D-Brane Probes in Melonic Matrix Quantum Mechanics

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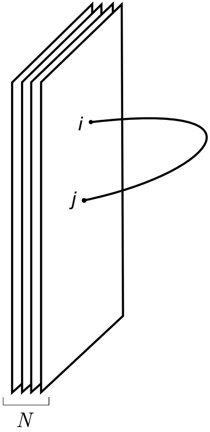
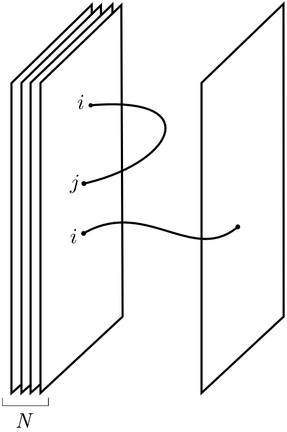
[Ferrari, Moskovic, Rovai]



We generalize the notion of D-brane to any U(N) gauge theory. For concreteness, let us start from a quartic $U(N)$ one-matrix model in zero dimensions.

$$S_N(M) = \frac{N}{\lambda}\mathrm{tr}\left(\frac{1}{2}M^2 + \frac{1}{4}M^4\right)$$

The matrix M is associated to open strings with endpoints attached to the D-branes, which are labeled by the U(N) indices.



We now distinguish between "background" and "probe" D-branes:

$$M^i_j \longrightarrow \begin{pmatrix} V^i_j & \bar{w}^i \\ w_j & v \end{pmatrix}$$

The action of the model is rewritten in terms of the new degrees of freedom:

$$\begin{aligned} S_{N+1}(M) \longrightarrow \quad & S_N(V) + (N+1)S_1(v) \\ & + S_{mix}(V, v, w, \bar{w}) \end{aligned}$$

Emergent dimensions

General feature: the resulting action is quartic in w and $\bar{w}$:

$$S_{mix}(V, v, w, \bar{w}) = \frac{N+1}{\lambda}\left(\bar{w}w(1+v^2) + wV^2\bar{w} + vwV\bar{w} + \frac{1}{2}(\bar{w}w)^2\right)$$

We introduce the auxiliary field ϕ:

$$\hat{S}_{mix}(V, v, w, \bar{w}, \phi) = \frac{N+1}{\lambda}\left(\bar{w}w(1+v^2+\phi) - \frac{1}{2}\phi^2 + wV^2\bar{w} + vwV\bar{w}\right)$$

The path integral over the vector fields results in the introduction of bosonic auxiliary fields, which at $N \to \infty$ are classical: good candidates for emergent space coordinates.

Effective probe brane action

The final goal of the procedure is to compute the effective probe brane action $\mathscr{A}(\phi)$ defined by

$$e^{-\mathscr{A}(\phi,v)} = \int \mathcal{D}V\mathcal{D}w\mathcal{D}\bar{w}\, e^{-(N+1)S_1(v)-S_N(V)-\hat{S}_{mix}(V,v,w,\bar{w},\phi)}$$

This is a hard task in general: summing over an infinite class of planar diagrams of the original model is required. On-shell, it satisfies the non-trivial relation

$$\mathscr{A}^* = 2F_0 + \lambda\partial_\lambda F_0$$

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The SYK model

N Majorana fermions in 0+1 dimensions, with quartic interaction term:

$$H = \sum_{i,j,k,l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l \qquad \{\chi_i, \chi_j\} = \delta_{ij}$$

[Kitaev 2016] [Maldacena, Stanford 2016]

Random couplings J_{ijkl}, drawn from Gaussian distribution

$$\overline{J_{ijkl}^2} = \frac{3!\lambda^2}{N^3} \qquad \overline{J_{ijkl}} = 0$$

The SYK model

Observables are computed by averaging over the random couplings (quenched disorder):

$$\overline{\langle O \rangle} = \int dJ_{ijkl}\, e^{-J_{ijkl}^2 N^3/12\lambda^2} \frac{\int \mathcal{D}\chi_i O\, e^{-\int dtL}}{\int \mathcal{D}\chi_i e^{-\int dtL}}$$

The SYK model

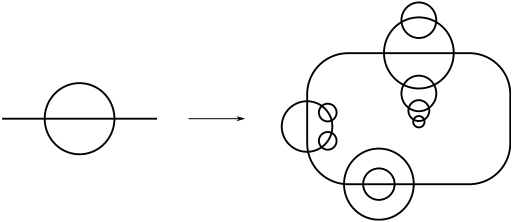
The SYK model has many non-trivial properties:

- Continuous energy spectrum
- Large entropy at zero temperature
- Quasi-normal behaviour of 2-pt functions
- Emergent SL$(2, \mathbb{R})$ symmetry in the IR
- Chaotic behaviour of out-of-time-order 4-pt functions

Suggesting a classical gravitational dual containing black holes.

The SYK model

The large N diagrammatic structure is dominated by “melons”:



Diagrammatics are simple enough to allow for solvability, non-trivial enough to reproduce interesting physics.

The SYK model

There are two main issues with the SYK model:

- Random couplings are hard to reconcile with holography
- Vector d.o.f. do not allow for a probe brane analysis

Both problems are solved by using matrix-vector models.

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SYK-like matrix-vector model

A model of $N \times N$ fermionic matrices with $\mathrm{O}(D)$ symmetry:

$$L = ND\mathrm{tr}\left(\psi_\mu^\dagger \dot{\psi}_\mu + \sqrt{D}\frac{\lambda}{2}\psi_\mu \psi_\nu^\dagger \psi_\mu \psi_\nu^\dagger\right)$$

[Ferrari 2017]

In the large N, large D limit it reproduces the SYK diagrammatics.

The $\sqrt{D}$ enhancement of the coupling is crucial: more diagrams are kept in the large D limit. Moreover, the large N and large D limit do not commute.

The large D limit

Basic variables: $N \times N$ complex matrices in the fundamental of $\mathrm{O}(D)$:

$$X_\mu \qquad\qquad \mu = 1, \cdots, D$$

Typical Lagrangian involving single-trace interaction terms:

$$L = ND\Big(\text{Kinetic Term} - \sum_B t_B I_B(X)\Big)\,,$$

with

$$I_B = \mathrm{tr}\left(X_{\mu_1} X^\dagger_{\mu_2} X_{\mu_3} \cdots X^\dagger_{\mu_{2s}}\right)$$

The old scaling

For example, we have two quartic interaction terms:

$$t_1 \mathrm{tr}(X_\mu X_\mu^\dagger X_\nu X_\nu^\dagger) + t_2 \mathrm{tr}(X_\mu X_\nu^\dagger X_\mu X_\nu^\dagger)$$

We can expand observables in powers of N and D:

$$F = \sum_{g,n} f_{g,n} N^{2-2g} D^{1-n}$$

At leading order, only the first vertex contributes!

New scaling

We now introduce couplings λ_a:

$$\lambda_B = D^{-g(B)} t_B$$

and keep them fixed. The new scaling is an enhancement: more diagrams are kept. On top of the usual large $1/N$ expansion

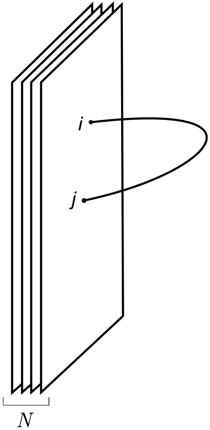
$$F = \sum_{g \in \mathbb{N}} F_g(D) N^{2-2g}$$

we have a $1/\sqrt{D}$ expansion:

$$F_g = \sum_{\ell \in \mathbb{N}} F_{g,\ell} D^{1+g-\ell/2}$$

The two limits do not commute!

Holographic picture



$$(X_\mu)^i_j \qquad 1 \leq \mu \leq D \qquad 1 \leq i, j \leq N$$

e.g.: D0-brane quantum mechanics ($D = 9$):

$$L_{BFSS} = \frac{1}{2\lambda} \mathrm{tr} \left[\dot{X}_\mu \dot{X}_\mu - \frac{1}{2} \left[X_\mu, X_\nu \right]^2 + \dots \right]$$

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Probe analysis of the quartic matrix-vector model

[Ferrari, Gregori 2019]

For the probe analysis, we consider the fermionic model

$$S_N = ND \int dt \, \mathrm{tr} \left(\psi_\mu^\dagger \dot{\psi}_\mu + m \psi_\mu^\dagger \psi_\mu + \frac{\lambda_1}{2} \psi_\mu^\dagger \psi_\mu \psi_\nu^\dagger \psi_\nu + \sqrt{D} \frac{\lambda_2}{2} \psi_\mu \psi_\nu^\dagger \psi_\mu \psi_\nu^\dagger \right)$$

with

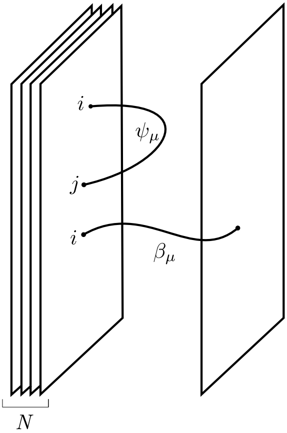
$$\left\{ \psi^a_{\mu b}, \left(\psi_\nu^\dagger \right)^c_d \right\} = \frac{1}{ND} \delta_{\mu\nu} \delta^a_d \delta^c_b \, .$$

In the large N, large D limit:

- Same diagrammatic structure as complex SYK
- Non-trivial IR regime with macroscopic low temperature entropy

Probe analysis of the quartic matrix-vector model

To perform the probe analysis, we distinguish the probe from the background



$$\Psi_\mu = \begin{pmatrix} \psi^a_{\mu b} & \alpha^a_\mu \\ \beta_{\mu b} & \chi_\mu \end{pmatrix} \qquad \text{and}$$

$$S_{N+1}(\Psi) \longrightarrow S_N(\psi) + (N+1)S_1(\chi) + S_{mix}(\psi, \alpha, \beta, \chi)$$

Probe analysis of the quartic matrix-vector model

We are interested in the computation of the probe effective action:

$$e^{-\mathscr{A}_N} = \frac{e^{-(N+1)S_1(\chi)}}{Z_N} \int \mathcal{D}\psi \mathcal{D}\alpha \mathcal{D}\beta e^{-S_N(\psi) - S_{mix}(\psi,\alpha,\beta,\chi)} .$$

Thanks to the summability of melon diagrams, we were able perform a first, non-trivial check:

$$\mathscr{A}_N^* = 2F_0 + \lambda_1 \partial_{\lambda_1} F_0 + \lambda_2 \partial_{\lambda_2} F_0 .$$

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Conclusions and outlook

The new large D limit opens the door to the study of many previously untractable models, with strong connections with holography. A couple of possible future directions are:

- Non-equilibrium probe analysis of the quartic matrix vector model
- Understanding the connection with the large D limit of General Relativity [Emparan, Suzuki, Tanabe; Bhattacharyya, Minwalla; ...]

Thank you!

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