Solvable Tensor Field Theory

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Based on:

R. Pascalie, C. I. Pérez-Sánchez, and R. Wulkenhaar, arXiv:1706.07358.

R. Pascalie, C. I. Pérez-Sánchez, A. Tanasa and R. Wulkenhaar, arXiv:1810.09867.

R. Pascalie, arXiv:1903.02907.

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Outline

The model

Complex rank-3 tensor field theory:

$$\mathcal{S}[\varphi,\bar{\varphi}] = \sum_{\mathbf{x}} \bar{\varphi}^{\mathbf{x}}(1+|\mathbf{x}|^2)\varphi^{\mathbf{x}} + \mathcal{S}_{\text{int}}[\varphi,\bar{\varphi}], \tag{1}$$

with $\mathbf{x} = (x_1, x_2, x_3) \in \{\frac{1}{N}, \frac{2}{N}, \dots, 1\}^3$, $|\mathbf{x}|^2 = x_1^2 + x_2^2 + x_3^2$.

Kinetic term $\to$ discrete Laplacian in Fourier.

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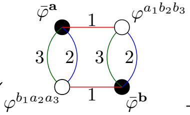
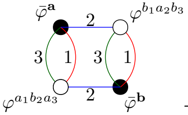
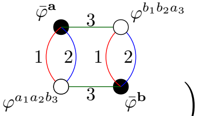
$\mathrm{U}(N)^3$-invariant "pillow" interactions:

$$\mathcal{S}_{\text{int}}[\varphi,\bar\varphi] = \frac{\lambda}{N^2}\sum_{\mathbf{a},\mathbf{b}} \Big(\quad + \quad + \quad \Big). \tag{2}$$







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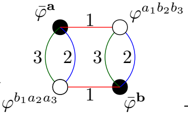

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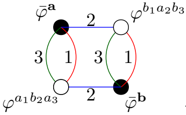

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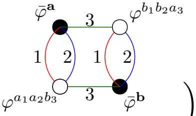


$$\Bigg). \tag{2}$$

Generating functional:

$$\mathrm{Z}[J,\bar J] = \int \mathcal{D}\varphi\mathcal{D}\bar\varphi \exp\left(-\mathcal{S}[\varphi,\bar\varphi] + \sum_{\mathbf{x}}(\bar J_{\mathbf{x}}\varphi^{\mathbf{x}} + J_{\mathbf{x}}\bar\varphi^{\mathbf{x}})\right). \tag{3}$$

Motivation

- Renormalization group flow induce an effective laplacian.
- Generalize to tensor models, technics from similar matrix model: $S[M] = \mathrm{Tr}(EM^2) + \lambda \mathrm{Tr}(M^4)$ (Grosse-Wulkenhaar model).
- Find solvable model.

Boundary graph

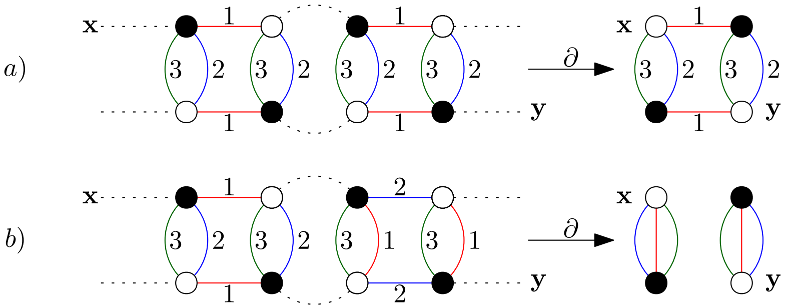
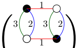


Figure: Two connected Feynman graphs and the associated boundary graphs. In figure a) the boundary graph is connected, in figure b) the boundary graph $\mathrm{m}|\mathrm{m}$ is disconnected.

Free energy

Boundary graphs expansion of the form:

$$\mathrm{W}[J,\bar{J}] = \sum_{k=1}^{\infty} \sum_{\mathcal{B}} \sum_{\mathbf{x^1},\dots,\mathbf{x}^k} N^{\alpha(\mathcal{B})} \mathrm{G}_{\mathcal{B}}^{(2k)}(\mathbf{x}^1,\dots,\mathbf{x}^k) \cdot \mathbb{J}(\mathcal{B})(\mathbf{x}^1,\dots,\mathbf{x}^k). \quad (4)$$

Examples: $\mathbb{J}\Big($ 

$\Big)(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{x_1 y_2 y_3} \bar{J}_{y_1 x_2 x_3}$, $\mathbb{J}(\mathrm{m}|\mathrm{m})(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{\mathbf{x}} \bar{J}_{\mathbf{y}}$.

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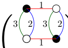
Examples: $\mathbb{J}\Big($  $\Big)(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{x_1 y_2 y_3} \bar{J}_{y_1 x_2 x_3}$, $\mathbb{J}(\mathrm{m}|\mathrm{m})(\mathbf{x},\mathbf{y}) = J_{\mathbf{x}} J_{\mathbf{y}} \bar{J}_{\mathbf{x}} \bar{J}_{\mathbf{y}}$.

The scaling is

$$\alpha(\mathcal{B}) = 3 - B - 2g - 2k, \quad (5)$$

$2k$ is the number of vertices of $\mathcal{B}$, B its number of connected components and g its genus.

In matrix model: $\alpha(\mathcal{B}) = 2 - B - 2g$ (H. Grosse, R. Wulkenhaar, arXiv:1402.1041.).

Ward-Takahashi Identity

Fields transform as

$$\varphi^{\mathbf{x}} \to \varphi^{\mathbf{x}} = \sum_{y_1} U^{(1)}_{x_1 y_1} \varphi^{y_1 x_2 x_3}, \quad \bar{\varphi}^{\mathbf{x}} \to \bar{\varphi}^{\mathbf{x}} = \sum_{y_1} \bar{U}^{(1)}_{x_1 y_1} \bar{\varphi}^{y_1 x_2 x_3}, \tag{6}$$

for $U^{(1)} \in \mathrm{U}(N)$, similarly for the other indices.

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for $U^{(1)} \in \mathrm{U}(N)$, similarly for the other indices.

The WTI for the colour 1 writes:

$$\sum_{q_2, q_3} \frac{\delta^2 \, \mathrm{Z}[J, \bar{J}]}{\delta J_{x_1 q_2 q_3} \delta \bar{J}_{y_1 q_2 q_3}} - \delta_{x_1 y_1} \mathrm{Y}^{(1)}_{x_1}[J, \bar{J}] \cdot \mathrm{Z}[J, \bar{J}]$$
$$= \frac{1}{x_1^2 - y_1^2} \sum_{q_2, q_3} \left(\bar{J}_{x_1 q_2 q_3} \frac{\delta}{\delta \bar{J}_{y_1 q_2 q_3}} - J_{y_1 q_2 q_3} \frac{\delta}{\delta J_{x_1 q_2 q_3}} \right) \mathrm{Z}[J, \bar{J}], \tag{7}$$

where

$$\mathrm{Y}^{(1)}_{x_1}[J, \bar{J}] = \sum_{q_2 q_3} \frac{\delta^2 \, \mathrm{W}[J, \bar{J}]}{\delta J_{x_1 q_2 q_3} \delta \bar{J}_{x_1 q_2 q_3}}. \tag{8}$$

2-point function 1

The 2-point function explicitly writes

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{\mathrm{Z}_0} \frac{\delta^2 \mathrm{Z}[J,\bar{J}]}{\delta \bar{J}_{\mathbf{x}} \delta J_{\mathbf{x}}}\bigg|_{J=\bar{J}=0} \tag{9}$$
$$= \frac{1}{1+|\mathbf{x}|^2} - \frac{1}{\mathrm{Z}_0} \frac{1}{1+|\mathbf{x}|^2} \left(\bar{\varphi}^{\mathbf{x}} \frac{\partial \mathcal{S}_{\text{int}}}{\partial \bar{\varphi}^{\mathbf{x}}} \right) \left[\frac{\delta}{\delta J}, \frac{\delta}{\delta \bar{J}} \right] \mathrm{Z}[J,\bar{J}] \bigg|_{J,\bar{J}=0},$$

$\bar{\varphi}^{\mathbf{x}} \frac{\partial \mathcal{S}_{\text{int}}}{\partial \bar{\varphi}^{\mathbf{x}}}$ contains 4 derivatives $\rightarrow$ trade 2 using the WTI.

2-point function 2

Using the WTI we get:

$$G^{(2)}(\mathbf{x}) = \frac{1}{1+|\mathbf{x}|^2} - \frac{2\lambda}{1+|\mathbf{x}|^2}\Bigg(\frac{1}{N^2}\sum_{q_2,q_3} G^{(2)}(x_1,q_2,q_3)G^{(2)}(\mathbf{x}) + \frac{1}{N^4}G_1^{(4)}(\mathbf{x},\mathbf{x})$$
$$+\frac{1}{N^5}\sum_{q_2,q_3} G^{(4)}_{m|m}(x_1,q_2,q_3,\mathbf{x}) + \frac{1}{N^4}\sum_{q_3} G_2^{(4)}(\mathbf{x},x_1,x_2,q_3)$$
$$+\frac{1}{N^4}\sum_{q_2} G_3^{(4)}(\mathbf{x},x_1,q_2,x_3) + \frac{1}{N^2}\sum_{q_1}\frac{G^{(2)}(q_1,x_2,x_3)-G^{(2)}(\mathbf{x})}{x_1^2-q_1^2} + \text{perm.}\Bigg), \tag{10}$$

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$$+\frac{1}{N^5}\sum_{q_2,q_3}\mathrm{G}^{(4)}_{m|m}(x_1,q_2,q_3,\mathbf{x}) + \frac{1}{N^4}\sum_{q_3}\mathrm{G}^{(4)}_2(\mathbf{x},x_1,x_2,q_3)$$
$$+\frac{1}{N^4}\sum_{q_2}\mathrm{G}^{(4)}_3(\mathbf{x},x_1,q_2,x_3) + \frac{1}{N^2}\sum_{q_1}\frac{\mathrm{G}^{(2)}(q_1,x_2,x_3)-\mathrm{G}^{(2)}(\mathbf{x})}{x_1^2-q_1^2} + \text{perm.}\Bigg), \tag{10}$$

In the large N limit the SDE is:

$$\mathrm{G}^{(2)}(\mathbf{x}) = \left(1+|\mathbf{x}|^2 + 2\lambda\Big(\int \mathrm{d}q_2\mathrm{d}q_3\ \mathrm{G}^{(2)}(x_1,q_2,q_3) + \text{perm.}\Big)\right)^{-1}. \tag{11}$$

Perturbative expansion 1

Plugging in the 2-point function SDE the following expansion

$$\mathrm{G}^{(2)}(\mathbf{x}) = \sum_{n\geq 0} \lambda^n \mathrm{G}_n^{(2)}(\mathbf{x}), \tag{12}$$

we obtain a recursive equation for $n \geq 1$:

$$\mathrm{G}_n^{(2)}(\mathbf{x}) = -\frac{2}{1+|\mathbf{x}|^2} \sum_{k=0}^{n-1} \Big(\int \mathrm{d}q_2 \mathrm{d}q_3 \big(\mathrm{G}_k^{(2)}(x_1, q_2, q_3) - \frac{\delta_{k0}}{1+q_2^2+q_3^2} \big) + \text{perm.} \Big) \mathrm{G}_{n-k-1}^{(2)}(\mathbf{x}), \tag{13}$$

with Taylor subtraction to regularise the UV divergences.

Perturbative expansion 2

The first few orders are:

$$\mathrm{G}_0^{(2)}(\mathbf{x}) = \frac{1}{1+|\mathbf{x}|^2}, \tag{14}$$

$$\mathrm{G}_1^{(2)}(\mathbf{x}) = \frac{\pi}{2(1+|\mathbf{x}|^2)^2} \sum_{c=1}^{3} \log\left(x_c^2+1\right), \tag{15}$$

$$\mathrm{G}_2^{(2)}(\mathbf{x}) = \frac{1}{(1+|\mathbf{x}|^2)^2} \left(\sum_{c=1}^{3}\sum_{d=1}^{3} \frac{\pi^2 \log\left(x_c^2+1\right)\log\left(x_d^2+1\right)}{4(1+|\mathbf{x}|^2)} - \sum_{c=1}^{3} \frac{\pi \log\left(x_c^2+1\right)}{2(x_c^2+1)} \right.$$
$$\left. - \pi^2 \sum_{c=1}^{3} \frac{x_c \log\left(\frac{1}{4}\left(x_c^2+1\right)\right) + 2\tan^{-1}(x_c)}{2\left(x_c^3+x_c\right)} \right). \tag{16}$$

Perturbative expansion 3

Last term in $G_2^{(2)}(\mathbf{x})$ is not a power of logarithms. Graphically:

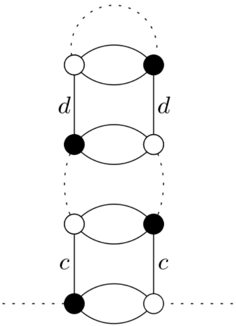

Figure: The only graphs at 2-loop order, giving contribution other than powers of logarithms for $d \neq c$.

If only 1 pillow interaction, such graphs cannot exist $\rightarrow$ only powers of logarithms.

Model with 1 pillow interaction

By computing up to order λ^9, we found the ansatz for any $n \geq 1$:

$$\mathrm{G}_n^{(2)}(\mathbf{x}) = \left(\frac{\pi}{2}\right)^n \Bigg(\frac{\log^n(1+x_1^2)}{(1+|\mathbf{x}|^2)^{n+1}} + (n-1)! \sum_{k=0}^{n} \sum_{j=1}^{n-1} \frac{s_{j,n-k}}{j!k!} \frac{(-1)^j(n-j)}{(1+|\mathbf{x}|^2)^{n+1-j}(1+x_1^2)^j} \log^k(1+x_1^2) \Bigg), \quad (17)$$

where $s_{n,k}$ are the Stirling numbers of the 1st kind.

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We first rewrite the 2-point function as

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{1+|\mathbf{x}|^2} - \sum_{n=1}^{\infty} \left(\frac{\pi}{2}\right)^n \frac{\lambda^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}(x_1^2)^{n-1}} \frac{(-\log(1+x_1^2))^n}{(1+|\mathbf{x}|^2)^2}. \quad (18)$$

Resummation 1

To sum the series we use:

Theorem (Lagrange-Bürmann inversion formula)

Let $\phi(\omega)$ be analytic at $\omega = 0$ with $\phi(0) \neq 0$ and $f(\omega) = \frac{\omega}{\phi(\omega)}$. Then the inverse function $g(z)$ of $f(\omega)$ with $z = f(g(z))$, is analytic at $z = 0$ and given by

$$g(z) = \sum_{n=1}^{\infty} \frac{z^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}\omega^{n-1}} \phi(\omega)^n \bigg|_{\omega=0} . \tag{19}$$

Moreover, for any analytic function $H(z)$ with $H(0) = 0$,

$$H(g(z)) = \sum_{n=1}^{\infty} \frac{z^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}\omega^{n-1}} \Big(H'(\omega)\phi(\omega)^n \Big) \bigg|_{\omega=0} . \tag{20}$$

Resummation 2

Setting $z = \frac{\pi}{2}\lambda$ and $\phi(\omega) = -\log(1+\omega+x_1^2)$,

$$g(x_1, z) = \sum_{n=1}^{\infty} \frac{z^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}(x_1^2)^{n-1}} (-\log(1+x_1^2))^n, \tag{21}$$

is the inverse function of $f(\omega) = \frac{\omega}{\phi(\omega)} = -\frac{\omega}{\log(1+x_1^2+\omega)}$:

$$z = f(g(x_1, z)) = -\frac{g(x_1, z)}{\log(1+x_1^2+g(x_1, z))}, \tag{22}$$

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which is solved by

$$g(x_1, z) = zW\left(\frac{1}{z} e^{\frac{1+x_1^2}{z}}\right) - 1 - x_1^2, \tag{23}$$

where $W(z)$ is the Lambert function defined by $z = W(ze^z)$.

Solution

With $H(\omega) = \frac{1}{1+\omega+|\mathbf{x}|^2} - \frac{1}{1+|\mathbf{x}|^2}$, the 2-point function is:

$$\begin{aligned} \mathrm{G}^{(2)}(\mathbf{x}) &= \frac{1}{1+|\mathbf{x}|^2} - \sum_{n=1}^{\infty} \frac{z^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}(x_1^2)^{n-1}} \frac{(-\log(1+x_1^2))^n}{(1+|\mathbf{x}|^2)^2} \\ &= \frac{1}{1+|\mathbf{x}|^2} + H(g(x_1,z)) = \frac{1}{1+|\mathbf{x}|^2+g(x_1,z)}. \end{aligned} \tag{24}$$

Solution

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$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{1+|\mathbf{x}|^2} - \sum_{n=1}^{\infty} \frac{z^n}{n!} \frac{\mathrm{d}^{n-1}}{\mathrm{d}(x_1^2)^{n-1}} \frac{(-\log(1+x_1^2))^n}{(1+|\mathbf{x}|^2)^2}$$
$$= \frac{1}{1+|\mathbf{x}|^2} + H(g(x_1,z)) = \frac{1}{1+|\mathbf{x}|^2+g(x_1,z)}. \quad (24)$$

To prove it is indeed a solution, we integrate:

$$\int \mathrm{d}q_2 \mathrm{d}q_3 \big(G(x_1,q_2,q_3) - \frac{1}{1+q_2^2+q_3^2} \big) = -\frac{\pi}{4} \log\left(1+x_1^2+g(x_1,z)\right), \quad (25)$$

plug it in the regularised SDE

$$\mathrm{G}^{(2)}(\mathbf{x}) = \Big(1+|\mathbf{x}|^2 + 2\lambda \int \mathrm{d}q_2 \mathrm{d}q_3 \big(G(x_1,q_2,q_3) - \frac{1}{1+q_2^2+q_3^2} \big) \Big)^{-1}, \quad (26)$$

and recover the equation solved by $g(x_1,z)$.

Solution 2

Sum up: the regularised SDE for the 2-point function

$$\mathrm{G}^{(2)}(\mathbf{x}) = \Big(1 + |\mathbf{x}|^2 + 2\lambda \int \mathrm{d}q_2\mathrm{d}q_3\big(G(x_1, q_2, q_3) - \frac{1}{1 + q_2^2 + q_3^2}\big)\Big)^{-1}, \tag{27}$$

is solved by

$$\mathrm{G}^{(2)}(\mathbf{x}) = \frac{1}{1 + |\mathbf{x}|^2 + g(x_1, z)}, \tag{28}$$

where

$$g(x_1, z) = zW\left(\frac{1}{z} e^{\frac{1+x_1^2}{z}}\right) - 1 - x_1^2, \tag{29}$$

with $z = \frac{\pi}{2}\lambda$ and $W(z)$ the Lambert function.

Higher point functions

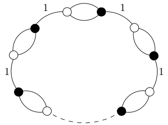

Figure: General form of a connected boundary graph in the model with 1 pillow interaction.

All higher point functions with **connected** boundary graph are obtained recursively:

$$\mathrm{G}^{(2k)}(\mathbf{x}^1, \ldots, \mathbf{x}^k) = 2\lambda \mathrm{G}^{(2)}(x_1^1, x_2^2, x_3^2) \sum_{\rho=2}^{k} \mathrm{G}^{(2k-2\rho+2)}(\mathbf{x}^\rho, \ldots, \mathbf{x}^k)$$
$$\frac{\mathrm{G}^{(2\rho-2)}(\mathbf{x}^1, \ldots, \mathbf{x}^{\rho-1}) - \mathrm{G}^{(2\rho-2)}(x_1^\rho, x_2^1, x_3^1, \ldots, \mathbf{x}^{\rho-1})}{(x_1^1)^2 - (x_1^\rho)^2}. \quad (30)$$

In particular,

$$\mathrm{G}^{(4)}(\mathbf{x}, \mathbf{y}) = 2\lambda \mathrm{G}^{(2)}(x_1, y_2, y_3) \mathrm{G}^{(2)}(\mathbf{y}) \frac{\mathrm{G}^{(2)}(\mathbf{x}) - \mathrm{G}^{(2)}(y_1, x_2, x_3)}{x_1^2 - y_1^2}. \quad (31)$$

Perspectives

- Proving the conjecture on the large N limit, using the SDE for disconnected boundary graphs.
- Finding the 2-point function in the model with the 3 pillow interactions.
- Implementing this methods for the study of SYK-like tensor models.