Holographic dual descriptions for gravity in finite regions

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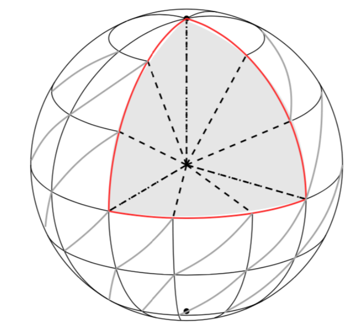
Introduction and Outline

Study holographic dual theories for general boundaries

- Regge calculus suitable yet computationally challenging
- construction of holographic duals in 4D
  - non-perturbative holography (quantum flat spacetimes)
  - restore diffeomorphism invariance in discrete theory

Holographic dual theories 3D results

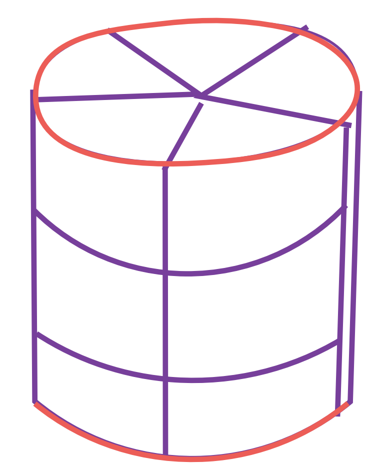


Get a quasi-local holographic dual boundary field theory for geodesic lengths

asymptotic and finite boundaries [*V* Bonzom, *B Dittrich, H Haggard, F Hopfmueller, SKA*] [*S Carlip, Skenderis et al,*]

- boundary dofs described by Liouville field action
- also one loop correction in terms of BMS characters [*J Oblak et al*] [*B Dittrich, A Castro*]

Also works for non-perturbative theories

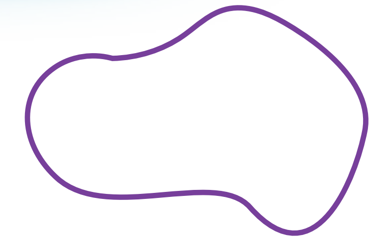


3D: Ponzano-Regge model [*B Dittrich, C Goeller, E Livine, A Riello*]

- spin chain models, non-linear sigma models, 6-vertex models, …

Holographic dual theories for gauge sector (linearized)

$$S_{\text{EH}} = -\frac{1}{\kappa}\int_{\mathscr{M}} d^d x\sqrt{g}(R-2\Lambda) - \frac{2}{\kappa}\int_{\partial\mathscr{M}} d^{d-1}y\sqrt{h}K$$



$\partial\mathscr{M} < \infty$

Background spacetime

$$ds^2 = dr^2 + h_{AB}dy^A dy^B$$

Linearize around background $\quad g_{ab} \to g^{\text{bg}}_{ab} + \gamma_{ab} \quad \gamma_{ab} = \nabla_a\xi_b + \nabla_b\xi_a \quad \xi_a = (\xi^\perp, \xi_A)$

For diffeomorphism generating vector fields

[B Dittrich, F Hopfmueller, SKA '19]

$$\gamma_{AB} = 2K_{AB}\xi^\perp + D_A\xi_B + D_B\xi_A$$

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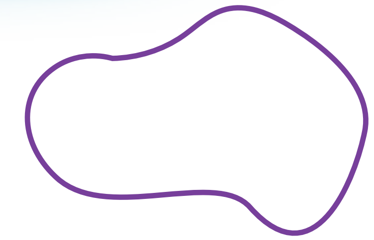

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✦ Hamilton-Jacobi action

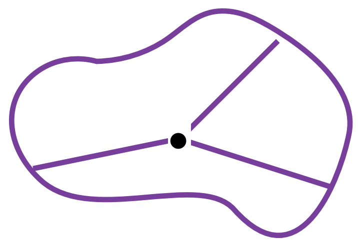
$$S^{(2)}_{\text{HJ}} = \int d^{d-1}x\sqrt{h}\left(\xi^\perp\Delta\xi^\perp - 2\frac{(d-3)}{(d-1)}(2\Lambda - {}^bR)\xi^\perp\left(K\xi^\perp + 2h^{AB}D_A\xi_B\right) - \xi^A\left(\mathfrak{D}_{AB} - 2\frac{(d-3)}{(d-1)(d-2)}{}^bRKh_{AB}\right)\xi_B\right)$$

$$\Delta = 2(K^{AB} - Kh^{AB})D_AD_B - \frac{1}{(d-2)}{}^bRK$$

$$\mathfrak{D}_{AB} = 2(K^{CD} - Kh^{CD})D_CD_Dh_{AB} - \frac{1}{(d-1)(d-2)}{}^bRK_{AB}$$

Effective action for geodesic lengths

- (first order) geodesic length $\quad l_{\rm g}(\gamma_{AB}) = \xi^\perp \Big|_{r_1}^{r_2}$ $\quad \Delta \xi^\perp = \delta({}^bR)$



For $\quad d=3 \quad$ and in general for $\quad {}^bR - 2\Lambda = 0$

- Hamilton Jacobi action splits into normal and tangential invariant parts

Integrate out bulk fields except geodesic lengths

- gives effective action for geodesic lengths

$$S^*_\rho = \int d^{d-1}y \sqrt{h}\,\left(\rho\Delta\rho - 2\rho\delta({}^bR)\right)$$

Liouville action

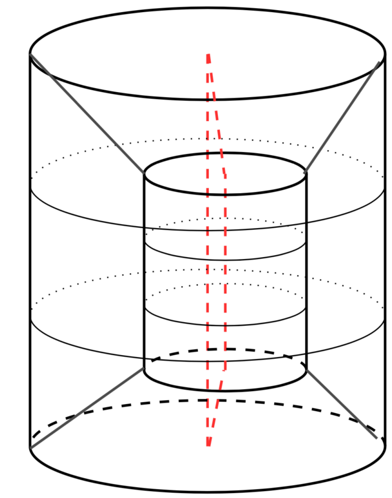
- reproduce normal part of Hamilton Jacobi functional

Example: 4D Regge calculus for thermal spinning flat spacetime

Background with topology of twisted solid torus

$$ds^2 = dr^2 + r^2 d\theta^2 + dy^2 + dz^2 \qquad (r, \theta, y, z) \sim (r, \theta + \gamma_i, y + \alpha_i, z + \beta_i)$$

[B Dittrich, H Haggard, SKA '19]



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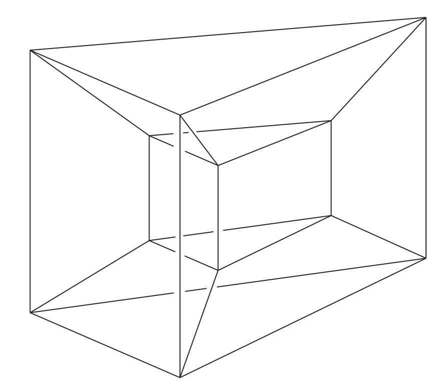

Choose coarse discretization

- radial edges serves as geodesic lengths normal to boundary
- boundary discretization allows continuum limit

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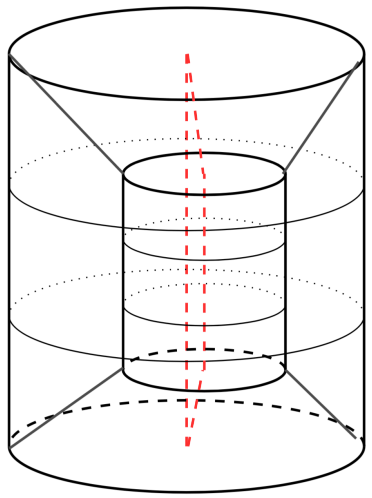

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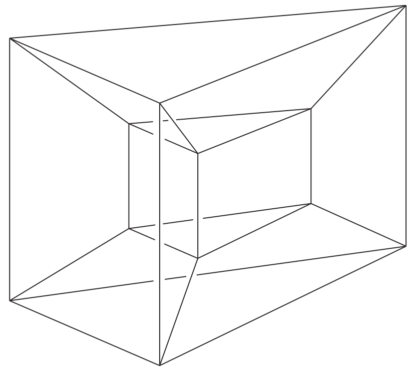

Discrete action

$$S_{\mathrm{R}}[l_e] = \sum_{t \in \mathscr{T}^\circ} A_t(l_e)\, \epsilon_t(l_e) + \sum_{t \in \partial\mathscr{T}} A_t(l_e)\, \omega_t(l_e)$$

Linearize around background $\quad l_e = L_e + \delta l_e \qquad \delta l_e(\theta, y, z) \to \delta l_e(k_\theta, k_y, k_z)$

δl_e for boundary perturbations admits flat or curvature solutions

Evaluate effection action for radial edges coupled to boundary geometry

Example: 4D Regge calculus… results

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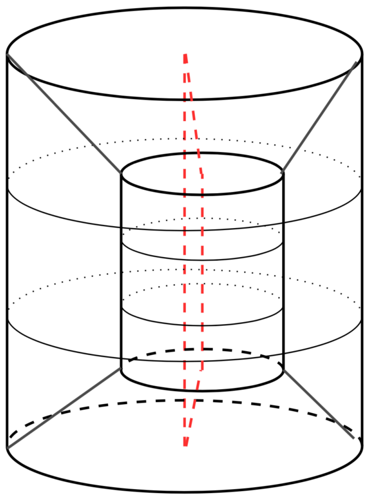

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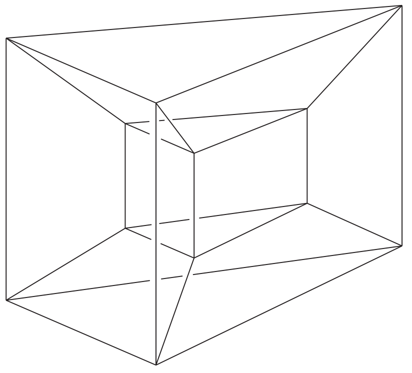

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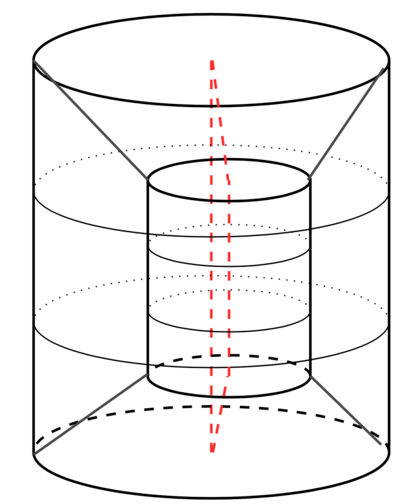

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second order Hamilton-Jacobi functional

$$S_{|_{\text{sol}}} = RS^{(R)} + R^{-1}S^{(R^{-1})} + \mathcal{O}(R^{-3})$$

$$S^{(R)}_{|_{\text{flat}}} \propto \int {}^3\mathfrak{R}(\Delta^{-1})\,{}^3\mathfrak{R}$$

dominates for large $R \gg 1$

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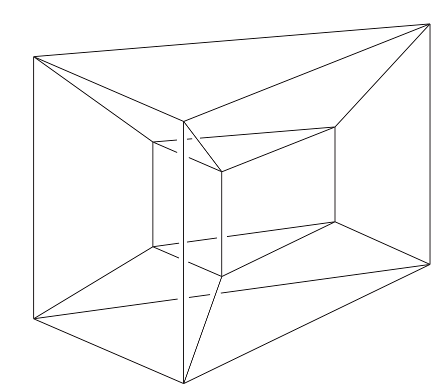

- no Lagrange multiplier method
- smoothness condition automatically implemented in bulk axis (one boundary)

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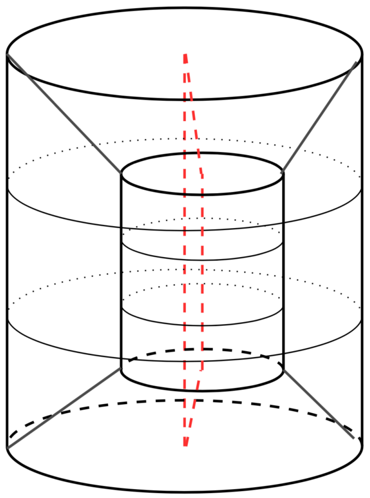

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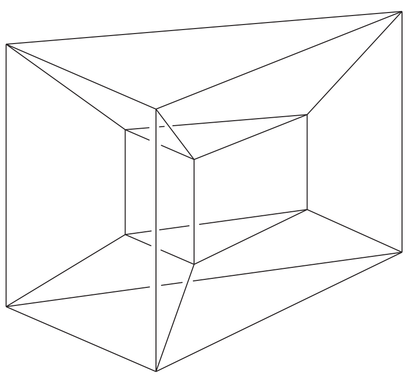

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effective action for radial lengths (continuum limit)

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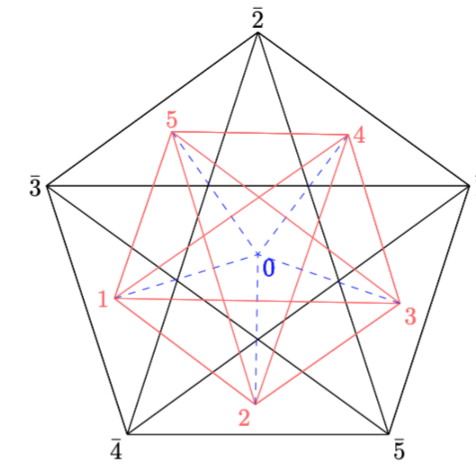
Liouville action

$$\Delta \equiv (K^{AB} - K h^{AB}) D_A D_B \sim (k_y - \frac{\gamma_y}{\alpha} k_\theta)^2 + (k_z - \frac{\gamma_z}{\beta} k_\theta)^2$$

Non perturbative version (flat spacetimes)

[I Korepanov, A Baratin, L Freidel]



4D: KBF (Korepanov-Baratin-Freidel) model

- spinfoam model for quantum flat spacetimes

$$Z_{\text{KBF}} = \int \mathscr{D} l_e \mathscr{D} s_t \prod_t 2 A_t(l_e) \prod_\sigma \frac{\cos(S_{\text{KBF}})}{V_\sigma(l_e)}$$

$$S_{\text{KBF}} = \sum_t s_t \, \epsilon_t(l_e) \qquad\qquad s_t \in \mathbb{Z}$$

- flat solutions
- topological (invariance under all Pachner moves)

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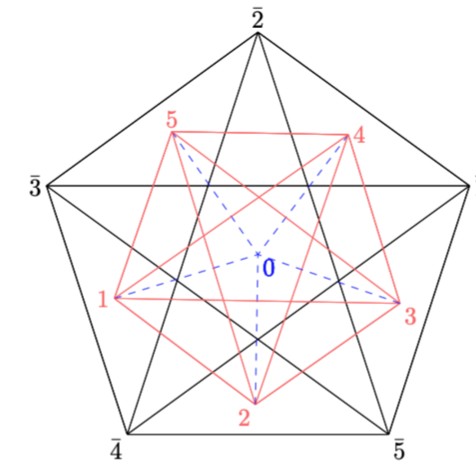

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Question

- what topological gauge (category) theory give rise to this state sum model ?

Higher gauge topological theory

[F Girelli et al, J Baez et al, A Mikovic et al ,.....

[A Baratin, D Wise, J Baez, L Freidel ,.....

BFCG model based on 2-group (Poincarè)

$$S_{\mathrm{BFCG}} = \int_M \mathrm{Tr}_{\mathfrak{g}}(\mathrm{B} \wedge \mathrm{F}) + \mathrm{Tr}_{\mathfrak{h}}(\mathrm{C} \wedge \mathrm{G}[\Sigma, \mathrm{A}])$$

tetrad

$$G[\Sigma, A] := d_A \Sigma$$

- 2-state sum model

$$Z_{\mathrm{BFCG}} = \int \mathscr{D}A \mathscr{D}\Sigma \; \delta(F[A]) \, \delta(G[\Sigma, A])$$

$F[A]$ $1-$curvature

$G[\Sigma, A]$ $2-$curvature

- new quantum geometry based on higher categories

- well adapted for 4D discretization on simplicial manifold

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$G[\Sigma, A]$ 2 – curvature

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Conjecture

KBF two-state sum model is equivalent to Yetter model for Poincare 2-group for BFCG gauge theory

stay tuned for proof .... ➞ B. Dittrich, F. Girelli, A. Riello, P. Tsimiklis, SKA to appear soon.

Graviton sector

Parametrize 'spatial' perturbations

$$\gamma_{AB} = \left\{ \underbrace{T^{\text{diff}}, \overbrace{V^{\perp}}^{\text{boundary graviton}}}_{\text{flat modes}}, \underbrace{x, w}_{\textit{bulk gravitons}} \right\}$$

diffeos + 2 graviton modes

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Bulk equations of motion describe two propagating degrees of freedom — graviton modes

Bessel functions for cylindrical symmetric spacetime

H-J action for two boundaries

- solve discrete recursion relations ⇒ 'perfect action'
- also get path integral measure for gravitons

Geodesic lengths

- normal geodesic lengths modified with graviton modes

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Is there a 'better' observable for graviton sector ?

Conclusion & Outlook

Holographic description allows finite boundaries

- ⋆ connection between discrete and continuum formulations
- ⋆ connection between perturbative and non-perturbative schemes

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Explore non-perturbative dual boundary theories in 4D [wip]

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Applications to H-J functionals and One loop partition functions

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Thank You !!

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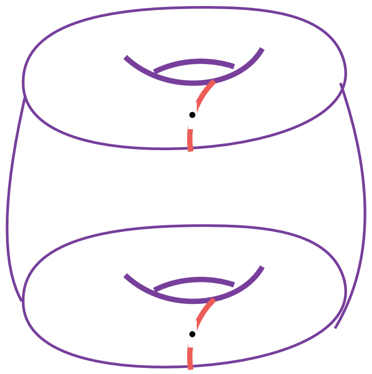
[B. Dittrich, SKA]

Example:

- consider background 'thermal spinning flat space time' in 4D

twisted solid torus



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Fourier modes

k_θ, k_y, k_z

Fourier transformation (twisted) on boundary $\gamma_{ab}(r, \theta, y, z) \to \gamma_{ab}(r, k_\theta, k_y, k_z)$

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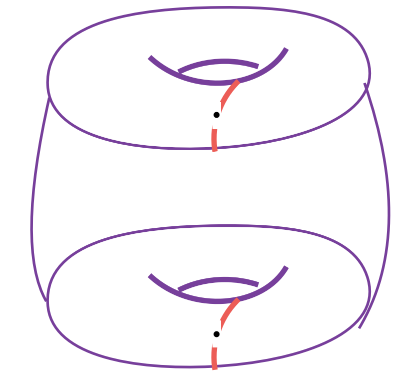

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Fourier modes
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Bulk equations of motion describe two propagating degrees of freedom — graviton modes

$$\mathscr{L}^{(2)} = w(r)\left(r w''(r) + w'(r) - \frac{1}{r}\left(k_\theta^2 + r^2 k_{yz}^2\right) w(r)\right) + \frac{x(r)}{2}\left[r x''(r) + x'(r) - \frac{1}{r}\left(r^2\Delta + \frac{k_{yz}^2}{\Delta} - \frac{3k_\theta^2 k_{yz}^2}{r^2\Delta^2}\right) x(r)\right]$$

$$\Delta = \frac{k_\theta^2}{r^2} + k_y^2 + k_z^2$$

Bulk solution

Solutions to the two graviton modes are given in terms of Bessel functions

$$w(r) = c_1\, I_{k_\theta}(k_{yz}\, r) + c_2\, K_{k_\theta}(k_{yz}\, r) \qquad x(r) = \frac{1}{\sqrt{\Delta}}\left(c_1\, I'_{k_\theta}(k_{yz}\, r) + c_2\, K'_{k_\theta}(k_{yz}\, r)\right)$$

Boundary effective action

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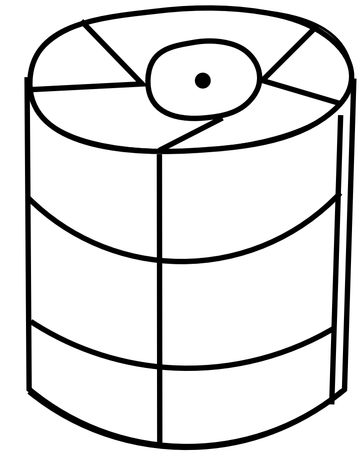

$$S^{(2)}_{\text{bdry}} = \int d^3y\,\sqrt{h}\left(\tilde{\gamma}_{AB}H_0^{ABCD}\tilde{\gamma}_{CD} + rw(r)\partial_r w(r) + \frac{r}{2}x(r)\partial_r x(r)\right)$$

Boundary conditions

Two boundaries— outer and inner boundaries at $r = R_{\text{in}}$ and $r = R_{\text{out}}$

One outer boundary $R_{\text{in}} \to 0$ use smoothness conditions at the origin

Boundary effective action

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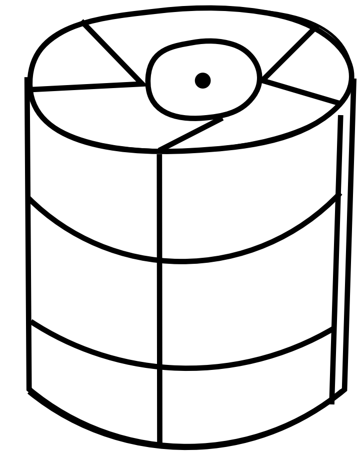

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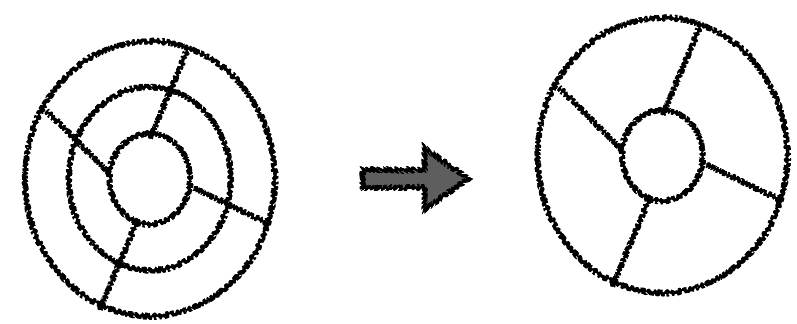
Renormalization procedure



Use H-J action for two boundaries as 'perfect' discrete action

$$S^{(0)}_{\rm disc}(q_1,q_2) = \alpha_1^{(0)}q_1^2 + \beta_1^{(0)}\,q_1q_2 + \alpha_2^{(0)}q_2^2$$

Discretization invariance of discrete action gives recursion relations for coefficients

$$\alpha_n^{(\tau+1)}(r_n,r_{n+1}) = \alpha_n^{(\tau)}(r_n,\tilde r) - \frac{(\beta_n^{(\tau)}(r_n,\tilde r))^2}{4\left(\alpha_n^{(\tau)}(\tilde r,r_{n+1}) + \alpha_{n+1}^{(\tau)}(r_n,\tilde r)\right)} \qquad \beta_n^{(\tau+1)}(r_n,r_{n+1}) = -\frac{\beta_n^{(\tau)}(r_n,\tilde r)\,\beta_n^{(\tau)}(\tilde r,r_{n+1})}{2\left(\alpha_n^{(\tau)}(\tilde r,r_{n+1}) + \alpha_{n+1}^{(\tau)}(r_n,\tilde r)\right)}$$

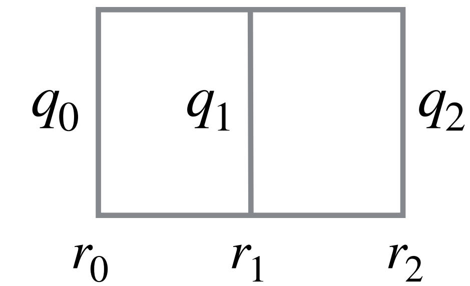
Perfect action solves recursion relations — coefficients are fixed points

Serves as refinement limit from Regge calculus

Renormalization

Discretization invariance for partition functions also gives recursion relations

$$Z(q_0, q_2) = \int \mathscr{D}q_1 \, Z(q_0, q_1) Z(q_1, q_2) \qquad \mathscr{D}q_1 = \mu \, dq_1$$



also get recurrence relations for measure

$$\mu_n^{(\tau+1)}(r_n, r_{n+1}) = \frac{\sqrt{\pi\hbar}\ \left(\mu_n^{(\tau)}(r_n, \tilde{r})\, \mu_n^{(\tau)}(\tilde{r}, r_{n+1})\right)}{2\sqrt{\left(\alpha_n^{(\tau)}(\tilde{r}, t_{n+1}) + \alpha_n^{(\tau)}(r_n, \tilde{r})\right)}}$$

solutions to measure recursion relations

$$\mu^{(*)}(r_i, r_j) = \sqrt{\frac{-2\beta^{(*)}(r_i, r_j)}{\pi\hbar}}$$

$$\beta_x^*(r_1, r_2) = \frac{2}{k_{yz}^2 \sqrt{\Delta_1 \Delta_2} \left(I_{k_\theta}(k_{yz} r_2) K_{k_\theta}(k_{yz} r_1) - I_{k_\theta}(k_{yz} r_1) K_{k_\theta}(k_{yz} r_2) \right)}$$

$$\beta_w^*(r_1, r_2) = \frac{2}{I_{k_\theta}(k_{yz} r_2) K_{k_\theta}(k_{yz} r_1) - I_{k_\theta}(k_{yz} r_1) K_{k_\theta}(k_{yz} r_2)}$$

Thus we get measure for the gravitons — non-vanishing β_n

For gauge sector: use gauge symmetry and Fadeev-Popov procedure to get measure factors

Compute one loop partition function