

Quantum general covariance and the trinity of relational quantum dynamics

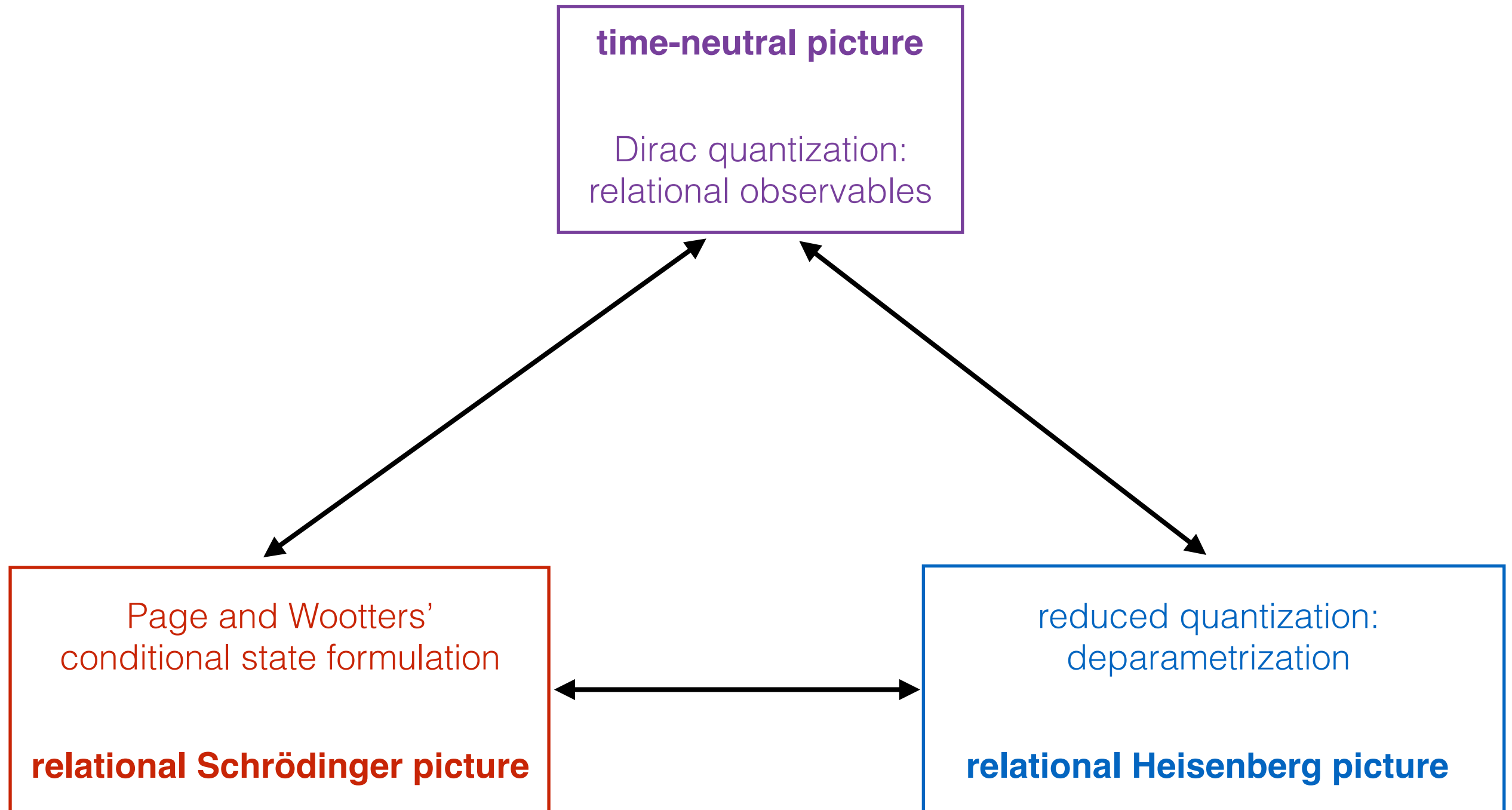
Philipp Höhn
Institute for Quantum Optics and Quantum Information
Vienna

“Quantum & Gravity in Okinawa” OIST,
July 27th, 2019

based on: Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556
Vanrietvelde, PH, Giacomini, arXiv:1809.05093
PH, Vanrietvelde, arXiv:1810.04153
PH, arXiv:1811.00611
PH, Lock, Smith, to appear

The trinity of relational quantum dynamics

PH, Lock, Smith, to appear



General covariance

“All the laws of physics are the same in all reference frames.”

General covariance

“All the laws of physics are the same in all reference frames.”



laws as tensor equations



reference-frame-neutral

General covariance

“All the laws of physics are the same in all reference frames.”

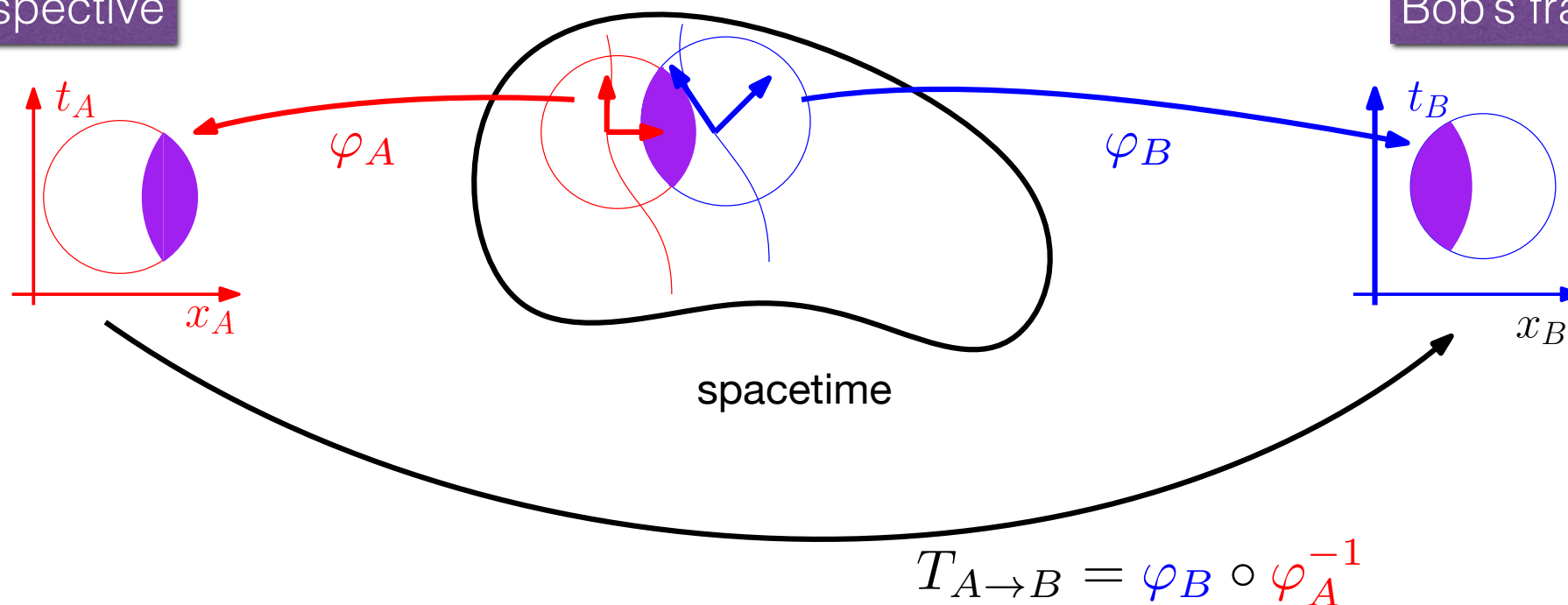
frames usually

- idealized
- external

frame-neutral

Alice's frame perspective

Bob's frame perspective



Towards quantum general covariance

Bojowald, PH, Tsobanjan, CQG 28, 035006 (2011)

Bojowald, PH, Tsobanjan, PRD 83, 125023 (2011)

PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)

Palmer, Girelli, Bartlett, PRA 89, 052121 (2014)

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Giacomini, Castro-Ruiz, Brukner *Nat. Comm.* **10**, 494 (2019)

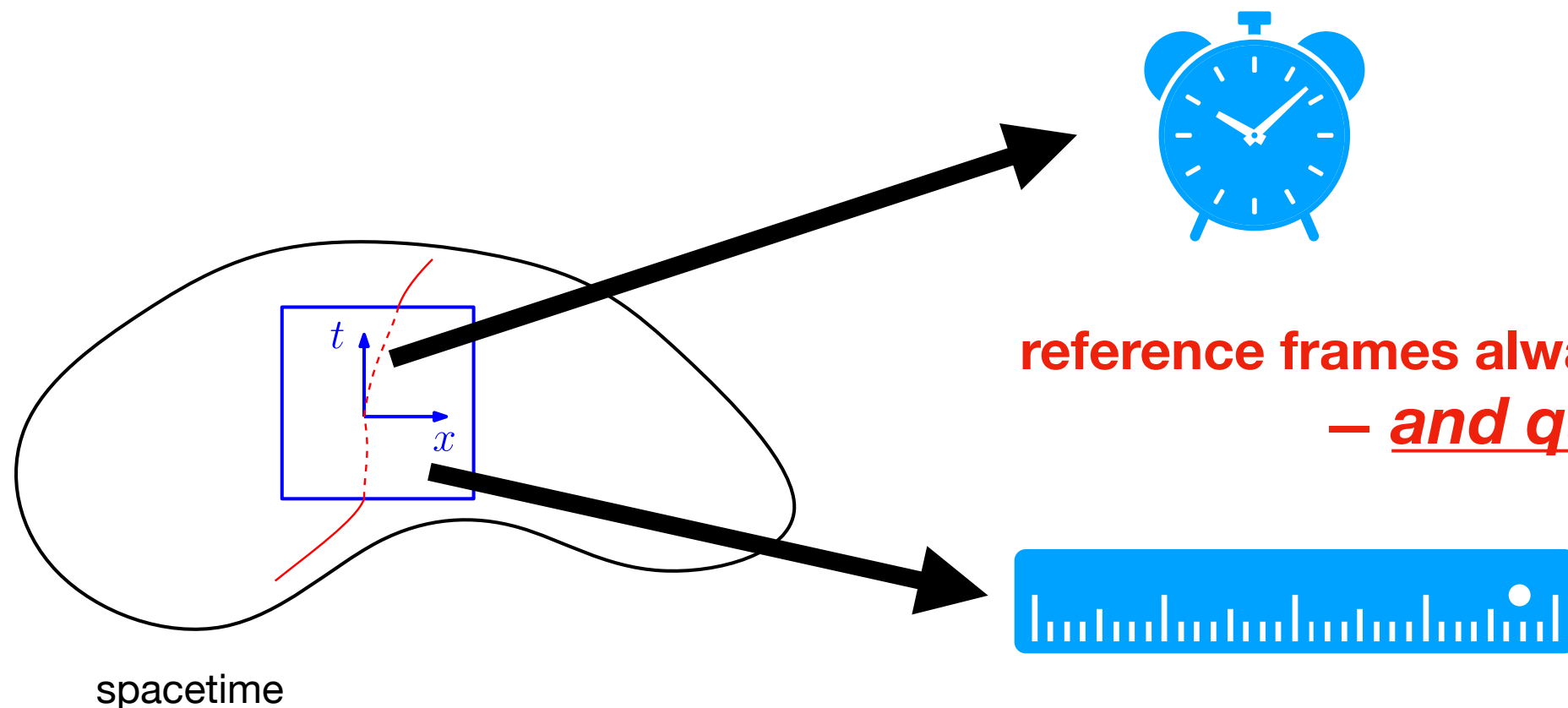
Vanrietvelde, PH, Giacomini, Castro Ruiz, arXiv:1809.00556

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“All the laws of physics are the same in all reference frames.”



**reference frames always physical systems
– and quantum**

Towards quantum general covariance

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“All the laws of physics are the same in all reference frames.”

How to make sense of general covariance when frames are quantum?

Multiple choice problem

- many possible choices for relational clocks $\Rightarrow$ inequivalent quantum dynamics

e.g., 2 clocks variables T_1, T_2

$T_1(T_2)$ vs $T_2(T_1)$

e.g. $T_1 = a, T_2 = \varphi$
in quantum cosmology

What if T_1, T_2 operators?

Kuchar (1992):

“The multiple choice problem is one of an embarrassment of riches: out of many inequivalent options, one does not know which one to select.”

Isham (1993):

*“Can these different quantum theories be seen to be part of an overall scheme that is covariant?...
It seems most unlikely that a single Hilbert space can be used for all possible choices of an internal time function.”*

AIM:
unifying framework for switches of
temporal and spatial
quantum reference systems

how?

symmetry principle



redundancy (constraints)

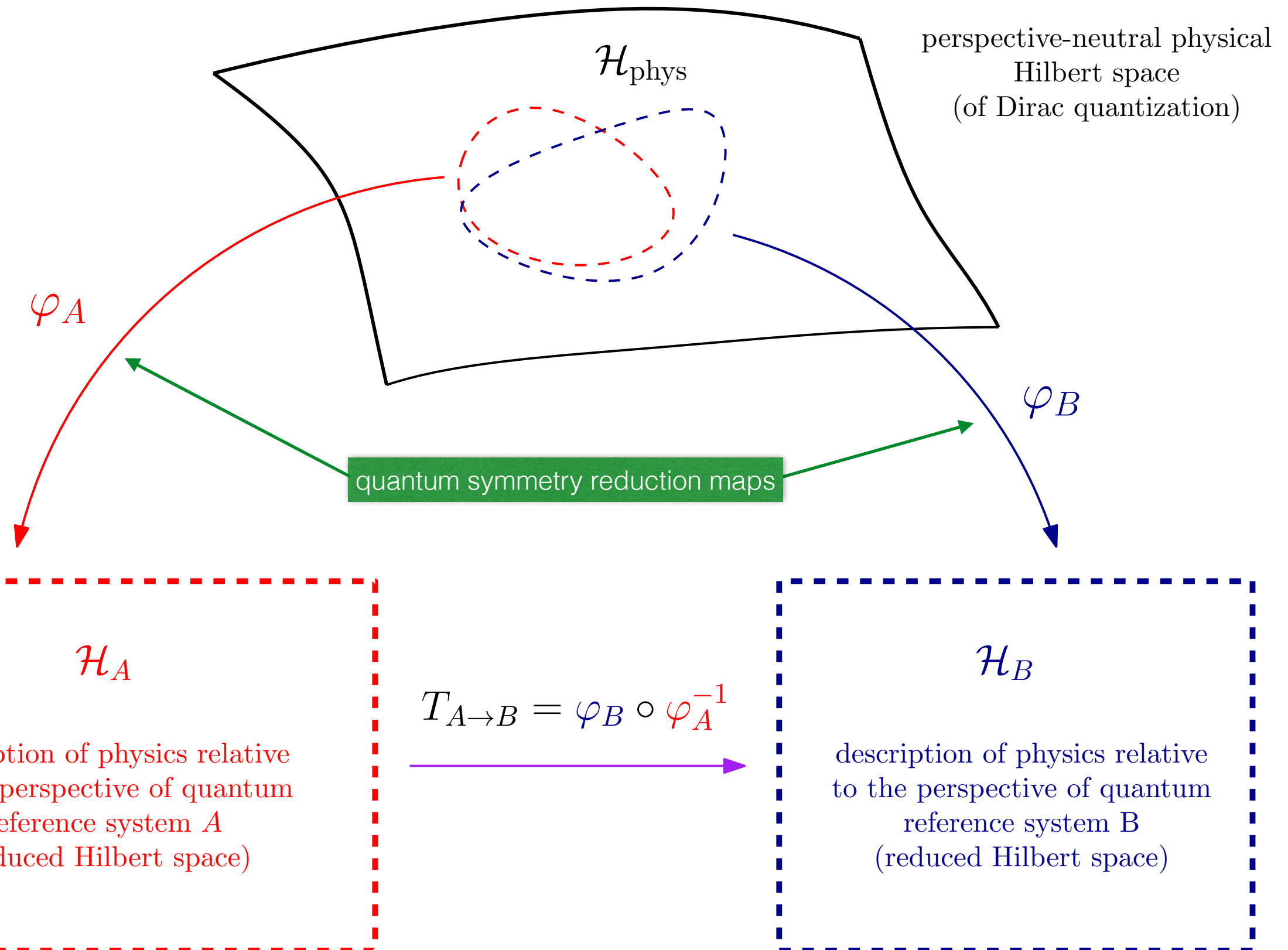


perspective-neutral framework

Vanrietvelde, PH, Giacomini, Castro Ruiz, arXiv:1809.00556
Vanrietvelde, PH, Giacomini, arXiv:1809.05093
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‘Quantum coordinate changes’:

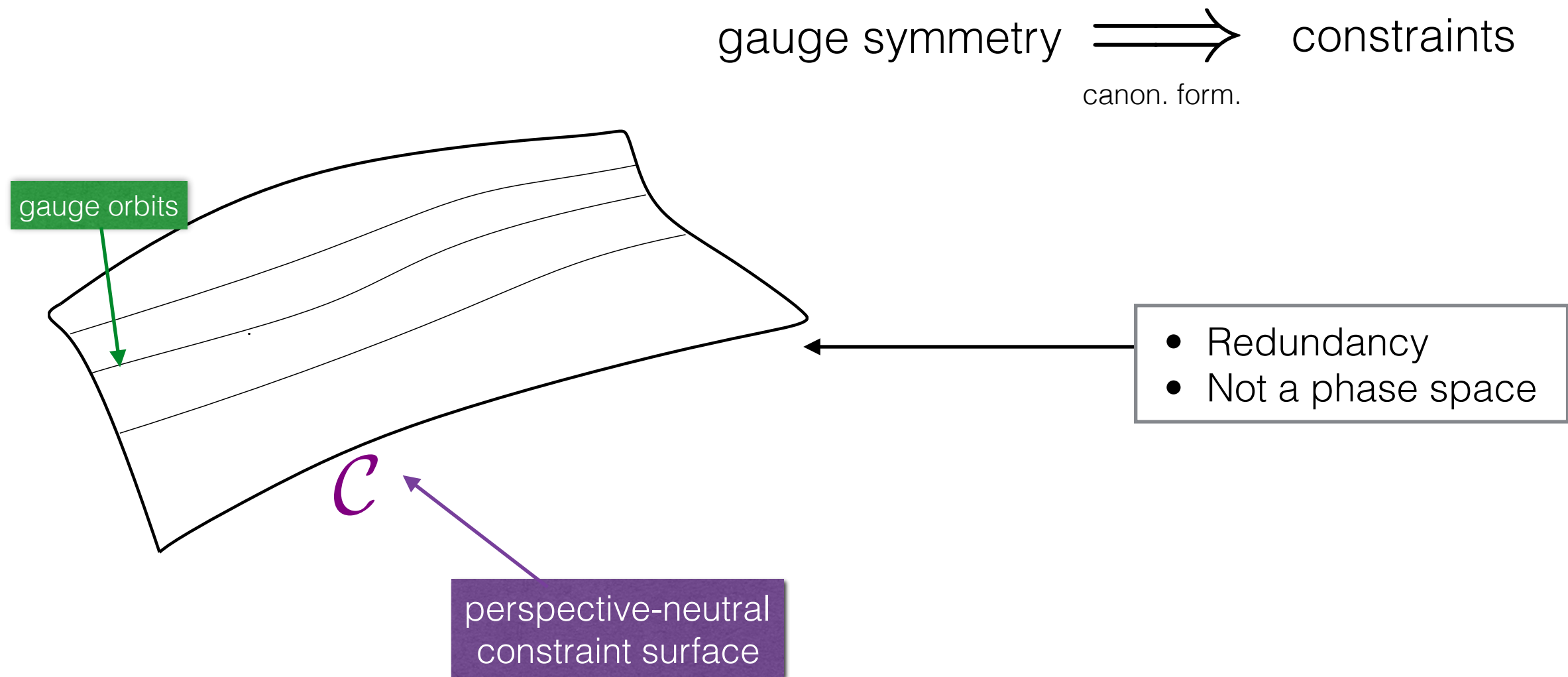
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PH, Vanrietvelde 1810.04153
PH 1811.00611



Plan

1. Illustration in toy model
2. Trinity
3. Clock switches
4. Conclusions

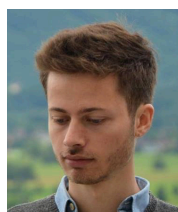
Symmetry & perspective-neutral structure



constraint surface encodes all frame choices at once

Example: relative motion in 1D

Constraint $P = p_A + p_E + p_F \approx 0$ translation generator



● q_A



● q_E



● q_F



gauge symmetry: $q_a(t)$ not physical

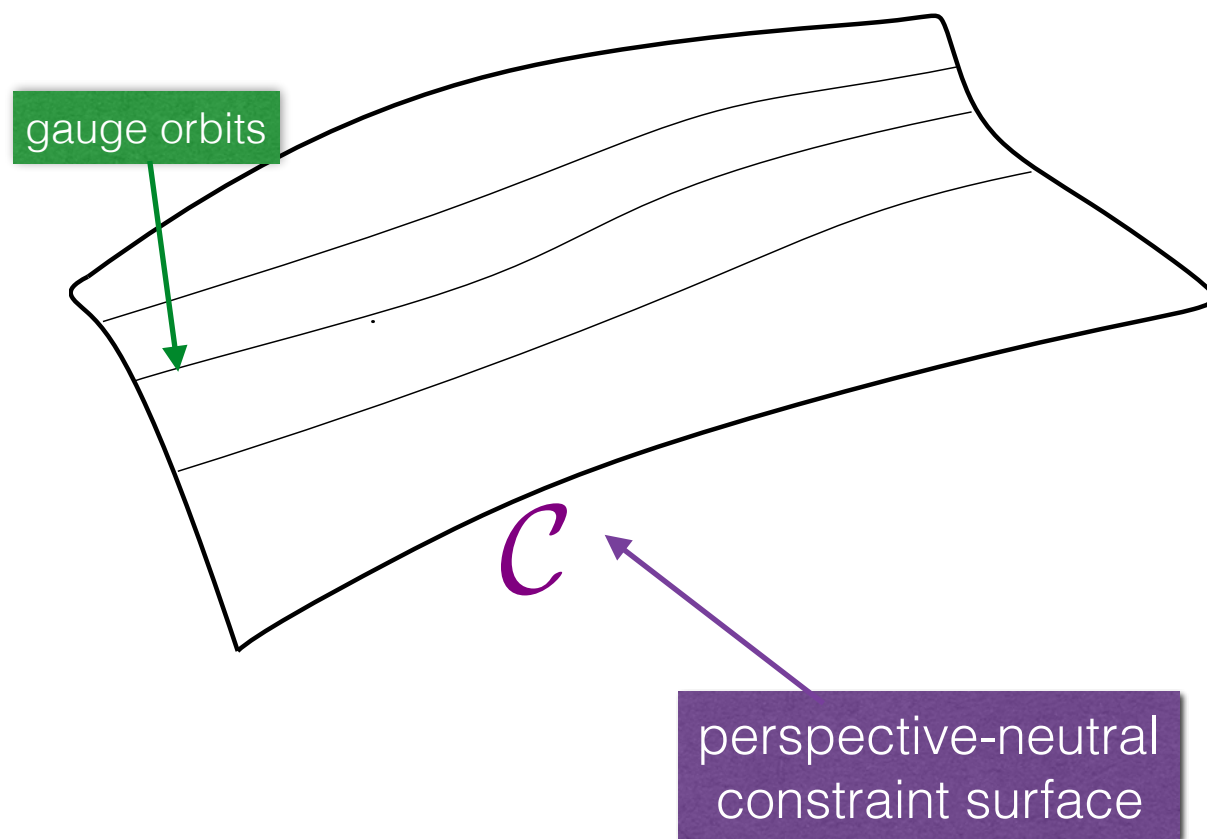
Localization in Newtonian
space unphysical,
only relational motion

gauge inv. observables: $(q_E - q_A), (q_F - q_A), (q_E - q_F), p_A, p_E, p_F$

redundancy: (only 4 indep. gauge inv. obs)

perspective-neutral description

From perspective-neutral to perspectival

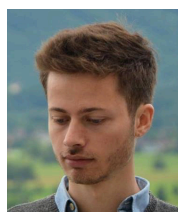


Classical reduction:

1. Choose reference system (e.g. A)

Example: relative motion in 1D

Constraint $P = p_A + p_E + p_F \approx 0$ translation generator



q_A



q_E



q_F

q

gauge symmetry: $q_a(t)$ not physical

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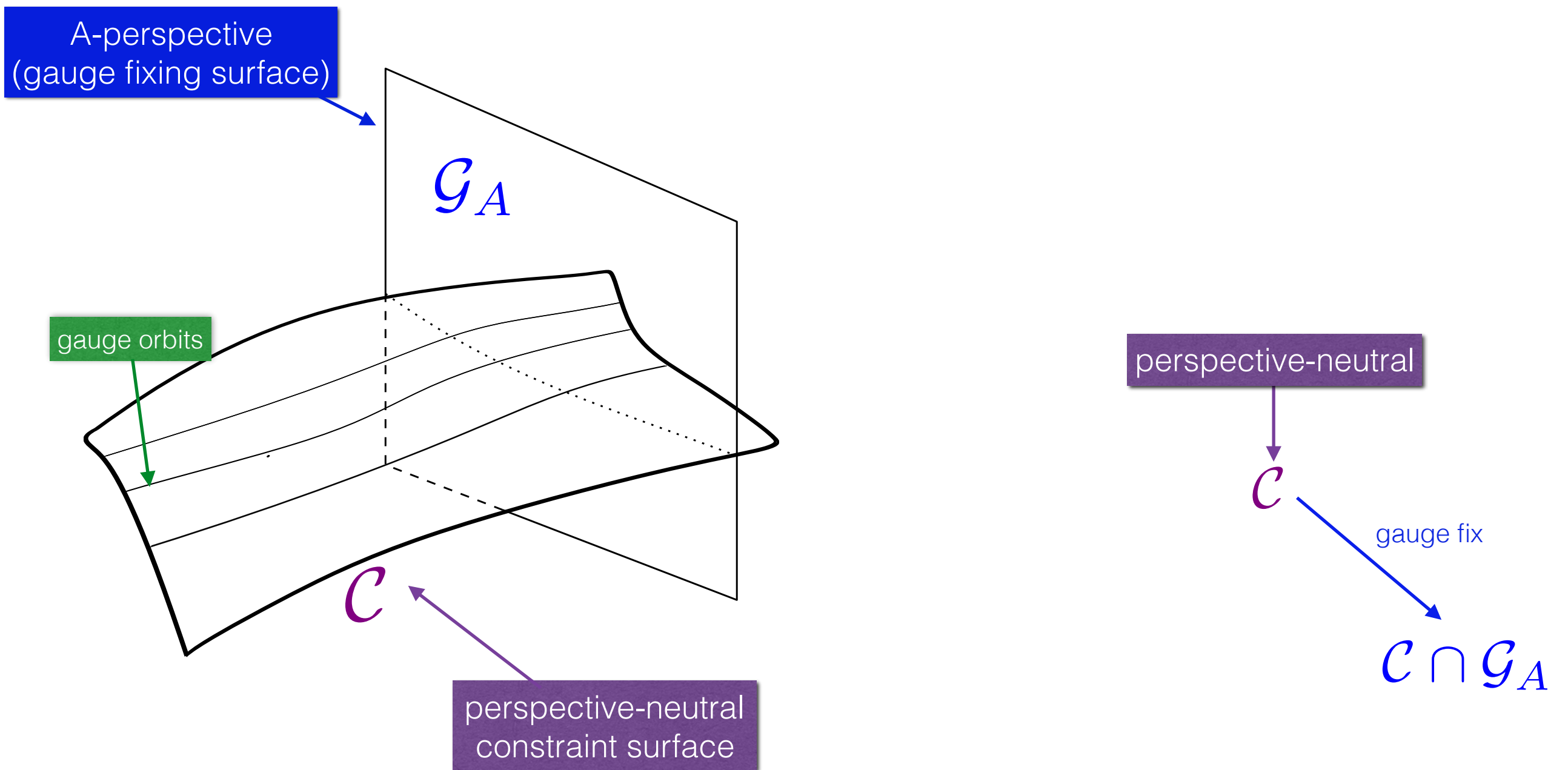
redundancy: (only 4 indep. gauge inv. obs)

perspective-neutral description

- fix symmetry, **take A perspective** $q_A = 0$

$(q_{E,F} - q_A) \mapsto q_{E,F}$ become rel. distance to A

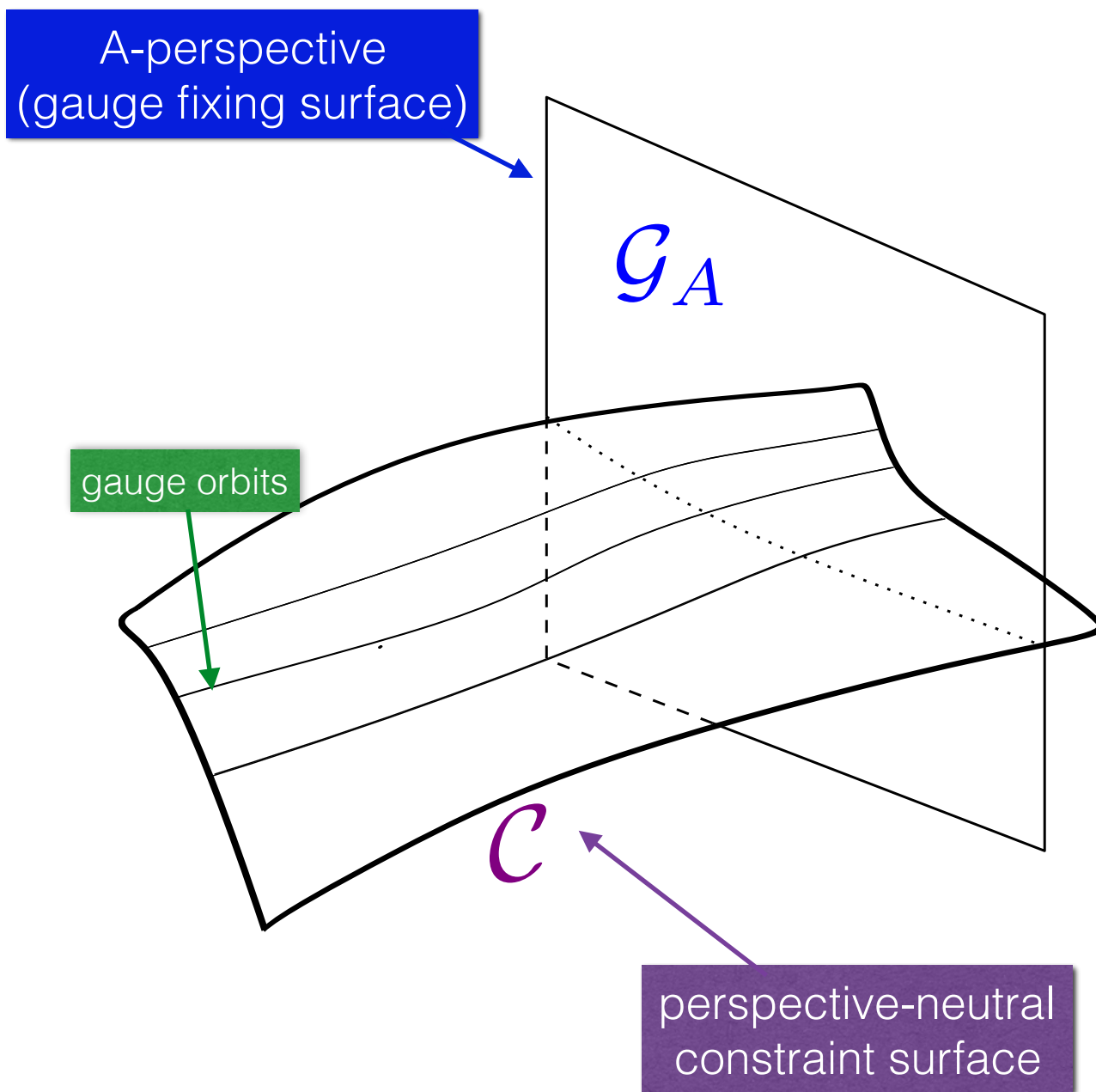
Frame choice as gauge



Classical reduction:

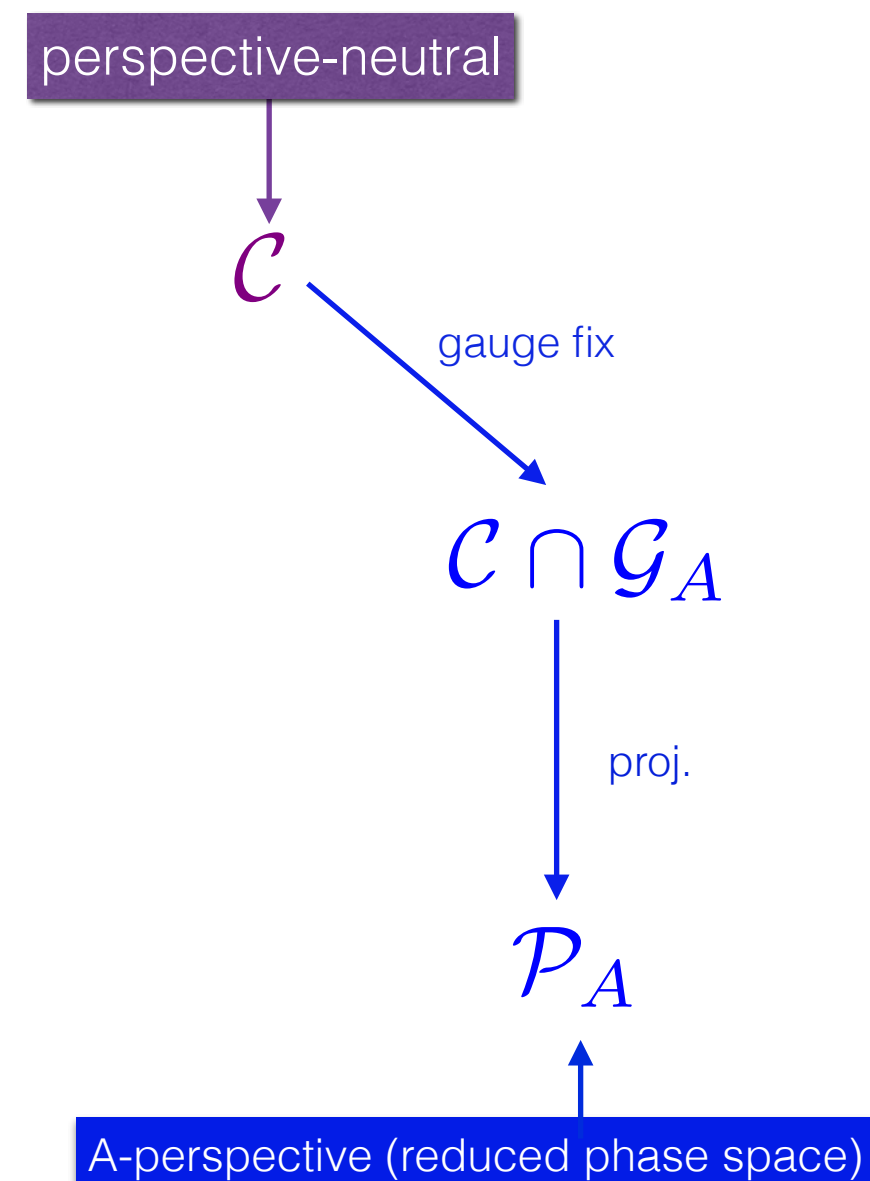
1. Choose reference system (e.g. A)
2. Gauge fix

Frame choice as gauge

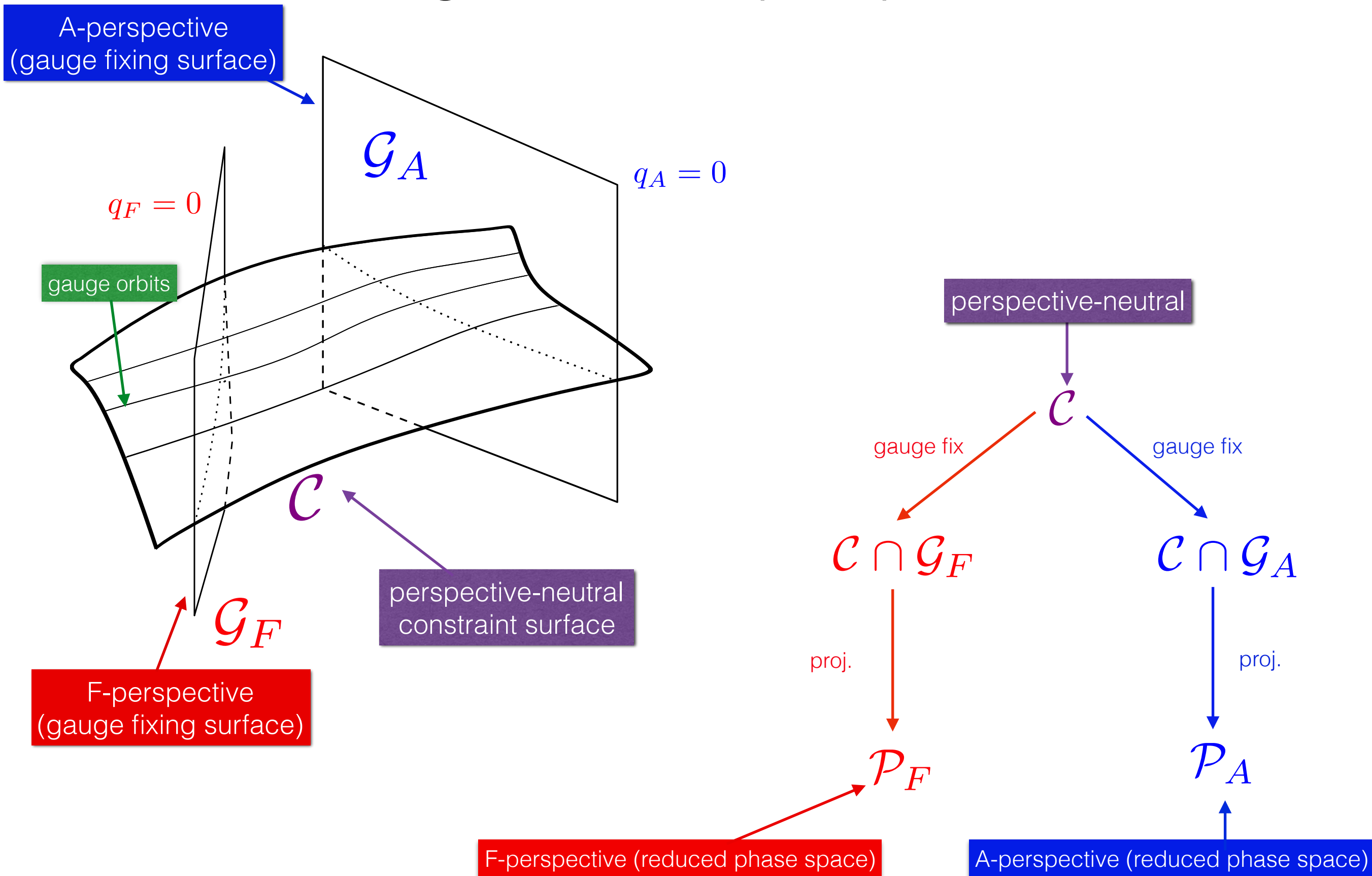


Classical reduction:

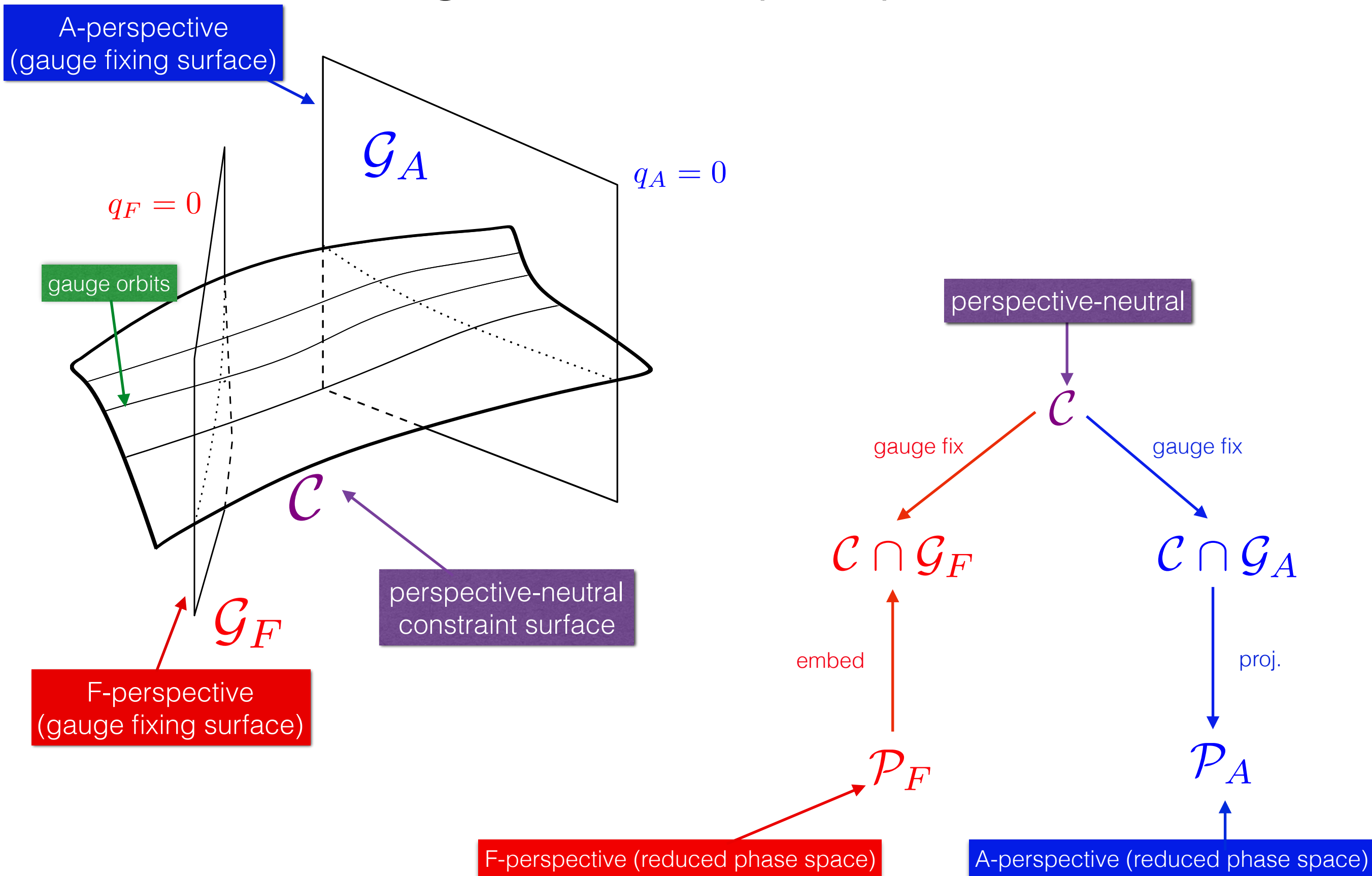
1. Choose reference system (e.g. A)
2. Gauge fix
3. remove A's redundant DoFs



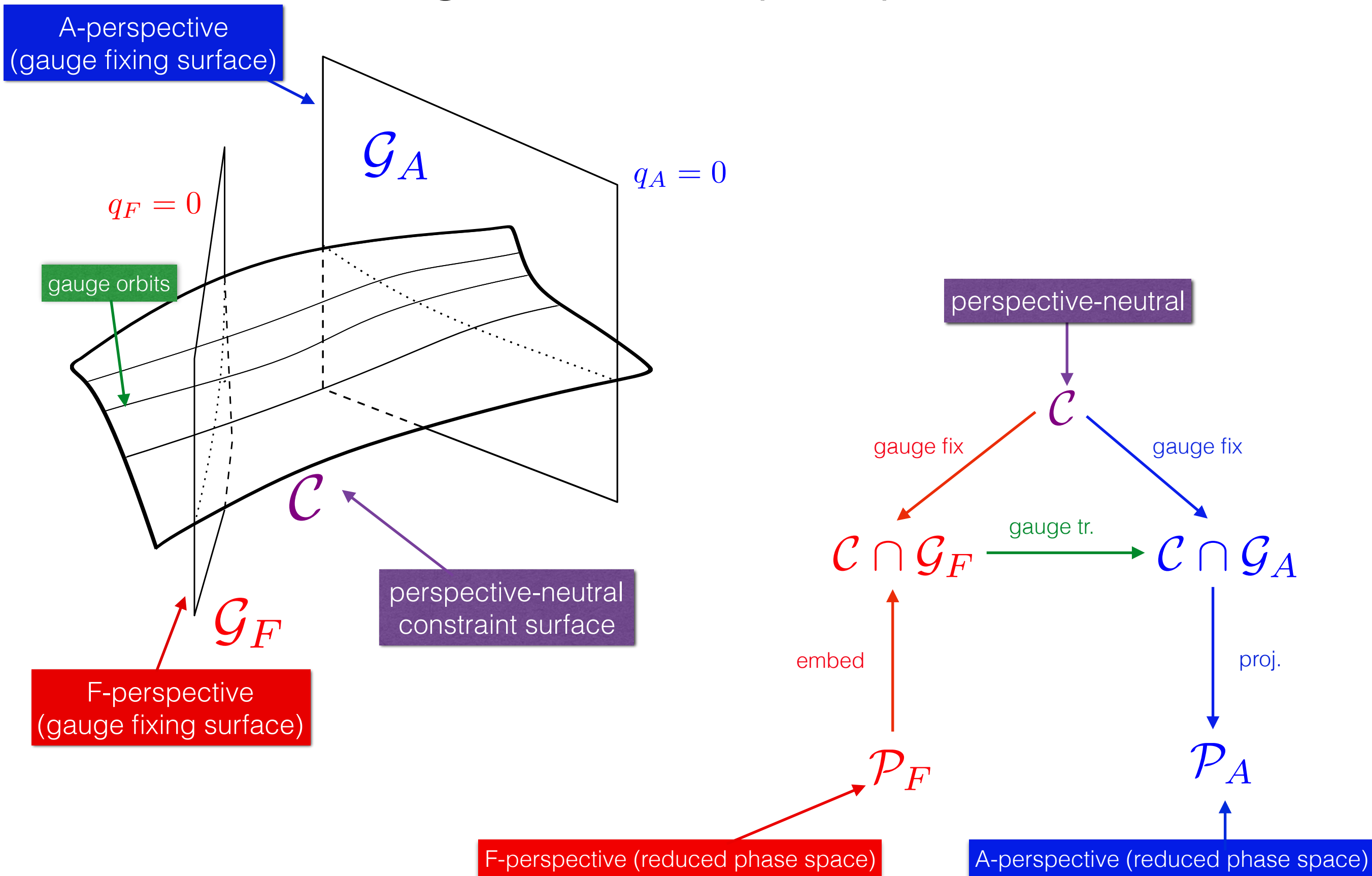
Change of frame perspective



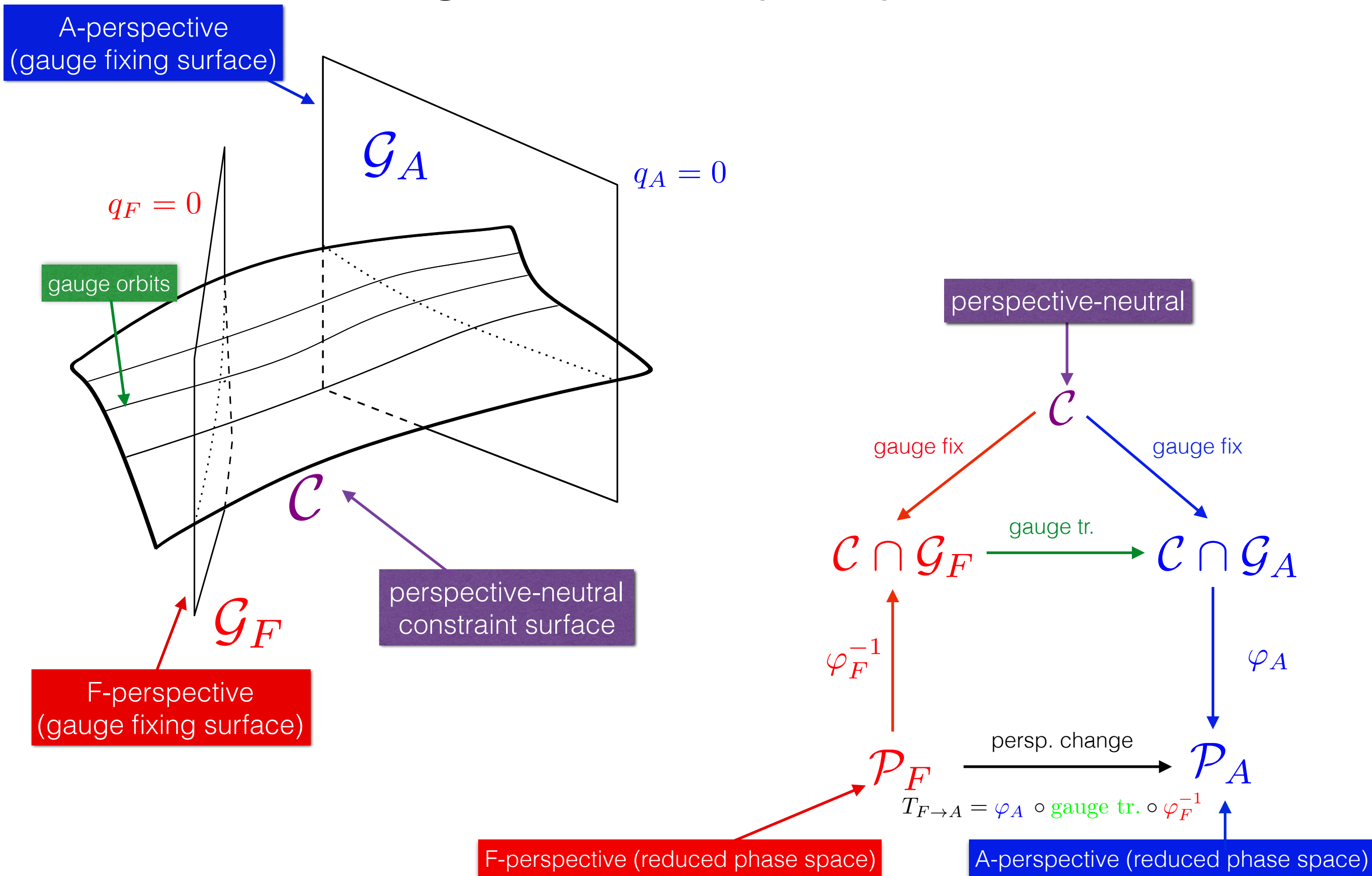
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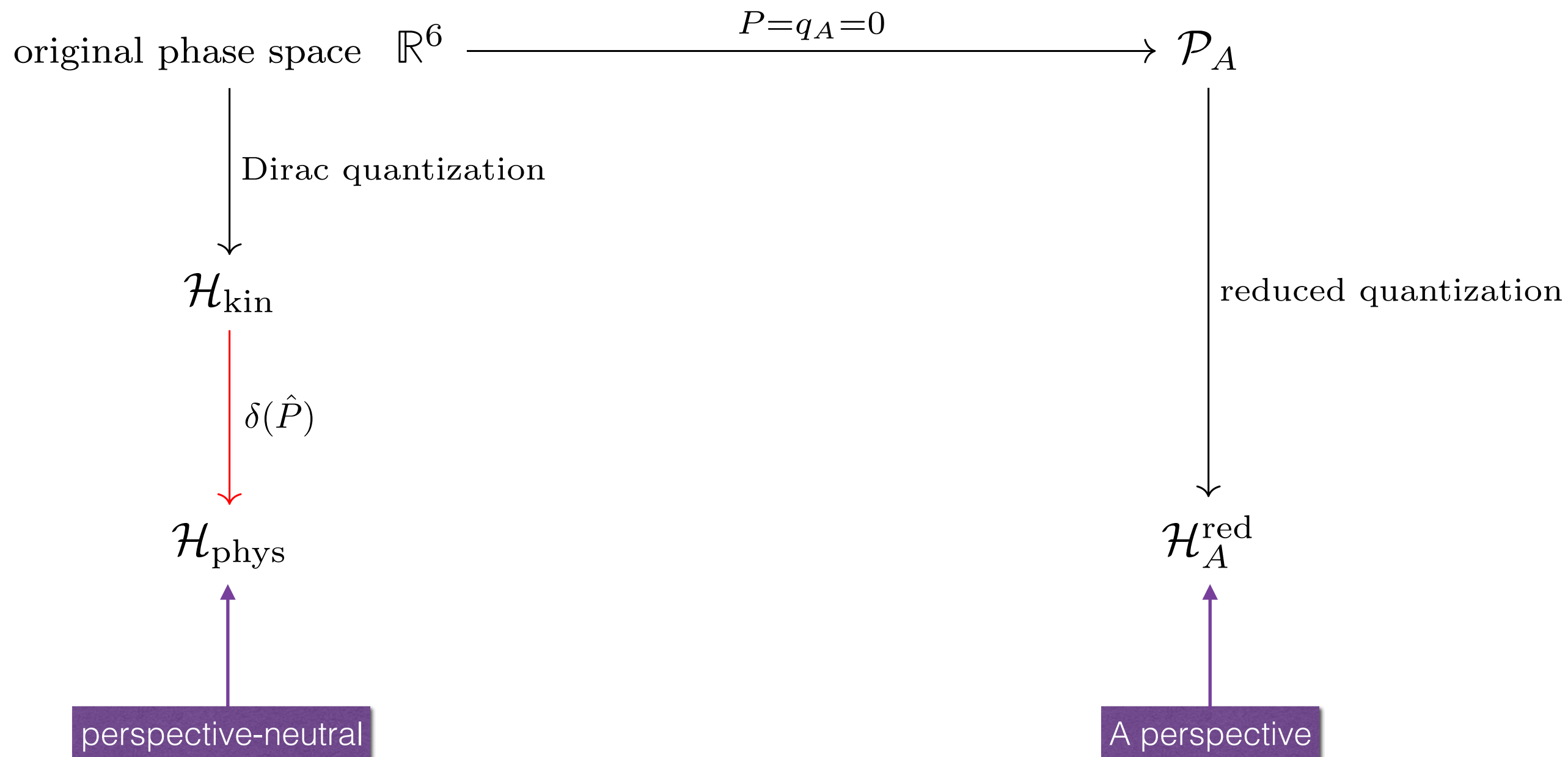


Change of frame perspective

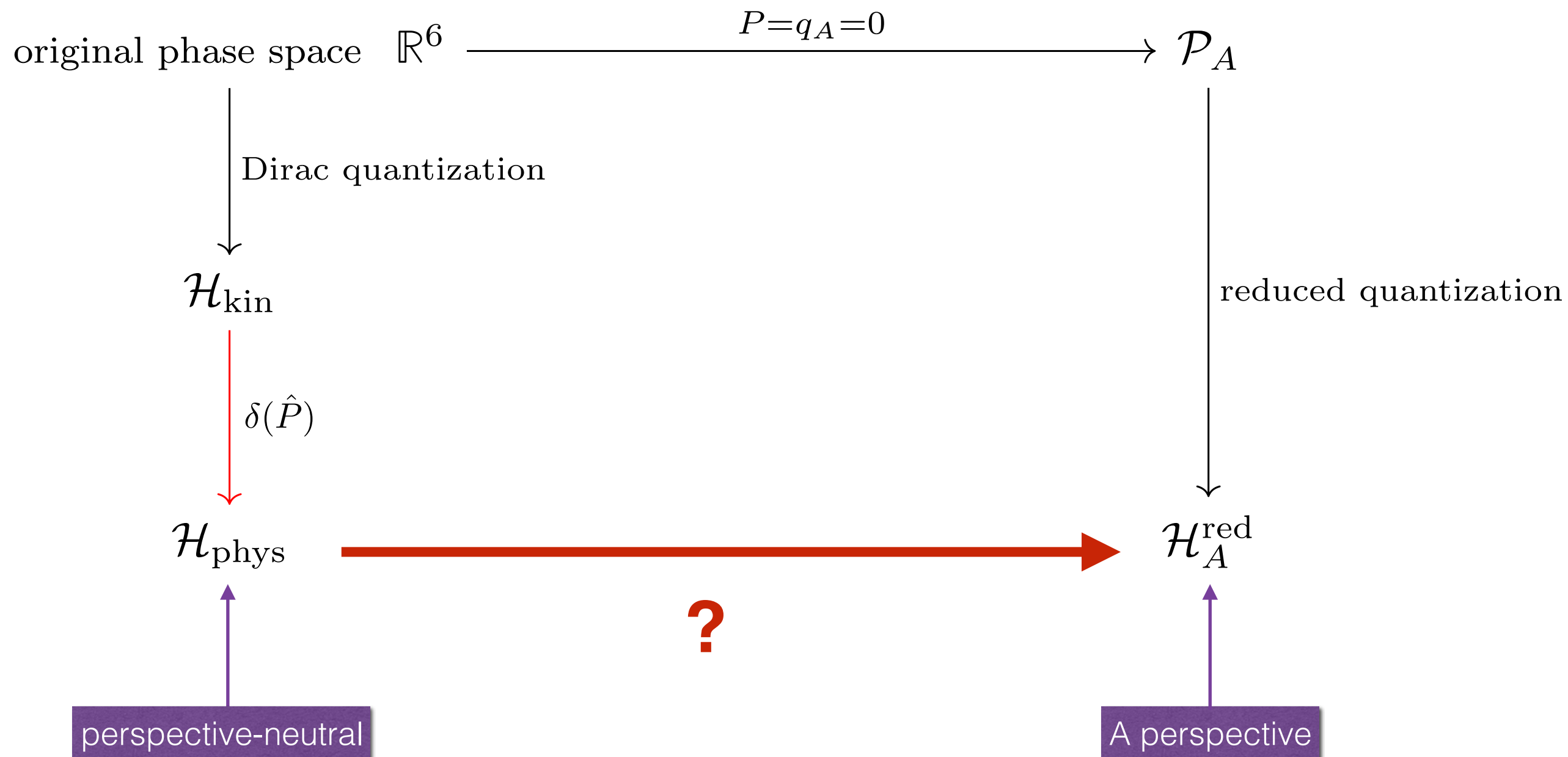


Quantum reference frame switches

Strategy in example



Strategy in example



Dirac quantization - perspective-neutral

quantize 1st on $\mathcal{H}_{\text{kin}} = L^2(\mathbb{R}^3)$ solve constraint in QT

$$\hat{P}|\psi\rangle_{\text{phys}} = (\hat{p}_A + \hat{p}_E + \hat{p}_F)|\psi\rangle_{\text{phys}} = 0$$

group averaging $\delta(\hat{P}) : \mathcal{H}_{\text{kin}} \rightarrow \mathcal{H}_{\text{phys}}$

$$|\psi_{\text{phys}}\rangle = \delta(\hat{P}) |\psi_{\text{kin}}\rangle$$

$$\delta(\hat{P}) := \int ds e^{i s \hat{P}}$$

- observables on $\mathcal{H}_{\text{phys}}$ $[\hat{O}, \hat{P}] = 0$ e.g. $\hat{q}_a - \hat{q}_b, \hat{p}_a$

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Again, redundancy!

quantum reduction procedure?

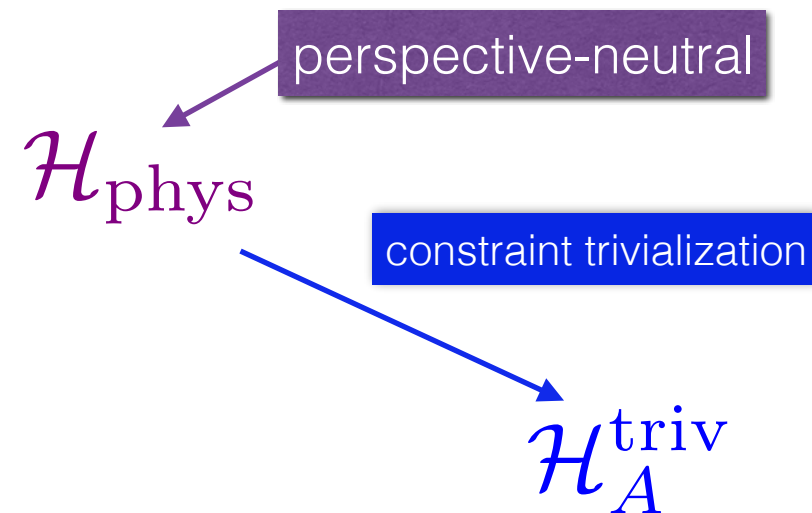
Want to remove redundancy!
→ **choice of QRF**

- debate on relation reduced vs Dirac quantization
- no gauge fixing in QT

$\mathcal{H}_{\text{phys}}$ perspective-neutral

1. Choose reference system (e.g. A)

Quantum reduction



1. Choose reference system (e.g. A)
2. Trivialize constraints: **transfer redundancy to QRF**

Trivialization to A in Example

- Trivialize constraint to A $\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$

$$\mathcal{T}_A \hat{P} \mathcal{T}_A^\dagger = \hat{p}_A$$

$$\mathcal{T}_A |\psi_{\text{phys}}\rangle = \underbrace{|p_A = 0\rangle}_{\text{redundant}} \otimes |\psi\rangle_{EF|A}$$

reduced QT



redundant

Trivialization to A in Example

- Trivialize constraint to A

$$\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$$

“ $-\hat{p}_A$ ”

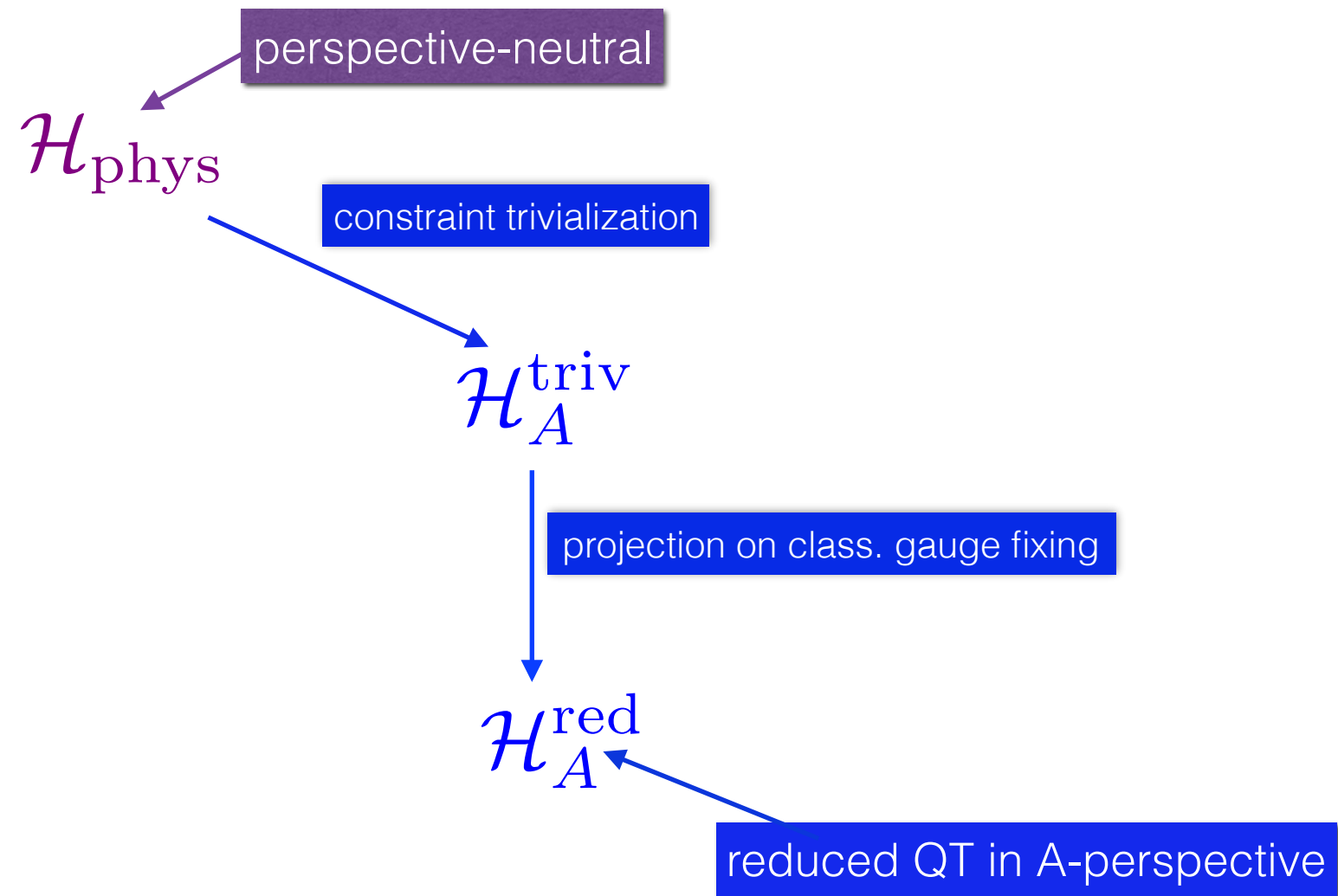
reduced QT

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$$\mathcal{T}_A |\psi_{\text{phys}}\rangle = |\underline{p_A = 0}\rangle \otimes |\psi\rangle_{EF|A}$$

redundant

Quantum reduction



1. Choose reference system (e.g. A)
2. Trivialize constraints to reference system
3. **Remove redundancy:** Project onto classical gauge fixing condition

Go to A-perspective in QT

- Trivialize constraint to A $\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$

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reduced QT

remove redundancy by projection on class. gauge fixing

Total symmetry reduction map

$$\varphi_A := \sqrt{2\pi} \langle q_A = 0 | \mathcal{T}_A$$

- isometry to reduced states in A perspective

$$\varphi_A |\psi_{\text{phys}}\rangle = |\psi\rangle_{EF|A}$$

Go to A-perspective in QT

- Trivialize constraint to A $\mathcal{T}_A := e^{i\hat{q}_A(\hat{p}_E + \hat{p}_F)}$

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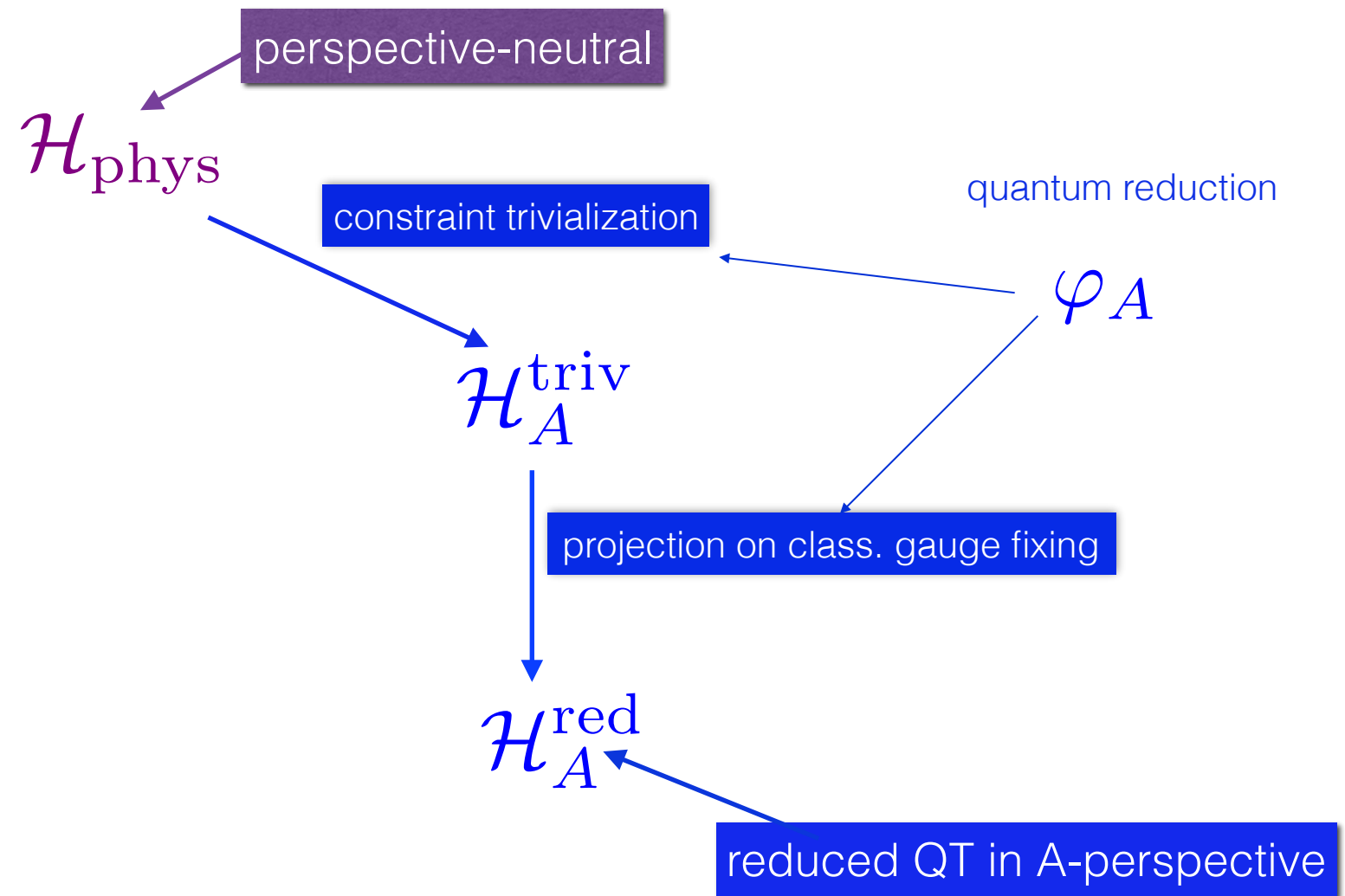
- observables transform correctly

$$\varphi_A (\hat{q}_{E,F} - \hat{q}_A) \varphi_A^{-1} = \hat{q}_{E,F}$$

$$\varphi_A \hat{p}_{E,F} \varphi_A^{-1} = \hat{p}_{E,F}$$

reduced QT in A gauge

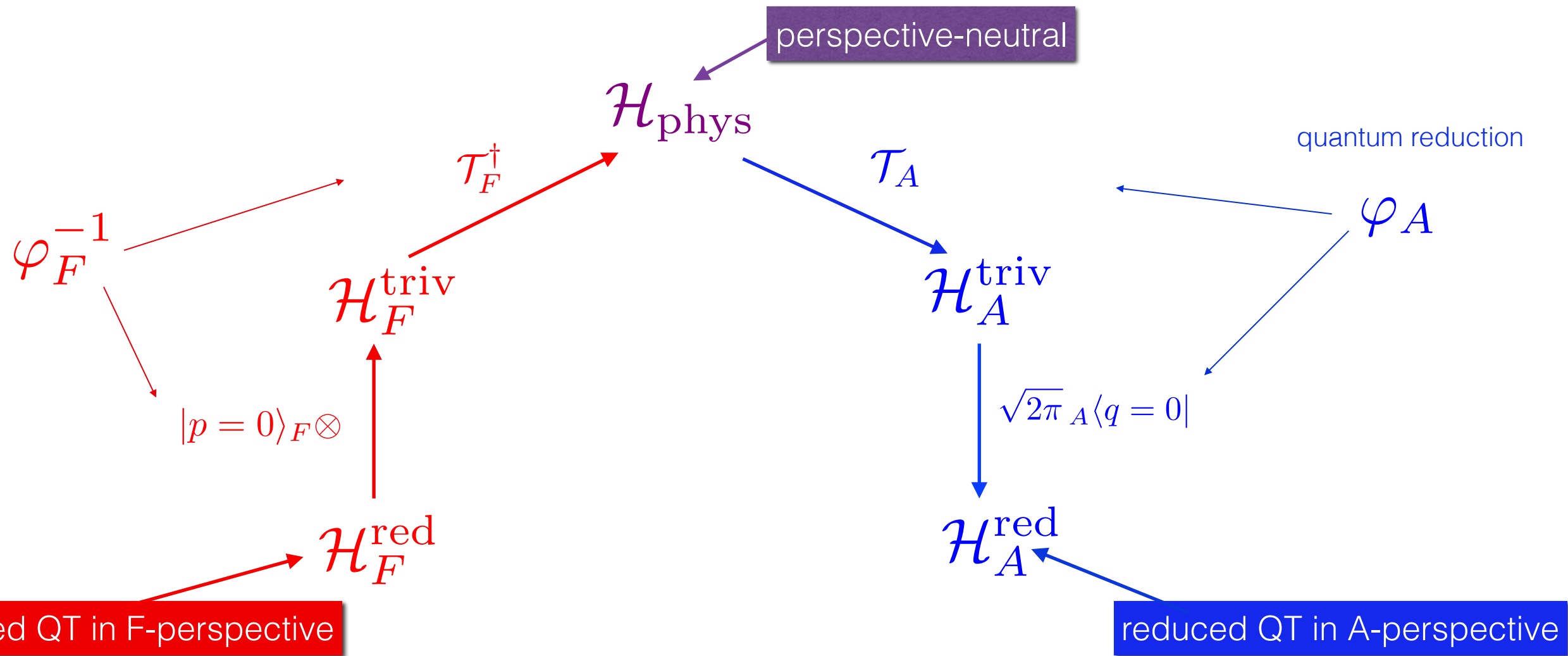
Switch from F- to A-perspective in QT



invert F-reduction φ_F^{-1} , concatenate
with A-reduction φ_A

Switch from F- to A-perspective in QT

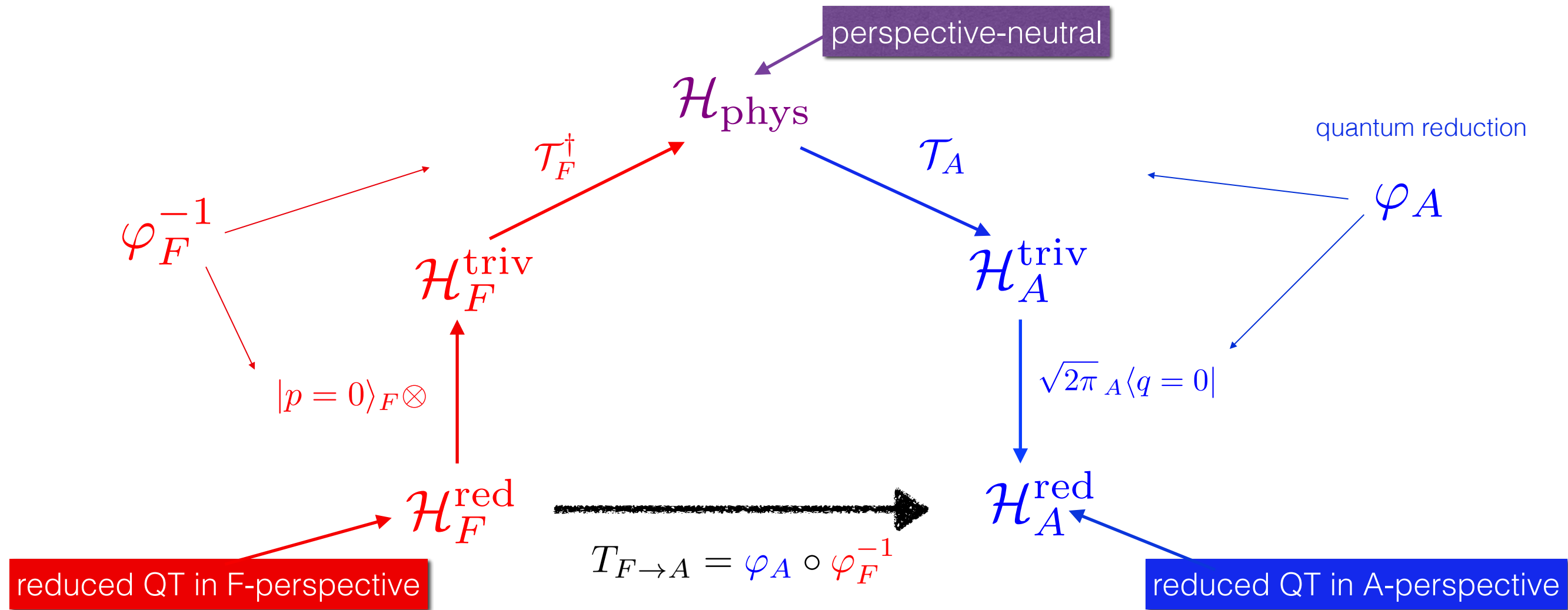
Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556



invert F-reduction φ_F^{-1} , concatenate
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Switch from F- to A-perspective in QT

Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556



concretely

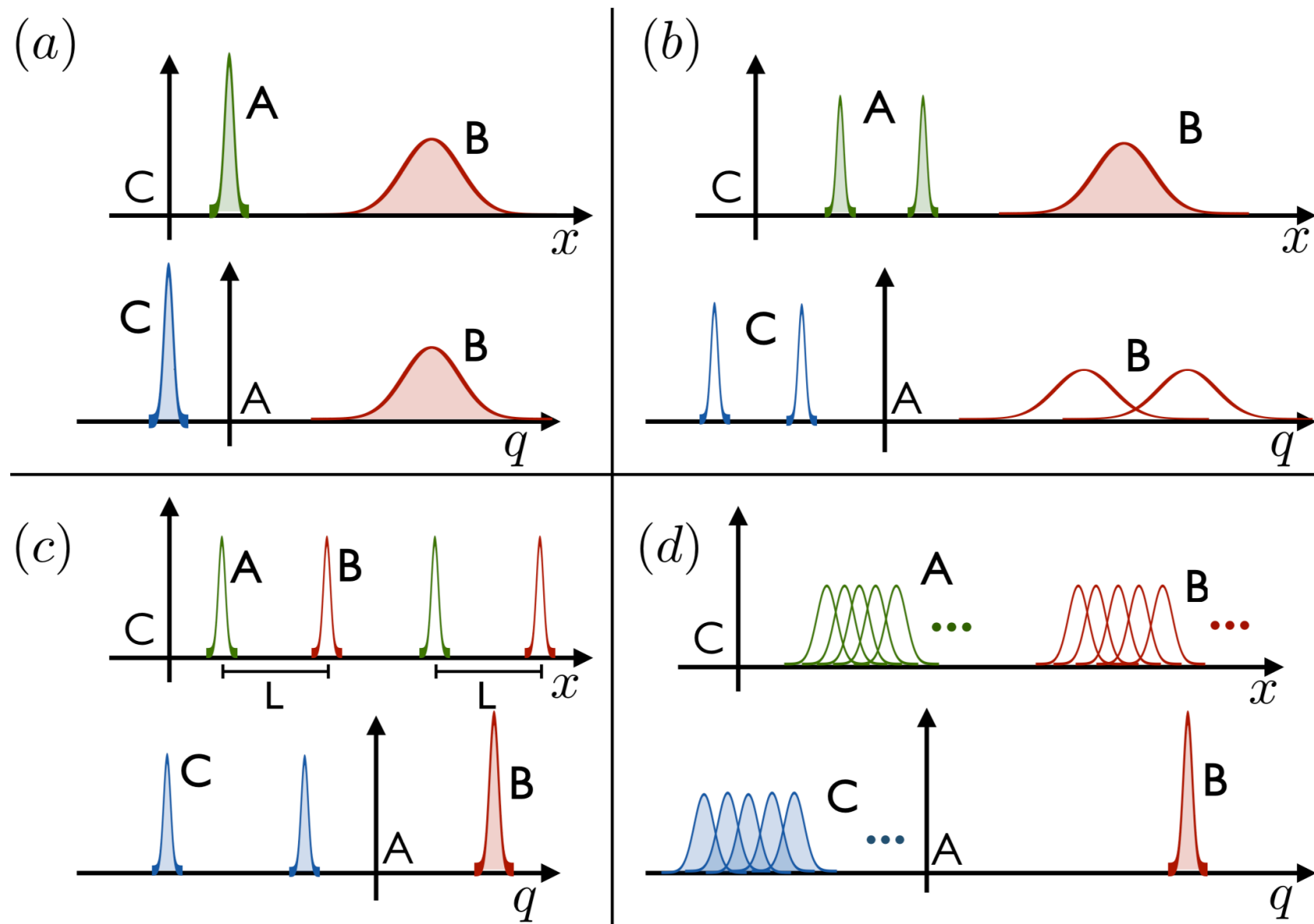
$$\begin{aligned}
 T_{F \rightarrow A} &= \sqrt{2\pi} {}_A\langle q=0| \mathcal{T}_A \mathcal{T}_F^\dagger |p=0\rangle_F \otimes \\
 &= \mathcal{P}_{AF} e^{i\hat{q}_A \hat{p}_E}
 \end{aligned}$$

parity swap

$\Rightarrow$

recover QRF transf. proposed by Giacomini, Castro-Ruiz, Brukner (Nat. Comm. **10**, 494 (2019))

Operational consequences



see Giacomini, Castro-Ruiz, Brukner *Nat. Comm.* **10**, 494 (2019)

Generalizations

- 3D relational N-body problem (spatial QRF)

Gribov problem arises
 $\Rightarrow$ no global relational perspectives

Vanrietvelde, PH, Giacomini, arXiv:1809.05093

- switching quantum clocks (temporal QRS)

- non-relativistic

PH, Vanrietvelde, arxiv:1810.04153
PH, Lock, Smith, to appear

- relativistic/quantum cosmology ($k=m=0$ FRW)

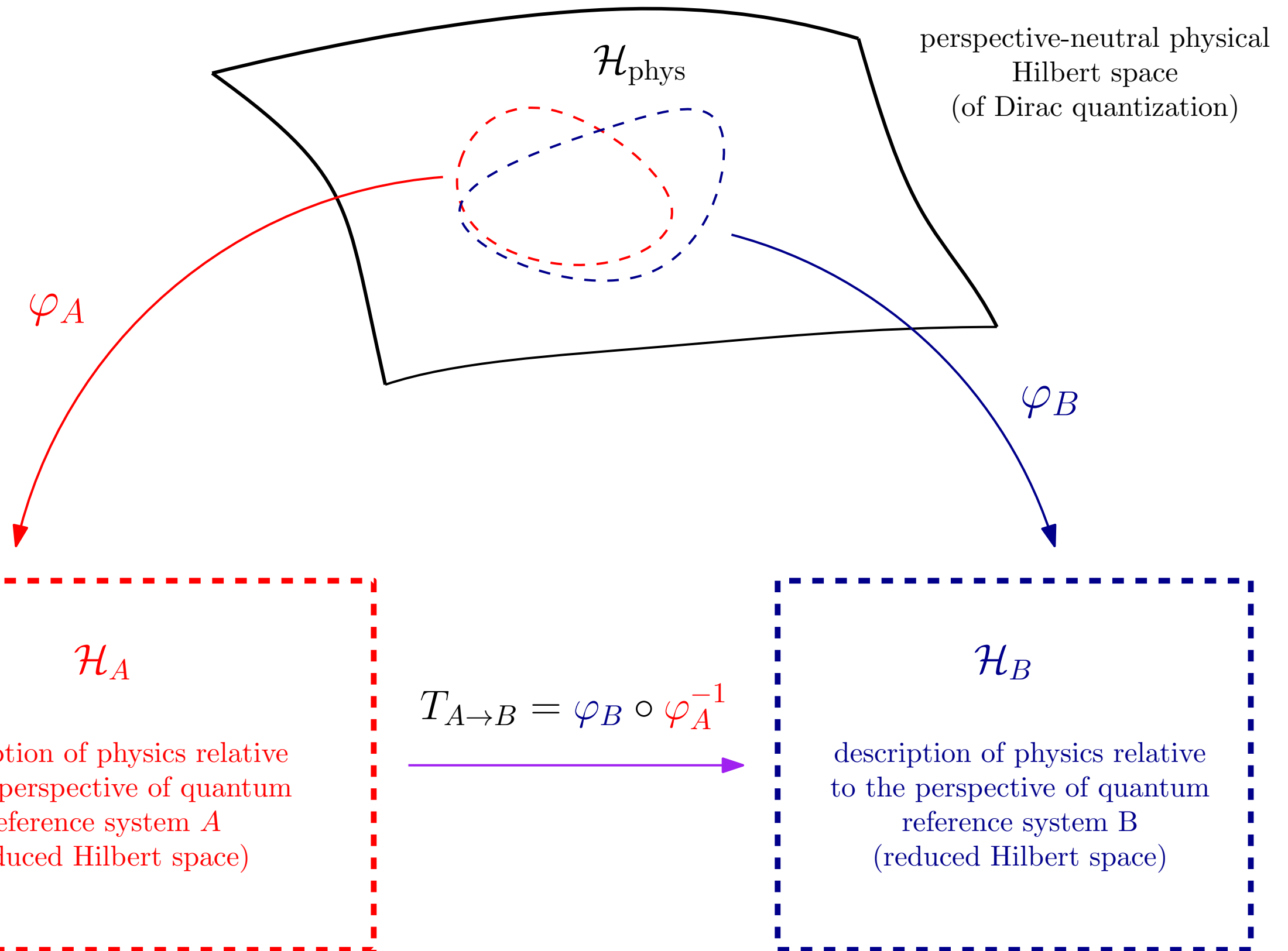
$\Rightarrow$ new perspective on “WF of Universe”

PH, arXiv:1811.00611
(also PH, Lock, Smith, to appear)

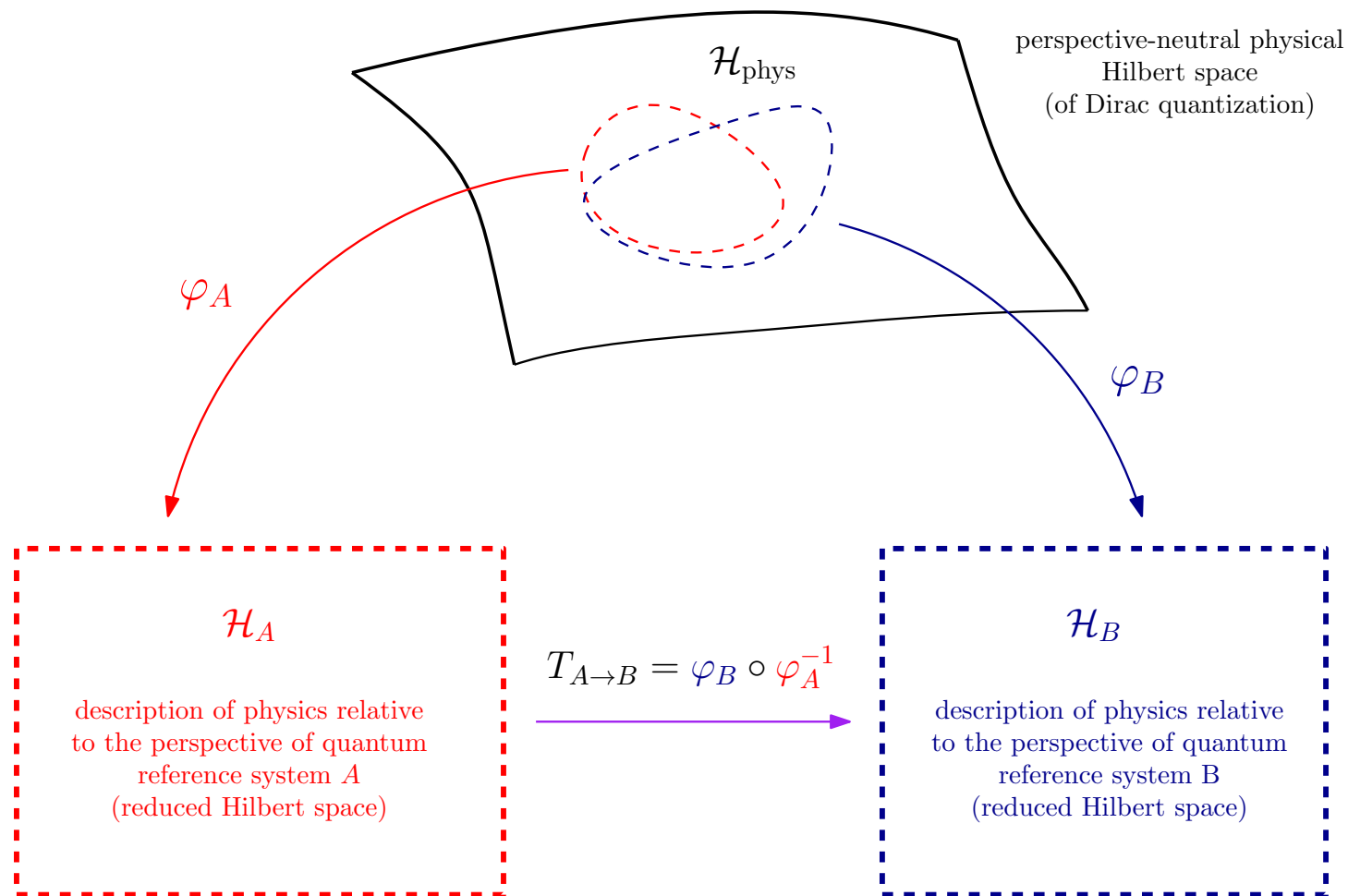
Quantum symmetry reduction procedure always:

1. Choose QRS
2. Transfer redundancy to QRS (trivialization)
3. Remove redundancy (project onto class. gauge fixing)

‘Quantum coordinate changes’



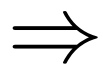
Conclusions so far:



systematic switches of quantum
reference system perspectives

- new: quantum reduction method
- both spatial and temporal

Not always global due to Gribov problem



**complete relational QT, admitting quantum general covariance:
conjunction of Dirac and red. quantized theories**

Application: Quantum Clocks

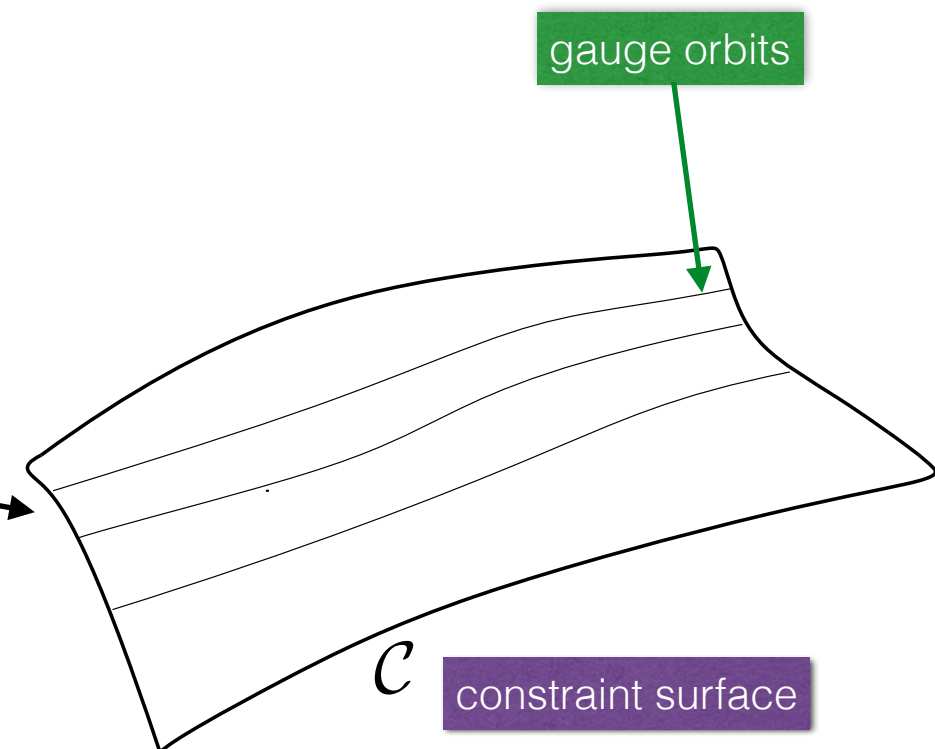
Hamiltonian constraints and time functions

- diffeo & reparametrization inv. lead to Hamiltonian constraints

$$C_H(\{\text{phase space variables}\}) \approx 0$$

$\Rightarrow$ generator of dynamics and symmetry

- Redundancy
- Not a phase space



Hamiltonian constraints and time functions

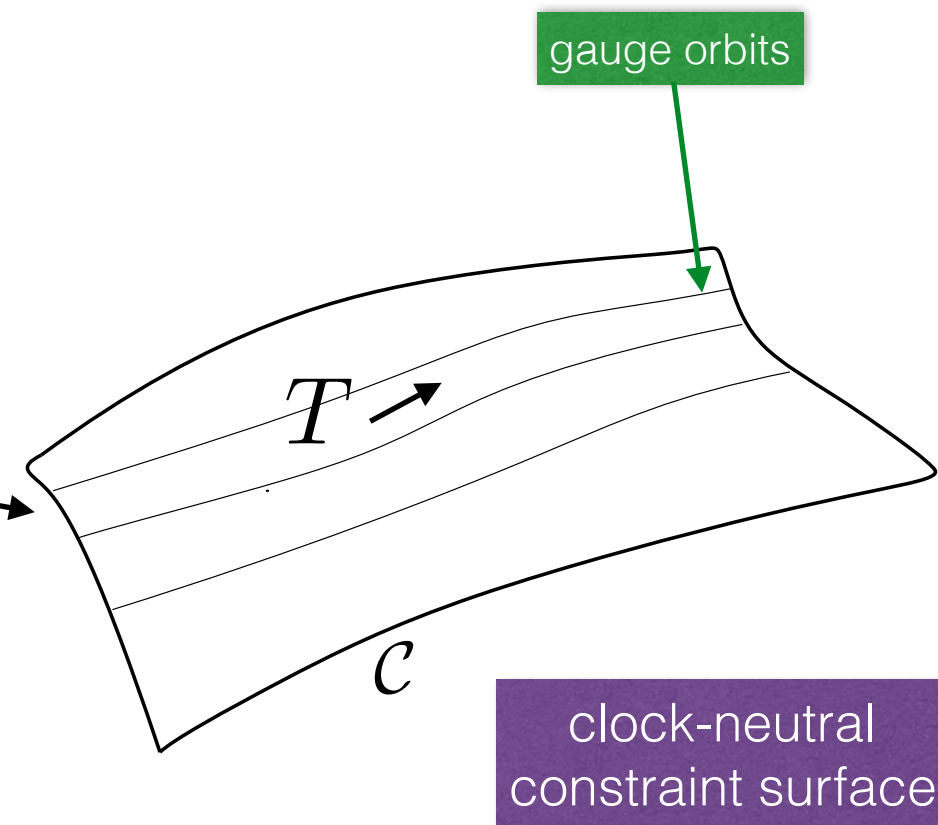
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- choose dynamical 'clock' T that parametrizes flow and evolve other DoFs w.r.t. it



[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich,...]

Restrict here to:

- diffeo & reparametrization inv. lead to Hamiltonian constraints

$$C_H(\{\text{phase space variables}\}) \approx 0$$

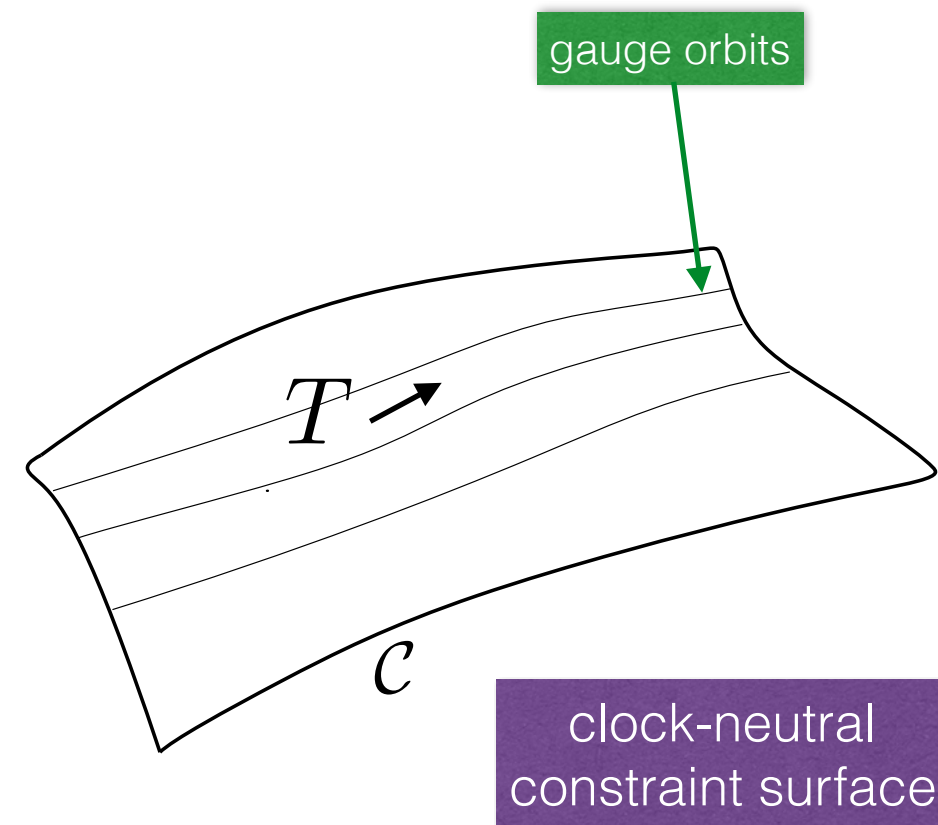
$\Rightarrow$ generator of dynamics and symmetry

$$C_H = H_C + H_S$$

clock system

- choose dynamical ‘clock’ T that parametrizes flow and evolve other DoFs w.r.t. it

[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich,...]



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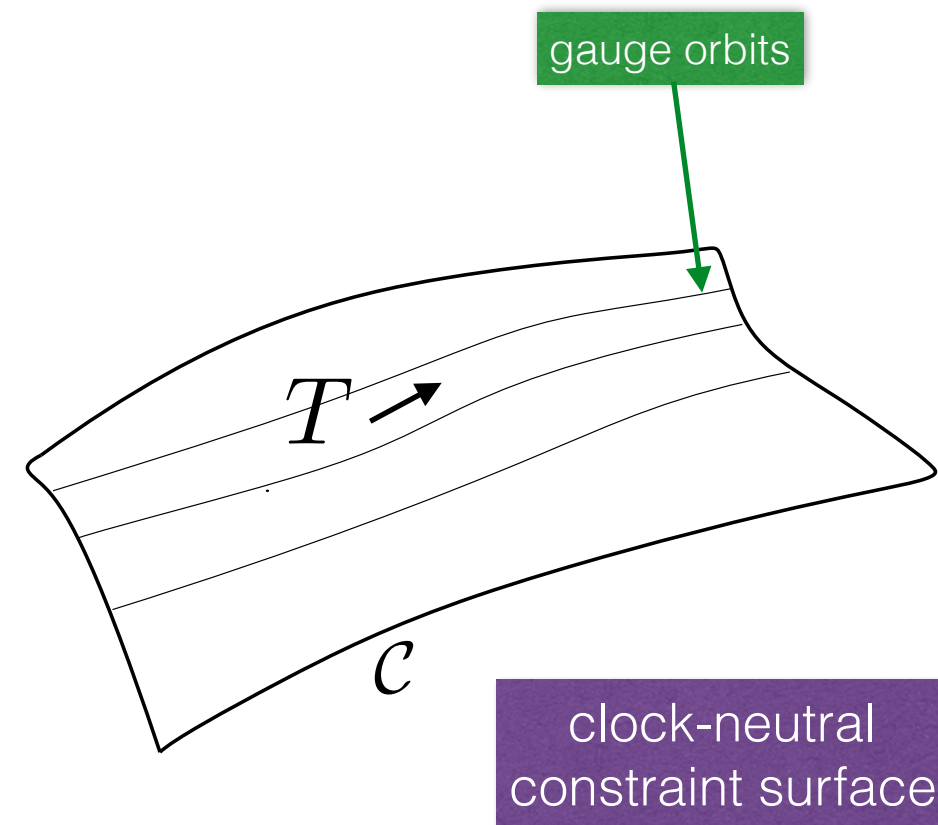
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[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich,...]

$$\{T, H_C\} = \{T, C_H\} = 1$$

Relational Dirac observables

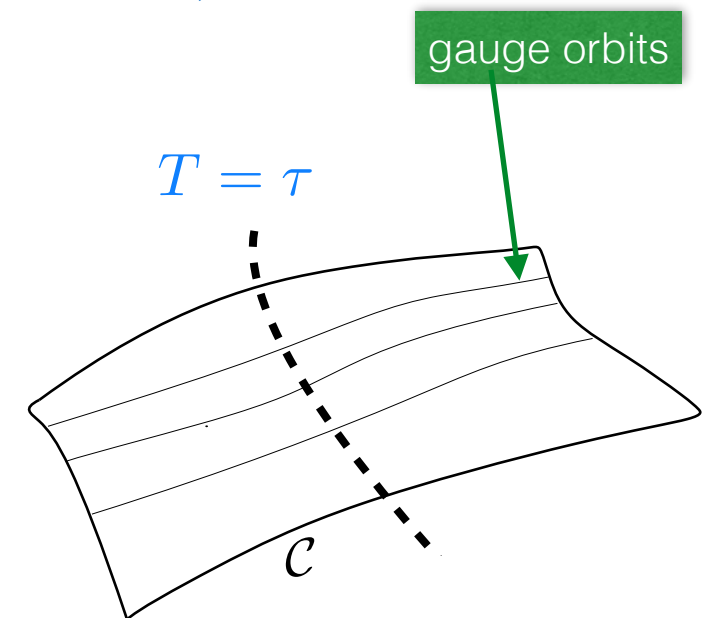
- under our restrictions:

$$F_{f_S, T} := \sum_{k=0} \frac{(\tau - T)^k}{k!} \{H_S, f_S\}_k$$

value of f when $T = \tau$

Dittrich '04; '05

- gauge-inv. $\{C_H, F_{f_S, T}(\tau)\} \approx 0$



- “gauge-inv. extensions of gauge fixed quantities”

Henneaux, Teitelboim 90's; Dittrich '04; '05

Problem of time

- Dirac quantization yields

$$\hat{C}_H |\psi_{\text{phys}}\rangle = (\hat{H}_C + \hat{H}_S) |\psi_{\text{phys}}\rangle = 0$$

Wheeler-DeWitt type equation, “frozen formalism”

Problem of time:

No more time coordinate, time has to be extracted from quantum DoFs

Problem of time

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⇒ **relational quantum dynamics**

[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich,...]

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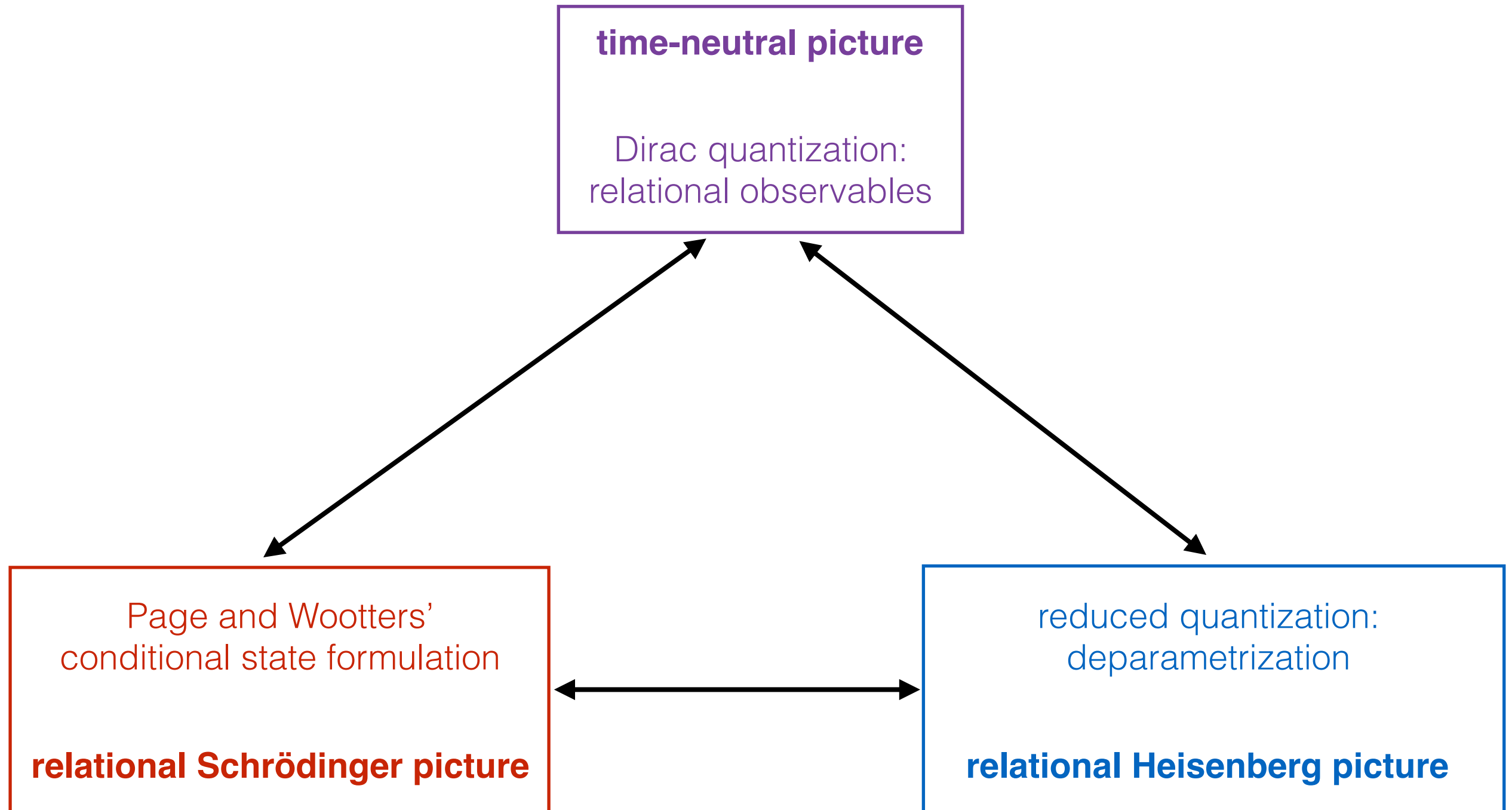
⇒ **relational quantum dynamics**

[DeWitt; Isham; Page & Wootters, Rovelli, Dittrich,...]

usually: ‘timeless’ state
better: clock-neutral state

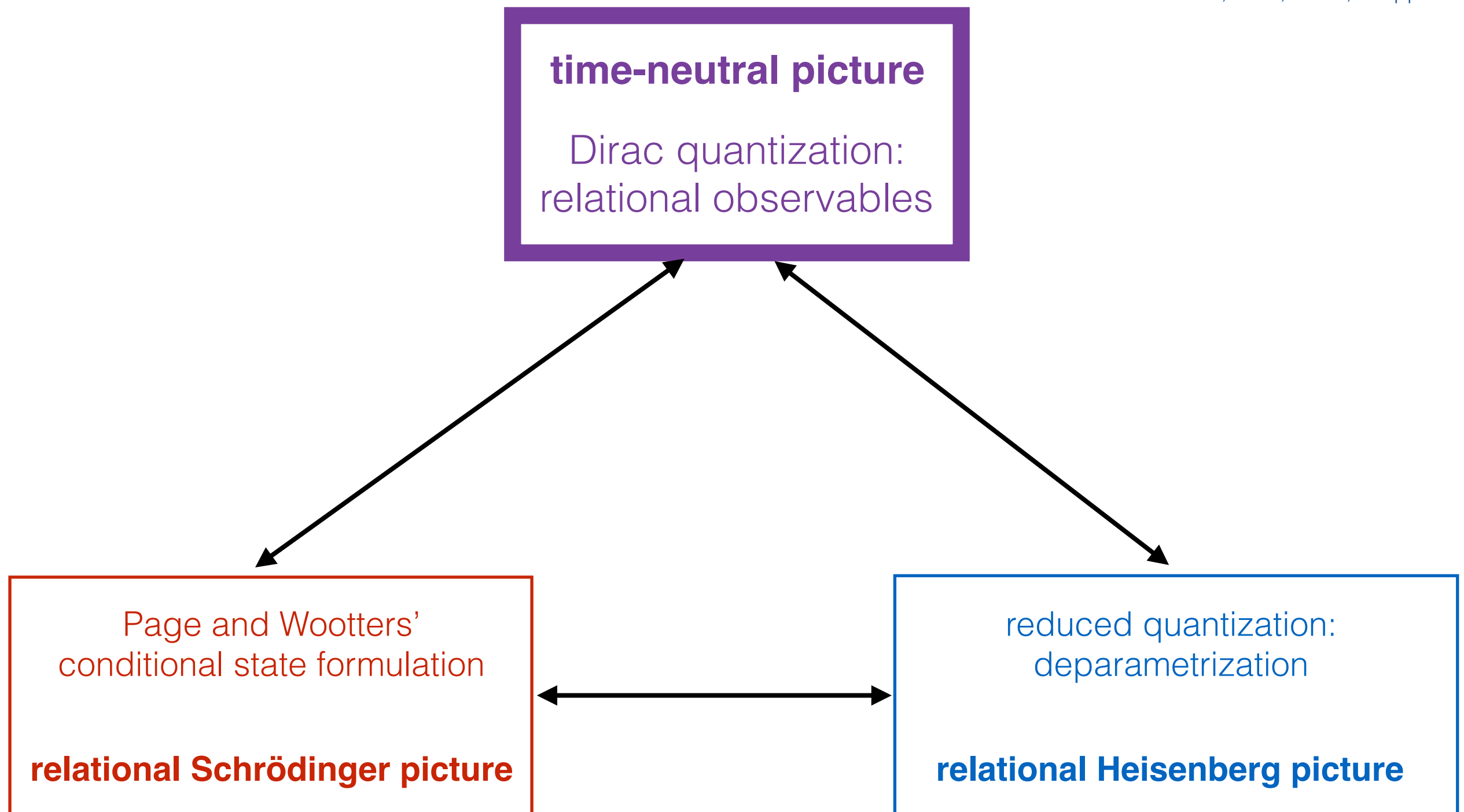
The trinity of relational quantum dynamics

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I: Quantum relational Dirac observables

‘clock-neutral’ states

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I: Quantum relational Dirac observables

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$$\hat{C}_H |\psi_{\text{phys}}\rangle = (\hat{H}_C + \hat{H}_S) |\psi_{\text{phys}}\rangle = 0$$

- choice of clock, covariant clock states

$$|t\rangle := e^{-i t \hat{H}_C} |t_0\rangle \quad \text{on clock Hilbert space } \mathcal{H}_C$$

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- dynamics via rel. observables

PH, Lock, Smith, to appear

$$\begin{aligned} \hat{F}_{f,T}(\tau) &:= \int_{\text{range}(T)} dt |t\rangle \langle t| \otimes \sum_{n=0}^{\infty} \frac{i^n}{n!} (\tau - t)^n [\hat{H}_S, \hat{f}_S]_n \\ &= \int_{\text{range}(T)} dt e^{-i \hat{C}_H t} \left(|\tau\rangle \langle \tau| \otimes \hat{f}_S \right) e^{i \hat{C}_H t} \end{aligned} \quad \longleftarrow \text{‘G-twirl’}$$

gauge-inv.

$$[\hat{F}_{f_S,T}, \hat{C}_H] = 0$$

- quantum analog of “gauge-inv. extensions of gauge fixed quantities”

I: Quantum relational Dirac observables

‘clock-neutral’ states

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PH, Lock, Smith, to appear

$$\begin{aligned} \hat{F}_{f,T}(\tau) &:= \int_{\text{range}(T)} dt |t\rangle \langle t| \otimes \sum_{n=0}^{\infty} \frac{i^n}{n!} (\tau - t)^n [\hat{H}_S, \hat{f}_S]_n \\ &= \int_{\text{range}(T)} dt e^{-i \hat{C}_H t} \left(|\tau\rangle \langle \tau| \otimes \hat{f}_S \right) e^{i \hat{C}_H t} \end{aligned} \quad \leftarrow \text{‘G-twirl’}$$

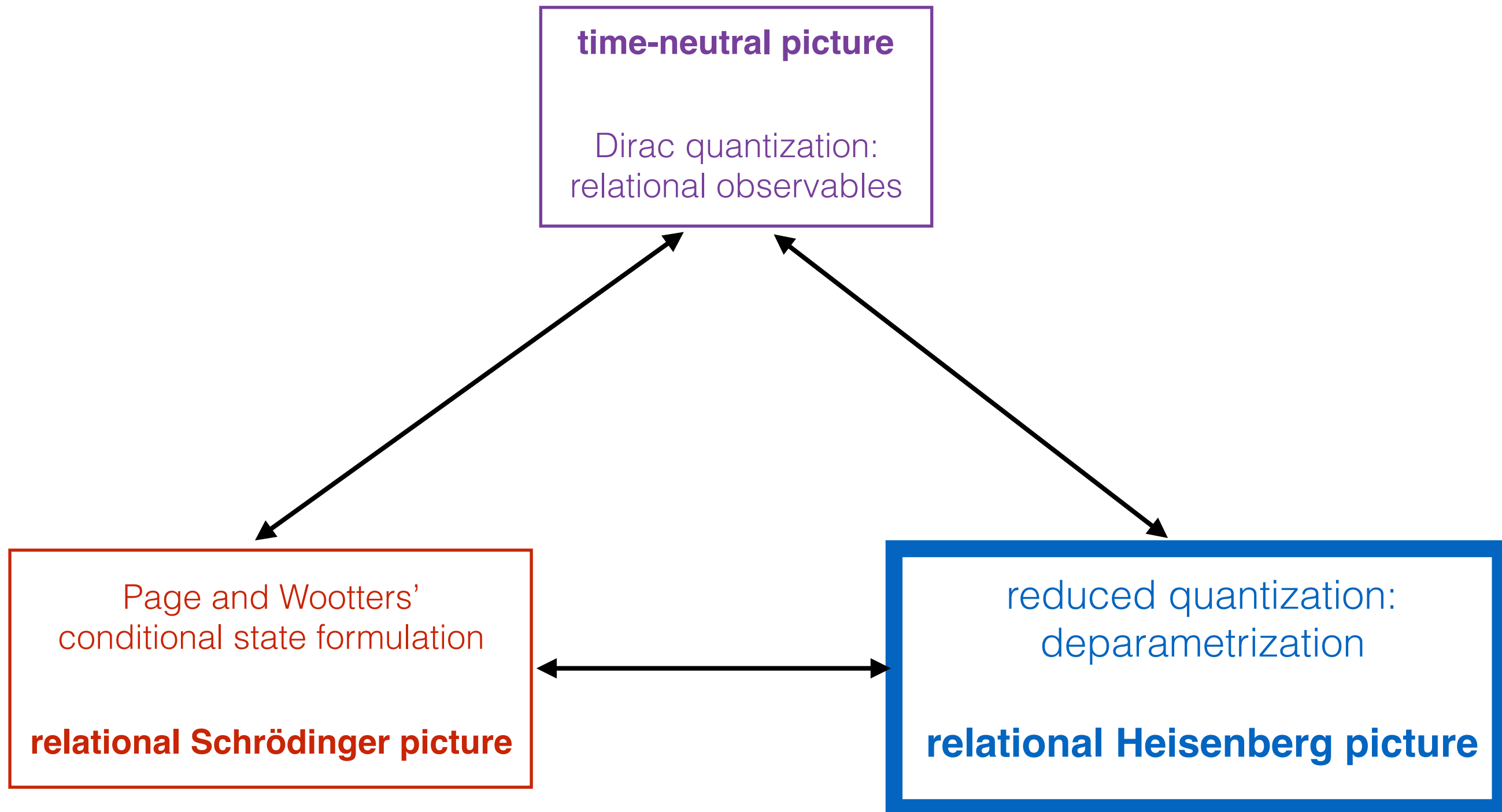
Physical inner product

$\Rightarrow$ gauge-inv. evolution: let $\mathcal{T}$ run in

$$\langle \psi_{\text{phys}} | \hat{F}_{f_S, T}(\tau) | \psi_{\text{phys}} \rangle_{\text{phys}}$$

The trinity of relational quantum dynamics

PH, Lock, Smith, to appear



II: deparametrization via reduced quantization

PH, Lock, Smith, to appear

1. classical reduction:

gauge fix flow of C_H via clock, e.g. $T = 0$

$$\Rightarrow \text{rel. Dirac obs. } F_{f_S, T}(\tau) = \sum_{n=0} \frac{(\tau - T)^n}{n!} \{H_S, f_S\}_n$$

reduce to

$$F_{f_S}(\tau) = \sum_{n=0} \frac{\tau^n}{n!} \{H_S, f_S\}_n^D$$

← Dirac bracket

relational EoMs $\frac{dF_f}{d\tau} = \{H_S, f_S\}^D$ on red. system phase space $\mathcal{P}_S$

II: deparametrization via reduced quantization

PH, Lock, Smith, to appear

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physics rel. to clock T

II: deparametrization via reduced quantization

PH, Lock, Smith, to appear

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$$\text{reduce to } F_{f_S}(\tau) = \sum_{n=0} \frac{\tau^n}{n!} \{H_S, f_S\}_n^D \quad \leftarrow \text{Dirac bracket}$$

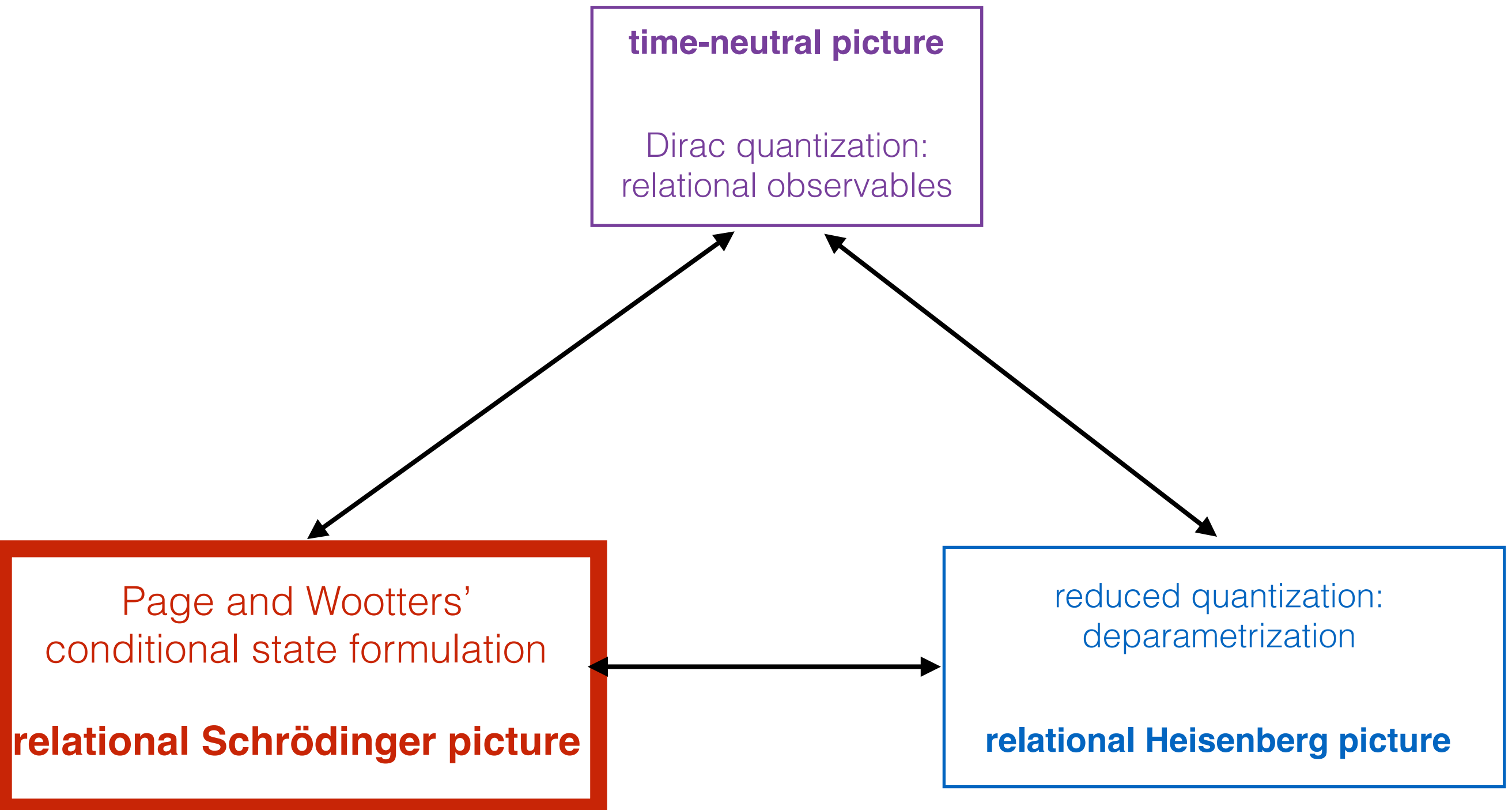
$$\text{relational EoMs } \frac{dF_f}{d\tau} = \{H_S, f_S\}^D \quad \text{on red. system phase space } \mathcal{P}_S$$

$$\begin{aligned} \text{2. reduced quantization: } \hat{F}_{f_S}(\tau) &:= \sum_{n=0} \frac{(i\tau)^n}{n!} [\hat{H}_S, \hat{f}_S]_n \\ &= e^{i\tau \hat{H}_S} \hat{f} e^{-i\tau \hat{H}_S} \quad \text{on system Hilbert space } \mathcal{H}_S \end{aligned}$$

$$\text{relational Heisenberg EoMs } \frac{d\hat{F}_{f_S}}{d\tau} = i [\hat{H}_S, \hat{f}_S]$$

The trinity of relational quantum dynamics

PH, Lock, Smith, to appear



III: Page and Wootters's formulation

Page, Wootters '83

'clock-neutral' states

$$\hat{C}_H |\psi_{\text{phys}}\rangle = (\hat{H}_C + \hat{H}_S) |\psi_{\text{phys}}\rangle = 0$$

- conditional state of system when clock reads τ

$$|\psi_S(\tau)\rangle := (\langle\tau| \otimes I_S) |\psi_{\text{phys}}\rangle \in \mathcal{H}_S$$

- solves **relational Schrödinger eq.**

$$i \frac{d}{d\tau} |\psi_S(\tau)\rangle = \hat{H}_S |\psi_S(\tau)\rangle$$

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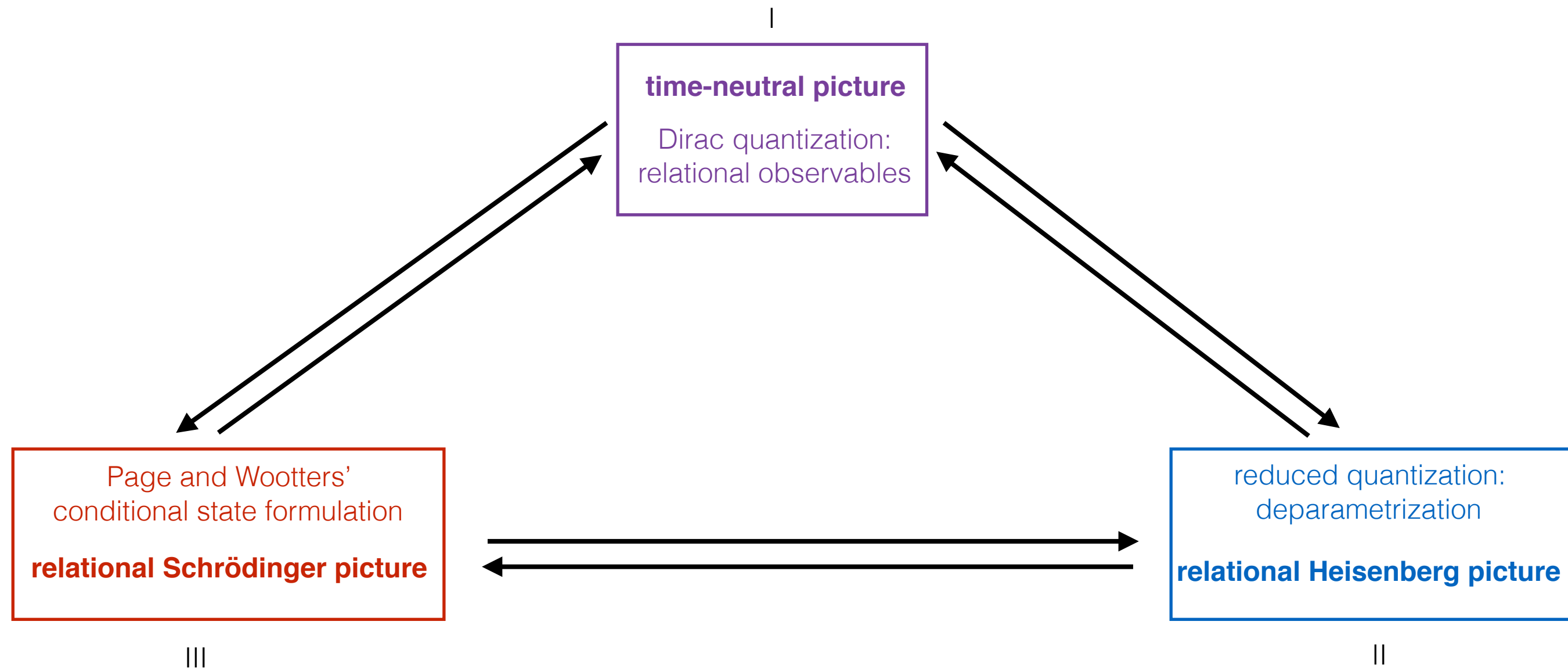
$\Rightarrow$ evaluate system observables $\hat{f}_S$ on evolv. states, e.g.

$$\frac{\langle\psi(\tau)| \hat{f}_S |\psi_S(\tau)\rangle}{\langle\psi_S|\psi_S\rangle} = \frac{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes \hat{f}_S)|\psi_{\text{phys}}\rangle_{\text{kin}}}{\langle\psi_{\text{phys}}|(|\tau\rangle\langle\tau| \otimes I_S)|\psi_{\text{phys}}\rangle_{\text{kin}}}$$

Trinity of relational quantum dynamics

I, II & III can be isometrically mapped into each other:

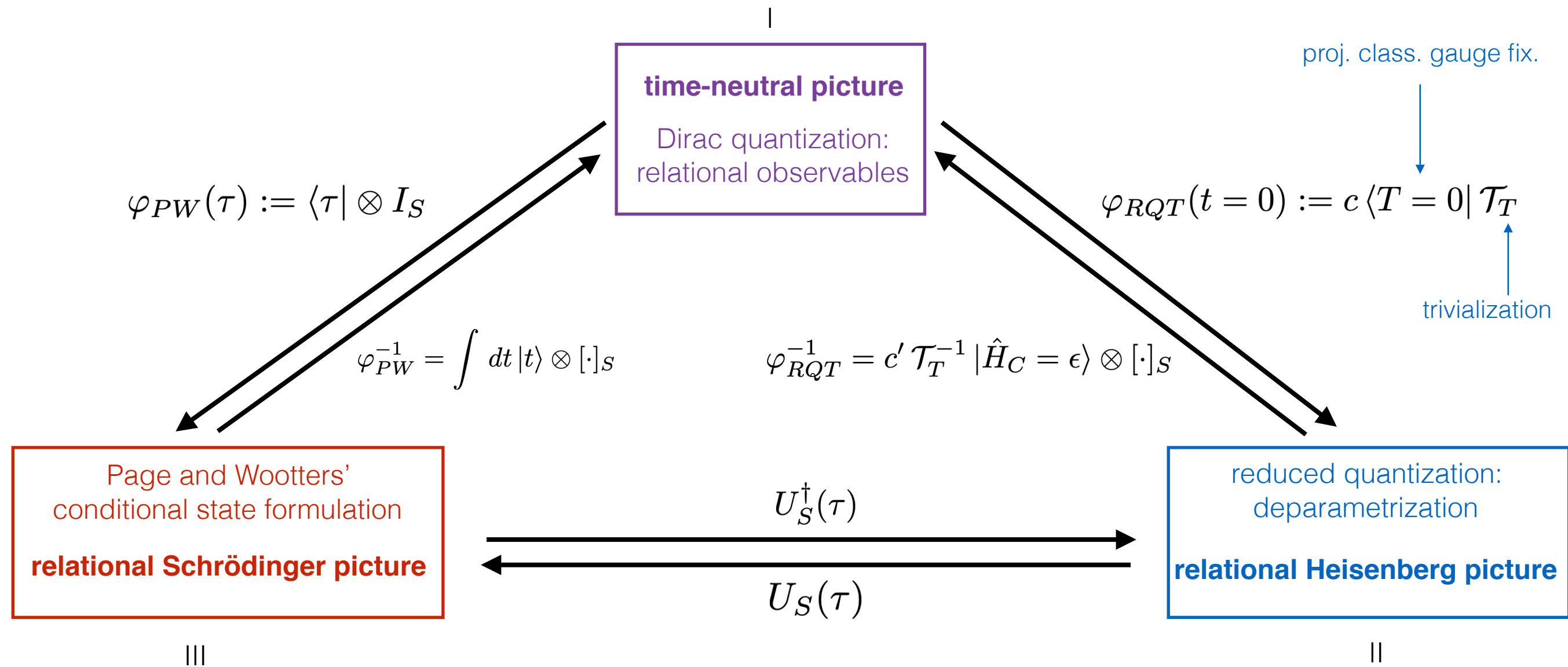
PH, Lock, Smith, to appear



Trinity of relational quantum dynamics

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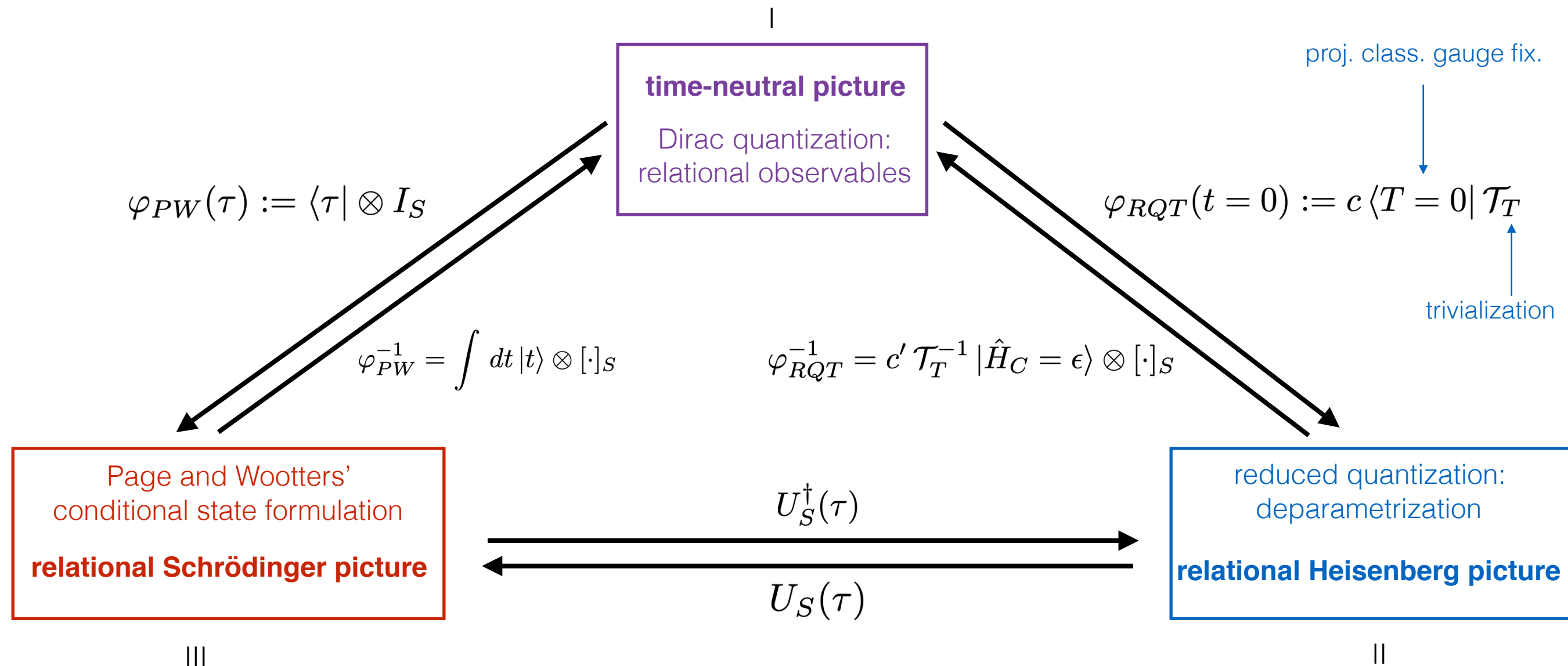
PH, Lock, Smith, to appear



Trinity of relational quantum dynamics

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PH, Lock, Smith, to appear



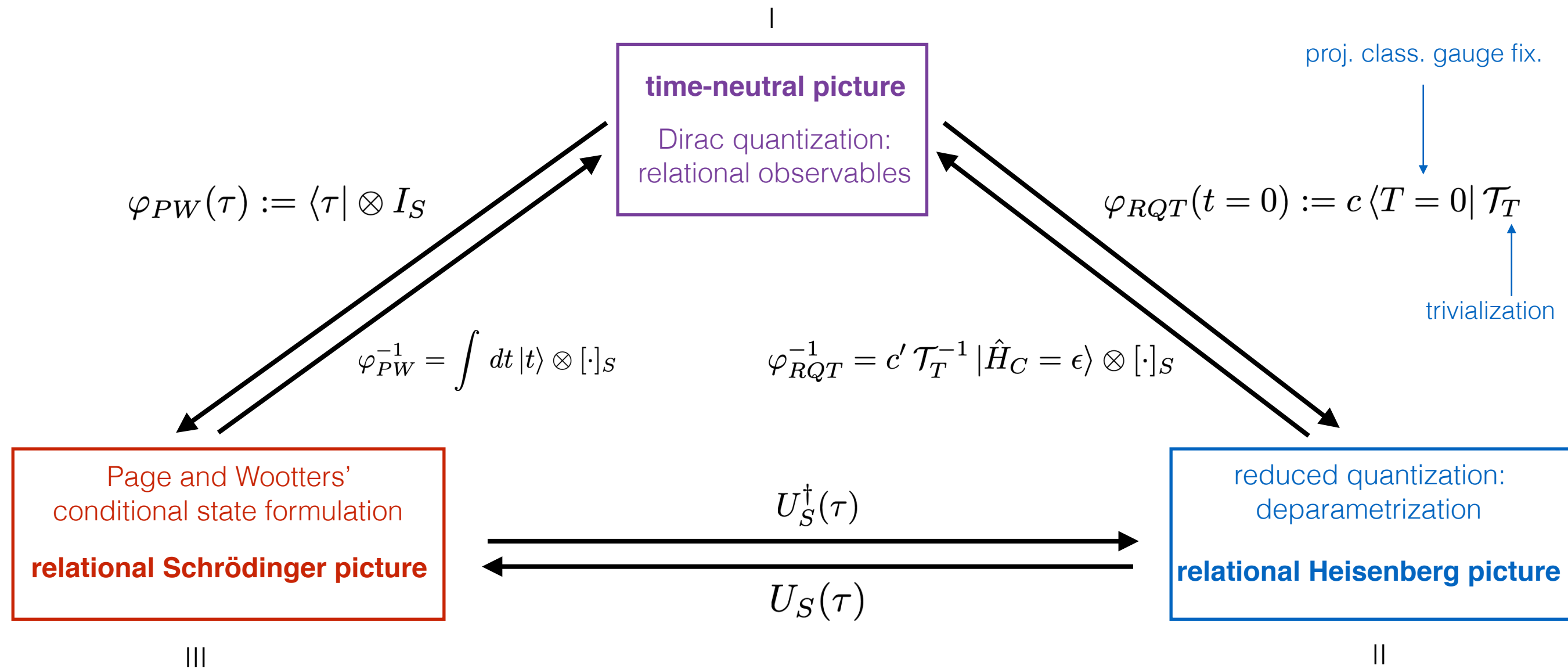
Fine print:

Equivalence shown for $\hat{H}_C$ non-deg. or $\hat{p}_T^2$ -like (in latter case: per +/- frequency sector), generalization: in progress

Trinity of relational quantum dynamics

I, II & III can be isometrically mapped into each other:

PH, Lock, Smith, to appear



- observables transform correctly, e.g., $\hat{F}_{f_S, T}(\tau) = \varphi_{RQT}^{-1} \hat{F}_{f_S}(\tau) \varphi_{RQT}$ and

$$\langle \psi_{\text{phys}} | \hat{F}_{f_S, T}(\tau) | \psi_{\text{phys}} \rangle_{\text{phys}} = \langle \psi_S(\tau) | \hat{f}_S | \psi_S(\tau) \rangle = \langle \psi_S | \hat{F}_{f_S}(\tau) | \psi_S \rangle$$

Switching quantum clocks

Bojowald, PH, Tsobanjan, CQG 28, 035006 (2011)
Bojowald, PH, Tsobanjan, PRD 83, 125023 (2011)
PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)

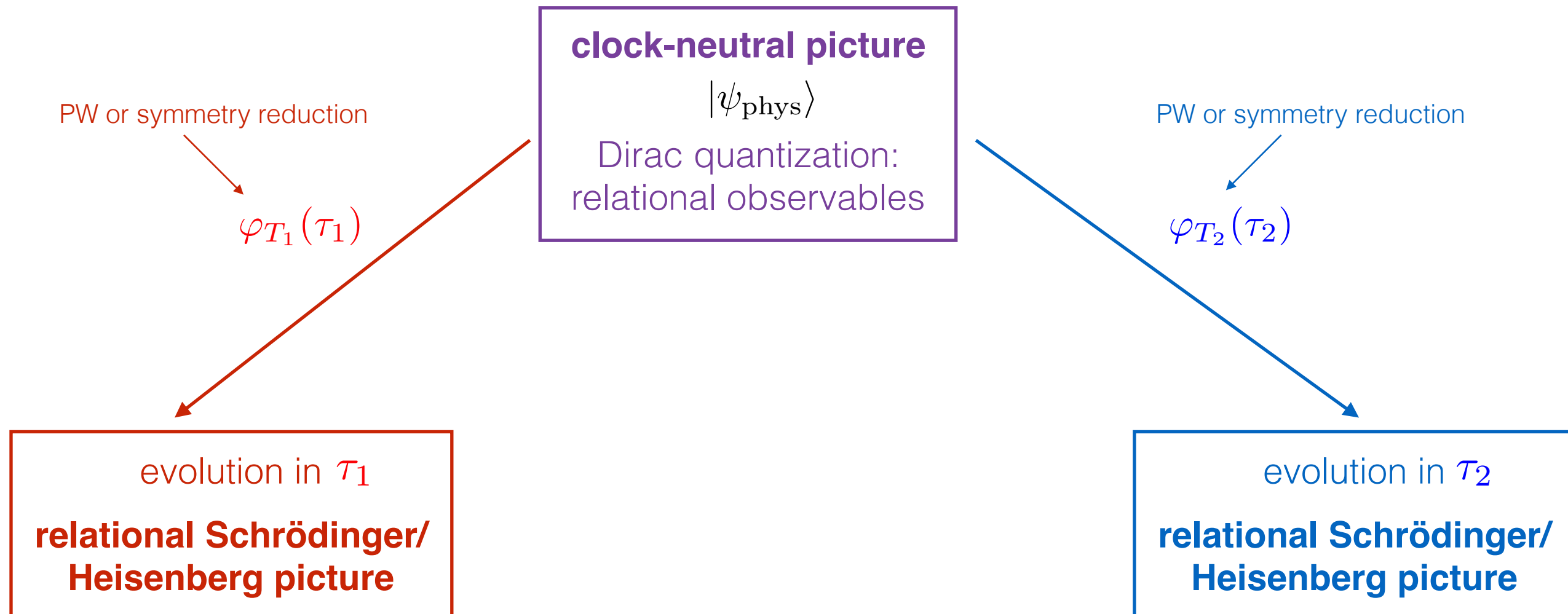
PH, Vanrietvelde, arxiv:1810.04153
PH, arXiv:1811.00611
PH, Lock, Smith, to appear

Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153

PH, arXiv:1811.00611

PH, Lock, Smith, to appear

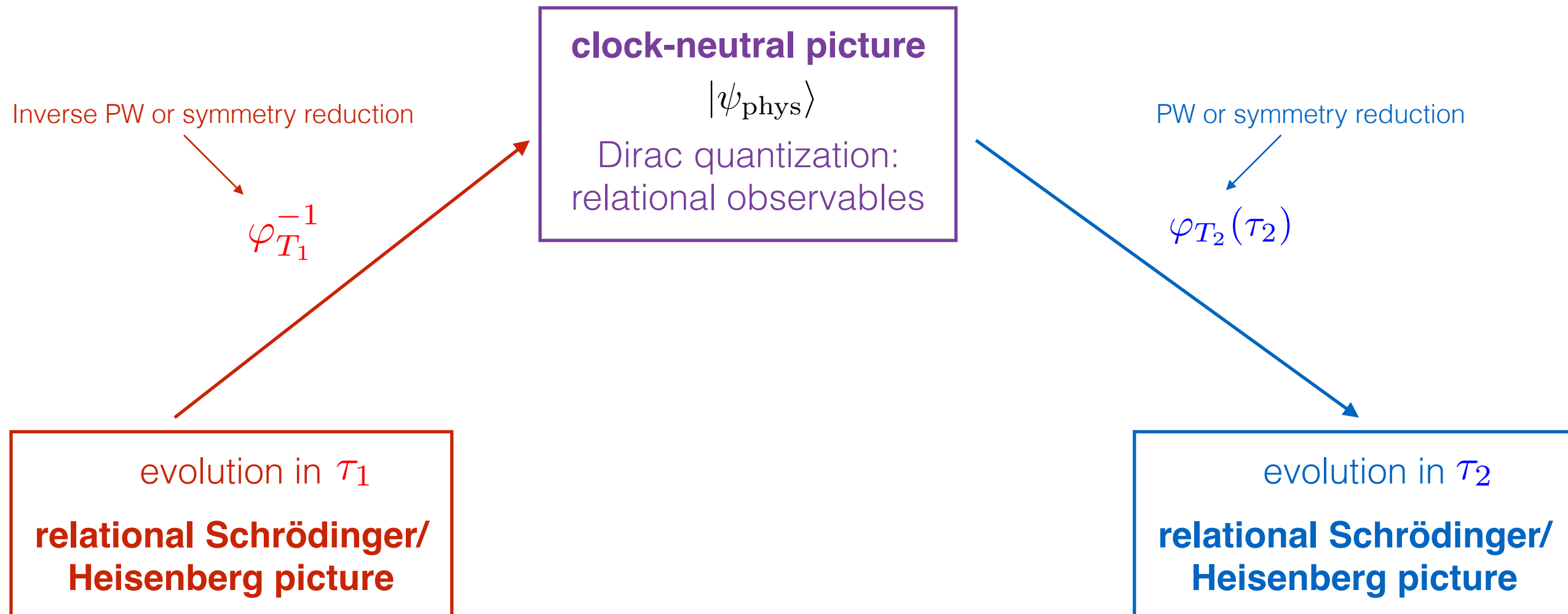


Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153

PH, arXiv:1811.00611

PH, Lock, Smith, to appear

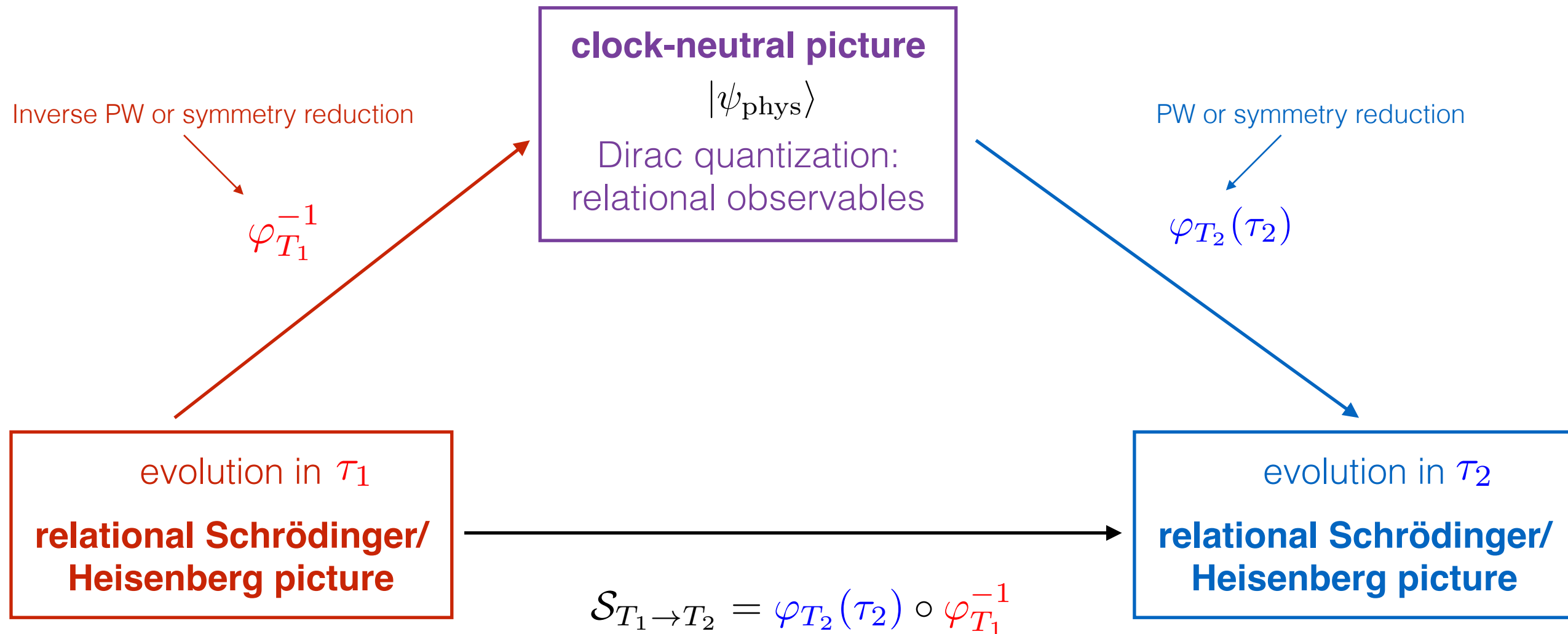


Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153

PH, arXiv:1811.00611

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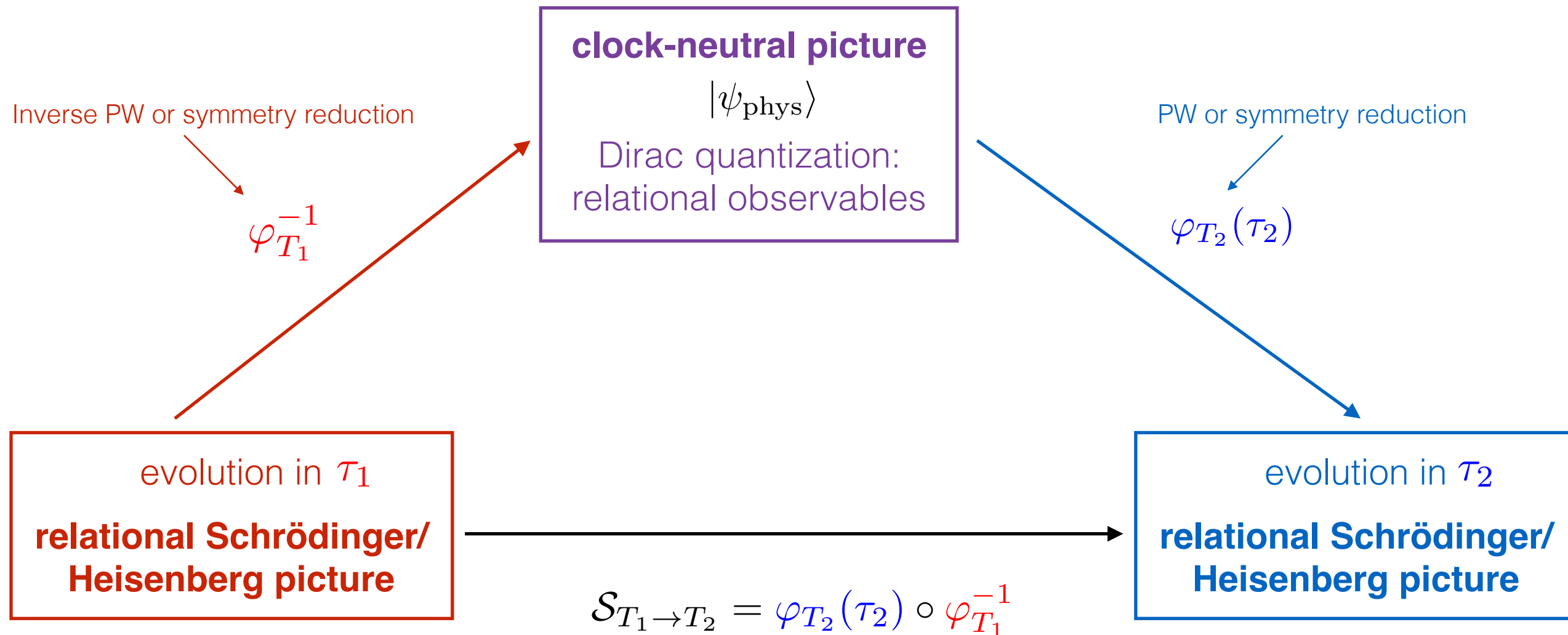


Scheme: switching quantum clocks

PH, Vanrietvelde, arxiv:1810.04153

PH, arXiv:1811.00611

PH, Lock, Smith, to appear



- observables + states transform correctly:

- $C_H = p_T + p_S^2$ PH, Vanrietvelde, arxiv:1810.04153

- k=0 FRW with m=0 scalar field (switching a, ϕ)

PH, arXiv:1811.00611

Multiple choice problem

- many possible choices for relational clocks $\Rightarrow$ inequivalent quantum dynamics

e.g., 2 clocks variables T_1, T_2

$T_1(T_2)$ vs $T_2(T_1)$

e.g. $T_1 = a, T_2 = \varphi$
in quantum cosmology

What if T_1, T_2 operators?

Kuchar (1992):

“The multiple choice problem is one of an embarrassment of riches: out of many inequivalent options, one does not know which one to select.”

Isham (1993):

“Can these different quantum theories be seen to be part of an overall scheme that is covariant?... It seems most unlikely that a single Hilbert space can be used for all possible choices of an internal time function.”

Multiple choice ~~problem~~ **feature?**

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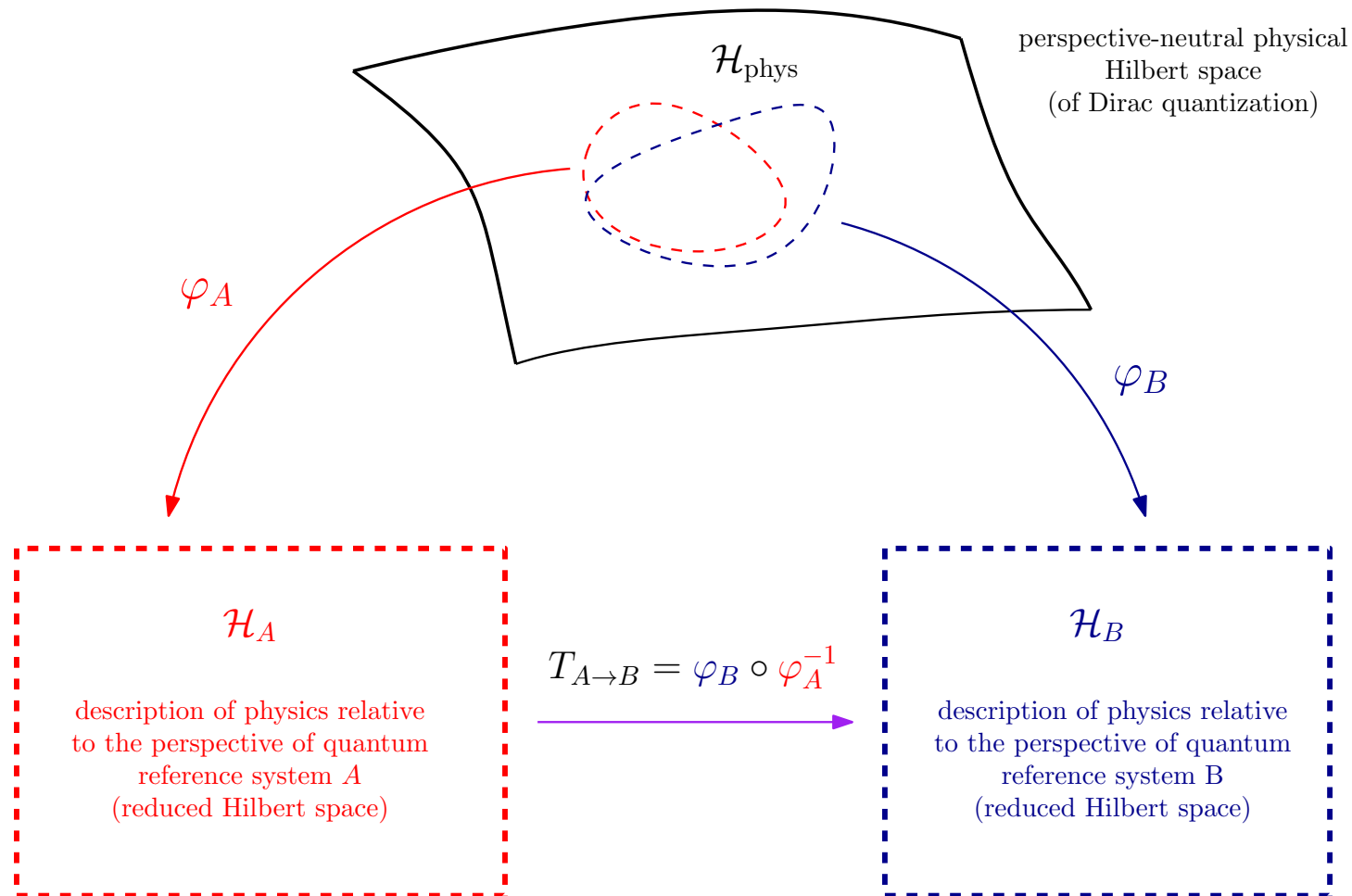
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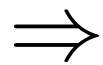
Conclusions



systematic switches of quantum
reference system perspectives

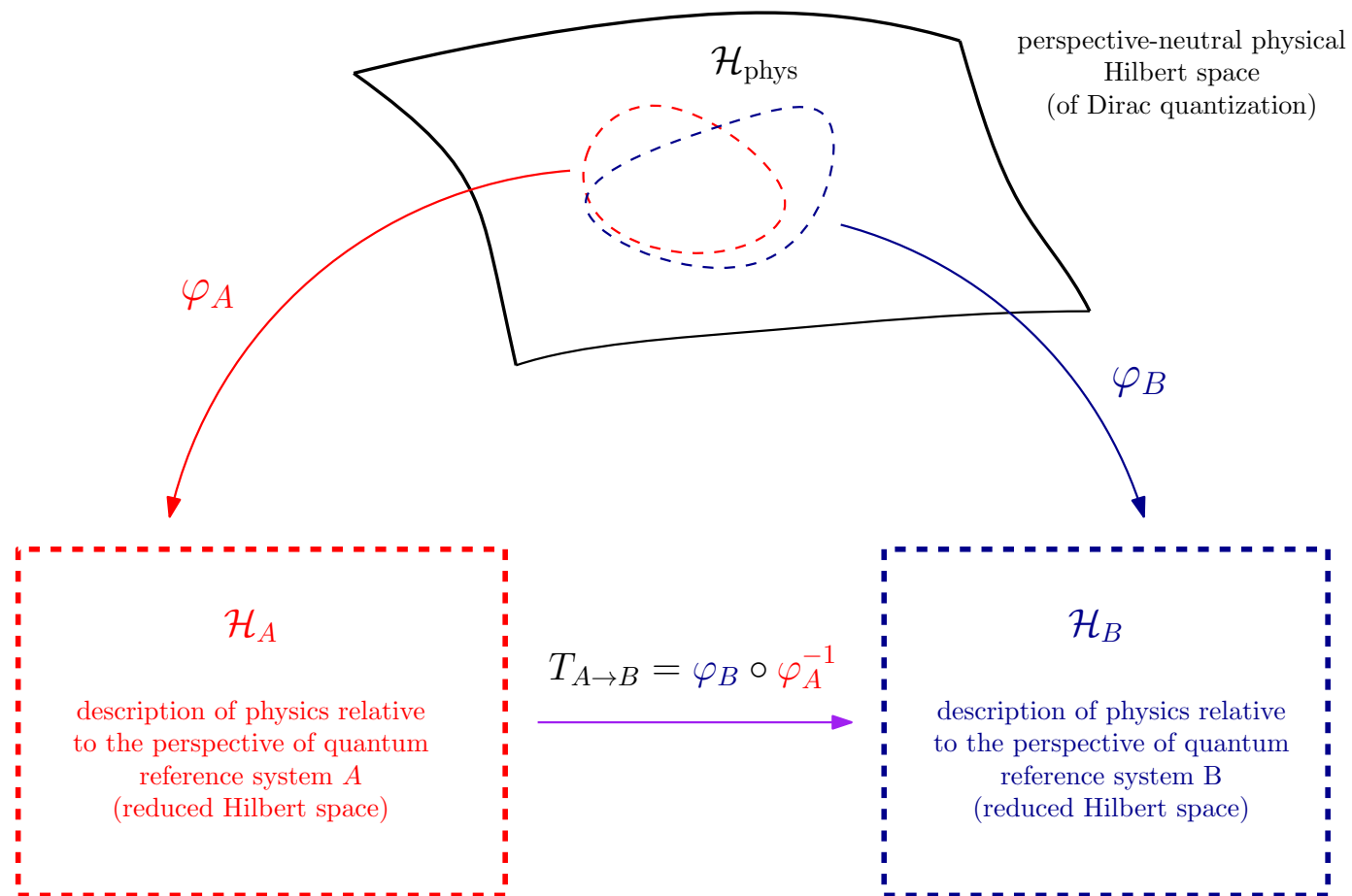
- new: quantum reduction method
- both spatial and temporal

Not always global due to Gribov problem



- **quantum general covariance**
- **Trinity:** Dirac, PaW, red. QT -> 3 faces of
same rel. quantum dynamics

Outlook



Systematic method for switching
(spatial and temporal)
quantum reference system perspectives

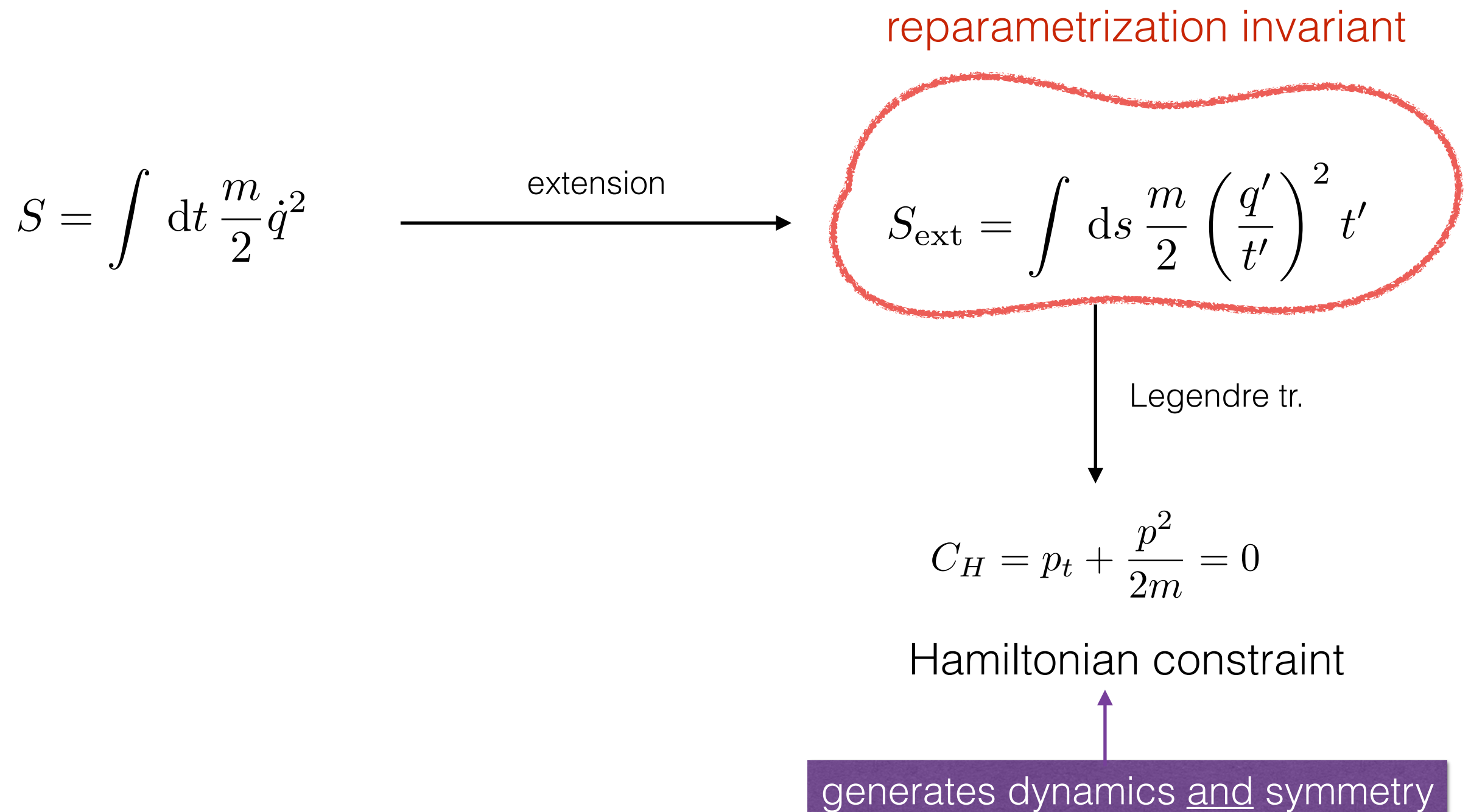
Vanrietvelde, PH, Giacomini, Castro-Ruiz, arXiv:1809.00556
 Vanrietvelde, PH, Giacomini, arXiv:1809.05093
 PH, Vanrietvelde, arXiv:1810.04153
 PH, arXiv:1811.00611
 PH, Lock, Smith, to appear

applications/extensions:

- QFT
- Measurement problem and Wigner's friend paradox
- Frame dependence of correlations in cosmology
- Import QRF machinery from QI into QG
- New method for constructing quantum rel. Dirac obs.

Appendix

Example: parametrized particle



Example: parametrized particle

$$S = \int dt \frac{m}{2} \dot{q}^2$$

extension

reparametrization invariant

$$S_{\text{ext}} = \int ds \frac{m}{2} \left(\frac{q'}{t'} \right)^2 t'$$

Legendre tr.

$$C_H = p_t + \frac{p^2}{2m} = 0$$

Hamiltonian constraint

generates dynamics and symmetry

clock	relational Dirac observable
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$
q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$

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Example: parametrized particle

Getting rid of redundancy through gauge-fixing

clock	relational Dirac observable on constraint surface
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Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m} (p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable	gauge fixing:
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\text{red}}(\tau) = 2p\tau + q$	
q	$T(X) = t(s) \big _{q(s)=X} = t_0 + \frac{X - q_0}{2p}$	$T_{\pm}(X) = t \mp \frac{X}{2\sqrt{-p_t}}$	

$$(t_0 = 0)$$

$$(q_0 = 0)$$

Example: parametrized particle

Getting rid of redundancy through gauge-fixing

$$C_H = p_t + \frac{p^2}{2m} = \frac{1}{2m} (p + \sqrt{-p_t})(p - \sqrt{-p_t})$$

clock	relational Dirac observable on constraint surface	reduced observable on reduced phase space	gauge fixing:
t	$Q(\tau) = q(s) \big _{t(s)=\tau} = 2p(\tau - t_0) + q_0$	$Q_{\text{red}}(\tau) = 2p\tau + q$	
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Example: parametrized particle

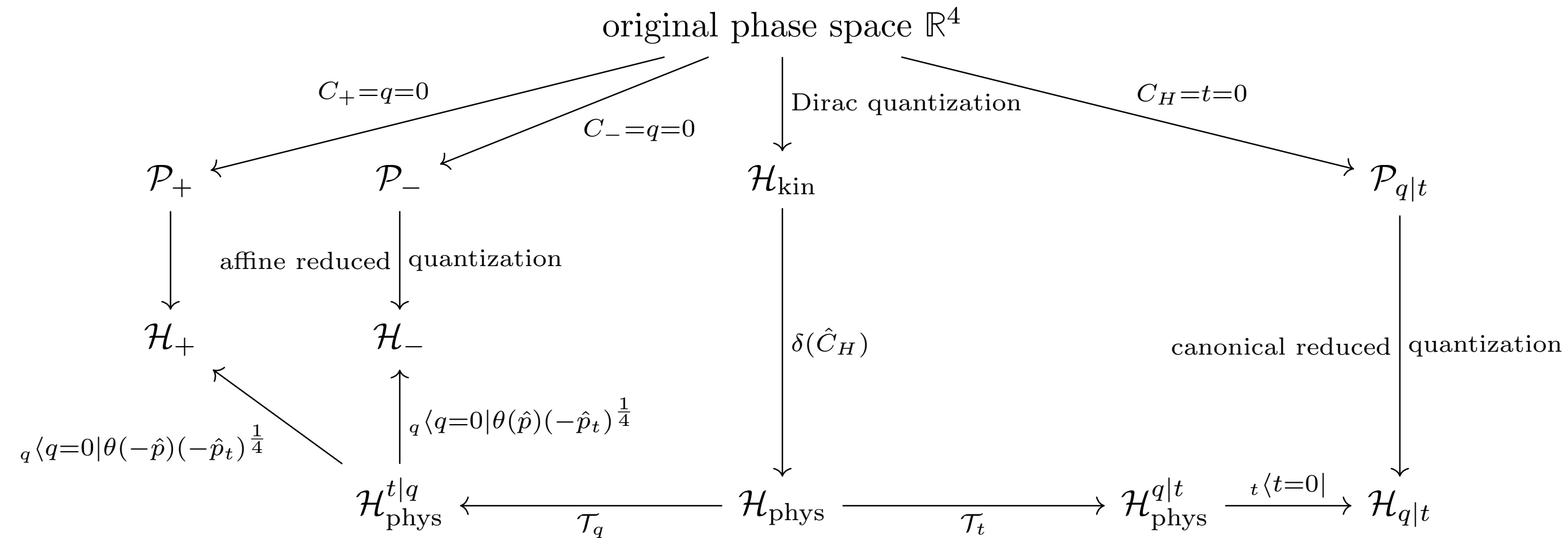
Getting rid of redundancy through gauge-fixing

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clock	relational Dirac observable on constraint surface	reduced observable on reduced phase space
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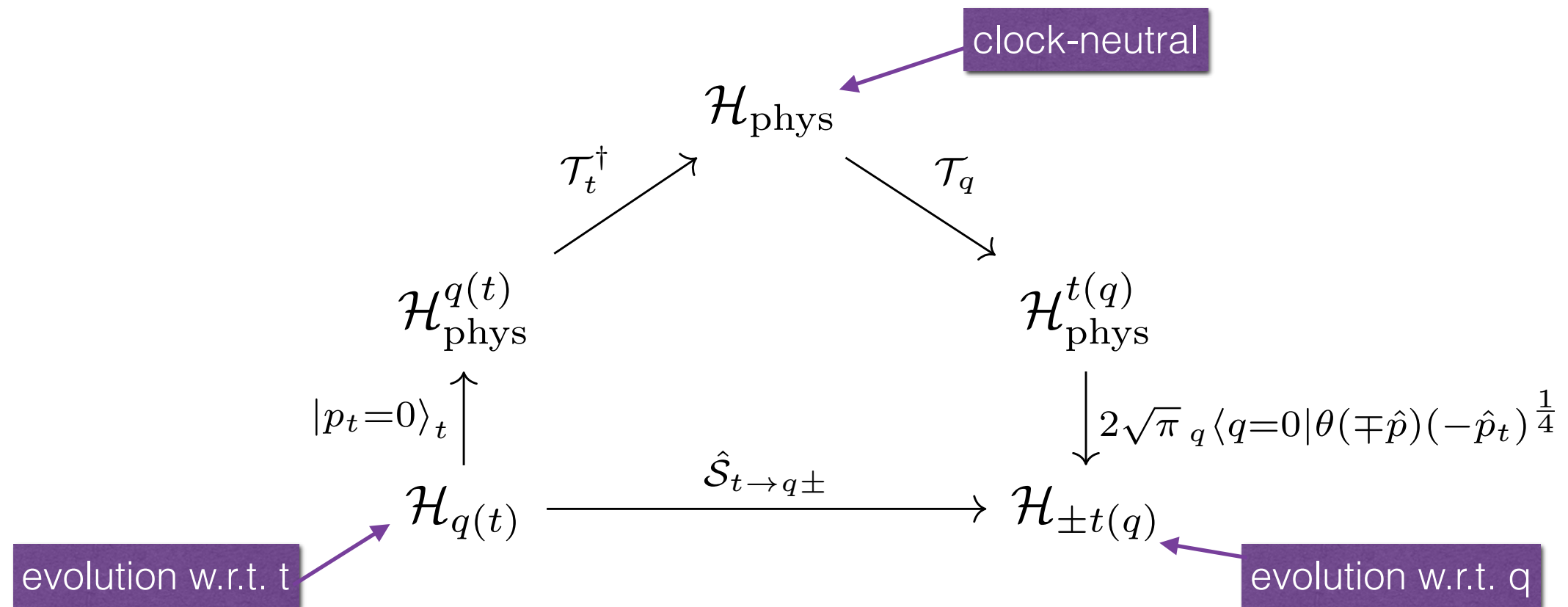
Summary of steps for parametrized particle

PH, Vanrietvelde, arxiv:1810.04153



Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153



where $\mathcal{T}_q := \mathcal{T}_{q+} + \mathcal{T}_{q-}$ $\mathcal{T}_{q\pm} := \exp \left(\pm i \hat{q} (\widehat{\sqrt{-p_t}} - \epsilon) \right) \theta(\mp \hat{p})$

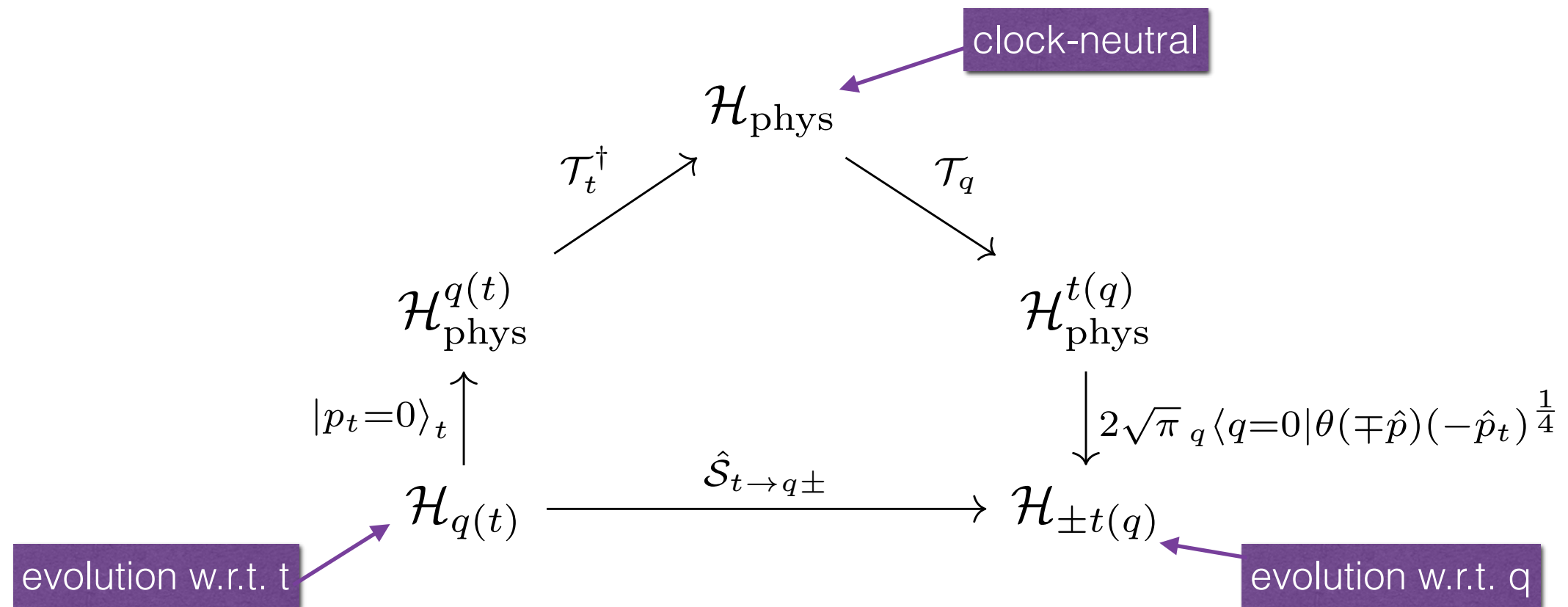
$$\mathcal{T}_t := \exp(i \hat{t} \hat{p}^2 / 2m) = \exp(i \hat{t} \hat{H})$$

$$\hat{\mathcal{S}}_{t \rightarrow q \pm} \equiv \sqrt{2} \hat{\mathcal{P}}_{q \rightarrow t} \theta(\mp \hat{p}) \widehat{\sqrt{|p|}}$$

clock swap

Quantum clock switch

PH, Vanrietvelde, arxiv:1810.04153



observables transform correctly

$$\mathcal{T}_t \hat{Q}(\tau) \mathcal{T}_t^\dagger = \hat{Q}_{\text{red}}(\tau)$$

$$\widehat{(-p_t)^{1/4}} \mathcal{T}_q \hat{T}_\delta(X) \mathcal{T}_q^{-1} \widehat{(-p_t)^{-1/4}} = \hat{T}_+^\delta(X) \theta(-\hat{p}) + \hat{T}_-^\delta(X) \theta(\hat{p})$$

Regularization details: time-of-arrival

Dirac:

affine reduced:

had:
$$\widehat{(-p_t)^{1/4}} \mathcal{T}_q \hat{T}_\delta(X) \mathcal{T}_q^{-1} \widehat{(-p_t)^{-1/4}} = \hat{T}_+^\delta(X) \theta(-\hat{p}) + \hat{T}_-^\delta(X) \theta(\hat{p})$$

Dirac
quantization

$$\hat{T}_\delta(X) := \hat{t}_\delta + \frac{1}{4} \left(\widehat{(p)_\delta^{-1}} (X - \hat{q}) + (X - \hat{q}) \widehat{(p)_\delta^{-1}} \right)$$

$$\hat{t}_\delta := \frac{1}{2} \left(\hat{t} \hat{p}_t \widehat{(p_t)_\delta^{-1}} + \hat{p}_t \widehat{(p_t)_\delta^{-1}} \hat{t} \right)$$

$$\widehat{(p_t)_\delta^{-1}} |p_t\rangle := \begin{cases} \frac{1}{p_t} |p_t\rangle & p_t \leq -\delta^2, \\ -\frac{1}{\delta^2} |p_t\rangle & -\delta^2 < p_t \leq 0, \end{cases}$$

$$\hat{T}_\pm^\delta(X) := \hat{t}_{\delta\pm} \mp \frac{X}{2} \widehat{(\sqrt{-p_t})_\delta^{-1}}$$

Affine
Reduced
Quantization

$$\hat{t}_{\delta\pm} := \frac{1}{2} \left(\widehat{(p_t)_\delta^{-1}} \hat{\mathfrak{t}} + \hat{\mathfrak{t}} \widehat{(p_t)_\delta^{-1}} \right)$$

$$\widehat{(\sqrt{-p_t})_\delta^{-1}} |p_t\rangle_\pm := \begin{cases} \frac{1}{\sqrt{-p_t}} |p_t\rangle_\pm & p_t \leq -\delta^2, \\ \frac{\sqrt{-p_t}}{\delta^2} |p_t\rangle_\pm & -\delta^2 < p_t \leq 0. \end{cases}$$

Relational dynamics in FRW

- homogeneous & isotropic universe

$$ds^2 = -dt^2 + a(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right)$$

- Hamiltonian constraint incl. homog. field

$$C_H = p_\phi^2 - p_\alpha^2 - 4k e^{4\alpha} + 4m^2 \phi^2 e^{6\alpha} \approx 0$$

$$\alpha = \ln a$$

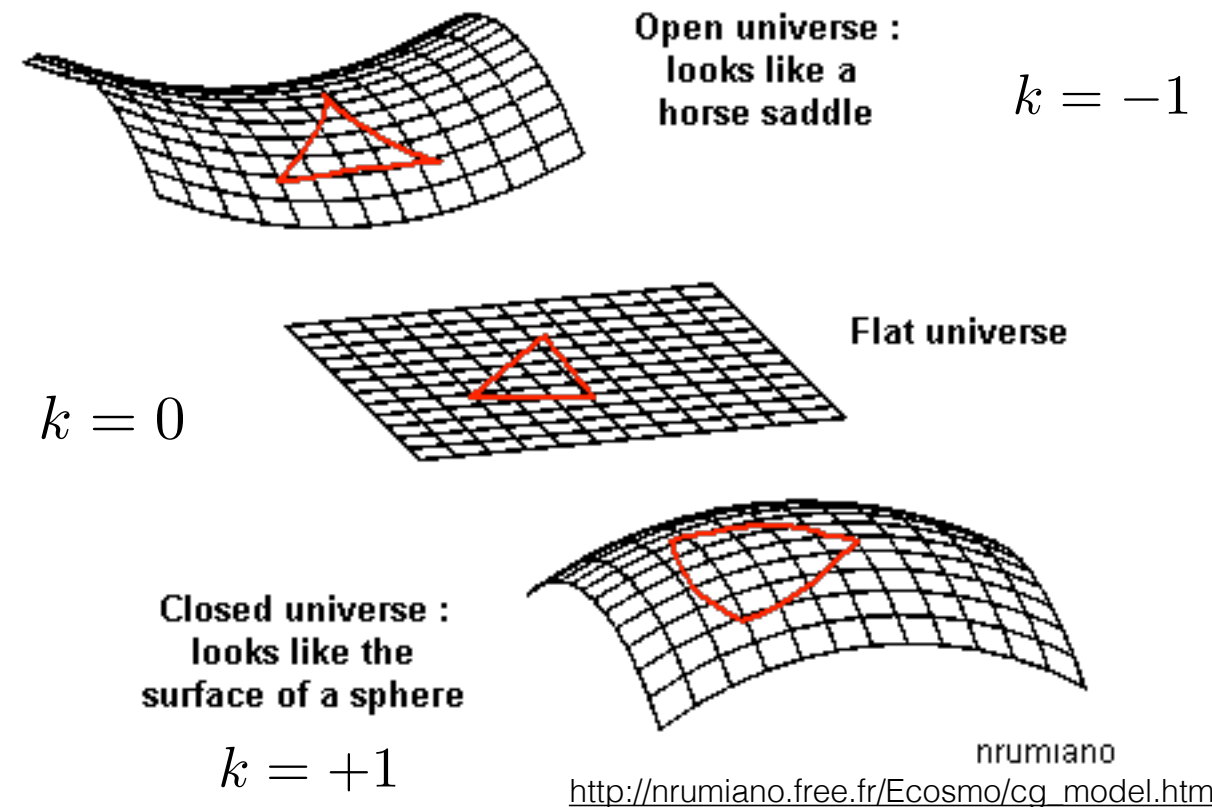
$\Rightarrow$ time evol. generated is gauge transf.

$$\dot{\phi} = \{\phi, C_H\} = 2p_\phi$$

$$\dot{\alpha} = \{\alpha, C_H\} = -2p_\alpha$$

can go through 0, thus
not necessarily monotonic

$\Rightarrow$ want to use α or ϕ as relational “clock”



Relational dynamics in FRW

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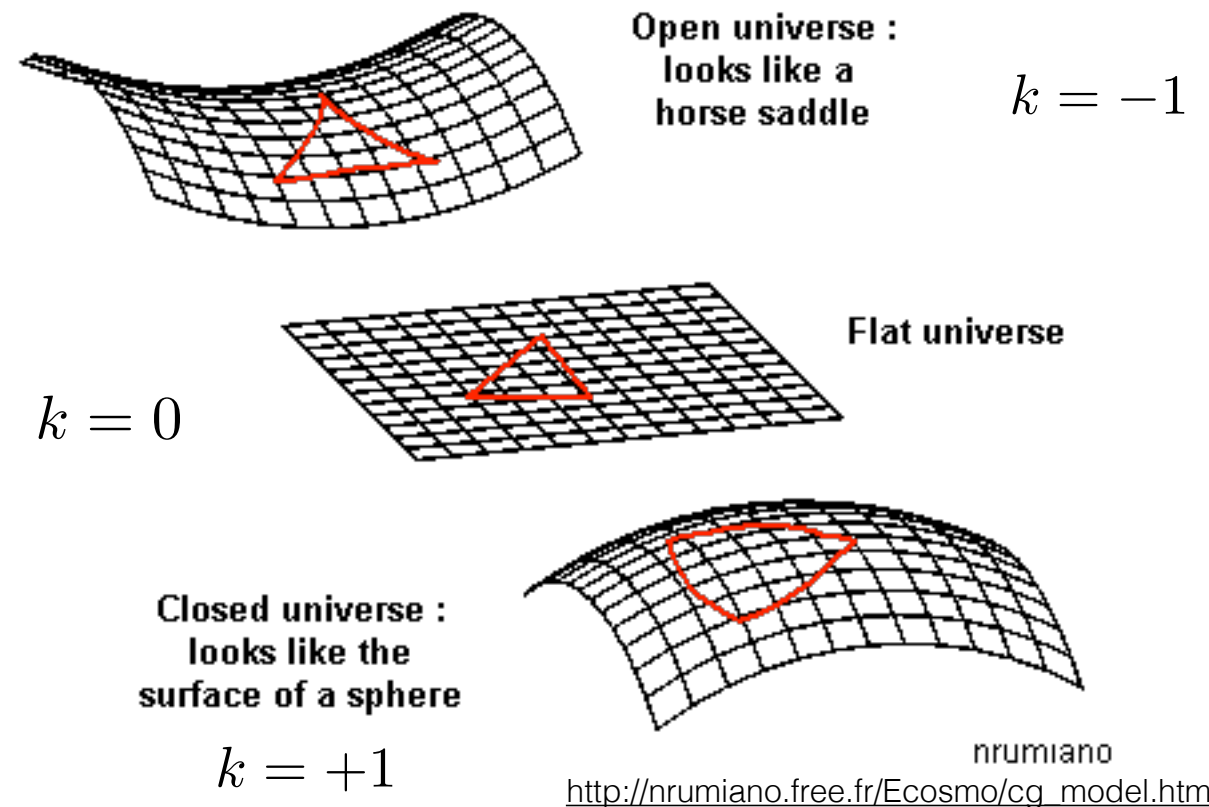
$$\alpha = \ln a$$

$\Rightarrow$ time evol. generated is gauge transf.

$$\dot{\phi} = \{\phi, C_H\} = 2p_\phi$$

$$\dot{\alpha} = \{\alpha, C_H\} = -2p_\alpha$$

$\Rightarrow$ want to use α or ϕ as relational “clock”



set

$$k = m = 0$$

$\Rightarrow$ want to use α or ϕ as relational “clock” $\leftarrow$ **both monotonic**

$k=0$ FRW with massless scalar

- Hamiltonian constraint of Klein-Gordon form

PH, arXiv:1811.00611

$$C_H = p_\phi^2 - p_\alpha^2 \approx 0$$

$$\Rightarrow \begin{aligned} \phi(t) &= 2p_\phi t + \phi_0 \\ \alpha(t) &= -2p_\alpha t + \alpha_0 \end{aligned}$$

- choose α as “clock” and affine evolving $\Phi = \phi p_\phi$

$$\Rightarrow \Phi(\tau) = -p_\alpha(\tau - \alpha) + \Phi \quad \text{rel. Dirac observable}$$

- get rid of redundant α, p_α

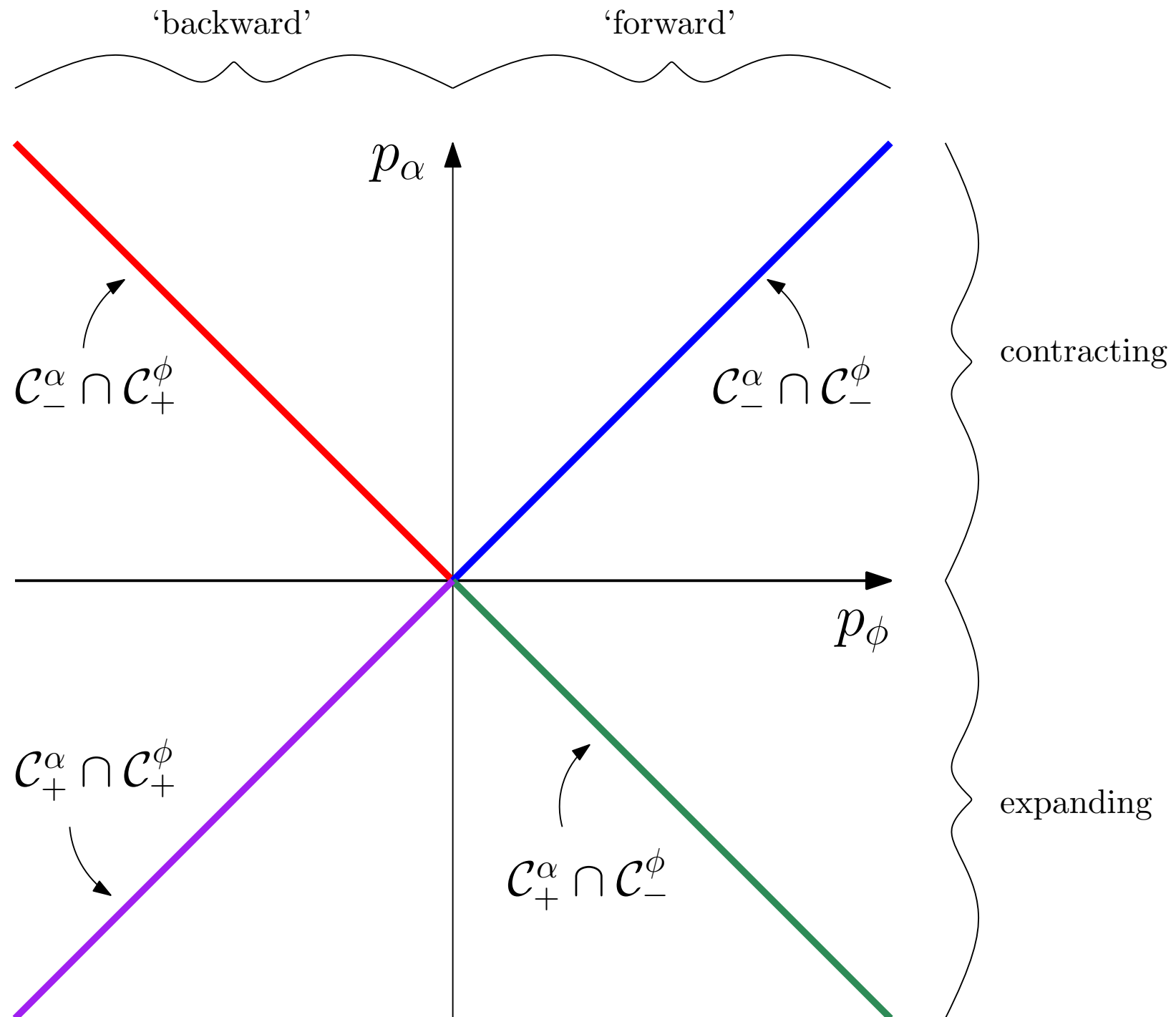
$$p_\alpha = \pm |p_\phi| = \pm H$$

generates fwd/bwd
evol.

$$\Phi_\pm(\tau) = \pm |p_\phi| \tau + \Phi$$

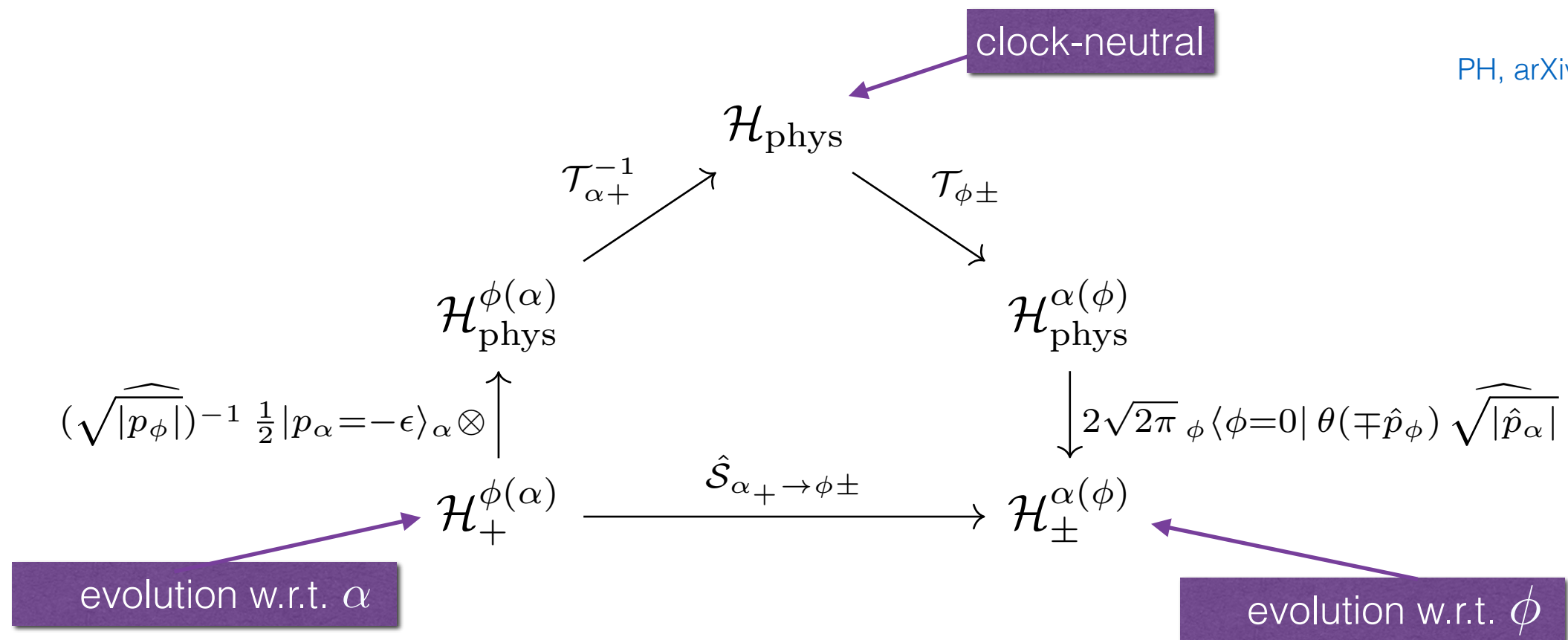
on reduced phase space

Positive and negative frequency sectors



Internal time switch in quantum cosmology

PH, arXiv:1811.00611



where

$$\mathcal{T}_\alpha = \mathcal{T}_{\alpha+} + \mathcal{T}_{\alpha-} \quad \mathcal{T}_{\alpha\pm} = e^{\pm i\hat{\alpha}(|\hat{p}_\phi| - \epsilon)} \theta(\mp \hat{p}_\alpha)$$

works the same way

Relational dynamics more generally

- can get arbitrarily complicated

$$C_H = p_\phi^2 - p_\alpha^2 - 4k e^{4\alpha} + 4m^2 \phi^2 e^{6\alpha} \approx 0$$

chaos for massive field

interacting clocks,
global problem of time,
non-unitarity, transient clocks

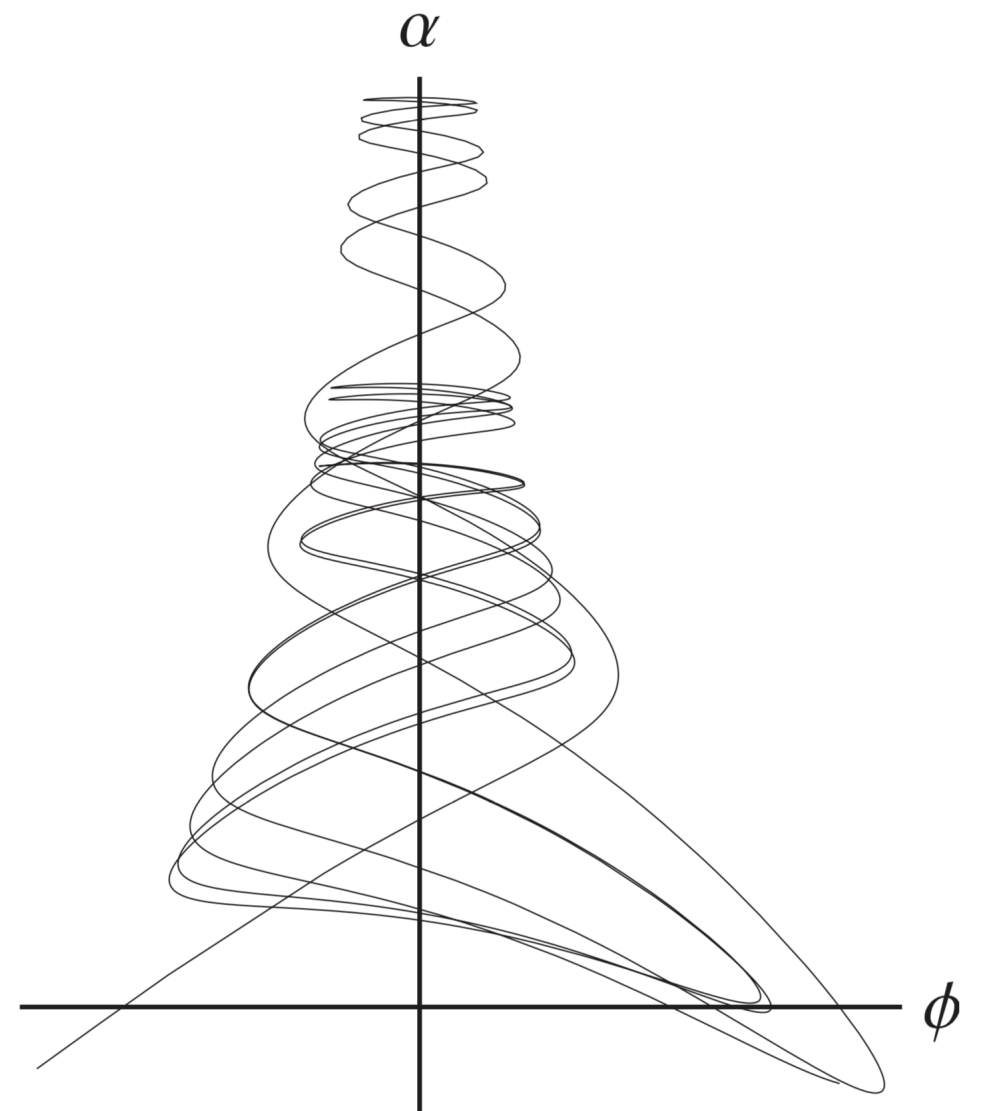
see

Bojowald, PH, Tsobanjan, CQG 28,035006, (2011)

Bojowald, PH, Tsobanjan, PRD 83,125023 (2011)

PH, Kubalova, Tsobanjan, PRD 86, 065014 (2012)

Dittrich, PH, Koslowski, Nelson, PLB 769, 554 (2017)



New perspective on “wave function of the universe”

PH arXiv:1811.00611

- no global operational state $\longrightarrow$ global state perspective neutral
- only relative operational states

“Wave function of the universe” as a perspective-neutral state

Operational interpretation from transformation to specific reduced theory