Remarks on Conformal Invariance of 3D Ising Model

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Based on: 1802.02319 w/ Meneses, Viana Parente Lopes,
Penedones, Yvernay
(v2 to appear)

Evidence for conf. invariance of 3D Ising

① CONFORMAL BOOTSTRAP

El-Showk, Paulos, Poland, SR, Simmons-Duffin, Vichi 2012 2014
Kos, Poland, Simmons-Duffin 2014
Kos, Poland, Simmons-Duffin, Vichi 2016
Komargodski, Simmons-Duffin 2016
Simmons-Duffin 2016

- Evidence for a <u>3d unitary CFT</u>
  - with $\mathbb{Z}_2$ global symmetry
  - one $\mathbb{Z}$-even and one $\mathbb{Z}_2$-odd scalar

$$\Delta_\varepsilon \approx 1.412625(10) \qquad \Delta_\sigma \approx 0.5181489(10)$$

- Exellent agreement (but more precise)
  - with lattice MC for ν, η Hasenbusch 2010
  - with RG determinations from ϕ^4 theory
    Guida, Zinn-Justin 1998

② LATTICE MONTE CARLO

Little work

→ Not much interest until recently

→ Not so simple

E.g. not easy to test Polyakov's triangle

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3) \rangle = \frac{\text{const}}{|x_{12}|^{h_{123}}\, |x_{13}|^{h_{132}}\, |x_{23}|^{h_{231}}}$$

$$h_{123} = \Delta_1 + \Delta_2 - \Delta_3$$

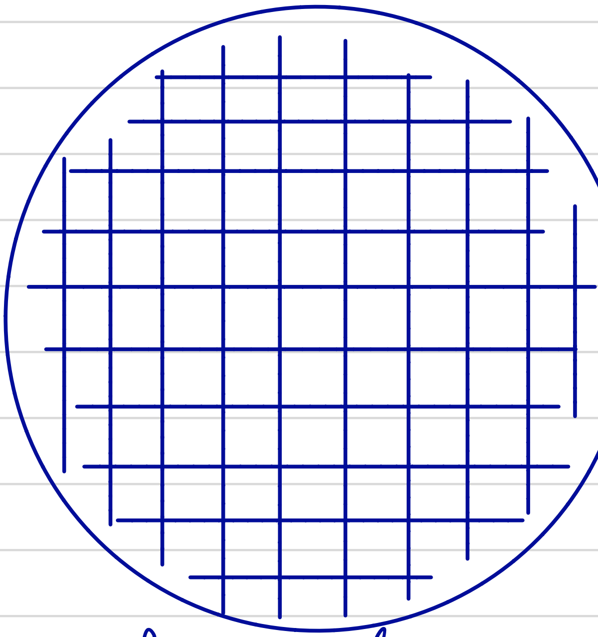
Existing tests use boundaries / defects

1. Cosme, Viana Parente Lopes, Penedones 2015

- 3d Ising inside a sphere $|x| \le R$

- Measure 1pt and 2pt functions

- Conf. inv. $\Longrightarrow$ $(R=1)$



free b.c.

$$\langle \varepsilon(x) \rangle = \frac{c}{(1-x^2)^{\Delta_\varepsilon}}$$

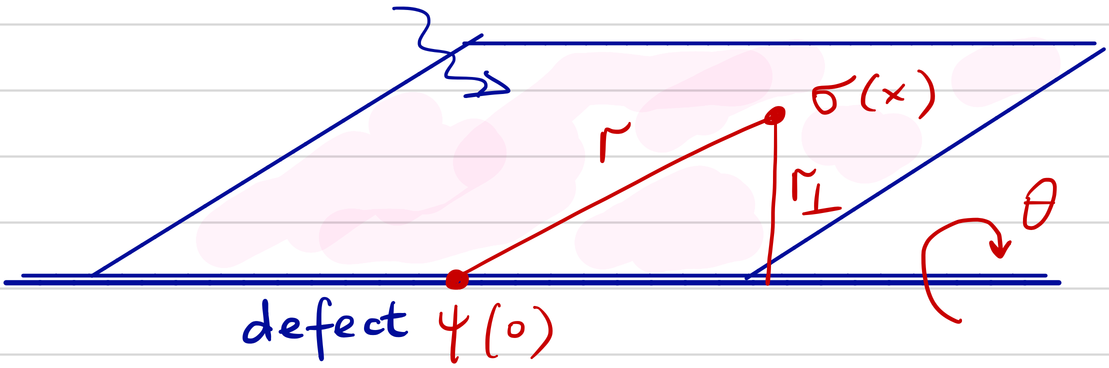
$$\langle \sigma(x_1)\,\sigma(x_2) \rangle = \frac{f(\zeta)}{(1-x_1^2)^{\Delta_\sigma}\,(1-x_2^2)^{\Delta_\sigma}} \qquad \left[\zeta = \frac{x_{12}^2}{(1-x_1^2)(1-x_2^2)}\right]$$

[Proved by mapping one point to the center]

Cf also Gori, Trombettoni 2015 for 3D percolation

(2) Billò, Caselle, Gaiotto, Gliozzi, Pellegrini 2013

Monodromy defect



Conf. inv.

$$\langle \sigma(x)\, \psi(0) \rangle = \text{const}\; e^{i\theta/2} \cdot \underbrace{\frac{1}{r_\perp^{\Delta_\sigma}} \left(\frac{r_\perp}{r}\right)^{\Delta_\psi}}_{\text{Conf. inv.}}$$

③ THEORETICAL ARGUMENTS

Scale invariance $\Longrightarrow$ conf. inv. ?

Review: Nakayama 2013

Classic argument [Polchinski 1988]

- To have (global) conformal inv. need

$$\delta S = \int T_{\mu\nu}\, \partial_\mu \varepsilon_\nu\, d^dx = 0 \qquad (*)$$

for ε_μ infinitesimal conformal

- $\partial_\mu \varepsilon_\nu + \partial_\nu \varepsilon_\mu \propto \delta_{\mu\nu} (\partial\cdot\varepsilon)$

- $\partial\cdot\varepsilon = 0$ for Poincaré
  $= \text{const}$ for scale
  $= \text{linear in } x$ for SCT

$$\boxed{\begin{aligned} T_\mu{}^\mu &= \partial_\nu \partial_\sigma L_{\nu\sigma} \Longrightarrow \text{conf. inv.} \\ T_\mu{}^\mu &= \partial_\nu W^\nu \Longrightarrow \text{scale inv.} \end{aligned}}$$

virial current

Careful: W may not be gauge inv.
[El-Showk, Nakayama, SR 2011]

So scale without conf. $\Longrightarrow$

$$T_\mu{}^\mu = \partial_\nu W^\nu$$

W is not a total derivative

$$[T] = d \Rightarrow [W] = d-1$$

Sufficient condition for conf. inv.

<u>NO VIRIAL CURRENT CANDIDATE</u>

[Polchinski 1988]

E.g. WF in $d = 4-\varepsilon$

The only vector op. of dim $3+O(\varepsilon)$ is $\varphi^2 \partial_\mu \varphi \propto \partial_\mu (\varphi^3)$

<u>Define</u> V_μ = lowest vector
$\mathbb{Z}_2$-even
parity-even
not a total derivative

- What is Δ_V in 3d Ising?

- If can show $\Delta_V > 2 \Rightarrow$ (physics) proof of conf. inv.

<u>Controversy</u> Delamotte, Tissier, Wschebor 2015

- "proof" that $\Delta_V > 2$
- I believe wrong
- Can say a few words if you're interested

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- showed $\Delta_V > 2$ using lattice MC

- a lattice test of conf. inv.

- unlike previous tests, we test the <u>underlying reason</u> rather then consequences.

- Not an easy measurement because Δ_V is large

- NB: nothing is known about Δ_V from the bootstrap, since

$$\sigma \times \sigma\,, \quad \sigma \times \varepsilon\,, \quad \varepsilon \times \varepsilon \not\ni V_\mu$$

Δ_V expectations from Wilson-Fischer

- 2D $\quad V \sim L_{-6}\bar{L}_{-7}\,\mathcal{E}$, $\Delta_V = 14$
- 4D free massless scalar

$$V \sim \varphi \text{ “}\partial^5 \varphi^5\text{”} \qquad \Delta_V = 11$$

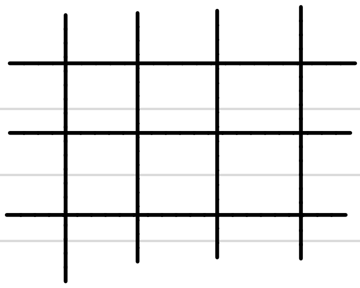
also [Meineri 2018]

$\Rightarrow$ So in 3D we expect $\Delta_V = O(10)$

Bad news for MC:

$$\langle O \rangle_{L^3} \sim \frac{\text{const}}{L^{\Delta_O}}$$

- E.g. $L = 10 \Rightarrow \langle O \rangle \sim 10^{-10}$
- Need 10^{20} configurations to resolve
- Cf. 10^{17} CPU cycles/year



$s(x) = \pm 1$ Ising spins

$$\nabla_\mu s = s(x+\hat{e}_\mu) - s(x-\hat{e}_\mu)$$

$$V_\mu^{lat} = s(x)\,\nabla_\mu s(x) \sum_{\nu=1}^{3} \left[\nabla_\nu s(x)\right]^2$$

simplest $\mathbb{Z}_2$-even lattice vector operator which is not a total lattice derivative

$$\boxed{\text{LATTICE} \to \text{CFT MATCHING}}$$

$V_\mu^{lat} \Rightarrow$ const $V_\mu + \ldots$

among which total derivatives!

1) of scalars $+\ \partial_\mu \varepsilon\ + \ldots$

$\Delta_\varepsilon \approx 1.41$

2) of tensors $+\ \partial^\nu T'_{\nu\mu}\ + \ldots$

$\Delta_{T'} \approx 5.5$ from bootstrap

- Expect

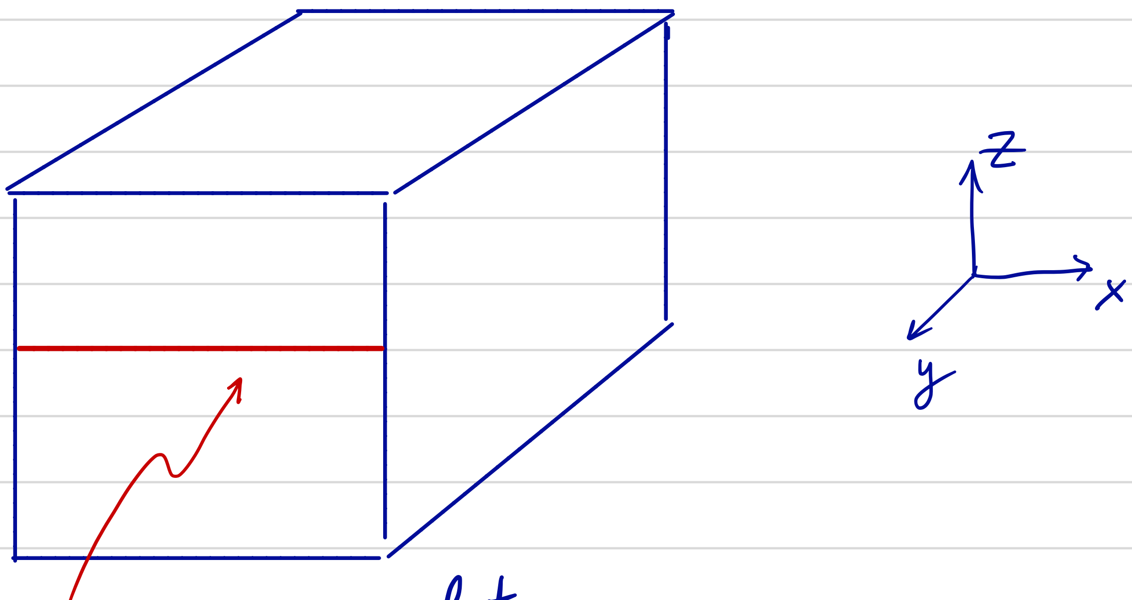
$$\langle V_\mu^{lat} \rangle_{L^3} = \frac{\text{const}}{L^{\Delta_0}}$$

O is the lowest of V_μ, $\partial_\mu \varepsilon$, $\partial^\nu T'_{\nu\mu}$

- Particularly annoying $\partial_\mu \varepsilon$

$\Delta(\partial_\mu \varepsilon) \approx 2.41$ close to 2

Our idea: integrate around a circle with periodic b.c.



$$A^{lat}(y,z) = \frac{1}{L} \sum_x V_x^{lat}(x,y,z)$$

- Kills $\partial_\mu(\text{scalar})$ contributions
  (but not $\partial^\nu T'_{\nu\mu}$)

- Must impose b.c. in y, z directions to break $x \to -x$ symmetry so that
$$\langle A^{lat}(y,z)\rangle \neq 0 .$$

- After some b.c. optimization we measured

$$\boxed{\langle A^{lat}\rangle = \frac{\text{const}}{L^{6\pm 1}}}$$

$(L \leq 16)$

300 CPU-years

$\Longrightarrow$

- $\Delta_V \gtrsim 5.0$
- $\Delta(\partial^\nu T'_{\nu\mu}) \approx 6.5$

  so most likely $\Delta_V > 5$

SUMMARY

- Revisited conf. inv. of 3d Ising
- A new lattice proof of conf. inv. via lower bound on virial candidate dimension $\Delta_V \geq 5.0$
- Can this be proven analytically?

"Proof" of Delamotte et al

- Work in discretized φ^4 theory

- $$V_\mu = \varphi \nabla_\mu \varphi \sum_{\nu=1}^{3} (\nabla_\nu \varphi)^2$$

$$= \sum_{e_i, e_j} \underbrace{\varphi(x) \nabla_\mu \varphi(x) \varphi(x+e_i) \varphi(x+e_j)}$$

$$\approx \nabla_\mu (\varphi^4)$$ bizarre claim

$$\Longrightarrow ???$$

- $$\langle V_\mu(x) V_\mu(0) \rangle \lesssim \langle \nabla_\mu(\varphi^4) \nabla_\mu(\varphi^4) \rangle$$

$$\sim \frac{1}{|x|^{2(\Delta_\varepsilon + 1)}}$$

- $$\Longrightarrow \Delta_V \geq \Delta_\varepsilon + 1 > 2$$

Can be also shown rigorously using Lebowitz inequalities

- The problem is the claim $\approx \nabla_\mu (\varphi^4)$

- They give incomprehensible (to me) arguments appealing to UV-completeness of $(\varphi^4)_3$

- Confused UV with IR ?

Need to set the record straight