Silver workshop 2020 : Flat Structure and Singularities

Date：6th March
Venue： OIST Lab3-B700 and B711

Invited speakers：

Kyoji Saito（IPMU）
Kostantin Aleshkin (Columbia Univ.)
Mitsuo Kato (Ryukyu Univ.)
Yoshihisa Saito (Rikkyo Univ.)
Shinichi Tajima （Niigata Univ.）
Motoko Kato (Ehime Univ.)
Shinobu Hikami (OIST)

Funding: OIST Mathematical and Theoretical Physics Unit.

Tentative schedule:

March 6 (Fri) (Lab3-B700): 10:00-10:45 Kyoji Saito
10:50-11:35 Kostantin Aleshkin
11:35-11:50 Coffee break
11:50-12:35 Yoshihisa Saito

(Lab3-B711)14:00-14:45 Mitsuo Kato
14:50-15:35 Shinichi Tajima
15:35-16:00 Coffee break
16:00-16:45 Motoko Kato
16:50-17:35 Shinobu Hikami
Contact: shiho.saito@oist.jp

Abstract :

Kyoji Saito: Integrable hierarchies arising from primitive forms (I)

Let $f(x)$ be a holomorphic function with an isolated critical point and let $F(x,t)$ be its universal unfolding. Consider the evolution map $exp((f-F)/z)$ from the twisted relative De Rham cohomology group $H^*(f_*(\Omega_X((z))), zd+df)$ for f to that $H^*((F,p)_*(\Omega_Z((z))), zd+dF)$ for F. Fixing a pair of a good opposite filtration of $H^*(f_*(\Omega((z))), zd+df)$ and a suitable volume form $\zeta = 0$, we make Birkhoff decomposition $exp((f-F)/z)\zeta_0 = \zeta_+ + \zeta_-$. Then, according as the section has the metric structure or not, ζ_+ is a primitive form with or without metric structure. It induces flat structure with or without metric on the deformation parameter space.

Kostantin Aleshkin: Integrable hierarchies arising from primitive forms (II)

Mitsuo Kato: Prepotentials derived from Painlevé VI solutions

It is known that, from a Painlevé VI solution $\varphi(\tau)$ with generic parameter $\theta = (\theta_x, \theta_y, \theta_z, \theta_\infty)$, we can construct an Okubo type differential equation of rank three generated by "potential vector"

$$\vec{g}(t_1,t_2,t_3) = (g_1(t_1,t_2,t_3), g_2(t_1,t_2,t_3), g_3(t_1,t_2,t_3)).$$

We give the condition satisfied by θ so that $\vec{g}(t_1,t_2,t_3)$ has a "prepotential" $F(t_1,t_2,t_3)$ satisfying

$$\frac{\partial F(t_1,t_2,t_3)}{\partial t_i} = g_{4-i}(t_1,t_2,t_3),\ 1 \le i \le 3.$$

We give prepotentials derived from

$$\begin{aligned}
&\theta = (0,0,0,2/3),\\
&\tau = -\frac{(2u+1)(u-1)^2}{(2u-1)(u+1)^2},\quad \varphi = -\frac{(2u+1)(u-1)}{u+1},\\
&\theta = (0,0,0,4/3),\\
&\tau = -\frac{(2u+1)(u-1)^2}{(2u-1)(u+1)^2},\quad \varphi = -\frac{(2u+1)(u-1)}{u+1},\\
&\theta = (0,0,0,8/3),\\
&\tau = -\frac{(2u+1)(u-1)^2}{(2u-1)(u+1)^2},\quad \varphi = -\frac{(4u^2-3)^2(2u+1)(u-1)}{(80u^4-40u^2+9)(u+1)}. \qquad (1)
\end{aligned}$$

S. Tajima: Computing torsion and logarithmic differential forms via local cohomology

We consider the module of germs of logarithmic differential forms associated to an isolated hypersurface singularity in the context of computational complex analysis. We show that the use of a result of A. Aleksandrov on torsion modules allows us to design an effective method for computing logarithmic differential forms. Based on the concept of local cohomology and Grothendieck local duality, we derive an algorithm of computation. The resulting algorithm computes a basis of torsion differential forms and a set of generators, over a local ring, of the module of germs of logarithmic differential forms associated to a hypersurface with an isolated singularity.

Yoshihisa Saito: Elliptic Artin Groups

In the study of representation theory of Lie groups and Lie algebras, the regular Weyl group orbit spaces and their fundamental groups (called Artin groups or generalized braid groups) have quite important roles.

In the middle of 80's, motivated by the study of singularity theory, Kyoji Saito introduced the notion of elliptic root systems, and study their basic properties. Especially, he introduced an "elliptic analogue" of the regular Weyl group orbit spaces, so-called the elliptic regular orbit spaces, and study their detailed structure in algebraic and differential geometrical point of view.

In this talk, we study the fundamental groups of the regular elliptic Weyl group orbit spaces. These groups are presented by a generator system associated with the elliptic diagrams, and we call them the elliptic Artin groups. Furthermore, some basic properties of these groups will be also discussed. Especially, the elliptic regular orbit space is defined over the moduli space of elliptic curves. This fact leads us to the description of the elliptic modular group actions on elliptic Artin groups. This talk is based on a joint work with Kyoji Saito.

Motoko Kato: On the acylindrical hyperbolicity of some Artin groups

Artin groups are defined by finite presentations, which are closely related to Coxeter groups. From the viewpoint of geometric group theory, we consider whether these groups have interesting actions on non-positively curved spaces. In particular, we consider a conjecture which states that the central quotient of every irreducible Artin group is either virtually cyclic or acylindrically hyperbolic. In this talk, we give some new examples of acylindrically hyperbolic Artin groups, by observing actions on CAT(0) spaces constructed by Brady and McCammond. This is a joint work with Shin-ichi Oguni (Ehime

University).

S. Hikami: Punctures and p-spin curves from matrix models

We investigate the intersection numbers of the moduli space of p-spin curves with the help of matrix models. The explicit integral representations that are derived for the generating functions of these intersection numbers exhibit p Stokes domains, labelled by a "spin"-component l taking values $l =$ -1, 0,1,2,...,p-2. Earlier studies concerned integer values of p, but the present formalism allows one to extend our study to half-integer or negative values of p, which turn out to describe new types of punctures or marked points on the Riemann surface. They fall into two classes : Ramond ($l = -1$), absent for positive integer p, and Neveu-Schwarz ($l \neq -1$). The intersection numbers of both types are computed from the integral representation of the n-point correlation functions in a large N scaling limit. We also consider a supersymmetric extension of the random matrix formalism to show that it leads naturally to an additional logarithmic potential. Open boundaries on the surface, or admixtures of R and NS punctures, may be handled by this extension. This is a joint work with E. Brezin , (arXiv:2001.09267).