# Asymptotic safety
# 40 years later

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## The continuum limit

In a discrete approach with "lattice spacing" $a$, let $a \to 0$.

In a continuum field theory with UV momentum cutoff $\Lambda$, let $\Lambda \to \infty$.

Continuum limit can be achieved by choosing an RG trajectory that ends at a Fixed Point.

## Asymptotic safety

For most trajectories some couplings blow up for $k \to \infty$.
Sometimes they even blow up at finite $k$ (Landau poles).
Generically leads to divergences in physical observables.

*"UV safe"* or *"renormalizable"* RG trajectories
= RG trajectories that reach a FP in the UV
= UV complete theories

Use as criterion to select theories

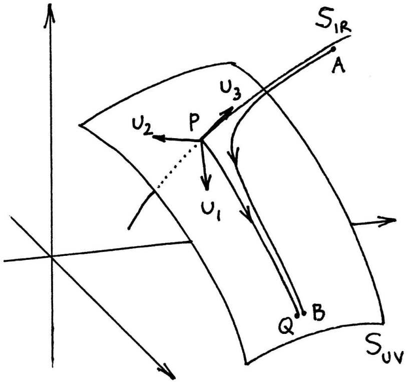
## Predictivity

Perturbations around FP can be

- attracted to FP in UV = repelled by FP in IR =**relevant**
- repelled by FP in UV = attracted to in IR =**irrelevant**

Every irrelevant direction leads to a prediction.
Generally expect only finitely many relevant directions.

## General picture



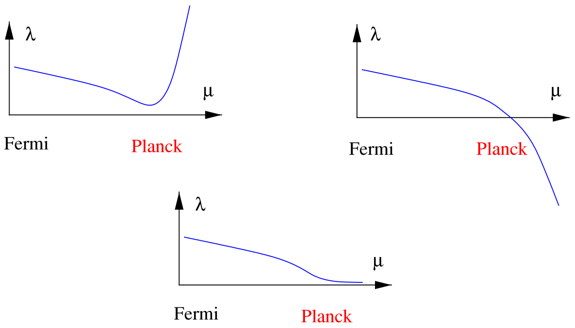
## Corrections to running of $\lambda$

$$\begin{aligned} V(\phi^2) &= \lambda_0 + \lambda_2\phi^2 + \lambda_4\phi^4 + \dots \\ F(\phi^2) &= \xi_0 + \xi_2\phi^2 + \dots \end{aligned}$$

$$\partial_t\lambda_4 = \frac{9\lambda_4^2}{2\pi^2} + f\tilde{G}\lambda_4 + \dots$$

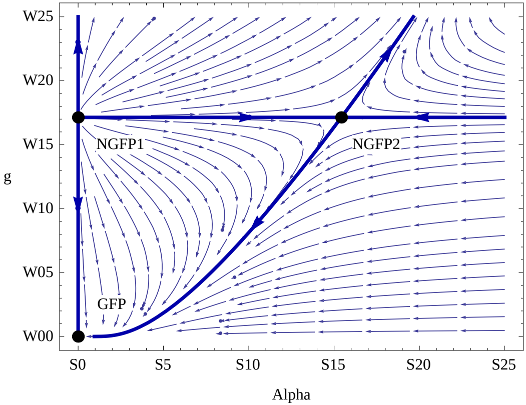
$f > 0$

## Prediction of the Higgs mass




[M. Shaposhnikov and C. Wetterich, Phys.Lett. B683, 196 (2010) ]

# Possible calculation of fine structure constant




$\alpha - \alpha_*$ irrelevant at NGFP2.

[U. Harst and M. Reuter, JHEP 1105, 119 (2011), arXiv:1101.6007 [hep-th]]

[A. Eichhorn and F. Versteegen, arXiv:1709.07252 [hep-th]]

## Examples of AS

QCD:

- Gaußian Fixed Point at $\tilde{g}_{i*} = 0$.
- relevant couplings=renormalizable couplings

Non-AF examples?

## General NLSM

$$\frac{1}{2} Z \int d^d x \, \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta h_{\alpha\beta}(\varphi)$$

$Z = \frac{1}{g^2}$
$Z \approx \mathrm{mass}^{d-2}$, $g \approx \mathrm{mass}^{\frac{2-d}{2}}$
nonrenormalizable in $d > 2$

Ricci flow:

$$\frac{d}{dt}\left(Z h_{\alpha\beta}\right) = 2 c_d k^{d-2} R_{\alpha\beta}$$

$c_d = \frac{1}{(4\pi)^{d/2}\Gamma(d/2+1)}$

## $O(N)$ NLSM

In $d = 2$:

$$\beta_{g^2} = -2c_2(N-2)\tilde{g}^4$$

In $d = 2 + \epsilon$: $\tilde{g}^2 = k^{d-2} g^2$ and

$$\beta_{g^2} = \epsilon \tilde{g}^2 - 2c_d(N-2)\tilde{g}^4$$

non gaussian FP at

$$\tilde{g}_*^2 = \frac{d-2}{2} \frac{1}{c_d(N-2)} \; .$$

$\left.\frac{d\beta}{d\tilde{g}}\right|_* = 2 - d.$
Confirmed by Monte-Carlo calculations in $d = 3$.

## Gauge theories in $d = 4$: one loop

$$
\begin{aligned}
L_{YM} &= -\frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu} \\
L_F &= \bar{\psi} i \not{D} \psi
\end{aligned}
$$

$$\alpha_g = \frac{g^2 N_c}{(4\pi)^2}$$

$$\beta_g = -B\alpha_g^2$$

$$B = -\frac{4}{3}\epsilon \; ; \qquad \epsilon = \frac{N_F}{N_c} - \frac{11}{2}$$

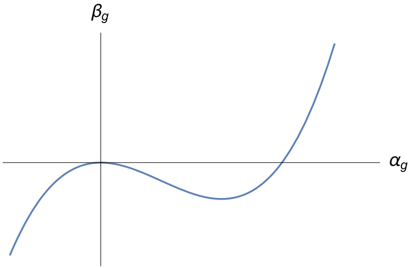
$$N_F < \tfrac{11}{2} N_c \Longrightarrow \epsilon < 0 \Longrightarrow B > 0 \Longrightarrow \text{AF}$$

## Gauge theories in $d = 4$: two loops

$$\beta_g = -B\alpha_g^2 + C\alpha_g^3$$

$$\alpha_{g*} = B/C \; ; \qquad \beta'(\alpha_{g*}) = B^2/C$$

AF+IR FP if $C > 0$.




[W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244 found $C \geq 0$]

[T. Banks and A. Zaks, Nucl. Phys. B 196 (1982)]

## From AF to AS: Yukawa couplings

$$
\begin{aligned}
L_H &= \mathrm{tr}(\partial^\mu H)^\dagger(\partial_\mu H) \\
L_Y &= y\,\mathrm{tr}(\bar{\psi}_L H \psi_R + \bar{\psi}_R H \psi_L)
\end{aligned}
$$

$$
\alpha_y = \frac{y^2 N_c}{(4\pi)^2}
$$

$$
\begin{aligned}
\beta_g &= \alpha_g^2 \left[\frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{3} + \epsilon\right)\alpha_y\right] \\
\beta_y &= \alpha_y \left[(13 + 2\epsilon)\alpha_y - 6\alpha_g\right]
\end{aligned}
$$

## Fixed points

$$(\alpha_{g*}, \alpha_{y*}) = \left(-\frac{4\epsilon}{75+26\epsilon}, 0\right)$$

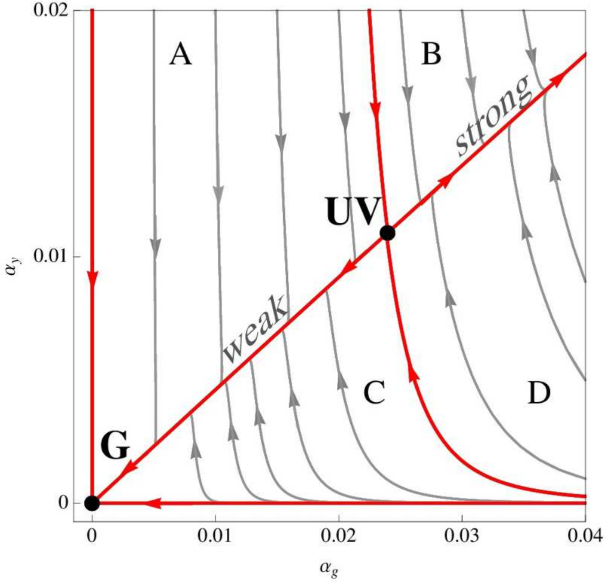
for $\epsilon < 0$, Banks-Zaks

$$\begin{aligned}(\alpha_{g*}, \alpha_{y*}) &= \left(\frac{2\left(13\epsilon + 2\epsilon^2\right)}{57-46\epsilon-8\epsilon^2}, \frac{12\epsilon}{57-46\epsilon-8\epsilon^2}\right) \\ &\approx (0.456\epsilon + O(\epsilon^2), 0.211\epsilon + O(\epsilon^2))\end{aligned}$$

for $\epsilon > 0$, Litim and Sannino

[D.F. Litim and F. Sannino, JHEP 1412 (2014) 178 ]

# Phase diagram

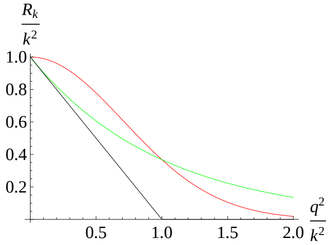


## EAA

$$e^{-W_k[J]} = \int (D\phi) \exp(-(S + \Delta S_k + \int J\phi))$$

Insert in the PI a term

$$\frac{1}{2}\int \phi \mathcal{R}_k(\Delta)\phi$$




$$\Gamma_k[\phi] = W_k[J] - \int J\phi - \Delta S_k$$

## ERGE

Think of $\Gamma_k$ as the result of having integrated out all modes with energy $> k$.
$k$ is the UV cutoff for the EFT describing the low momentum modes.

Dependence of $\Gamma_k$ on $k$ is such that the PI on modes from $k$ to 0 is fixed.

$\Gamma_0$ is the standard EA

FRGE

$$k\frac{d\Gamma_k}{dk} = \mathrm{Tr}\left(\frac{\delta\Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k\right)^{-1} k\frac{d\mathcal{R}_k}{dk}$$

[C. Wetterich, Phys. Lett. B **301** (1993) 90.]

## Beta functions

$$\Gamma_k(\phi) = \sum_i g_i(k)\mathcal{O}_i(\phi)$$

$$\partial_t \Gamma_k = \sum_i \partial_t g_i \mathcal{O}_i = \sum_i \beta_{g_i} \mathcal{O}_i$$

compare with

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{Tr}\left(\frac{\delta^2 \Gamma_k}{\delta\phi\delta\phi} + \mathcal{R}_k\right)^{-1} \partial_t \mathcal{R}_k$$

read off beta functions.

## Theory space

Beta functions

$$\beta_i(g_j, k) = k\frac{dg_i}{dk}$$

Dimensionless combinations $\tilde{g}_i = k^{-d_i} g_i$
are coordinates in theory space.

$$\tilde{\beta}_i(\tilde{g}_j) \equiv k\frac{d\tilde{g}_i}{dk} = -d_i\tilde{g}_i + k^{-d_i}\beta_i$$

and from dimensional analysis $\tilde{\beta}_i(\tilde{g}_j)$.
Well-defined flow on theory space.

## Scale invariance

$$\delta_\epsilon g_{\mu\nu} = 2\epsilon g_{\mu\nu}$$
$$\delta_\epsilon \phi = -\frac{d-2}{2}\epsilon\phi$$

$$T^{\mu\nu} = -\frac{2}{\sqrt{g}}\frac{\delta S}{\delta g_{\mu\nu}}\,.$$

Classical scale invariance

$$\delta_\epsilon S = \epsilon \int T^{\mu}{}_{\mu} = 0$$

implies

$$V(\phi) = \frac{\lambda}{4!}\,\phi^4\,,$$

## The anomaly in perturbatively renormalizable QFT

However

$$\delta_\epsilon \Gamma = \epsilon \int_x \langle T^\mu{}_\mu \rangle \equiv -\mathcal{A}(\epsilon)$$

$$\mathcal{A}(\epsilon) = \epsilon\, \beta \int_x \frac{1}{4!} \phi^4 \ ,$$

$$\beta = \frac{3\lambda^2}{16\pi^2} \, .$$

# WI of scale transformations

For the EAA

$$\delta_\epsilon \Gamma_k = -\mathcal{A}(\epsilon) + \epsilon \partial_t \Gamma_k \ ,$$

[T.Morris, R.P. Phys.Rev. D99 (2019) 105007 arXiv:1810.09824 [hep-th]]

$$\Gamma_k = \sum_i \lambda_i(k)\,\mathcal{O}_i \ ,$$

$\mathcal{O}_i$ has mass dimension $\Delta$ We can also write

$$\lambda_i\,\mathcal{O}_i = \tilde{\lambda}_i\,\tilde{\mathcal{O}}_i$$

where

$$\tilde{\lambda}_i = k^{\Delta}\lambda_i \ ; \qquad \tilde{\mathcal{O}}_i = k^{-\Delta}\mathcal{O}_i \,,$$

which thus implies that $\tilde{\lambda}_i$ is dimensionless. The l.h.s. of the WI is

$$\delta_\epsilon\left(\lambda_i\,\mathcal{O}_i\right) = -\epsilon\,\Delta\,\lambda_i\,\mathcal{O}_i$$

On the other hand

$$\partial_t \left(\lambda_i \, \mathcal{O}_i\right) = \partial_t \lambda_i \, \mathcal{O}_i \ ,$$

Thus the WI gives

$$-\epsilon\Delta \, \lambda_i \mathcal{O}_i = -\mathcal{A}(\epsilon) + \epsilon \partial_t \lambda_i \, \mathcal{O}_i$$

Bringing the l.h.s. to the r.h.s. it reconstructs the derivative of $\tilde{\lambda}_i$, times $k^{-\Delta}$, which can be rewritten as

$$\mathcal{A}(\epsilon) = \epsilon \, \partial_t \tilde{\lambda}_i \, \tilde{\mathcal{O}}_i \ .$$

Thus the WI implies

$$\mathcal{A}(\epsilon) = \epsilon \sum_i \tilde{\beta}_i \, \tilde{\mathcal{O}}_i$$

The anomaly vanishes at a FP

This does not imply that $\Gamma_k$ is scale invariant according to the definition given above.

However, define $\hat{\delta}_\epsilon$

$$\hat{\delta}_\epsilon k = -\epsilon\, k \ ,$$

and the same as the action of $\delta_\epsilon$ on all other quantities. Then

$$\hat{\delta}_\epsilon \Gamma_k = \delta_\epsilon \Gamma_k - \epsilon k \partial_k \Gamma_k = \mathcal{A}(\epsilon)$$

This implies that at a fixed point one has scale invariance in the sense of $\hat{\delta}_\epsilon$.

## RG equation for gravity

Using background field method

$$\partial_t \Gamma_k = \frac{1}{2} \mathrm{Tr} \frac{\partial_t \mathcal{R}_k}{\mathcal{H} + \mathcal{R}_k} + \text{ghost term}$$

where

$$\frac{\delta^2 \Gamma_k}{\delta g_{\mu\nu} \delta g_{\alpha\beta}} = \mathcal{H}^{\mu\nu,\alpha\beta},$$

## EH truncation

$$\Gamma_k(g_{\mu\nu}) \approx \int dx \sqrt{g} Z_N (2\Lambda - R) \ ; \quad Z_N = \frac{1}{16\pi G}$$

$$\mathcal{H} = Z_N \mathcal{O}$$

$$\mathcal{O} = \Delta + U$$

with $\Delta = -\bar{\nabla}^2$ and $U \sim R$.

It is therefore very convenient to choose

$$\mathcal{R}_k = Z_N R_k(\Delta) \ ,$$

so that

$$\dot{\Gamma}_k = \frac{1}{2}\mathrm{Tr}\frac{\partial_t[Z_N R_k(\Delta^2)]}{Z_N[\mathcal{O}+R_k(\Delta)]} = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k(\Delta) - \eta_N R_k(\Delta)}{\mathcal{O}+R_k(\Delta)},$$

where

$$\eta_N = -\frac{d\log Z_N}{dt} = \frac{\partial_t G}{G} = \frac{\partial_t \tilde{G}}{\tilde{G}} - (d-2)$$

$\tilde{G} = Gk^{-(d-2)}$

## One loop beta functions

$$k\frac{d}{dk}\frac{1}{16\pi G(k)} = B_1 k^{d-2}$$

$$k\frac{dG}{dk} = -16\pi B_1 G^2 k^{d-2}$$

$$\tilde{G} = G k^{d-2}$$

$$k\frac{d\tilde{G}}{dk} = (d-2)\tilde{G} - 16\pi B_1 \tilde{G}^2$$

fixed point at $\tilde{G} = (d-2)/16\pi B_1$

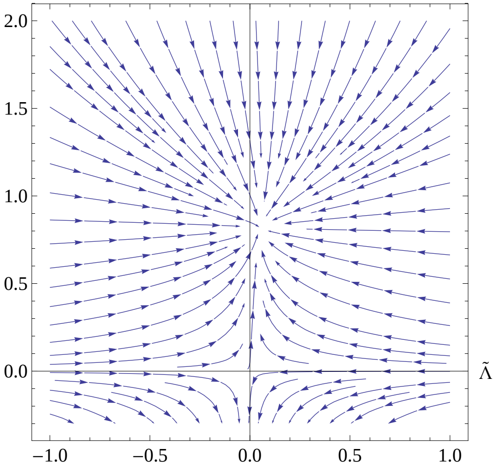
# One loop beta functions including $\Lambda$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{46\tilde{G}^2}{6\pi},$$

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{4\pi} - \frac{16\tilde{G}\tilde{\Lambda}}{6\pi}$$

$$\tilde{\Lambda}_* = \frac{3}{62} \qquad \tilde{G}_* = \frac{12\pi}{46}$$

## One loop flow



# With anomalous dimension

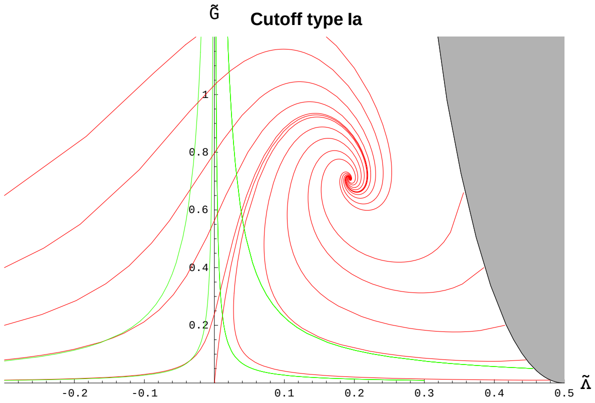
$$\beta_{\tilde{\Lambda}} = \frac{-2(1-2\tilde{\Lambda})^2\tilde{\Lambda} + \frac{36-41\tilde{\Lambda}+42\tilde{\Lambda}^2-600\tilde{\Lambda}^3}{72\pi}\tilde{G} + \frac{467-572\tilde{\Lambda}}{288\pi^2}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

$$\beta_{\tilde{G}} = \frac{2(1-2\tilde{\Lambda})^2\tilde{G} - \frac{373-654\tilde{\Lambda}+600\tilde{\Lambda}^2}{72\pi}\tilde{G}^2}{(1-2\tilde{\Lambda})^2 - \frac{29-9\tilde{\Lambda}}{72\pi}\tilde{G}}$$

[M. Reuter, Phys. Rev. D **57** 971(1998)]

[D. Dou and R. Percacci, Class. and Quantum Grav. **15** 3449 (1998)]

# Einstein–Hilbert truncation III



## Higher Derivative Gravity

$$
\begin{aligned}
\Gamma_k &= \int d^4x \sqrt{g} \left[ 2Z_N\Lambda - Z_N R + \left( \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu} + \gamma R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \right) \right] \\
&= \int d^4x \sqrt{g} \left[ 2Z_N\Lambda - Z_N R + \frac{1}{2\lambda} \left( C^2 - \frac{2\omega}{3} R^2 + 2\theta E \right) \right]
\end{aligned}
$$

[K.S. Stelle, Phys. Rev. **D16**, 953 (1977).]

[J. Julve, M. Tonin, Nuovo Cim. **46B**, 137 (1978).]

[E.S. Fradkin, A.A. Tseytlin,Phys. Lett. **104 B**, 377 (1981).]

[I.G. Avramidi, A.O. Barvinski,Phys. Lett. **159 B**, 269 (1985).]

[G. de Berredo–Peixoto and I. Shapiro, Phys.Rev. **D71** 064005 (2005).]

[A. Codello and R. P., Phys.Rev.Lett. **97** 22 (2006).]

[M. Niedermaier, Nucl. Phys. B833, 226-270 (2010).]

[N. Ohta and R.P. Class. Quant. Grav. **31** 015024 (2014); arXiv:1308.3398]

## Beta functions I

$$\beta_\lambda = -\frac{1}{(4\pi)^2}\frac{133}{10}\lambda^2$$

$$\beta_\omega = -\frac{1}{(4\pi)^2}\frac{25 + 1098\omega + 200\omega^2}{60}\lambda$$

$$\beta_\theta = \frac{1}{(4\pi)^2}\frac{7(56 - 171\theta)}{90}\lambda$$

$$\lambda(k) = \frac{\lambda_0}{1 + \lambda_0 \frac{1}{(4\pi)^2}\frac{133}{10}\log\left(\frac{k}{k_0}\right)}$$

$$\omega(k) \to \omega_* \approx -0.0228$$

$$\theta(k) \to \theta_* \approx 0.327$$

## Beta functions II

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{1}{(4\pi)^2}\left[\frac{1+20\omega^2}{256\pi\tilde{G}\omega^2}\lambda^2 + \frac{1+86\omega+40\omega^2}{12\omega}\lambda\tilde{\Lambda}\right] - \frac{1+10\omega^2}{64\pi^2\omega}\lambda + \frac{2\tilde{G}}{\pi} - q(\omega)\tilde{G}\tilde{\Lambda}$$

$$\beta_{\tilde{G}} = 2\tilde{G} - \frac{1}{(4\pi)^2}\frac{3+26\omega-40\omega^2}{12\omega}\lambda\tilde{G} - q(\omega)\tilde{G}^2$$

where $q(\omega) = (83 + 70\omega + 8\omega^2)/18\pi$

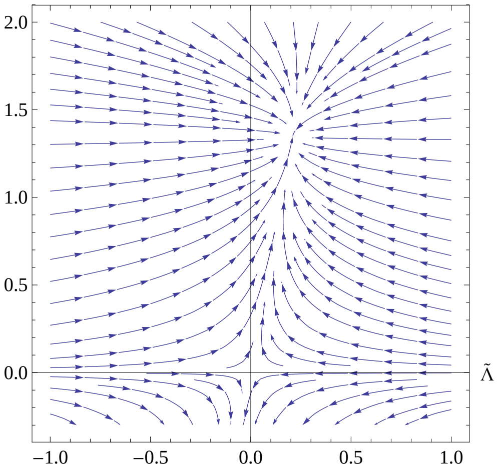
## Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane I

$$\beta_{\tilde{\Lambda}} = -2\tilde{\Lambda} + \frac{2\tilde{G}}{\pi} - q_* \tilde{G}\tilde{\Lambda}$$
$$\beta_{\tilde{G}} = 2\tilde{G} - q_* \tilde{G}^2$$

where $q_* = q(\omega_*) \approx 1.440$

$$\tilde{\Lambda}_* = \frac{1}{\pi q_*} \approx 0.221 \ , \qquad \tilde{G}_* = \frac{2}{q_*} \approx 1.389 \ .$$

# Flow in $\tilde{\Lambda}$–$\tilde{G}$ plane II



## From curvature-square truncation of ERGE

- $R^2$ O.Lauscher, M. Reuter, Phys. Rev. D 66, 025026 (2002) arXiv:hep-th/0205062
- $R^2 + C^2$ D. Benedetti, P.F. Machado, F. Saueressig, Mod. Phys. Lett. A24, 2233-2241 (2009) arXiv:0901.2984 [hep-th] Nucl. Phys. B824, 168-191 (2010), arXiv:0902.4630 [hep-th]

find nontrivial FP for $\lambda$.

# HDG revisited (with K.Falls and N.Ohta)

$$\frac{\delta^2\Gamma_k}{\delta g_{\mu\nu}\delta g_{\alpha\beta}} = \mathcal{H}^{\mu\nu,\alpha\beta},$$

where

$$\mathcal{H} = K\Delta^2 + D_{\rho\lambda}\nabla^\rho\nabla^\lambda + W.$$

where $\Delta = -\bar{\nabla}^2$,

$$K_{\mu\nu,\alpha\beta} = \frac{\beta+4\gamma}{4}\left(\bar{g}_{\mu\alpha}\bar{g}_{\nu\beta} + \frac{4\alpha+\beta}{4(\gamma-\alpha)}\bar{g}_{\mu\nu}\bar{g}_{\alpha\beta}\right),$$

Write $\mathcal{H} = K\mathcal{O}$, where

$$\begin{aligned}\mathcal{O} &= K^{-1}\mathcal{H} \\ &= \Delta^2 + K^{-1}D_{\rho\lambda}\bar{\nabla}^\rho\bar{\nabla}^\lambda + K^{-1}W \\ &= \Delta^2 + V_{\rho\lambda}\bar{\nabla}^\rho\bar{\nabla}^\lambda + U\ .\end{aligned}$$

## First cutoff

$$\mathcal{R}_k = K R_k(\Delta^2) \ ,$$

where $R_k(\Delta^2) = (k^4 - \Delta^2)\theta(k^4 - \Delta^2)$

$$T_g = \frac{1}{2}\mathrm{Tr}\frac{\partial_t[K R_k(\Delta^2)]}{K[\mathcal{O} + R_k(\Delta^2)]} = \frac{1}{2}\mathrm{Tr}\frac{\partial_t R_k(\Delta^2) + \eta_K R_k(\Delta^2)}{\mathcal{O} + R_k(\Delta^2)},$$

where we defined

$$\eta_K = K^{-1}\frac{dK}{dt}.$$

Note that $\eta_K$ is a tensor. We have $\eta_K = \eta_1 \mathbf{1} + \eta_P \mathbb{P}$, where

$$\begin{aligned}
\eta_1 &= \frac{\dot\beta + 4\dot\gamma}{\beta + 4\gamma}, \\
\eta_P &= \frac{(4\gamma + \beta)\dot\alpha}{(\gamma - \alpha)(3\alpha + \beta + \gamma)} + \frac{\dot\beta}{(3\alpha + \beta + \gamma)} - \frac{(4\alpha + \beta)\dot\gamma}{(\gamma - \alpha)(3\alpha + \beta + \gamma)} \ ,
\end{aligned}$$

## FP with first cutoff

Since at a FP

$$\dot{\alpha} = \dot{\beta} = \dot{\gamma} = 0$$

the FPs are the same as in the one loop approximation.

## Second cutoff

Extract a term $\Delta$ from $D^{\mu\nu}\nabla_\mu\nabla_\nu$

$$\mathcal{K} = K\Delta^2 + K_N\Delta + D'_{\rho\lambda}\nabla^\rho\nabla^\lambda + W,$$

where

$$(K_N)_{\mu\nu,\alpha\beta} = \frac{Z_N}{4}(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\nu}g_{\alpha\beta}) = \frac{Z_N}{4}(\mathbf{1} - 4\mathbb{P})_{\mu\nu,\alpha\beta},$$

## Second cutoff

$$\mathcal{R}_k = KR_k(\Delta^2) + K_N R_k(\Delta),$$

which implies

$$\mathcal{H} + \mathcal{R}_k = K(\Delta^2 + R_k(\Delta^2)) + K_N(\Delta + R_k(\Delta)) + D'_{\rho\lambda}\nabla^\rho\nabla_\lambda + W$$

Then we define

$$V'_{\rho\lambda} = K^{-1}D'_{\rho\lambda}, \qquad X = K^{-1}K_N = x_N(\mathbf{1} - \tau_P\mathbb{P}),$$

with

$$x_N = \frac{Z_N}{\beta + 4\gamma}, \qquad \tau_P = \frac{\beta + 4\gamma}{3\alpha + \beta + \gamma},$$

and

$$\eta_N = \mathrm{tr}\, K_N^{-1}\partial_t K_N,$$

## FP with second cutoff

Since $\eta_N \neq 0$ at a FP, the FP are different from those of one loop approximation.

Old work of BMS indeed used a cutoff that contained $Z_N$.

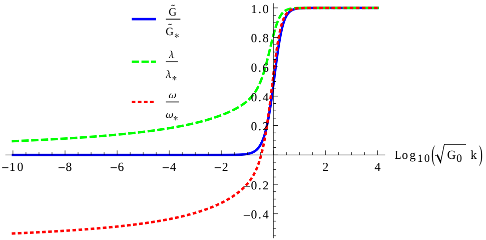
# $f(R)$ gravity

Can flow from

$$\tilde{\Lambda}_* = 0.33 \ , \quad \tilde{G}_* = 1.44 \ , \quad \lambda_* = 33 \quad \omega_* = 1.004$$

to

$$\tilde{\Lambda}_* = 0 \ , \quad \tilde{G}_* = 0 \ , \quad \lambda_* = 0 \quad \omega_* = -0.614$$



## $f(R)$ gravity

$$\Gamma_k(g_{\mu\nu}) = \int d^4x \sqrt{g} f(R)$$

$$f(R) = \sum_{i=0}^{n} g_i(k) R^i$$

[ A. Codello, R.P. and C. Rahmede Int.J.Mod.Phys.A23:143-150 arXiv:0705.1769 [hep-th];
A. Codello, R.P. and C. Rahmede Annals Phys. 324 414-469 (2009) arXiv: arXiv:0805.2909;
P.F. Machado, F. Saueressig, Phys. Rev. **D** arXiv: arXiv:0712.0445 [hep-th]
K. Falls, D.F. Litim, K. Nikolakopoulos, C. Rahmede, arXiv:1301.4191 [hep-th]
Dario Benedetti, Francesco Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
Juergen A. Dietz, Tim R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th]
Dario Benedetti, arXiv:1301.4422 [hep-th] ]

## Functional truncations

ERGE well suited to study flow of potential

$$\Gamma_k[\phi] = \int d^d x \left( V(\phi^2) + \frac{1}{2}(\partial\phi)^2 \right)$$

Successfully reproduces properties of many critical models.

## $f(R)$ gravity, functional treatment

Do not expand $f(R)$ but write flow equation for $f$

$$\partial_t \tilde{f}(\tilde{R}) = \beta(\tilde{f}, \tilde{f}', \tilde{f}'', \tilde{f}''')$$

where $\tilde{R} = R/k^2$, $\tilde{f} = f/k^4$.
For large $\tilde{R}$

$$\tilde{f}(\tilde{R}) = A\tilde{R}^2 \left(1 + \sum_{n>0} d_n \tilde{R}^{-n}\right)$$

[ D. Benedetti, F. Caravelli, JHEP 1206 (2012) 017, Erratum-ibid. 1210 (2012) 157 arXiv:1204.3541 [hep-th]
J.A. Dietz, T.R. Morris,JHEP 1301 (2013) 108 arXiv:1211.0955 [hep-th] ]

# $f(R)$ gravity, functional treatment

Theorem 1: $\Gamma_*(g_{\mu\nu}) = A_* \int d^4x \sqrt{g} R^2$, $A_* \neq 0$

Theorem 2: if $\tilde{f}_*$ exists, the spectrum of perturbations is discrete, real, and there are at most finitely many relevant direction.

[D. Benedetti, arXiv:1301.4422 [hep-th]]

Analytic and numerical solutions have been found.

[N. Ohta, R.P., G.P. Vacca, Eur. Phys. J. C (2016) 76:46 arXiv:1511.09393 [hep-th]]

## Alternative approach

Expand

$$\Gamma_k(h;\bar{g}) = \Gamma_k(0;\bar{g}) + \int \Gamma_k'(0;\bar{g})h + \int \Gamma_k''(0;\bar{g})h^2 + \int \Gamma_k'''(0;\bar{g})h^3 + \int \Gamma_k''''(0;\bar{g})h^4 + \dots$$

from FRGE obtain

$$\dot{\Gamma}_k(h;\bar{g}) = \dot{\Gamma}_k(0;\bar{g}) + \int \dot{\Gamma}_k'(0;\bar{g})h + \int \dot{\Gamma}_k''(0;\bar{g})h^2 + \int \dot{\Gamma}_k'''(0;\bar{g})h^3 + \int \dot{\Gamma}_k''''(0;\bar{g})h^4 + \dots$$

can read off beta functions of vertex functions.

# Bi-field truncation I

$$\Gamma_k(\bar{g}_{\mu\nu}, h_{\mu\nu}) = S_{EH}(\bar{g}_{\mu\nu}+h_{\mu\nu})+S_{GF}(\bar{g}_{\mu\nu}, h_{\mu\nu})+S_{ghost}(\bar{g}_{\mu\nu}, \bar{C}^{\mu}, C_{\nu})$$

$h_{\mu\nu} \to Z_h^{1/2} h_{\mu\nu}$, $c^{\mu} \to Z_c^{1/2} c^{\mu}$ and $c_{\nu} \to Z_c^{1/2} c_{\nu}$
Anomalous dimensions $\eta_h = -\frac{\partial_t Z_h}{Z_h}$ $\eta_c = -\frac{\partial_t Z_c}{Z_c}$
computed from $\langle h_{\mu\nu} h_{\rho\sigma} \rangle$, $\langle \bar{C}^{\mu} C_{\nu} \rangle$

FP at $\tilde{\Lambda} = -0.00812786$, $\tilde{G} = 1.44627$
scaling exponents $-3.32286$, $-1.95403$
anomalous dimensions $\eta_h = 0.0720708$, $\eta_c = -1.50325$.

[A. Codello, G. d'Odorico, G. Pagani, arXiv:1304.4777 [gr-qc] ]

## Vertex expansion

Flat space expansion.

Flow extracted from 3- and 4-point functions.

FP again present.

[N. Christiansen, J. Pawlowski, A. Rodigast,
Phys.Rev. D93 (2016) no.4, 044036 arXiv:1403.1232 [hep-th]]
[N. Christiansen, B. Knorr, J. Meibohm, J. Pawlowski, M. Reichert, Phys.Rev. D92 (2015) no.12, 121501 arXiv:1506.07016 [hep-th]]
[T. Denz, J. Pawlowski and M. Reichert, arXiv:1612.07315 [hep-th]]

## Split symmetry

Because

$$S = S(g_{\mu\nu}) = S(\bar{g}_{\mu\nu} + h_{\mu\nu})$$

the bare action is invariant under

$$\begin{aligned} \delta \bar{g}_{\mu\nu} &= \epsilon_{\mu\nu} \ , \\ \delta h_{\mu\nu} &= -\epsilon_{\mu\nu} \ . \end{aligned}$$

but the EAA $\Gamma_k(\mathbf{h}; \bar{\mathbf{g}})$ is not.

$$\frac{\delta^{(n)} \Gamma_k(h; \bar{g})}{\delta h^n} \neq \frac{\delta^{(n)} \Gamma_k(h; \bar{g})}{\delta \bar{g}^n}$$

## Split symmetry

Expanding the Hilbert action

$$S(g) = S(\bar{g}) + \int S'(\bar{g})h + \int S''(\bar{g})h^2 + \int S'''(\bar{g})h^3 + \int S''''(\bar{g})h^4 + \dots$$

all contain the same Newton constant.

Due to violation of split symmetry each term of the expansion gives a different "beta function"

## Dealing with the split symmetry violation

- Write the anomalous Ward identity for the split symmetry or a subgroup thereof
- Solve it to eliminate from the EAA a number of fields equal to the number of parameters of the transformation
- Write the flow equation for the EAA depending on the remaining variables

Carried through for $\epsilon_{\mu\nu} = \epsilon \bar{g}_{\mu\nu}$

[T.R. Morris, JHEP 1611 (2016) 160 arXiv:1610.03081 [hep-th]]

[R.P., G.P. Vacca, Eur.Phys.J. C77 (2017) no.1, 52 arXiv:1611.07005 [hep-th]]

## Other techniques could play a role

- $\epsilon$ expansion
- large $N$ expansion
- higher loop calculations
- generalization of CFT techniques

## Summary

- use powerful QFT tools
- bottom up approach (guaranteed to give correct IR limit)
- highly predictive: only few parameters undetermined (canonical dimension seems to be a reasonably good guide)
- inclusion of matter straightforward
- fixed point appears in all approximations tried so far

but

- some uncertainty on the number of relevant deformations
- truly functional truncations can be studied, but are hard (less tolerant of bad approximations.)
- use of background field method makes effective action depend on two fields
- no reliable calculation of physical observable yet