**

But radiation **behaves** quantitatively **different** than matter in realtivity.

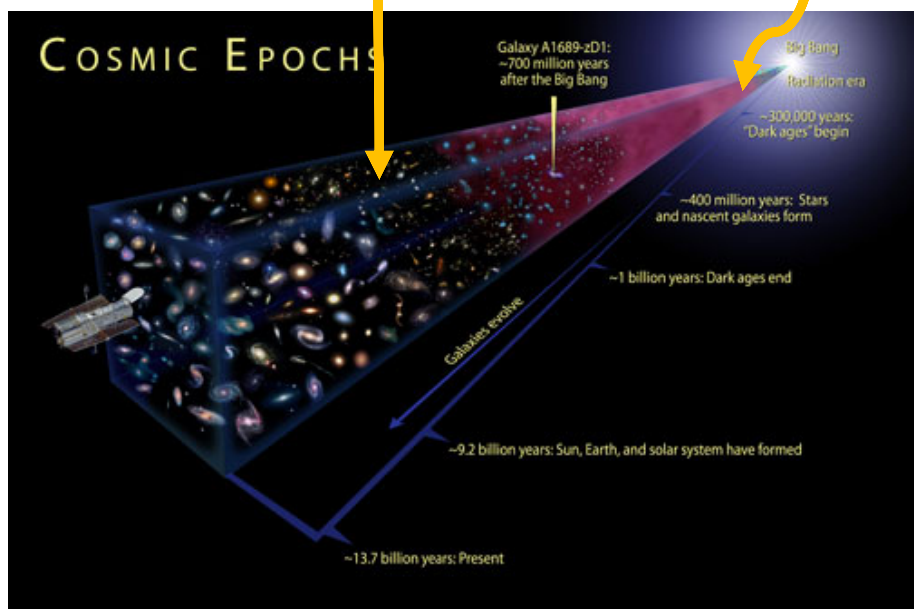
∴ expects $P(k)$ to behave differencely **beyond critical scale** $\boldsymbol{k_{eq}}$.



t_{eq}

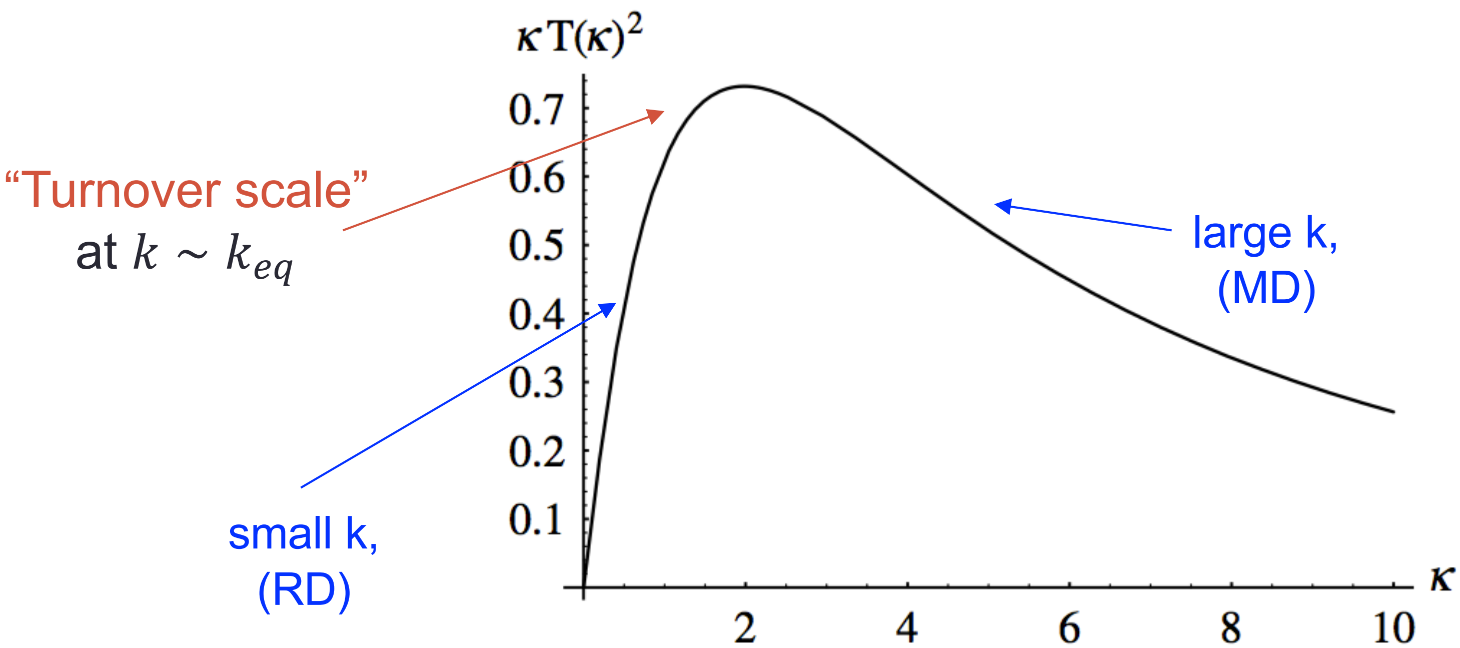
Matter Dom.

Radiation Dom.



Standard Method Extending the prediction to small k

- Classical GR ⇒ *"Transfer function"*
- Relates matter dynamics to radiation dynamics **via Boltzmann Eqns**.

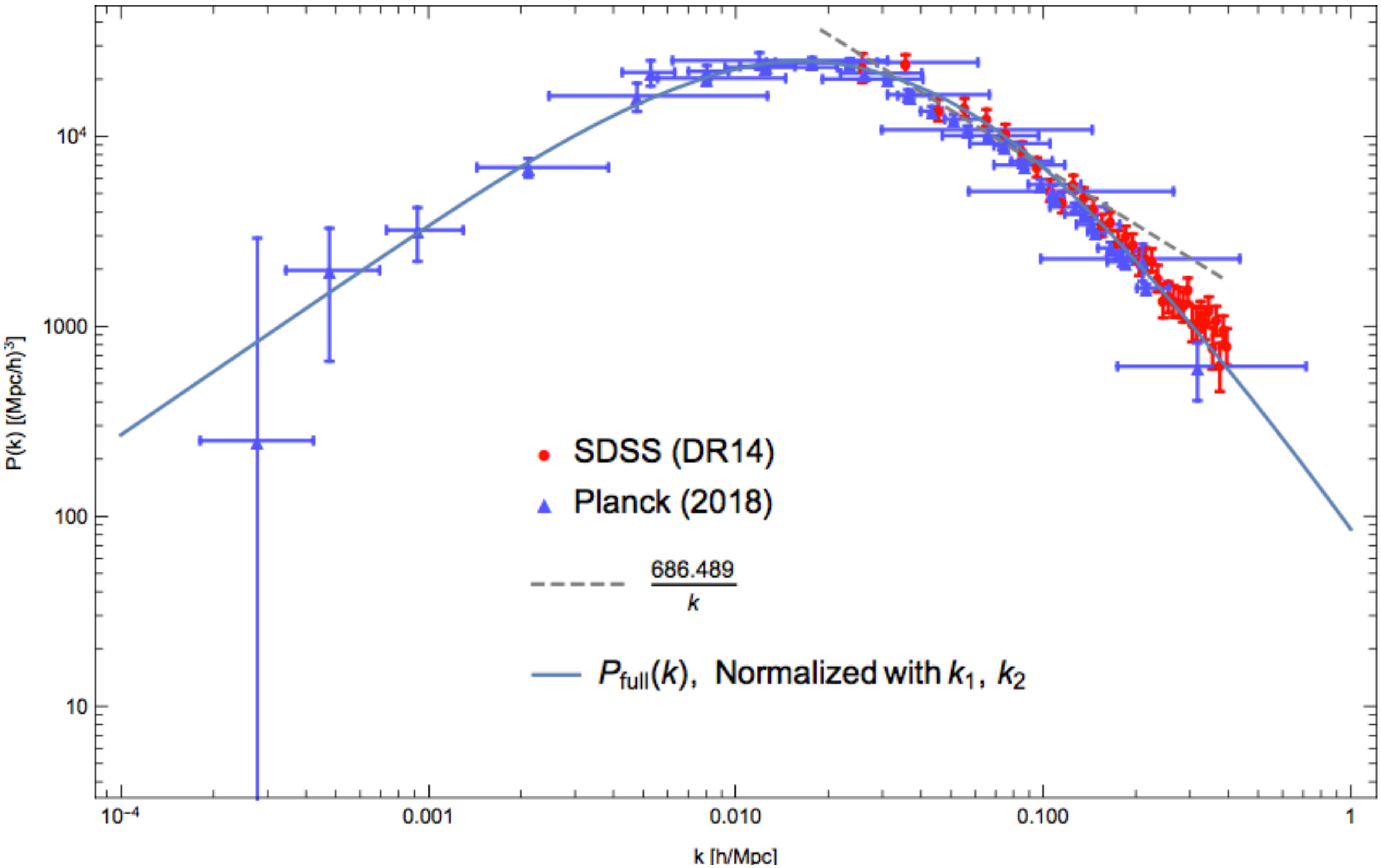


$$\mathcal{T}(\kappa) \simeq \frac{\ln[1+(0.124\kappa)^2]}{(0.124\kappa)^2}\left[\frac{1+(1.257\kappa)^2+(0.4452\kappa)^4+(0.2197\kappa)^6}{1+(1.606\kappa)^2+(0.8568\kappa)^4+(0.3927\kappa)^6}\right]^{1/2}$$

Weinberg p.412

Full P(k) from QG

- Normalized result for full $P(k)$, extrapolated to small k: (blue curve)



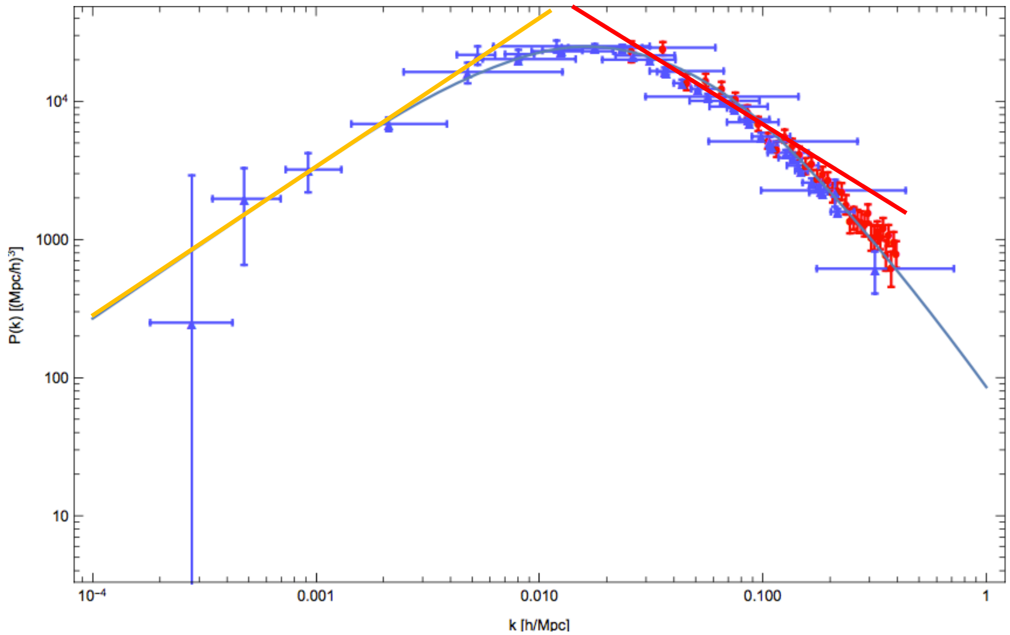
Comparison with Inflation $\phi(x)$

Inflation – the original picture

- Proposes → scalar field ϕ
- Fluctuations from $\langle\phi\phi\rangle$
- Calculated with *pert-qft* methods
- Relates $\langle\phi\phi\rangle \xrightarrow{M-S} \langle\delta\rho\delta\rho\rangle$
- Normalizes on CMB scales (small k)
- Extrapolate to SDSS scale (large k)

QG – an *alternative* picture

- Curvature/Gravitational field R
- Fluctuations from $\langle RR\rangle$
- Calculated with *nonpert-* methods
- Relates $\langle RR\rangle \xrightarrow{EFE} \langle\delta\rho\delta\rho\rangle$
- Normalizes on SDSS scales (large k)
- Extrapolate to CMB scale (small k)



Comparison with Inflation $\phi(x)$

- Radically different perspective!
  → *Arguable more grounded?*
- 1. No new fields required
- 2. Standard techniques based on Wilson's RG analysis
- 3. Very limited free/adjustable parameters in the theory

- Or, Possibly coexists??

Additional Quantum Effects 1 – *IR* regulators

Nonperturbative scale ξ $\rightarrow$ regulates expressions in *IR*-limit

QCD case:

$$\Lambda_{QCD}$$

$$(\Lambda_{QCD} \simeq 200 \text{ MeV})$$

$$\int dk \rightarrow \int_{\Lambda_{QCD}} dk$$

$$\frac{1}{k^2} \rightarrow \frac{1}{k^2 + \Lambda_{QCD}{}^2}$$

Gravity case:

$$m \sim \frac{1}{\xi}$$

$$(\xi \simeq \sqrt{\frac{3}{\lambda_{\cos}}} \simeq 5300 \text{ Mpc })$$

$$\int dk \rightarrow \int_{m} dk$$

$$\boxed{\frac{1}{k^2} \rightarrow \frac{1}{k^2 + m^2}}$$

Additional Quantum Effects 2 – *RG* running

Renormalization Group Eqns (RGE) ⇒ running of coupling "constants"

QCD case:

$$\alpha_s \rightarrow \alpha_s(k)$$

$$\alpha_s(k) = \frac{1}{\beta_0 \ln\left(\frac{k^2}{\Lambda_{qcd}^2}\right)}$$

β_0 (Wilczek, Gross and Politzer)

$\Lambda_{qcd} \simeq 200$ MeV

Gravity case:

$$G \rightarrow G(k)$$

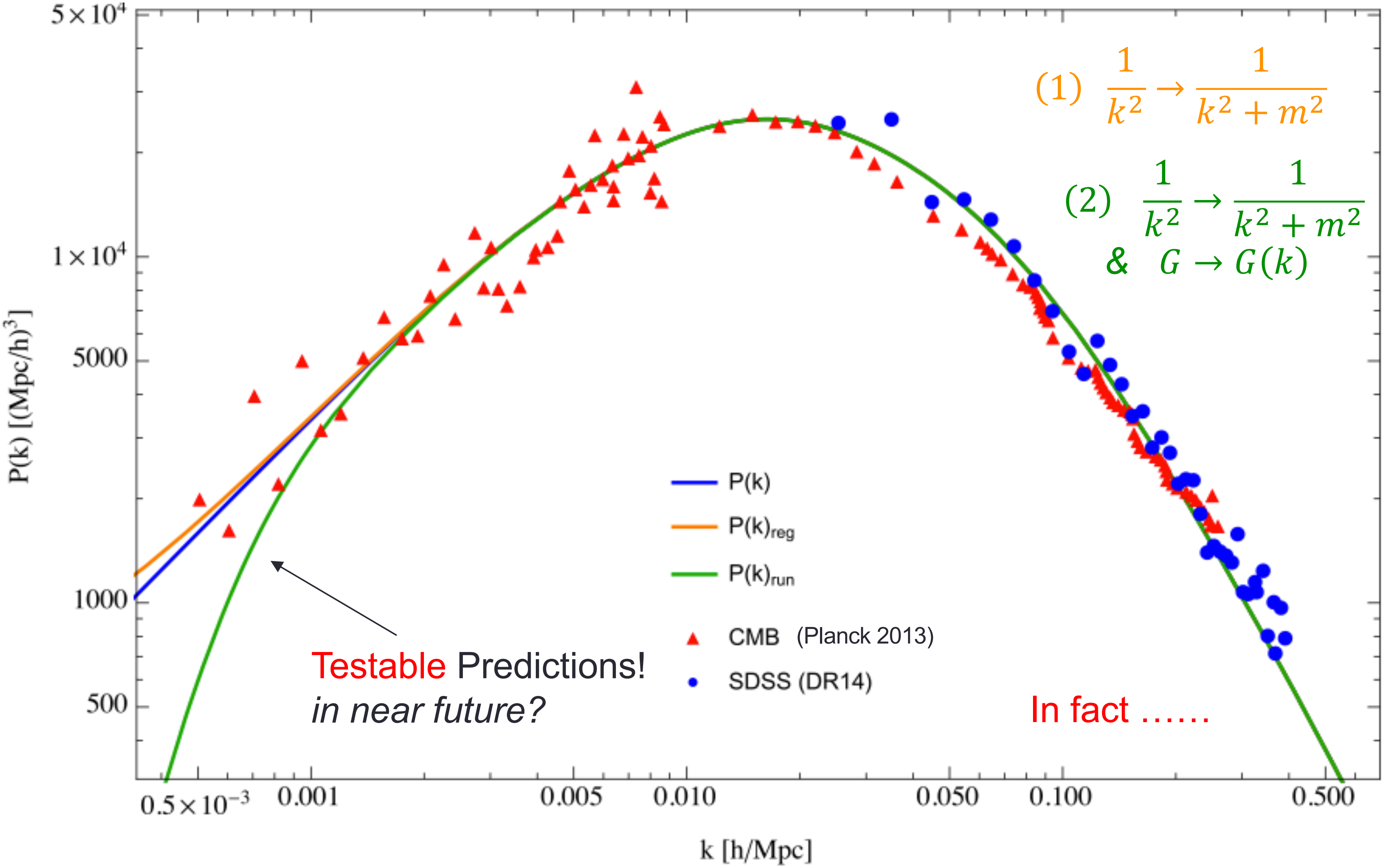
$$G(k) = G\left[1 + c_0\left(\frac{m^2}{k^2}\right)^{\frac{1}{2\nu}} + \mathcal{O}\left(\frac{m^2}{k^2}\right)^{1/\nu}\right]$$

$c_0 \approx 16.04$ (HWH)

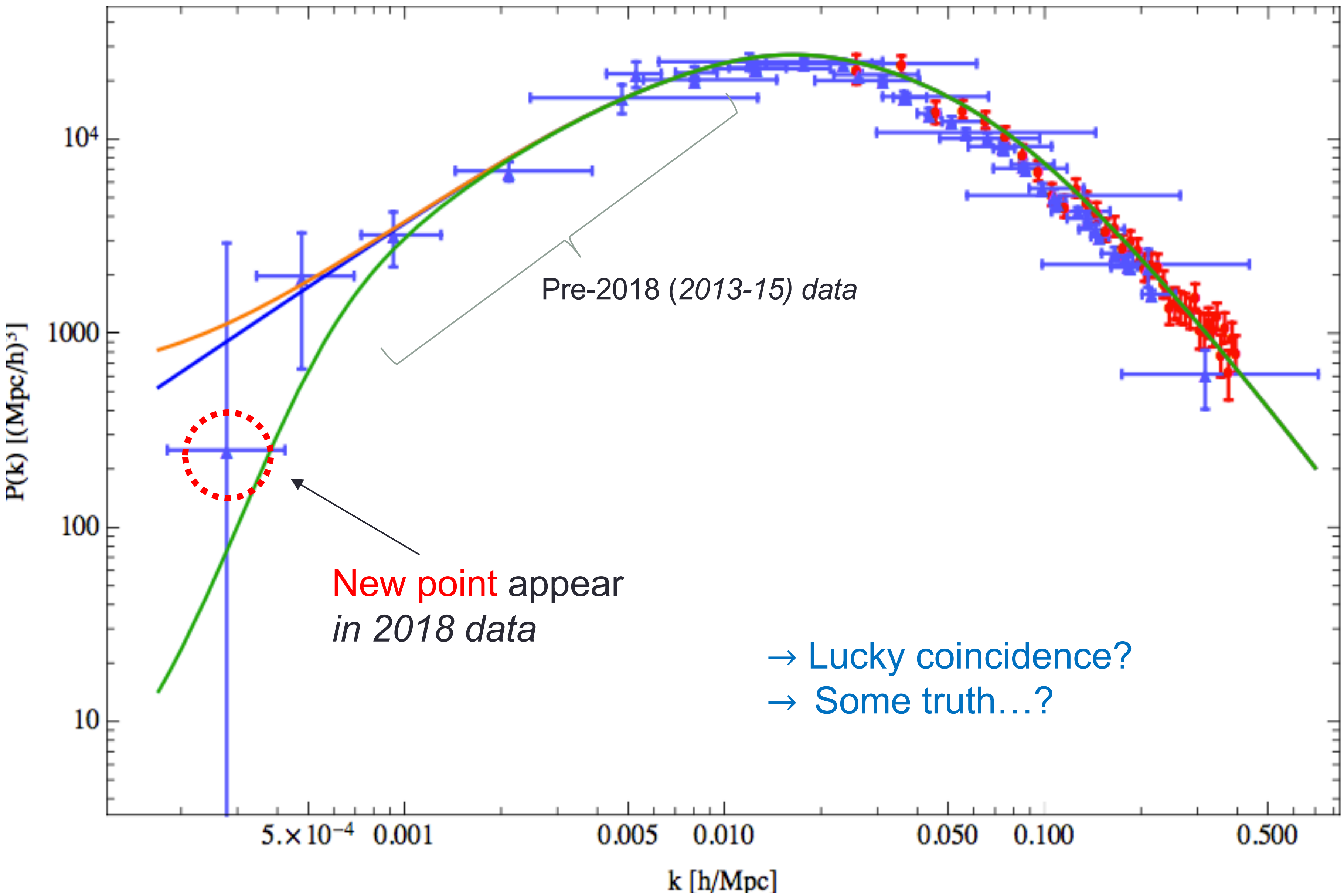
$m \sim$ 1/ 5300Mpc ~ $2{\times}10^{-4}$ Mpc^{-1}

$1/\nu \simeq 3$

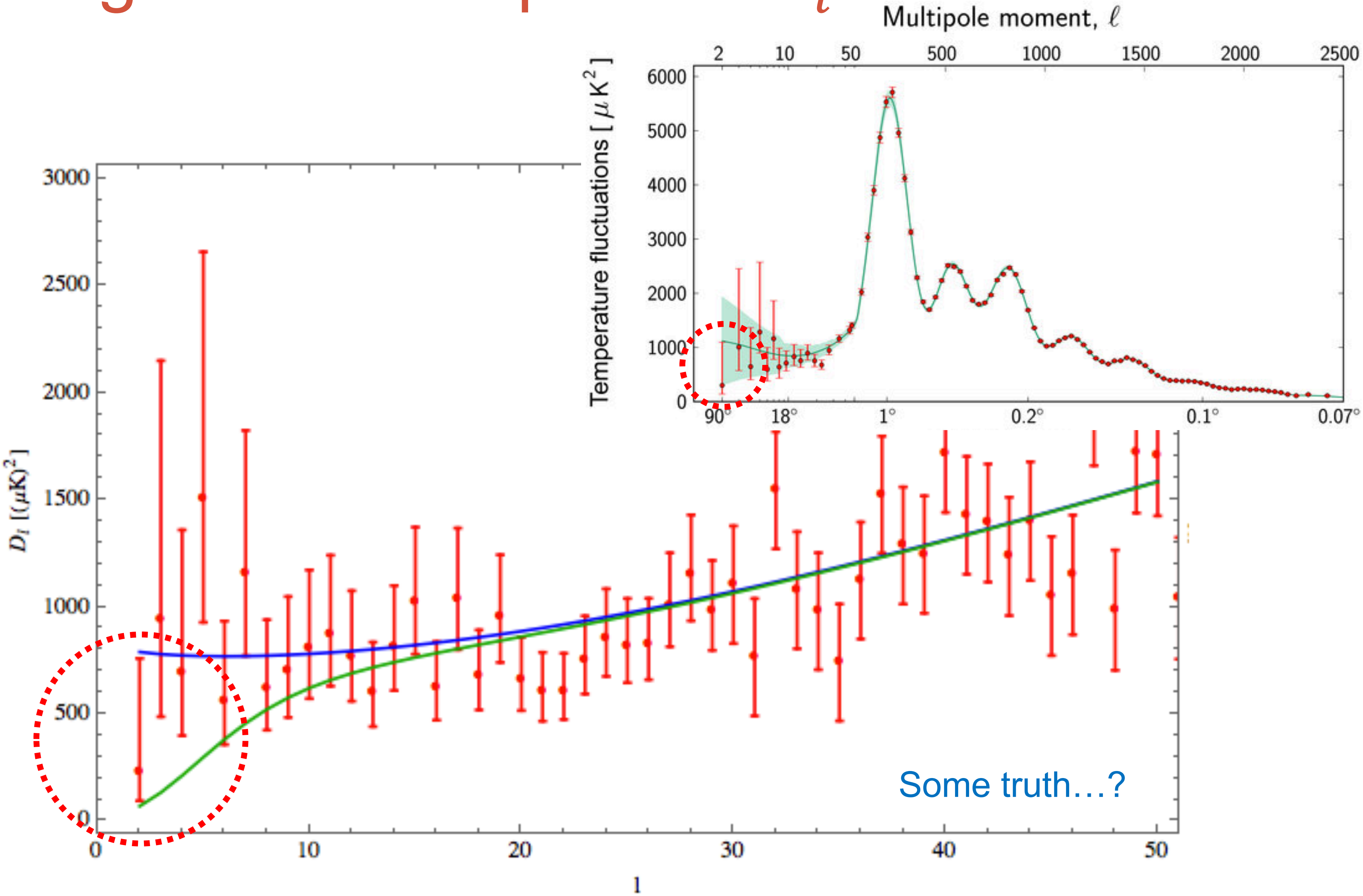
Further Quantum Effects (in small k)



Further Quantum Effects (in small k)



Angular Power Spectrum C_l



Comparison with Inflation $\phi(x)$

- Radically different perspective!
  → *Arguable more grounded?*
- 1. No new fields required
- 2. Standard techniques based on Wilson's RG analysis
- 3. Very limited free/adjustable parameters in the theory

* Makes testable predictions → *hopefully testable in near future!*

Conclusion & Summary

- 1) Nonperturbative methods provide a useful theory of QG
- 2) Provides alternative picture to Inflation
- 3) Provides testable predictions for future

  - *Exciting Future work:*
    - Improving / narrowing value and errors on n_s
    - Tensor-to-scalar ratio r
    - Bispectrums, Tri-spectrums? (Explore current measurement?)

References

- ***H. W. Hamber and L.H.S. Yu, "Gravitational Fluctuations as an alternative to Inflation", 2018***
- *H. W. Hamber, Vacuum Condensate Picture of Quantum Gravity, 2017.*
- *K. G. Wilson, Feynman-graph expansion for critical exponents, Phys. Rev. Lett. 28, 548 (1972); Quantum field-theory models in less than 4 dimensions, Phys. Rev. D 7, 2911 (1973).*
- *G. Parisi, On the Renormalizability of not Renormalizable Theories, 1973*