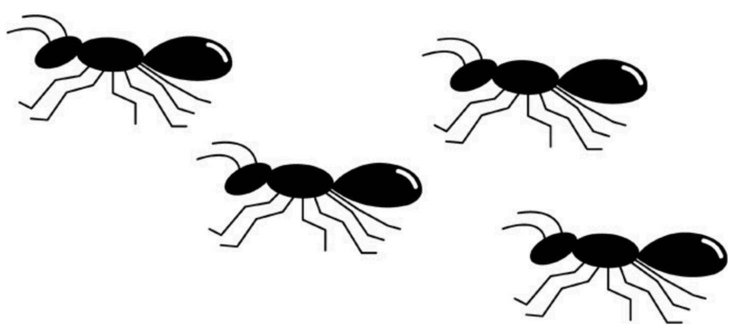
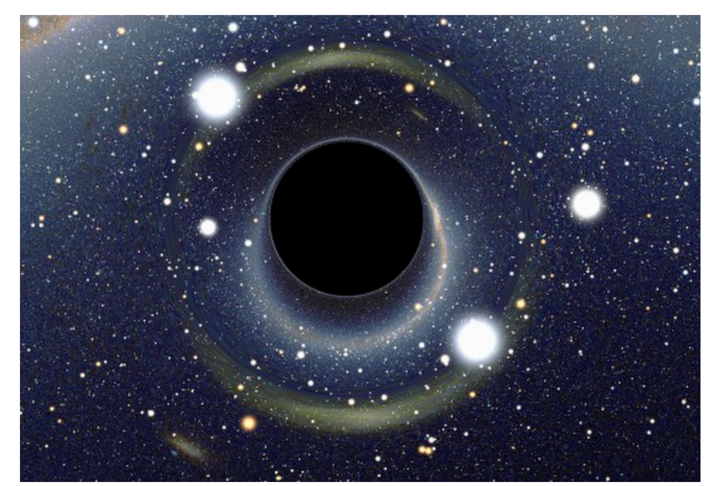
What the ants are telling us about Black Hole and QGP

Masanori Hanada
University of Southampton

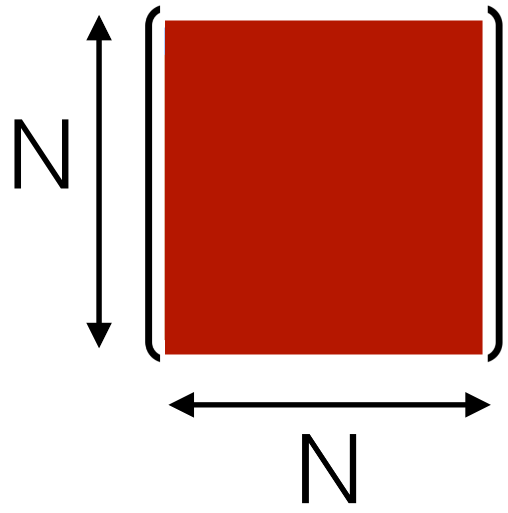




Berkowitz-M.H.-Maltz, 2016, PRD
M.H.-Malts, 2016, JHEP
M.H.-Ishiki-Watanabe, 2018, JHEP
M.H.-Jevicki-Peng-Wintergerts, in preparation
+ work in progress w/ Robinson, O'Bannon, …

24 July 2019 @ OIST

- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$



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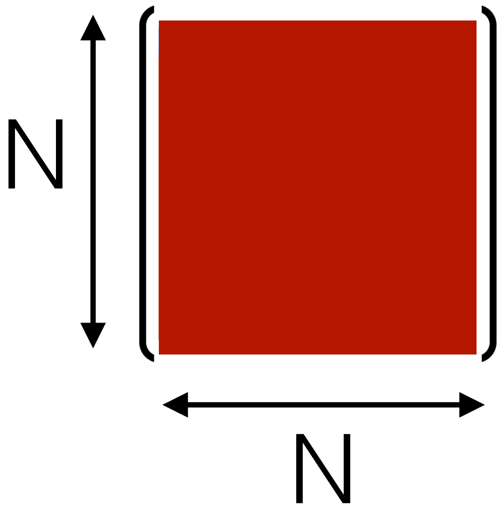
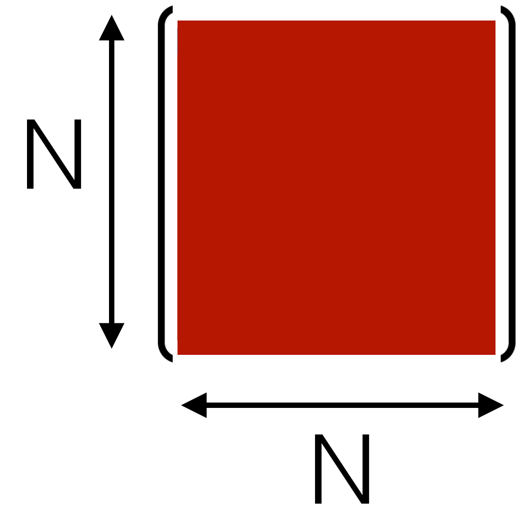
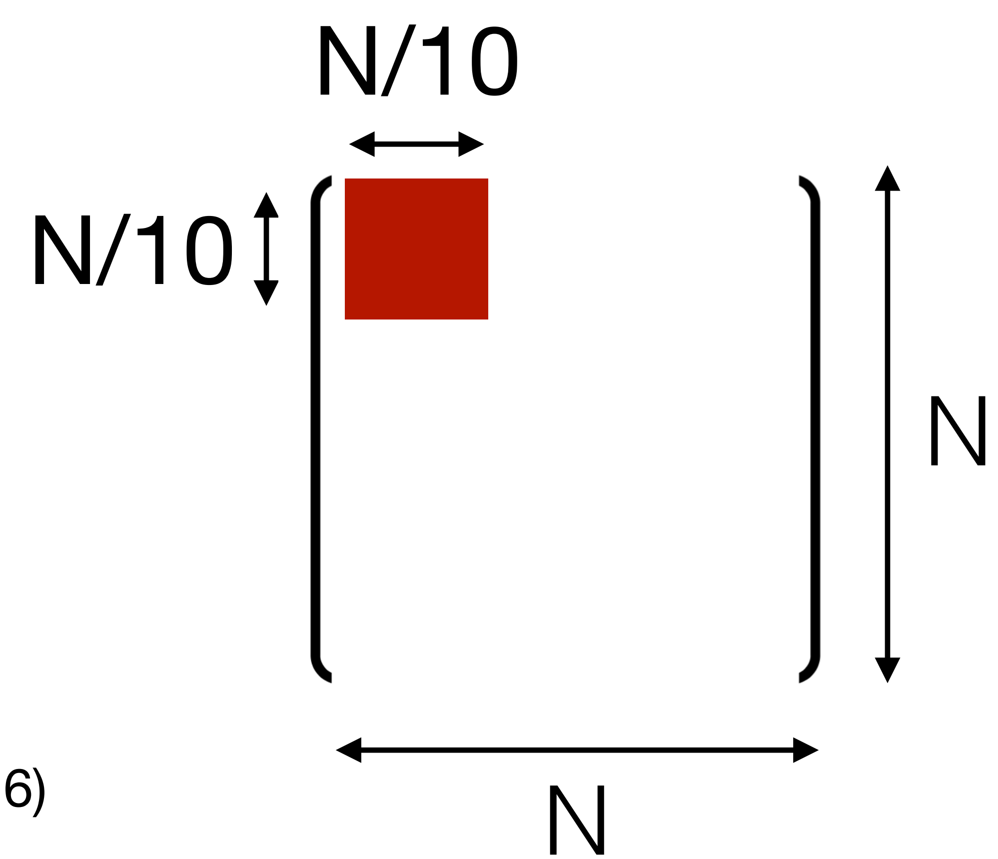


What if $E \sim N^2/100$?

- Confinement phase: $E \sim N^0$
- Deconfinement phase: $E \sim N^2$



What if $E \sim N^2/100$?

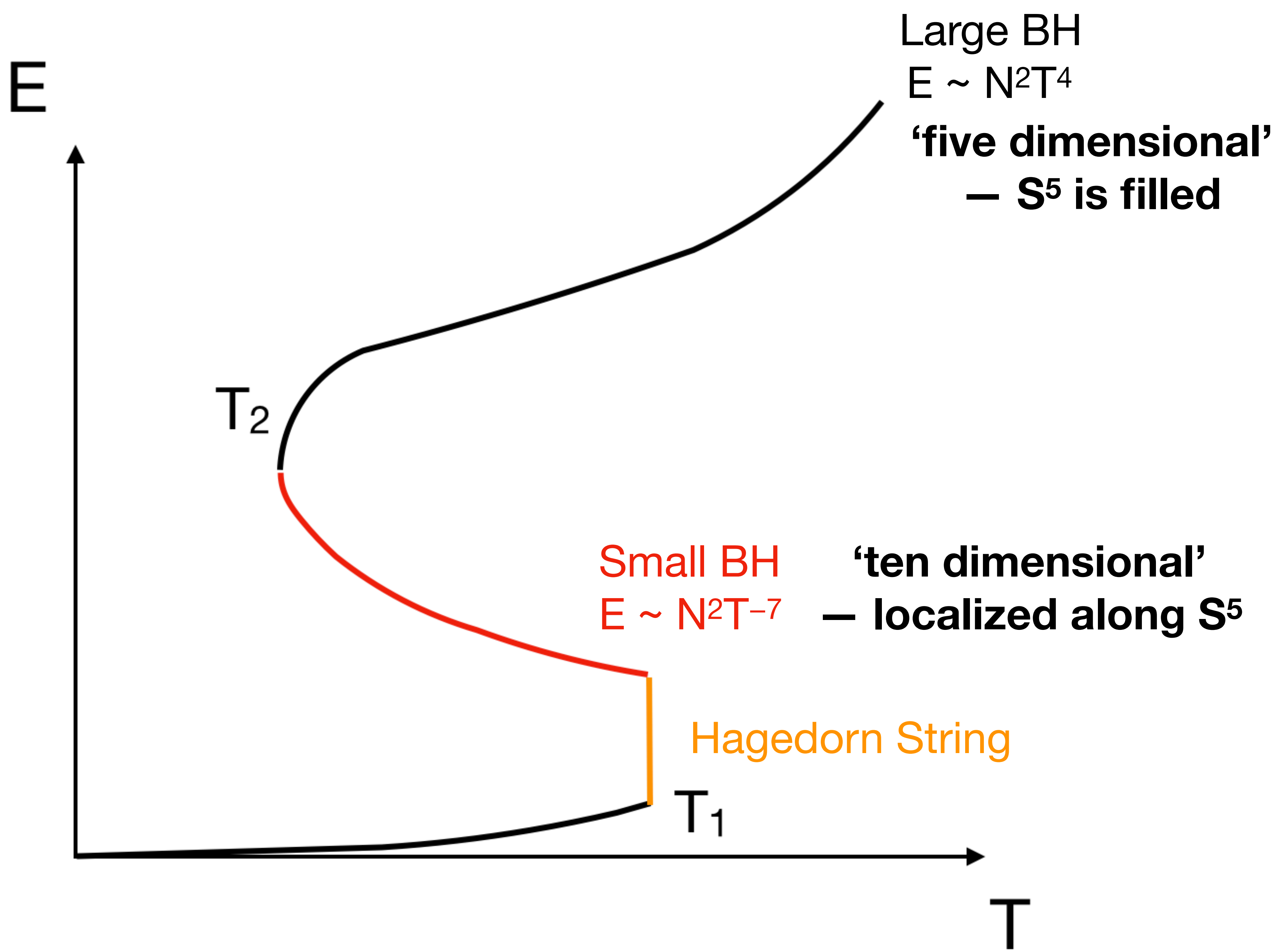


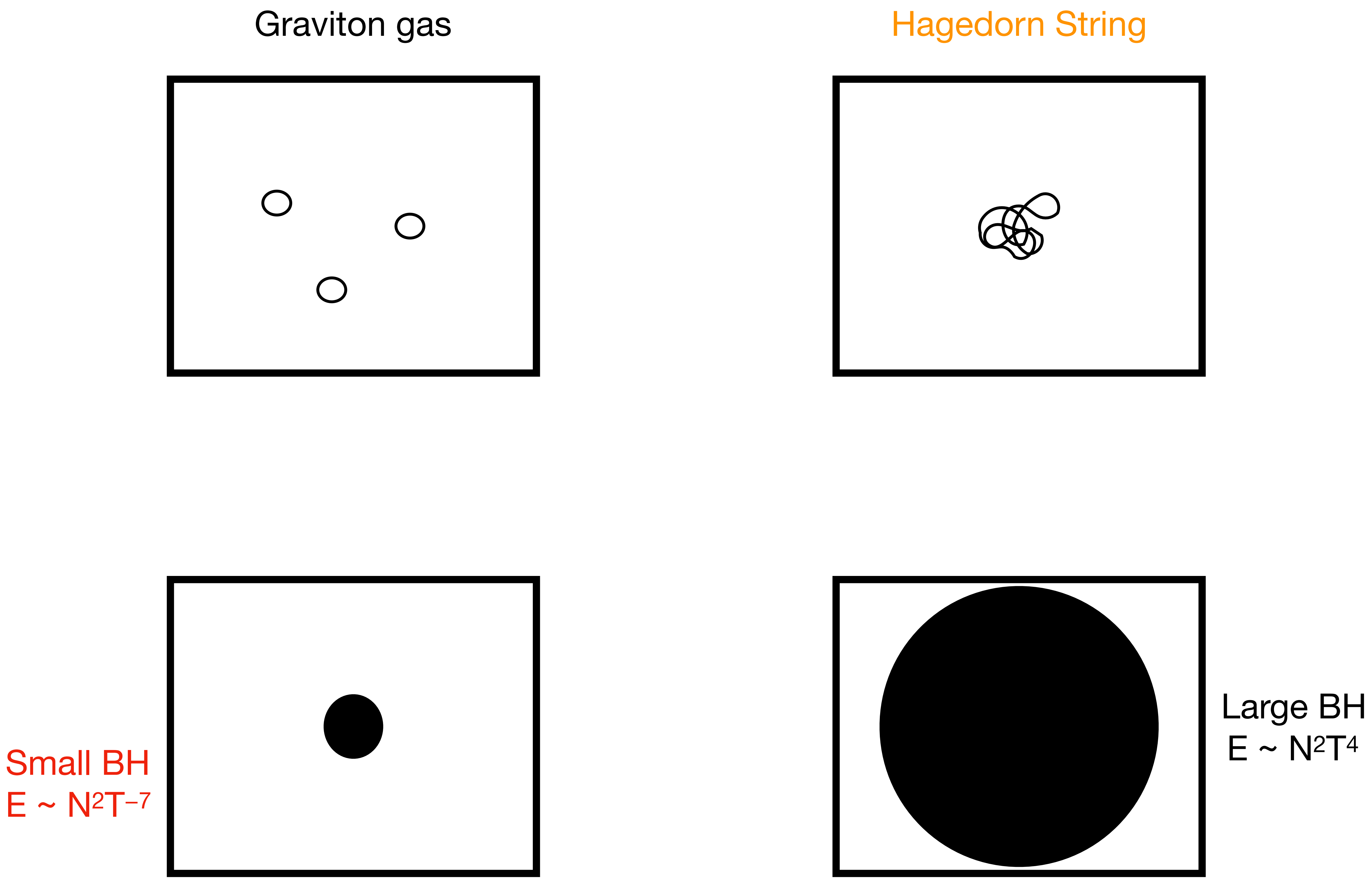
<u>'partially' deconfine</u>

(MH-Maltz, 2016)

Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

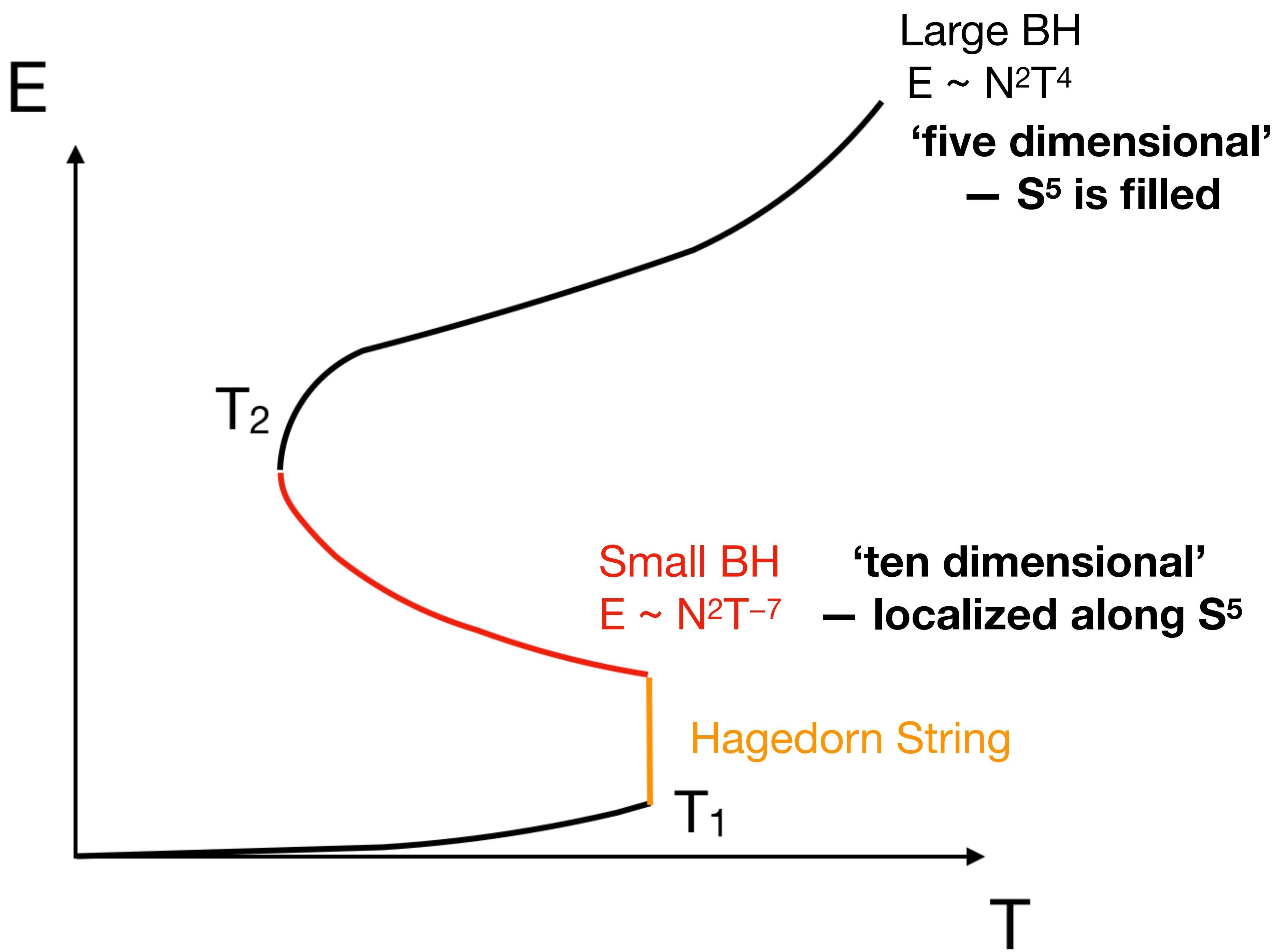
microcanonical
ensemble
(E fix)




Graviton gas
Hagedorn String
Small BH
E ~ N²T⁻⁷
Large BH
E ~ N²T⁴

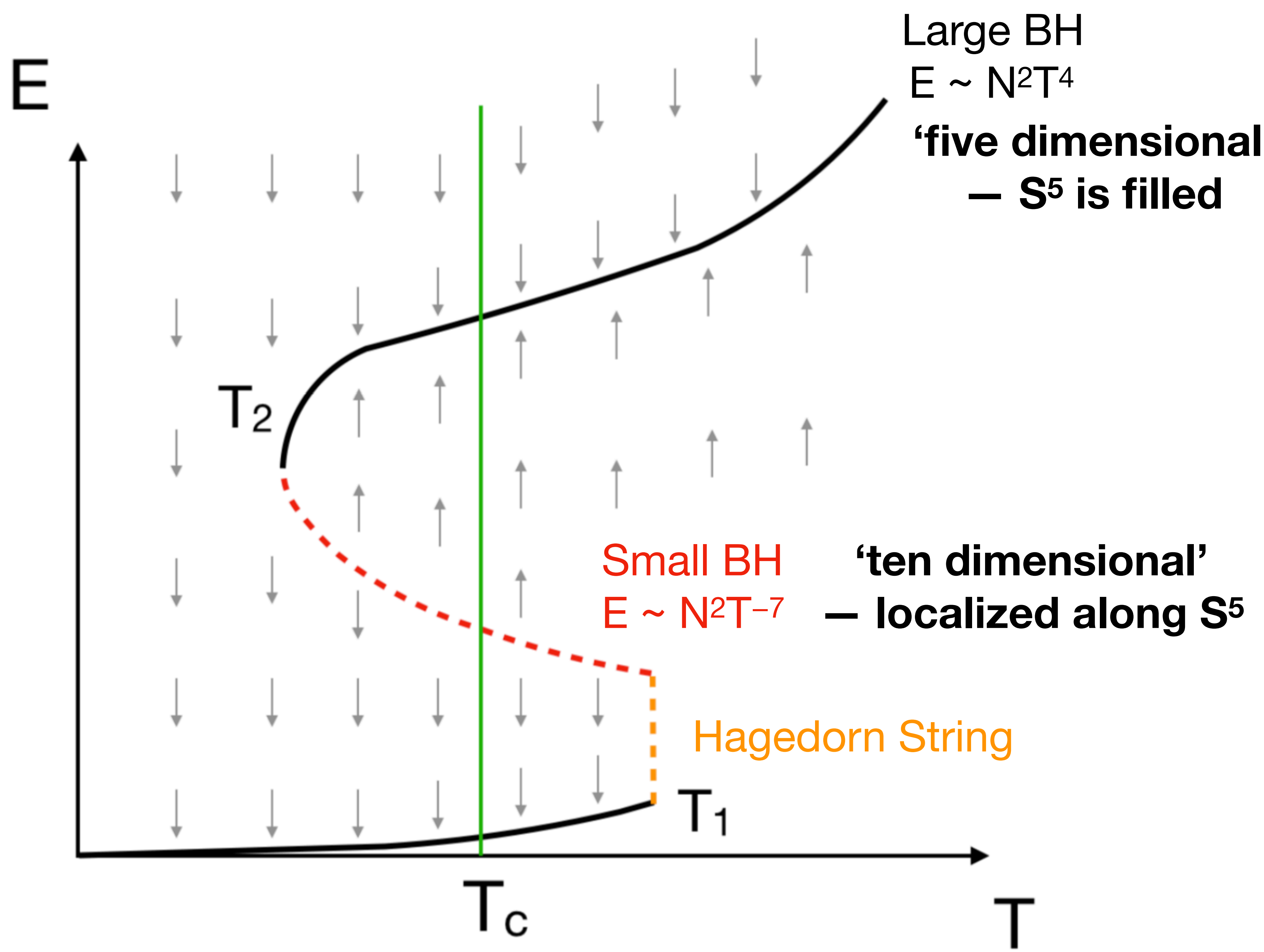
Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

microcanonical ensemble (E fix)

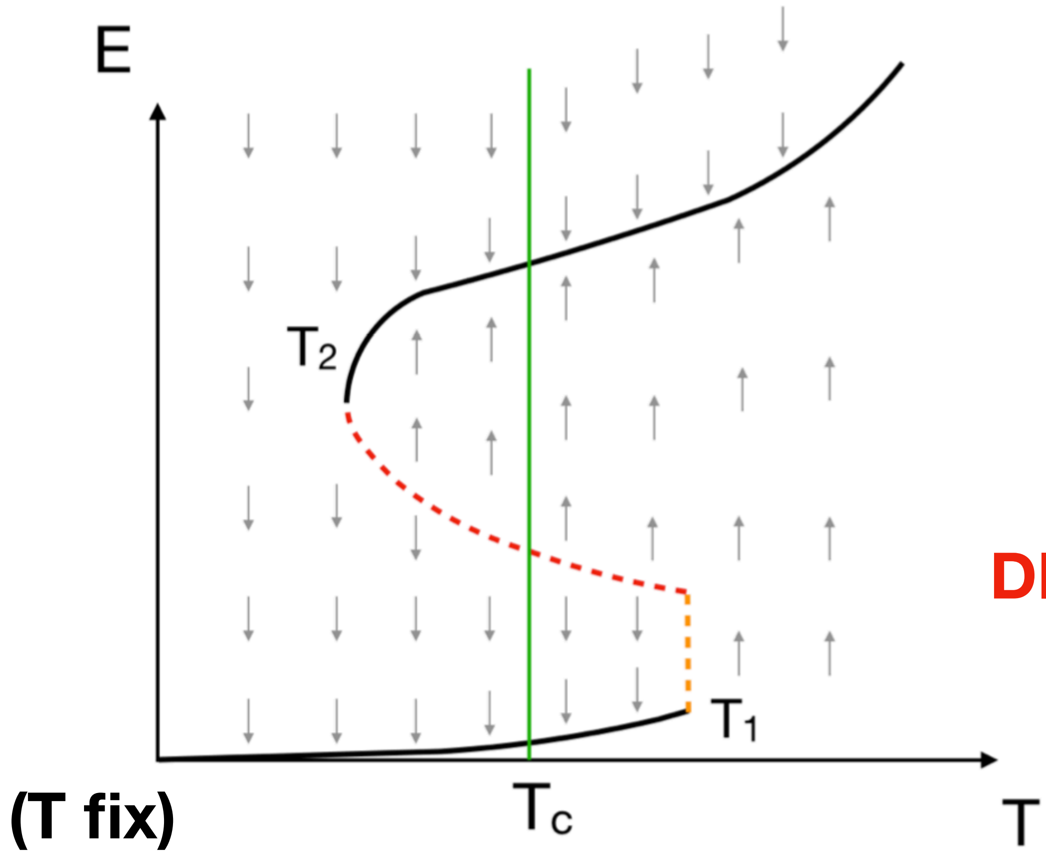


Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

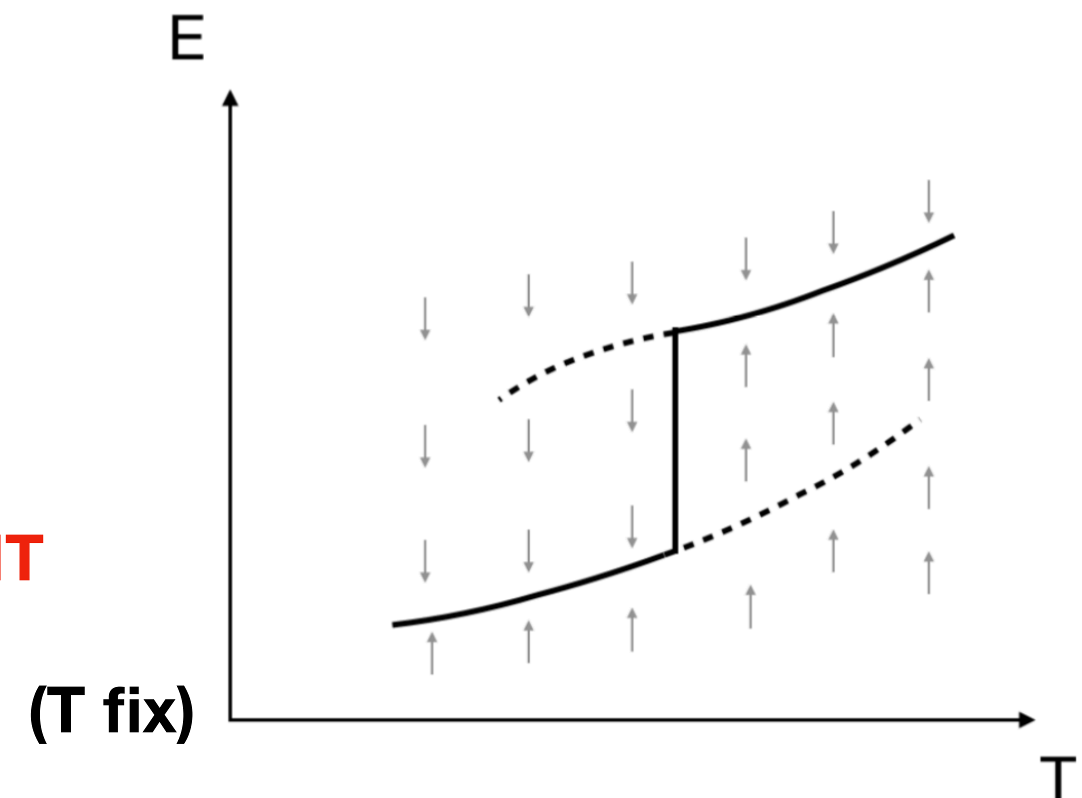
canonical ensemble (T fix)



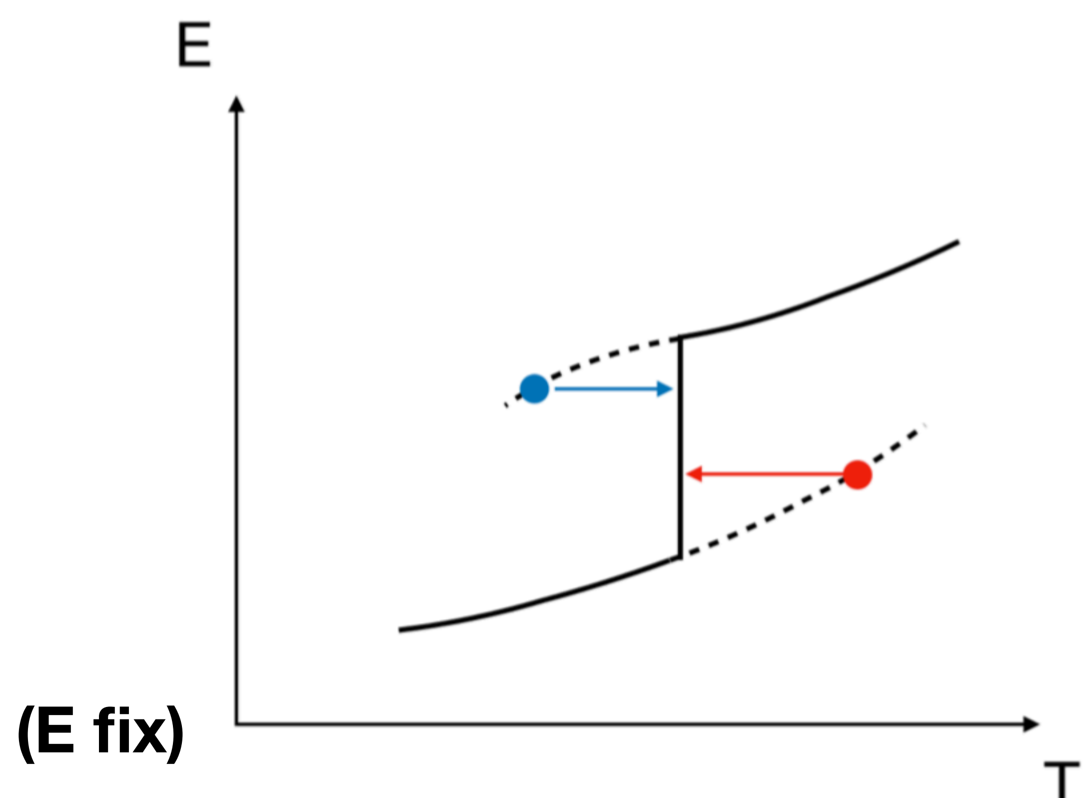
strongly coupled
4d SYM



water/ice

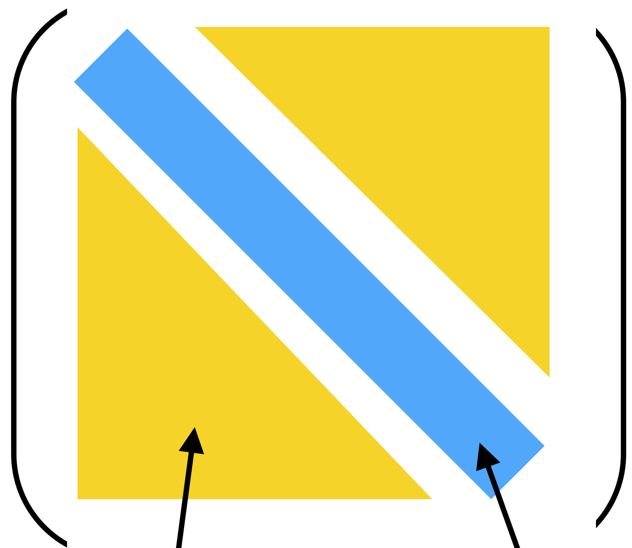


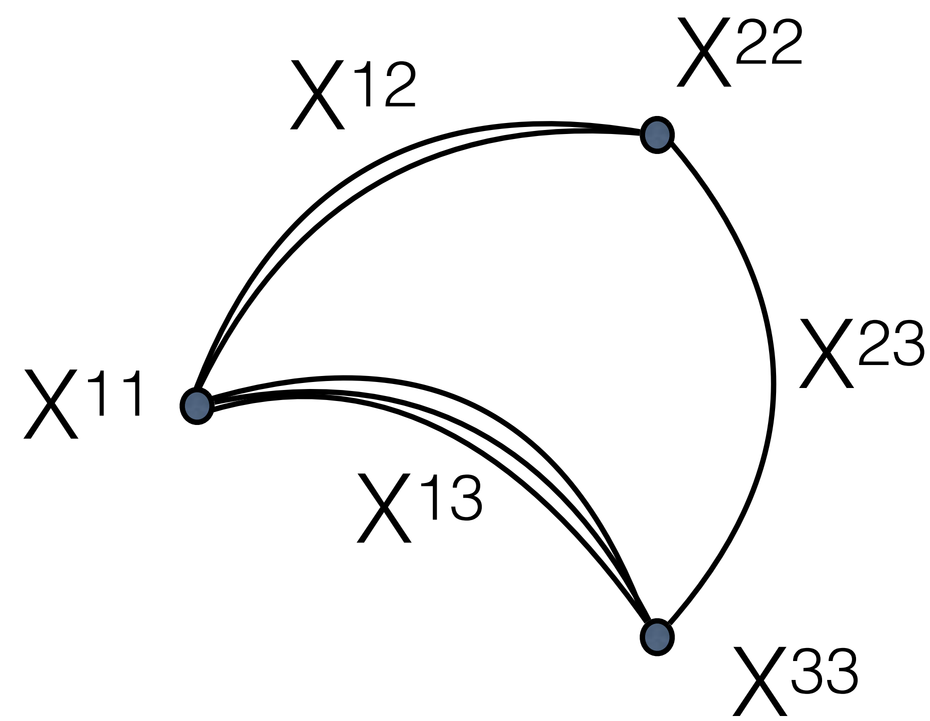
VERY DIFFERENT



How can we explain such difference?

D-brane bound state and Gauge Theory

$X_M =$ 



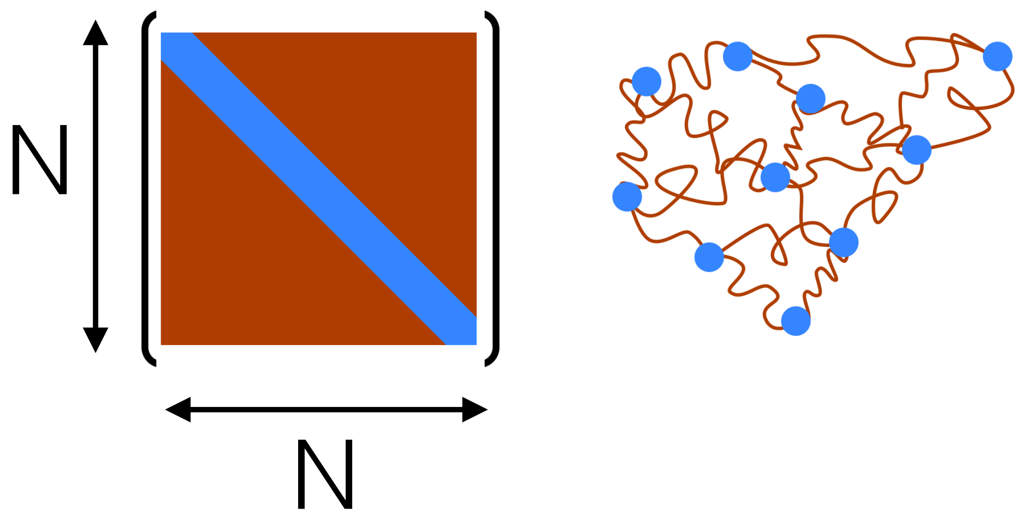
$(X_1^{ii}, X_2^{ii}, \ldots, X_6^{ii})$

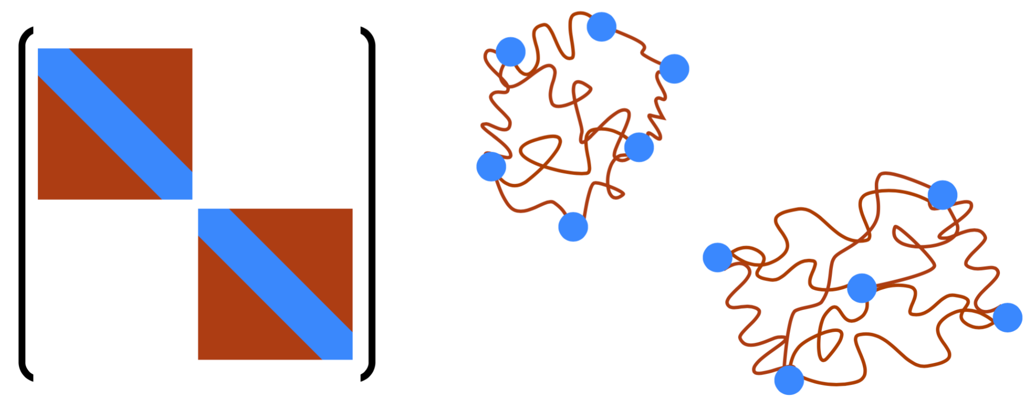
location of i-th D-brane

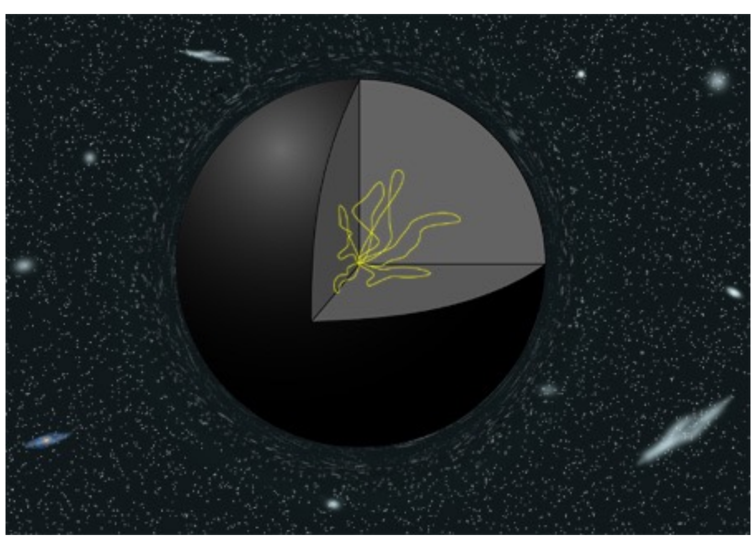
X_M^{ij} : open strings connecting i-th and j-th D-branes.

large value → a lot of strings are excited

(Witten, 1994)







diagonal elements = particles (D-branes)

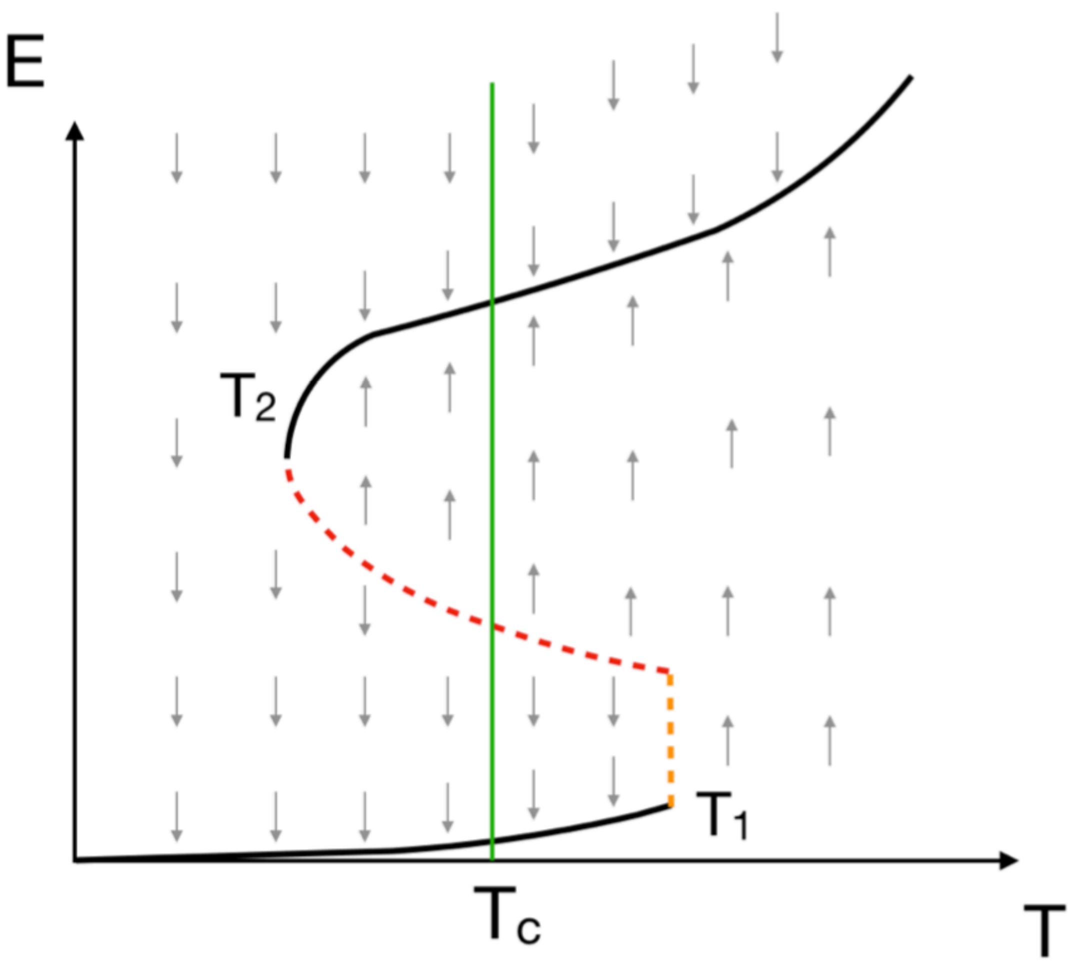
off-diagonal elements = open strings

(Witten, 1994)

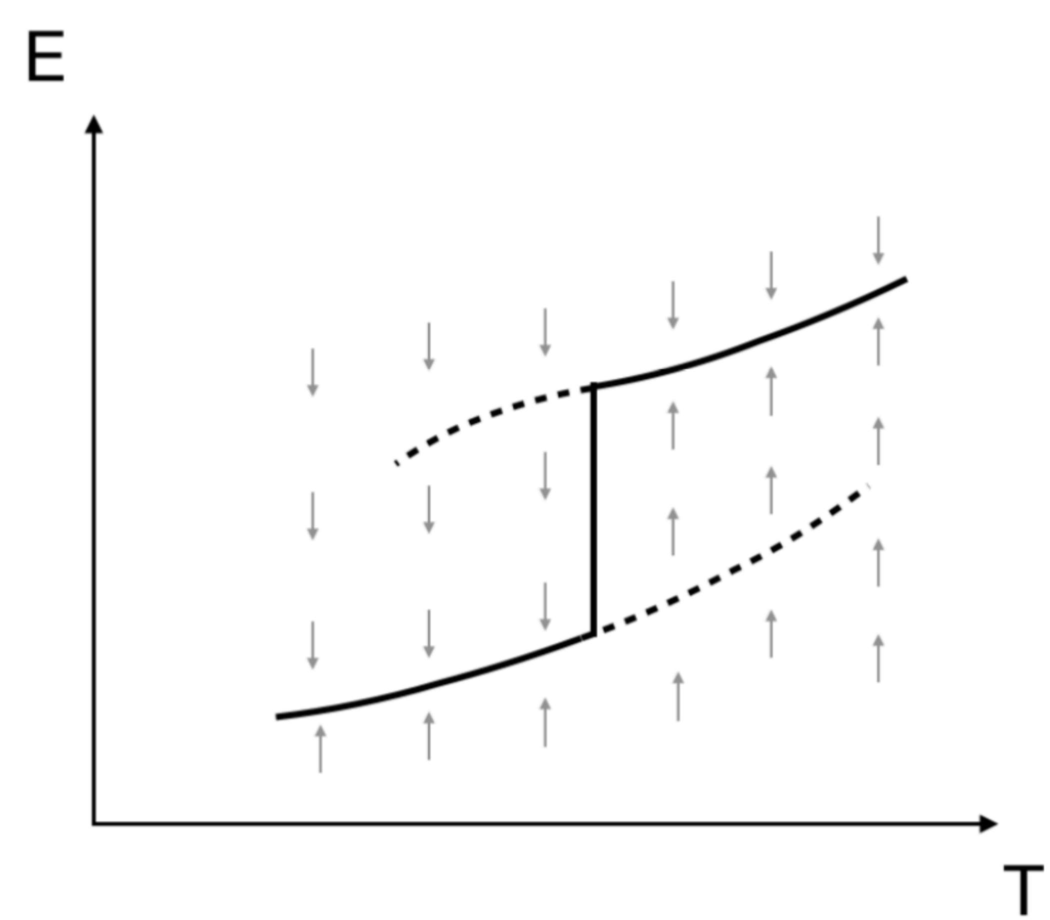
black hole = bound state of D-branes and strings

strongly coupled
4d SYM

water/ice

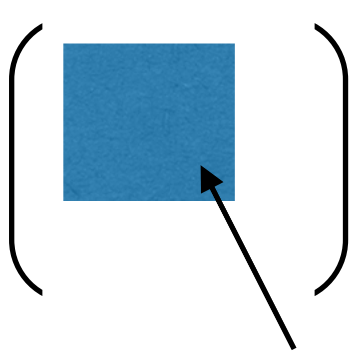


VERY DIFFERENT



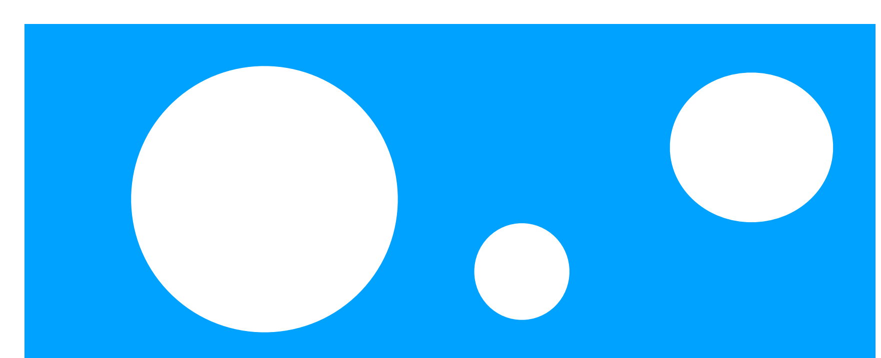
separation in color d.o.f

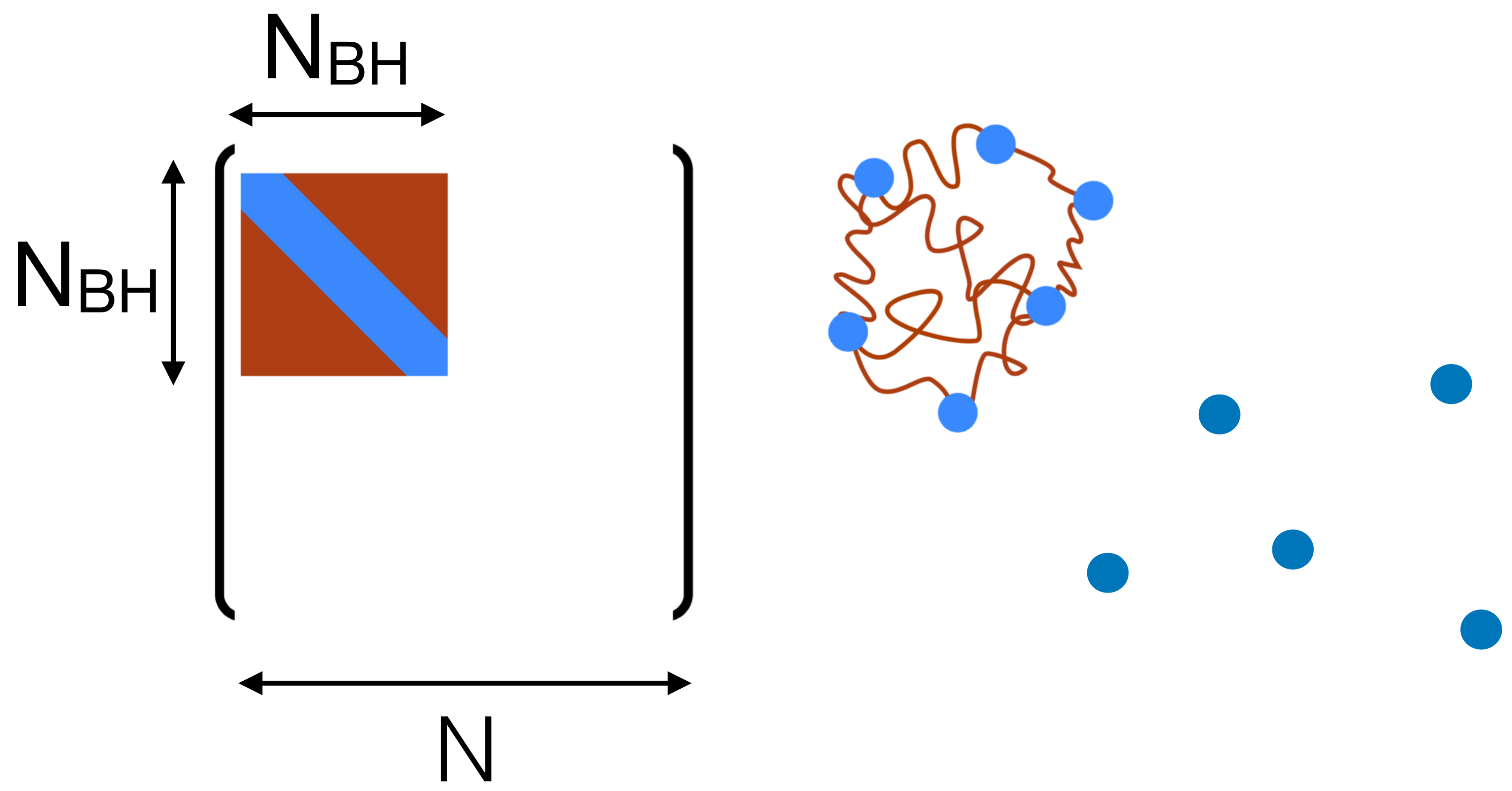
separation in space



partially deconfined

(MH-Malts, 2016)





N_{BH} D-branes form the bound state

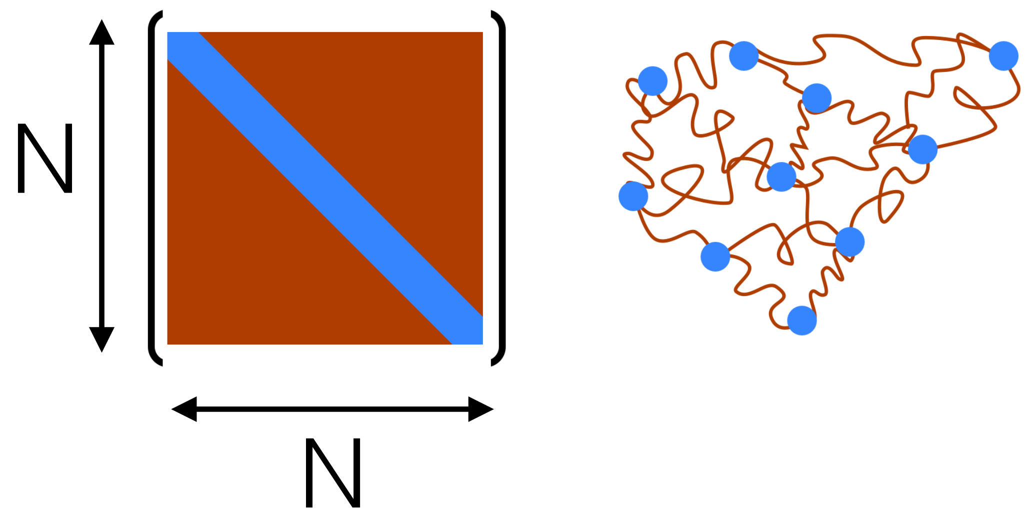
$U(N_{BH})$ is deconfined — ‘partial deconfinement’

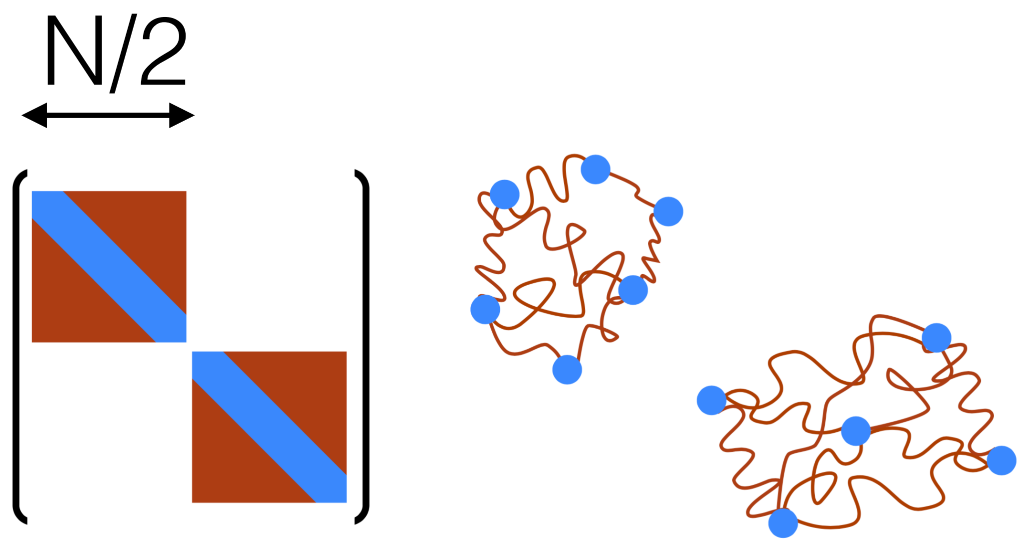
Can explain $E \sim N^2 T^{-7}$ for 4d SYM, $N^{3/2} T^{-8}$ for ABJM

(String Theory → 10d) (M-Theory → 11d)

(MH-Maltz, 2016)

Why can negative specific heat appear?





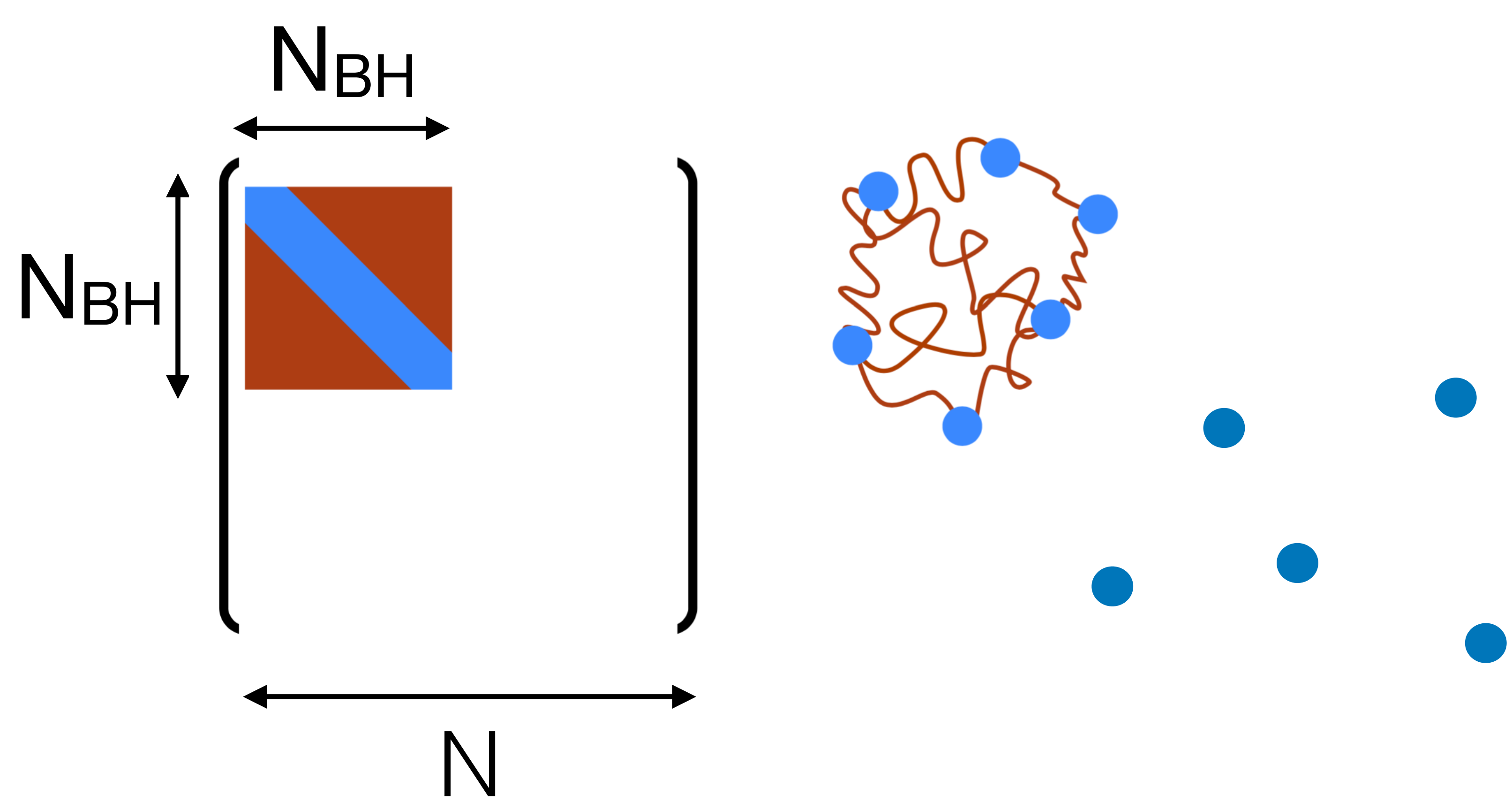
$T \sim E/N^2$

$T' \sim E'/[2\times(N/2)^2]$

$T' > T$ if $E' > E/2$

Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

Why can negative specific heat appear?



$T \sim E_{BH}/(N_{BH})^2$

N_{BH} is a function of E_{BH}

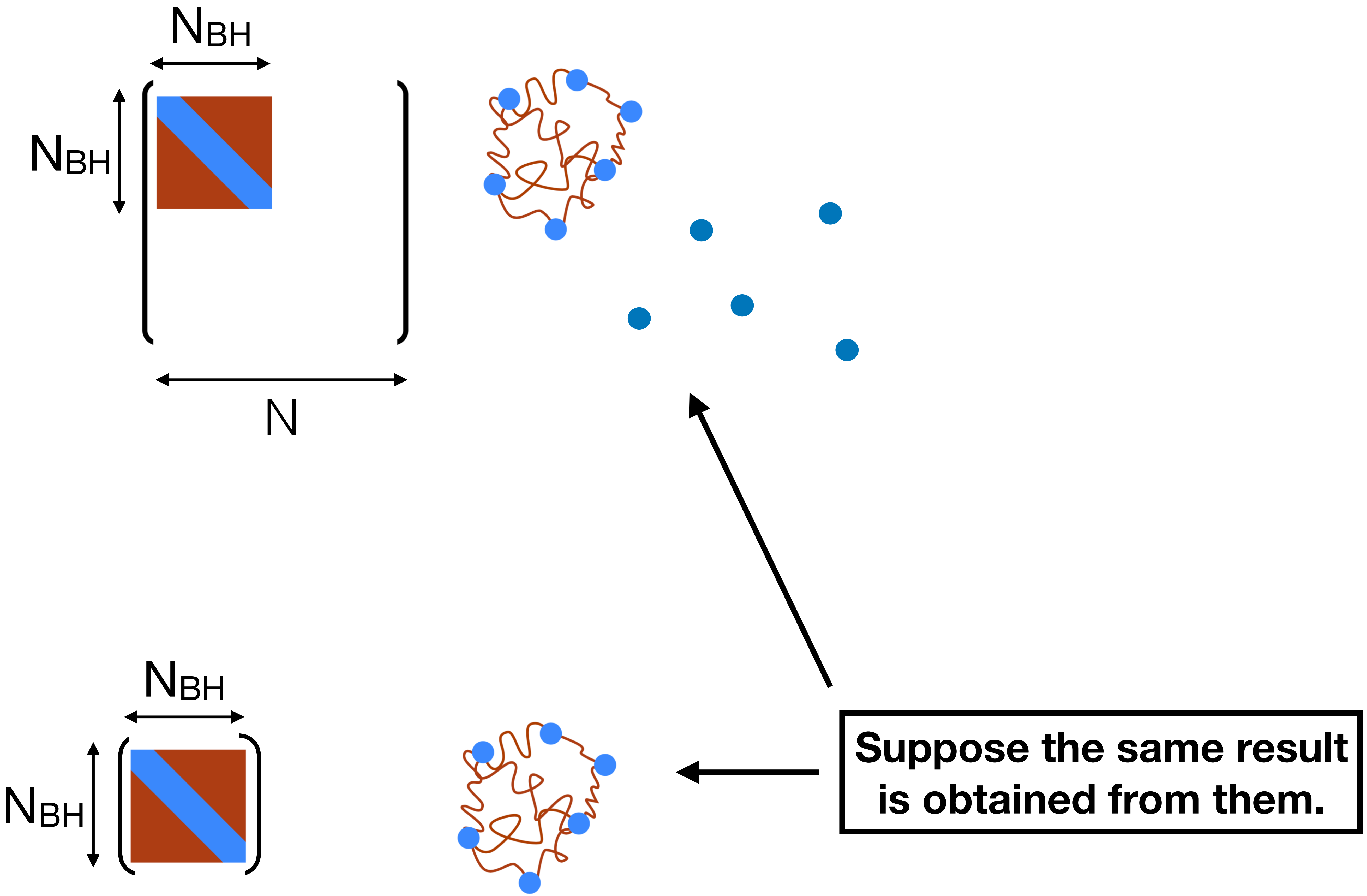
Berkowitz-M.H.-Maltz, 2016, PRD (Higgsing version)

10d Schwarzschild from 4d SYM

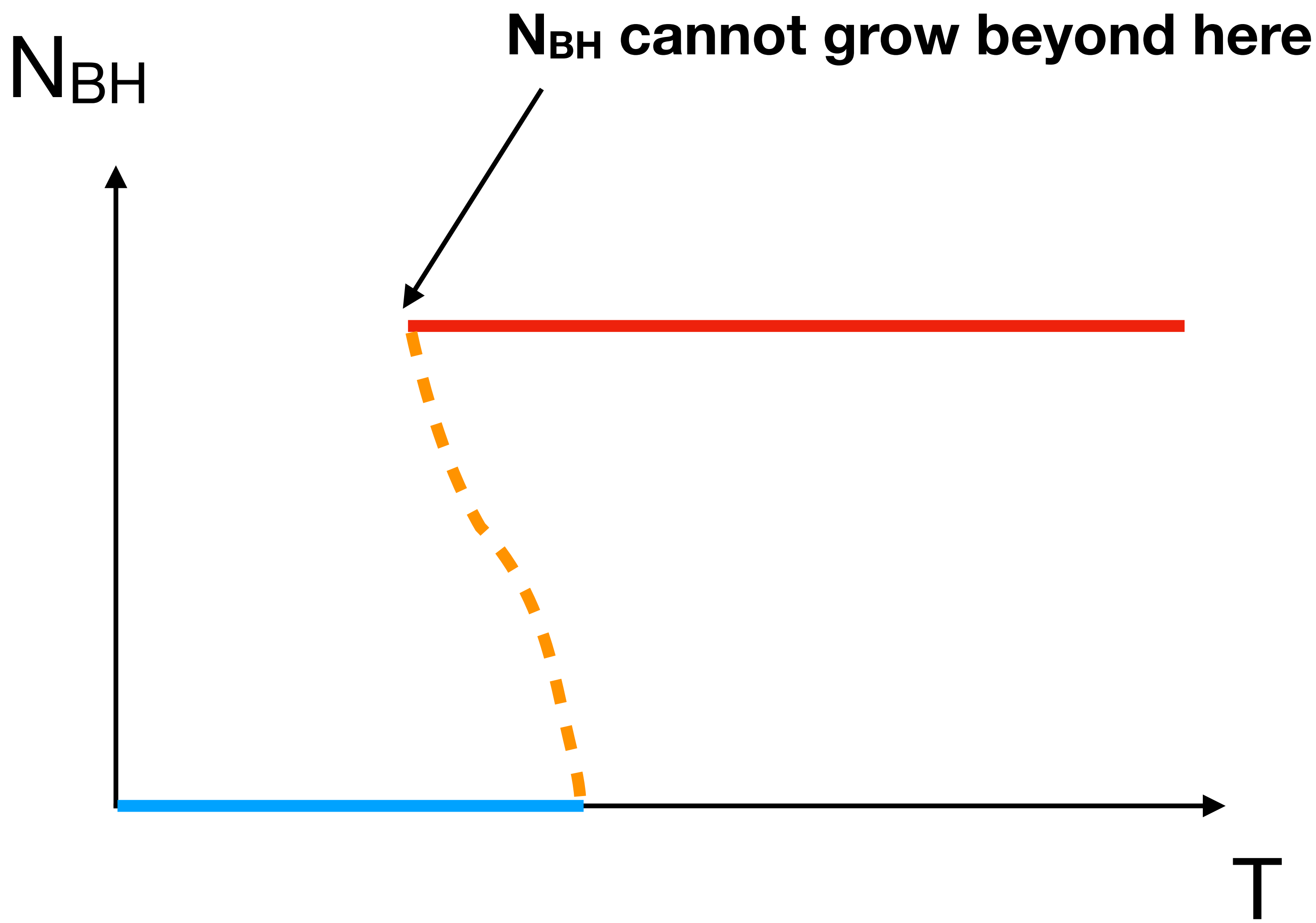
via

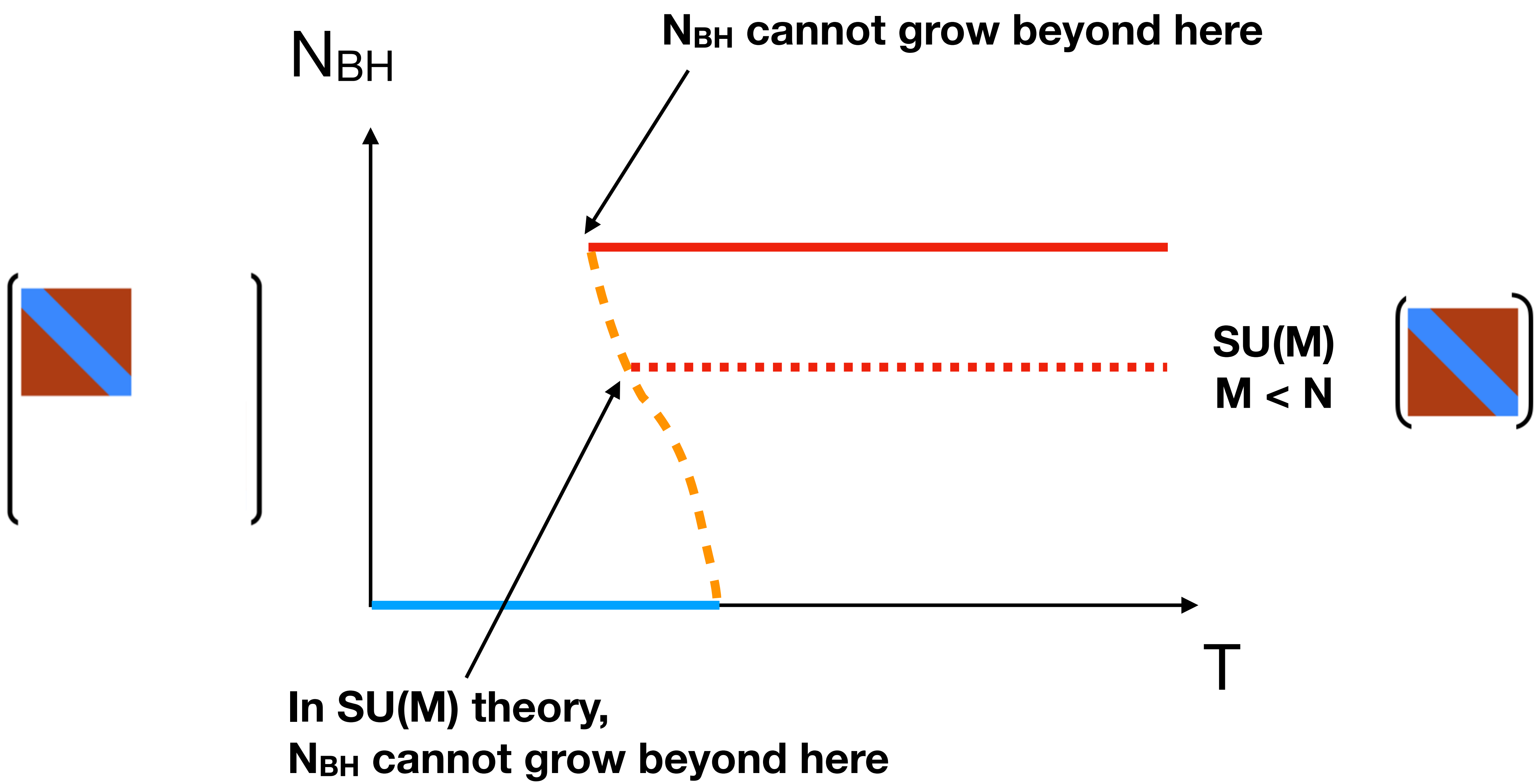
Partial Deconfinement

M.H., Maltz, 2016

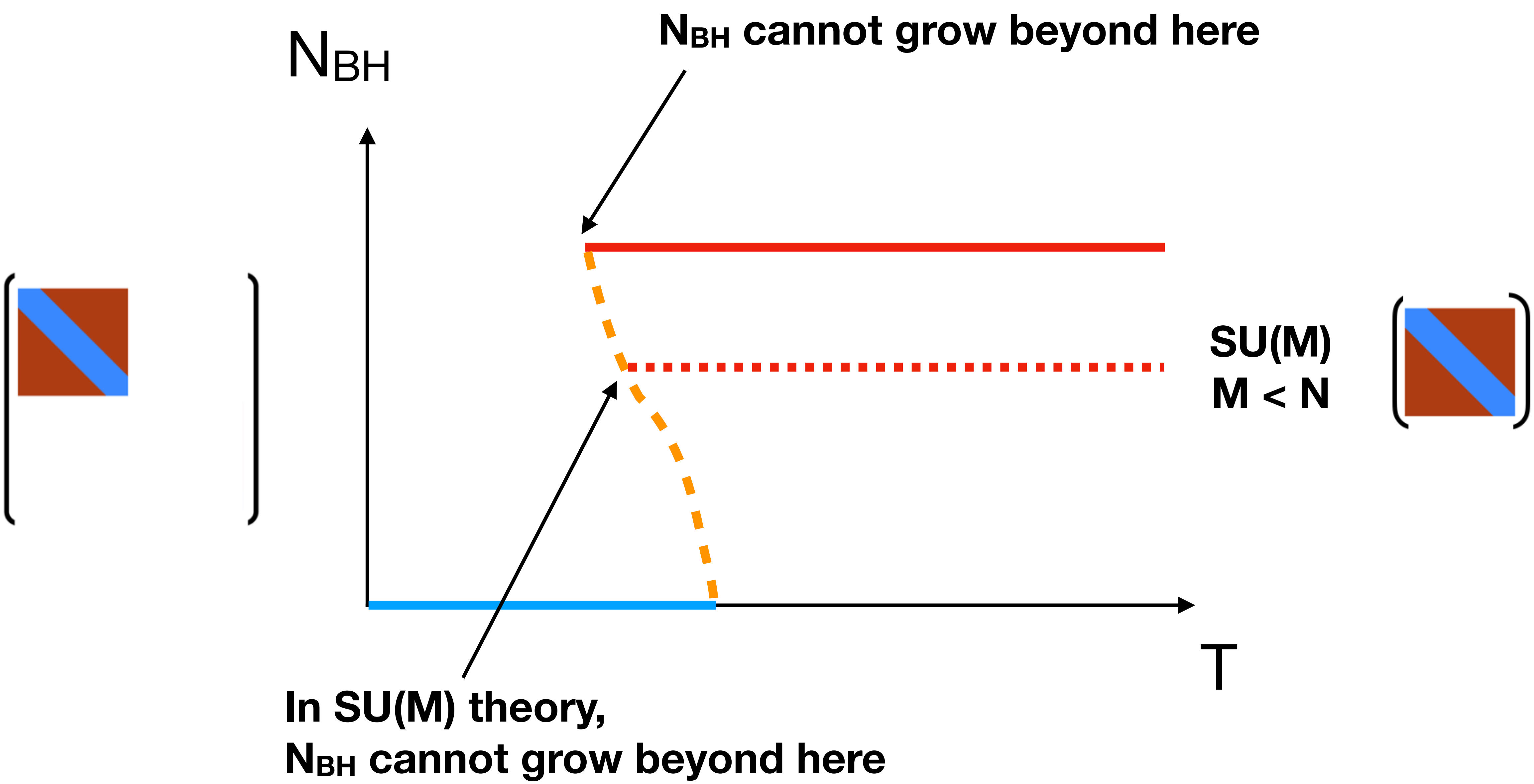


N_{BH}
N_{BH}
N
N_{BH}
N_{BH}
Suspose the same result is obtained from them.


N_BH cannot grow beyond here
N_BH
T



N_{BH} cannot grow beyond here
N_{BH}
SU(M)
M < N
T
In SU(M) theory,
N_{BH} cannot grow beyond here



We should determine ‘typical energy scale’ of SU(M) theory.

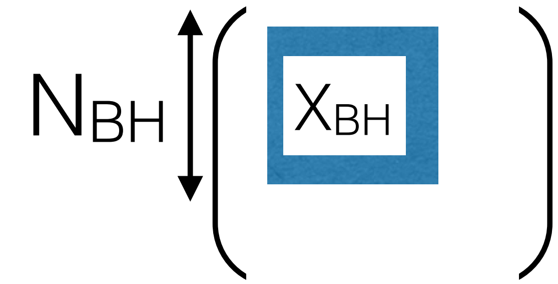
Heuristic Gauge Theory 'Derivation' (1)

- Take radius of S^3 to be 1.
- At strong coupling, the interaction term $(N/\lambda)*\mathrm{Tr}[X_I,X_J]^2$ is dominant.

$\lambda=g_{YM}^2N$

- Eigenvalues of $Y = \lambda^{-1/4}X$ are $O(1)$ because the interaction is simply $N*\mathrm{Tr}[Y_I,Y_J]^2$.
- Hence eigenvalues of X are $O(\lambda^{1/4})$.

Heuristic Gauge Theory 'Derivation' (2)



- When bunch size shrinks to $N_{BH}<N$, 't Hooft coupling effectively becomes $\lambda_{BH}=g_{YM}^2 N_{BH}$

$\lambda=g_{YM}^2 N$

- Hence eigenvalues of X_{BH} are $O(\lambda_{BH}^{1/4}) = O(g_{YM}^{1/2} N_{BH}^{1/4})$.

't Hooft counting
+
dimensional analysis

- $E_{BH} \sim N_{BH}^2 (N_{BH}/N)^{-1/4}$, $S_{BH} \sim N_{BH}^2$

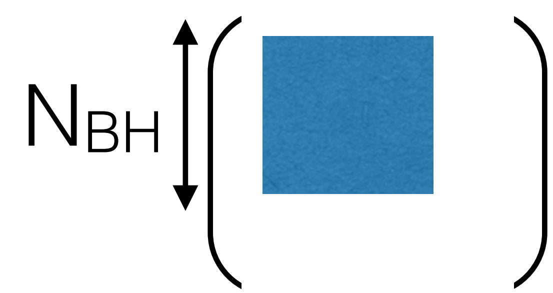
- $T_{BH} \sim (N_{BH}/N)^{-1/4}$

Heuristic Gauge Theory 'Derivation' (3)

- $E_{BH} \sim N_{BH}^2 (N_{BH}/N)^{-1/4}$, $S_{BH} \sim N_{BH}^2$

- $T_{BH} \sim (N_{BH}/N)^{-1/4}$

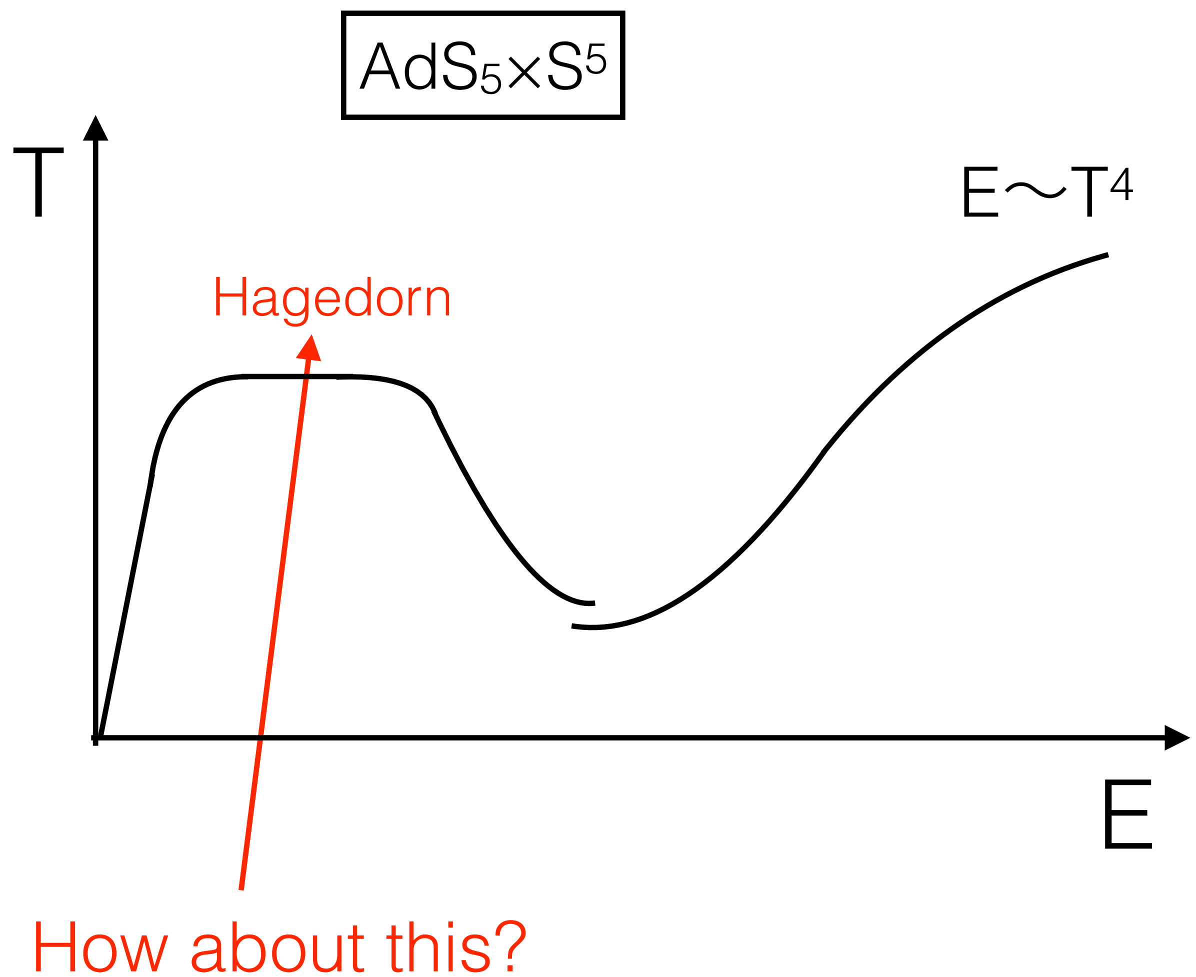
't Hooft counting
+
dimensional analysis

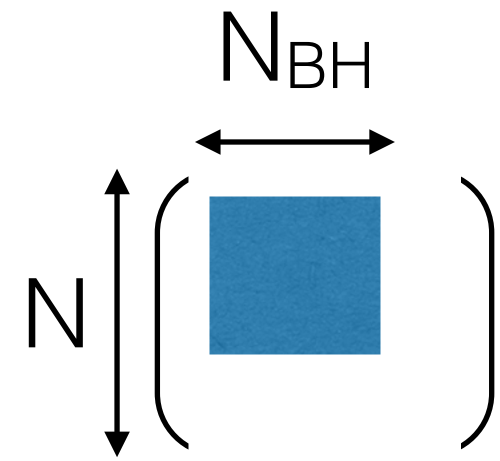


- $E_{BH} \sim N^2 (N_{BH}/N)^{7/4} \sim 1/(G_{N,10} T_{BH}^7)$

- $S_{BH} \sim N^2 (N_{BH}/N)^2 \sim 1/(G_{N,10} T_{BH}^8)$

10d Schwarzschild





$E < E_{min}$

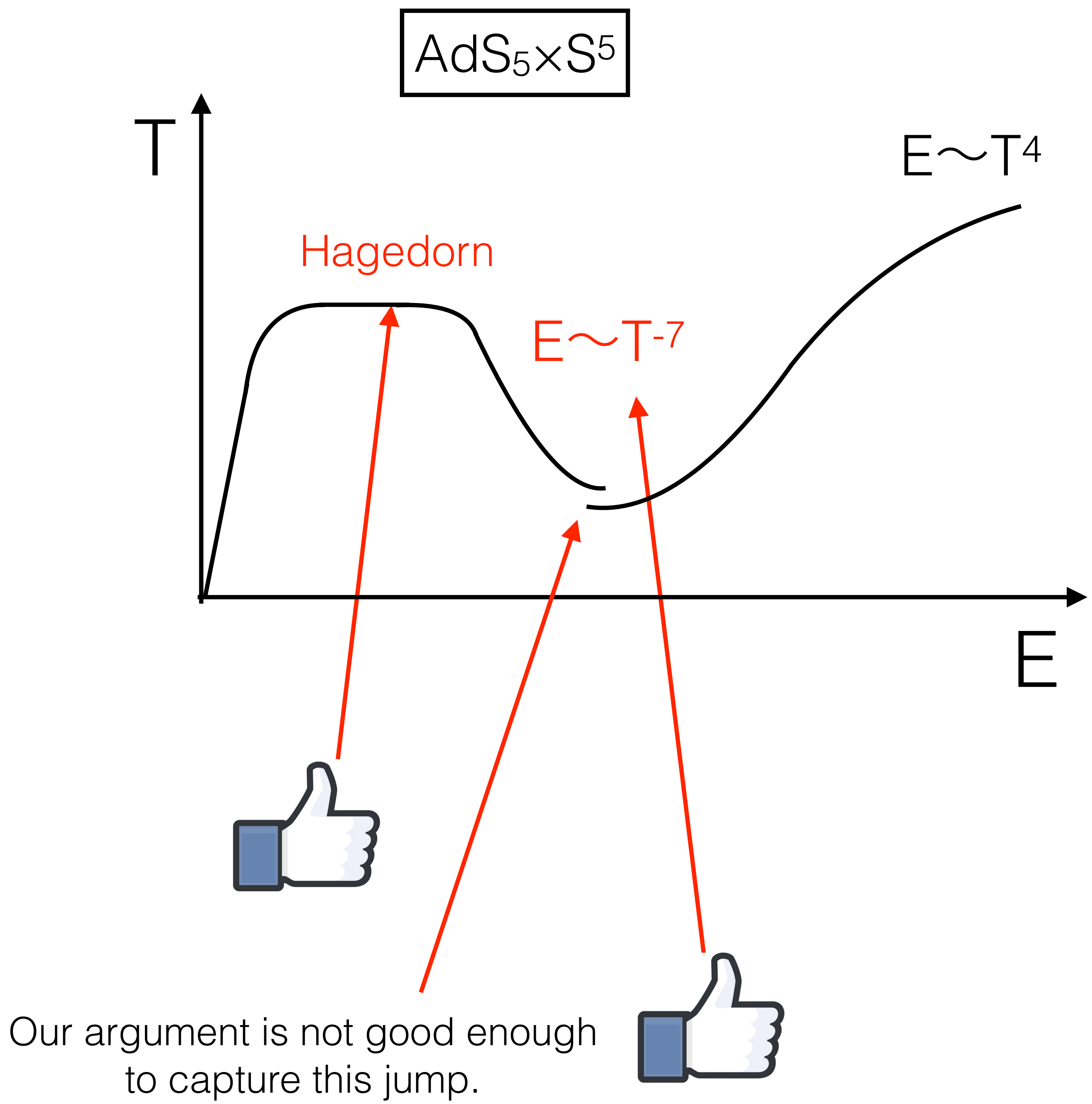
$T_{BH} = T_{Hagedorn} \sim 1$

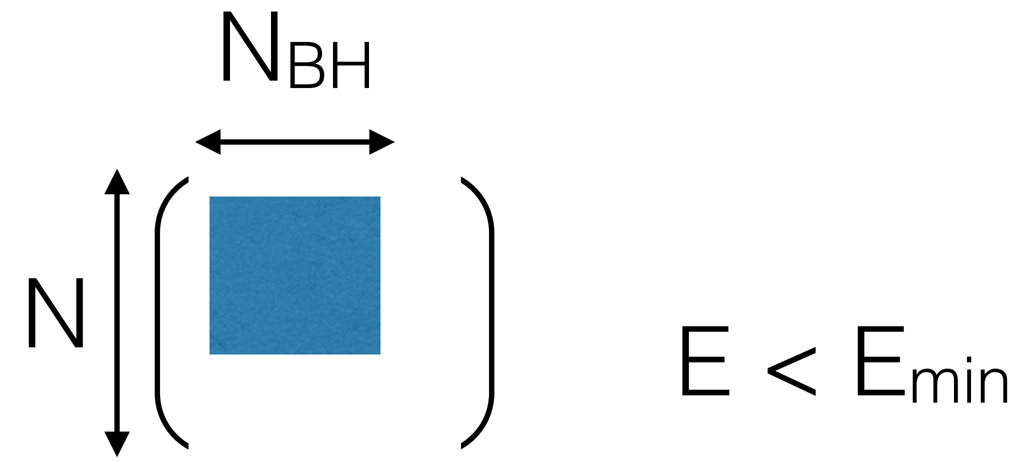
$E_{BH} \sim S_{min} \sim N_{BH}^2$

when $g_{YM}^2 N_{BH} << 1$

Just perturbative SYM.

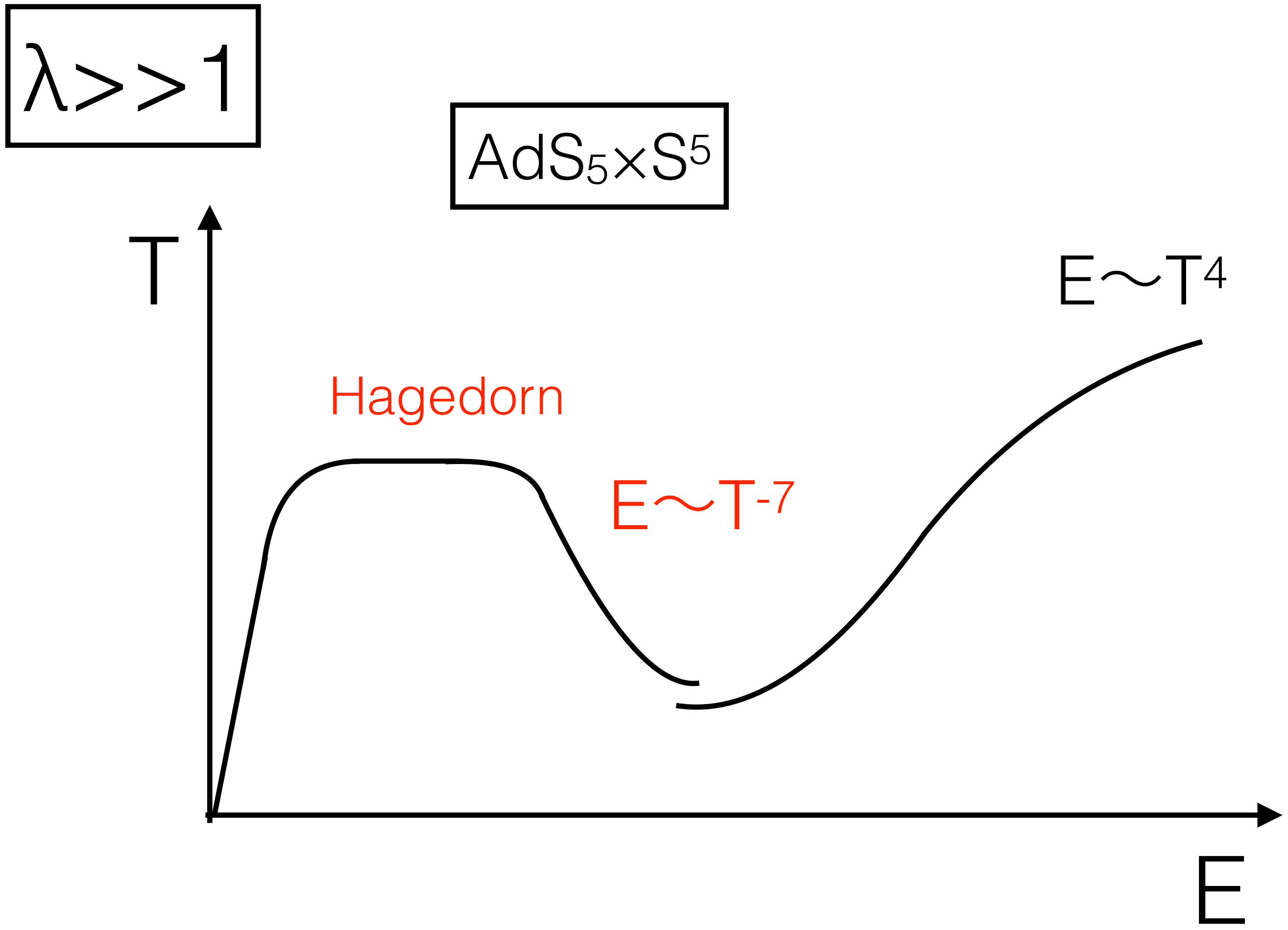
$g_{YM}^2 N_{BH} << 1$

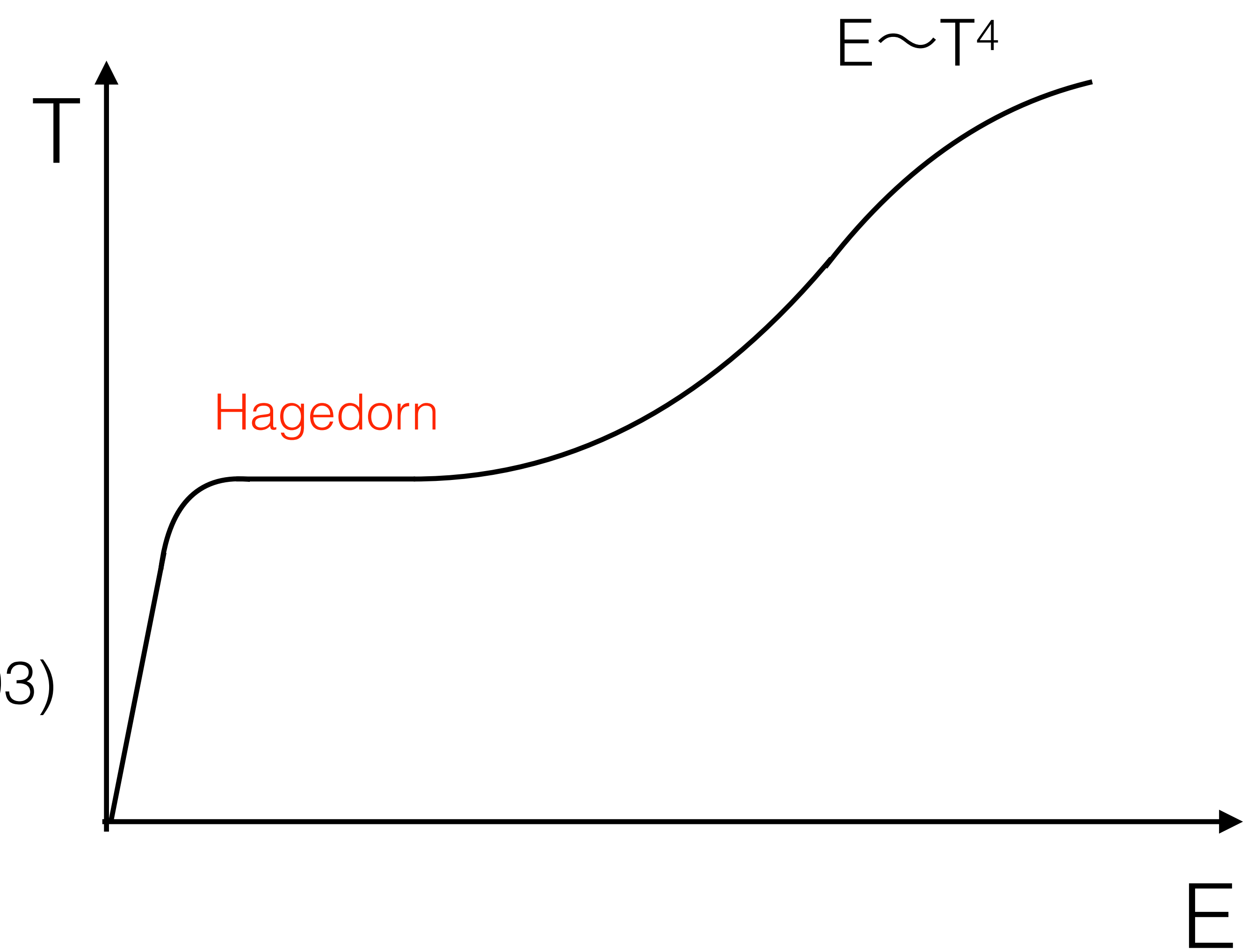




Large BH = 'Large' Matrices

Small BH = 'Small' Matrices

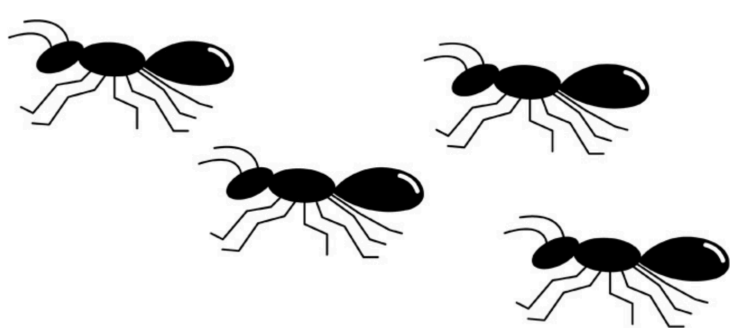


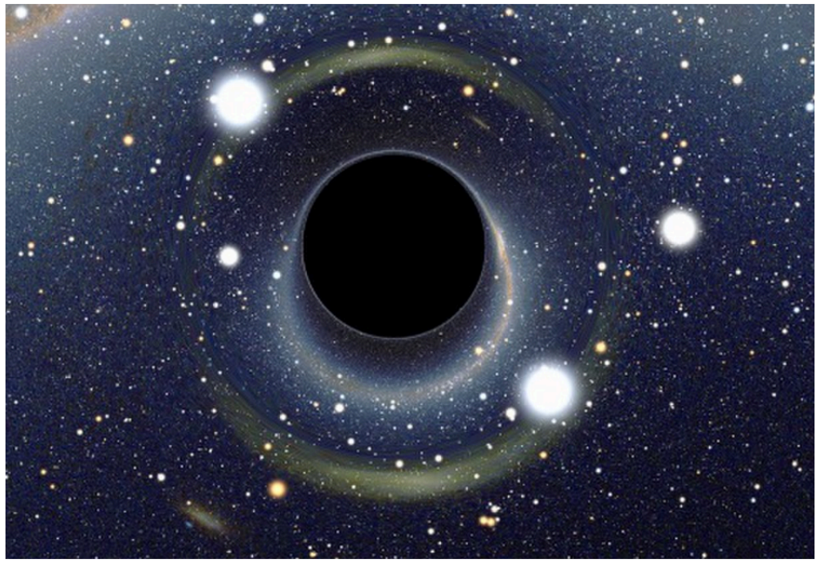


λ<<1

(see also Aharony et al 2003)

Ant trail/black hole correspondence
and
mechanism behind partial deconfinement





MH-Ishiki-Watanabe, arXiv:1812.05494 [hep-th] though it was a microscope of forty or fifty power. With care you could hold the focus. So I could go around and look at things right out in the street.

So when I was in graduate school at Princeton, I once took it out of my pocket to look at some ants that were crawling around on some ivy. I had to exclaim out loud, I was so excited. What I saw was an ant and an aphid, which ants take care of -- they carry them from plant to plant if the plant they're on is dying. In return the ants get partially digested aphid juice, called "honeydew." I knew that; my father had told me about it, but I had never seen it.

So here was this aphid and sure enough, an ant came along, and patted it with its feet -- all around the aphid, pat, pat, pat, pat, pat. This was terribly exciting! Then the juice came out of the back of the aphid. And because it was magnified, it looked like a big, beautiful, glistening ball, like a balloon, because of the surface tension. Because the microscope wasn't very good, the drop was colored a little bit from chromatic aberration in the lens -- it was a gorgeous thing!

The ant took this ball in its two front feet, lifted it off the aphid, and *held* it. The world is so different at that scale that you can pick up water and hold it! The ants probably have a fatty or greasy material on their legs that doesn't break the surface tension of the water when they hold it up. Then the ant broke the surface of the drop with its mouth, and the surface tension collapsed the drop right into his gut. It was *very* interesting to see this whole thing happen!

In my room at Princeton I had a bay window with a U-shaped windowsill. One day some ants came out on the windowsill and wandered around a little bit. I got curious as to how they found things. I wondered, how do they know where to go? Can they tell each other where food is, like bees can? Do they have any sense of geometry?

This is all amateurish; everybody knows the answer, but *I* didn't know the answer, so the first thing I did was to stretch some string across the U of the bay window and hang a piece of folded cardboard with sugar on it from the string. The idea of this was to isolate the sugar from the ants, so they wouldn't find it accidentally. I wanted to have everything under control.

Next I made a lot of little strips of paper and put a fold in them, so I could pick up ants and ferry them from one place to another. I put the folded strips of paper in two places: Some were by the sugar (hanging from the string), and the others were near the ants in a particular location. I sat there all afternoon, reading and watching, until an ant happened to walk onto one of my little paper ferries. Then I took him over to the sugar. After a few ants had been ferried over to the sugar, one of them accidentally walked onto one of the ferries nearby, and I carried him back.

I wanted to see how long it would take the other ants to get the message to go to the "ferry terminal." It started slowly, but rapidly increased until I was going mad ferrying the ants back and forth.

But suddenly, when everything was going strong, I began to deliver the ants from the sugar to a *different* spot. The question now was, does the ant learn to go back to where it just came from, or does it go where it went the time before?

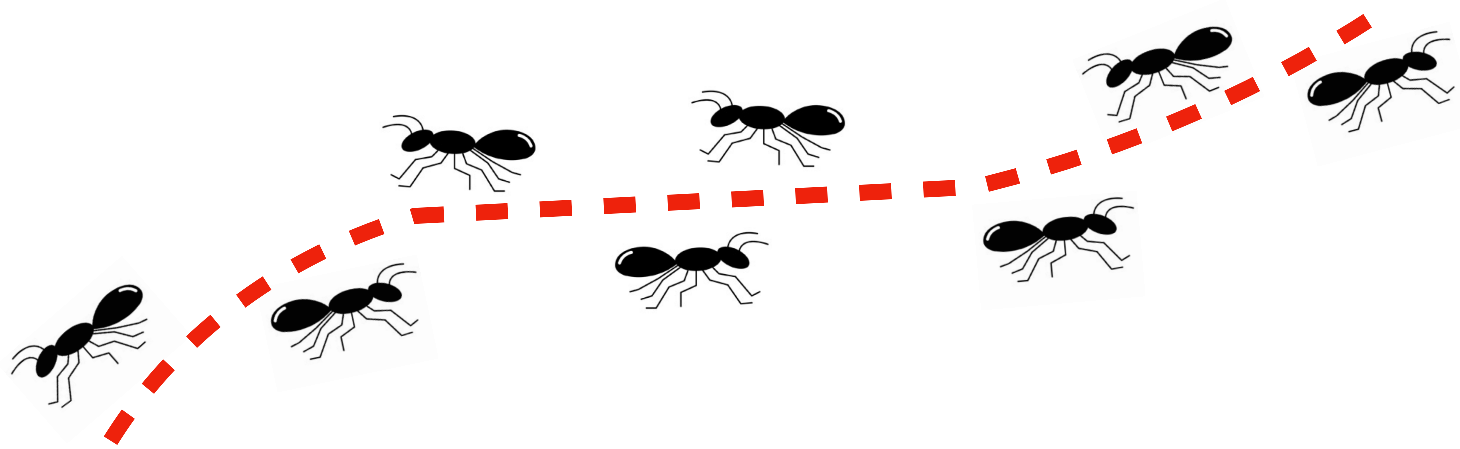
After a while there were practically no ants going to the first place (which would take them to the sugar), whereas there were many ants at the second place, milling around, trying to find the sugar. So I figured out so far that they went where they just came from.

Study of ants has long history.

And useful for gauge theory and quantum gravity.

From ‘Surely you are joking, Mr. Feynman!’

Heuristic Derivation

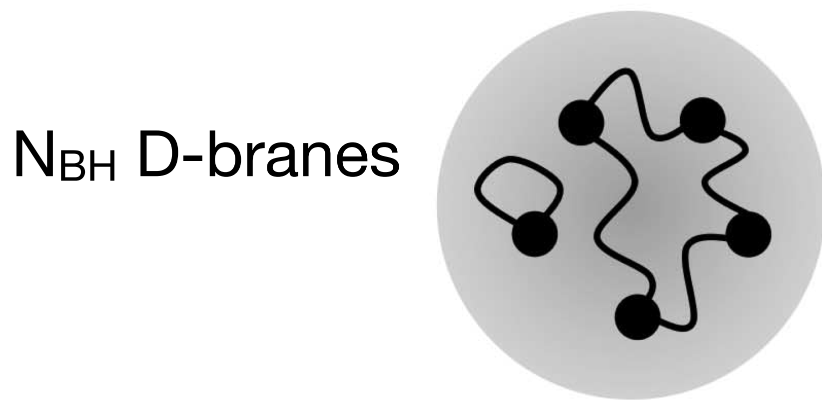


- Ant ‘trail’ is called 行列 in Japanese/Chinese.
- ‘Matrix’ is called 行列 in Japanese/Chinese.
- Gauge/gravity duality says BH is matrix (行列).

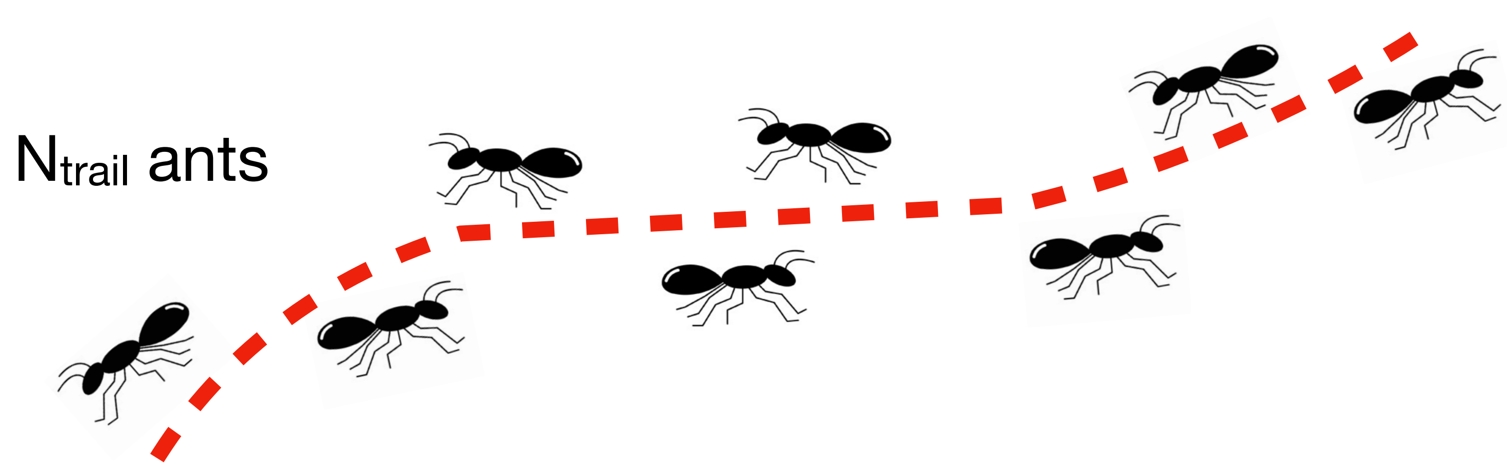
black hole = ant trail?

Hidehiko Shimada, 2018, private communication

Black hole = D-brane bound by open strings

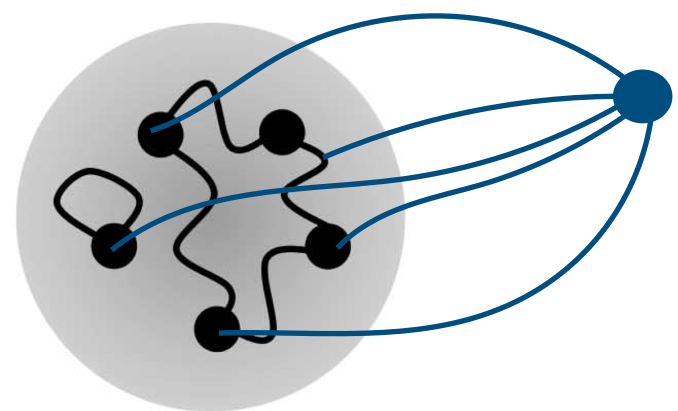


Ant trail = ants bound by pheromone



Black hole = D-brane bound by open strings

N_{BH} D-branes



N_{BH} open strings try to capture the other D-brane

$$N_{\mathrm{trail}} \longleftrightarrow N_{\mathrm{BH}}$$

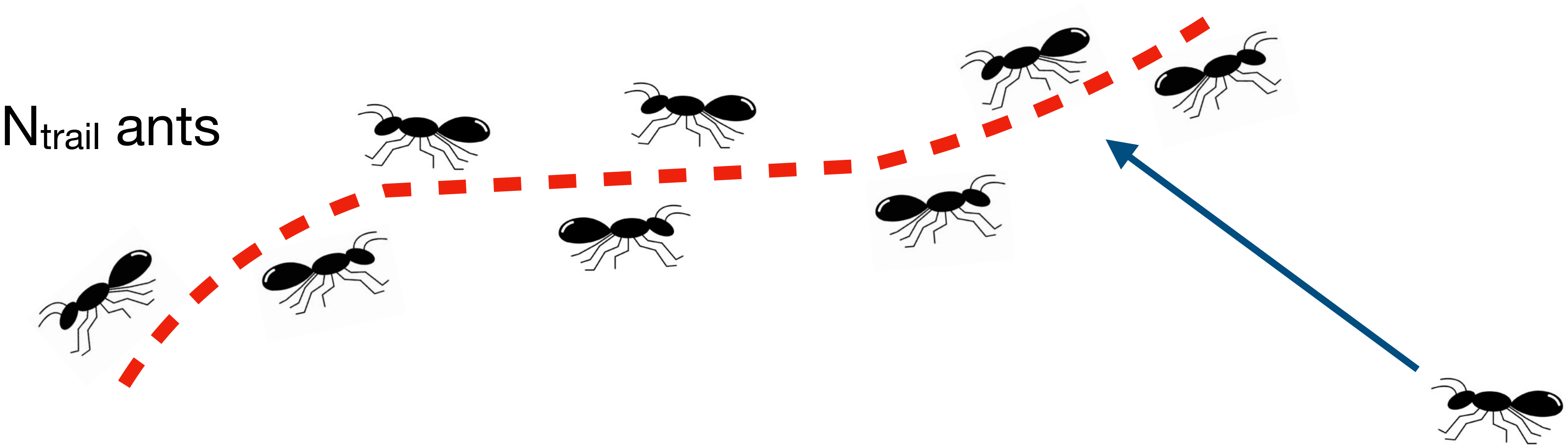
(new light modes
→ more entropy
→ entropic force)

Ant trail = ants bound by pheromone

pheromone strength = $p \times N_{trail}$

p: pheromone from each ant

N_{trail} ants



Meaning of Temperature

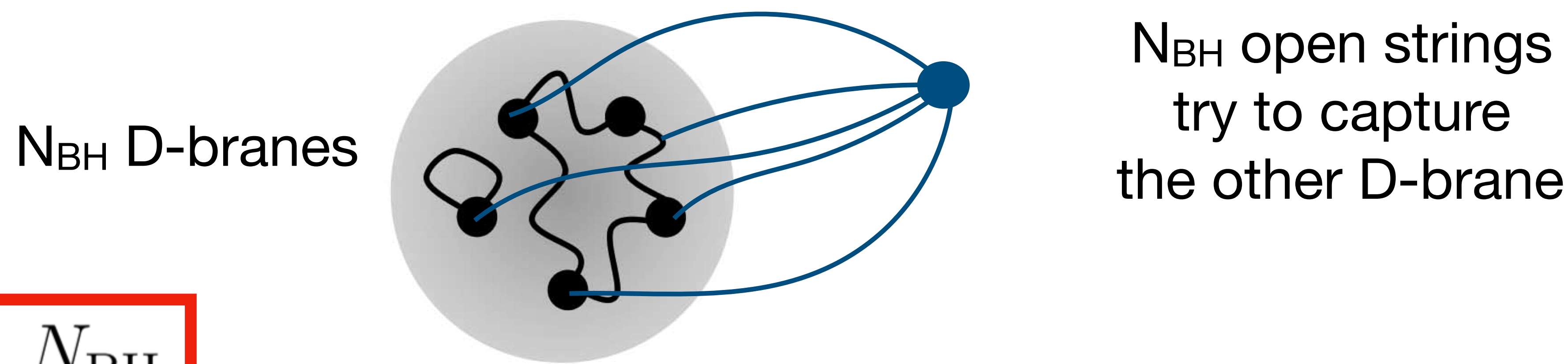
‘Hot ant’?

Meaning of Temperature

‘Hot ant’?

Maybe releases a lot of pheromones
and attracts many other ants.

Black hole = D-brane bound by open strings



$$N_{\text{trail}} \longleftrightarrow N_{\text{BH}},$$
$$p \longleftrightarrow T.$$

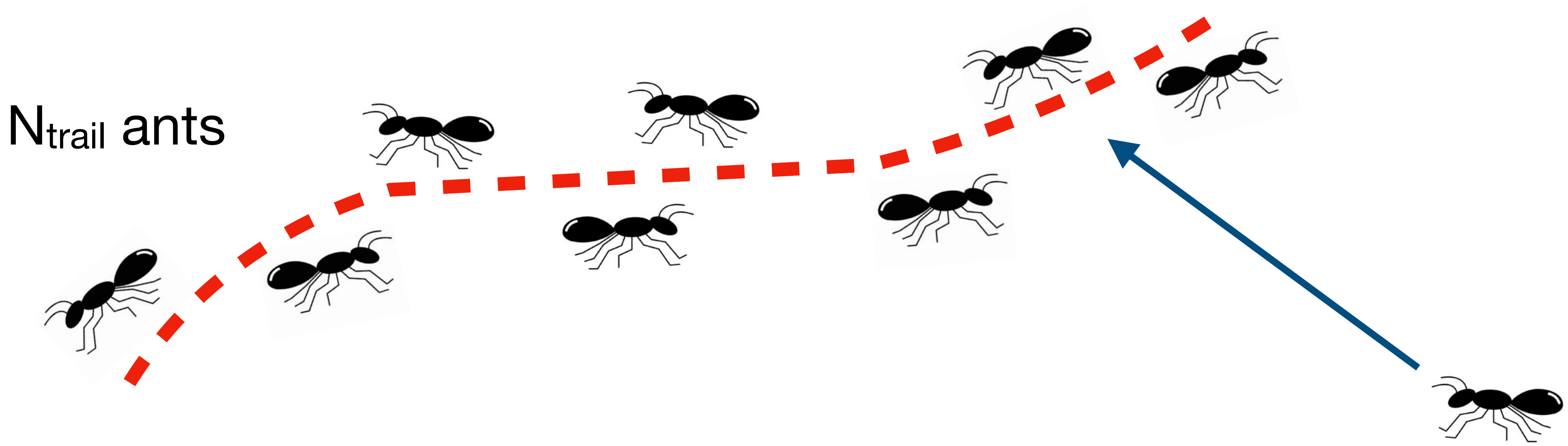
high T ∼ each mode is excited more

∼ stronger pheromone from each ant

Ant trail = ants bound by pheromone

pheromone strength = p × N_{trail}

p: pheromone from each ant



The ant equation

Phase transition between disordered and ordered foraging in Pharaoh's ants

Madeleine Beekman*†, David J. T. Sumpter‡, and Francis L. W. Ratnieks*

*Laboratory of Apiculture and Social Insects, Department of Animal and Plant Sciences, Sheffield University, Sheffield S10 2TN, United Kingdom; and ‡Centre for Mathematical Biology, Mathematical Institute, Oxford University, 24-29 St. Giles, Oxford OX1 3LB, United Kingdom

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PNAS Proceedings of the National Academy of Sciences of the United States of America

$$\frac{dN_{\text{trail}}}{dt} = \text{(ants coming into the trail)} - \text{(ants leaving the trail)}$$

$$= (\alpha + pN_{\text{trail}})(N - N_{\text{trail}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}} = 0$$

stringy interaction

penalty from Boltzmann factor

Natural large-N limit: $\alpha \sim N^0, p \sim N^0, s \sim N^1$

(many-ant limit)

The ant equation

Phase transition between disordered and ordered foraging in Pharaoh's ants

Madeleine Beekman*†, David J. T. Sumpter‡, and Francis L. W. Ratnieks*

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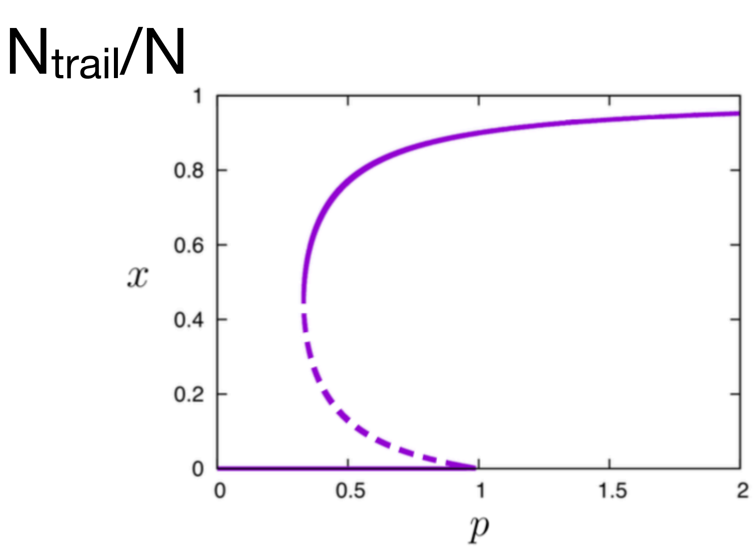
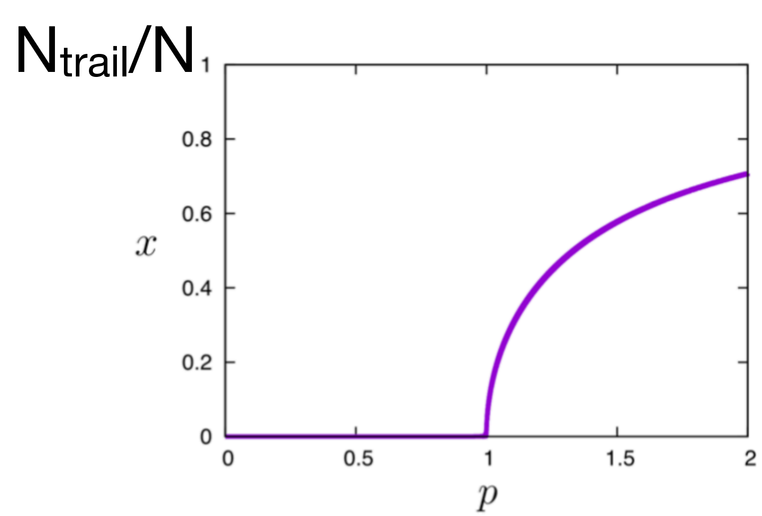
$$\frac{dN_{\text{trail}}}{dt} = (\text{ants coming into the trail}) - (\text{ants leaving the trail})$$

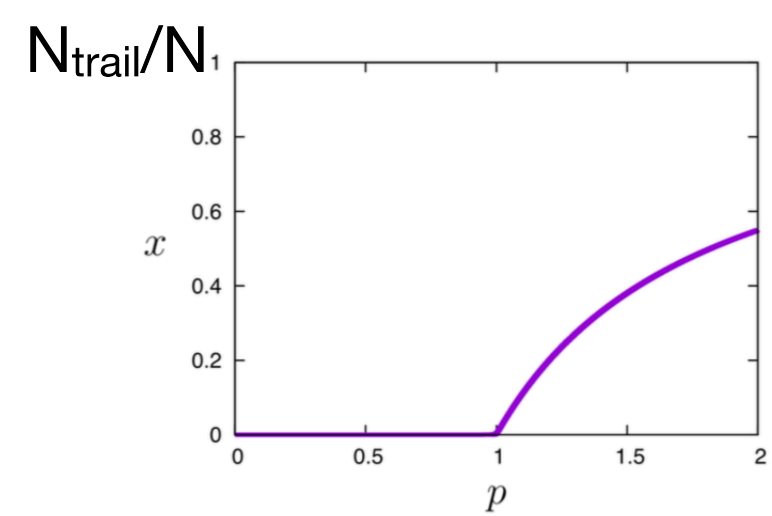
$$= (\alpha + \underline{pN_{\text{trail}})(N - N_{\text{trail}}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}} = \mathbf{0}$$

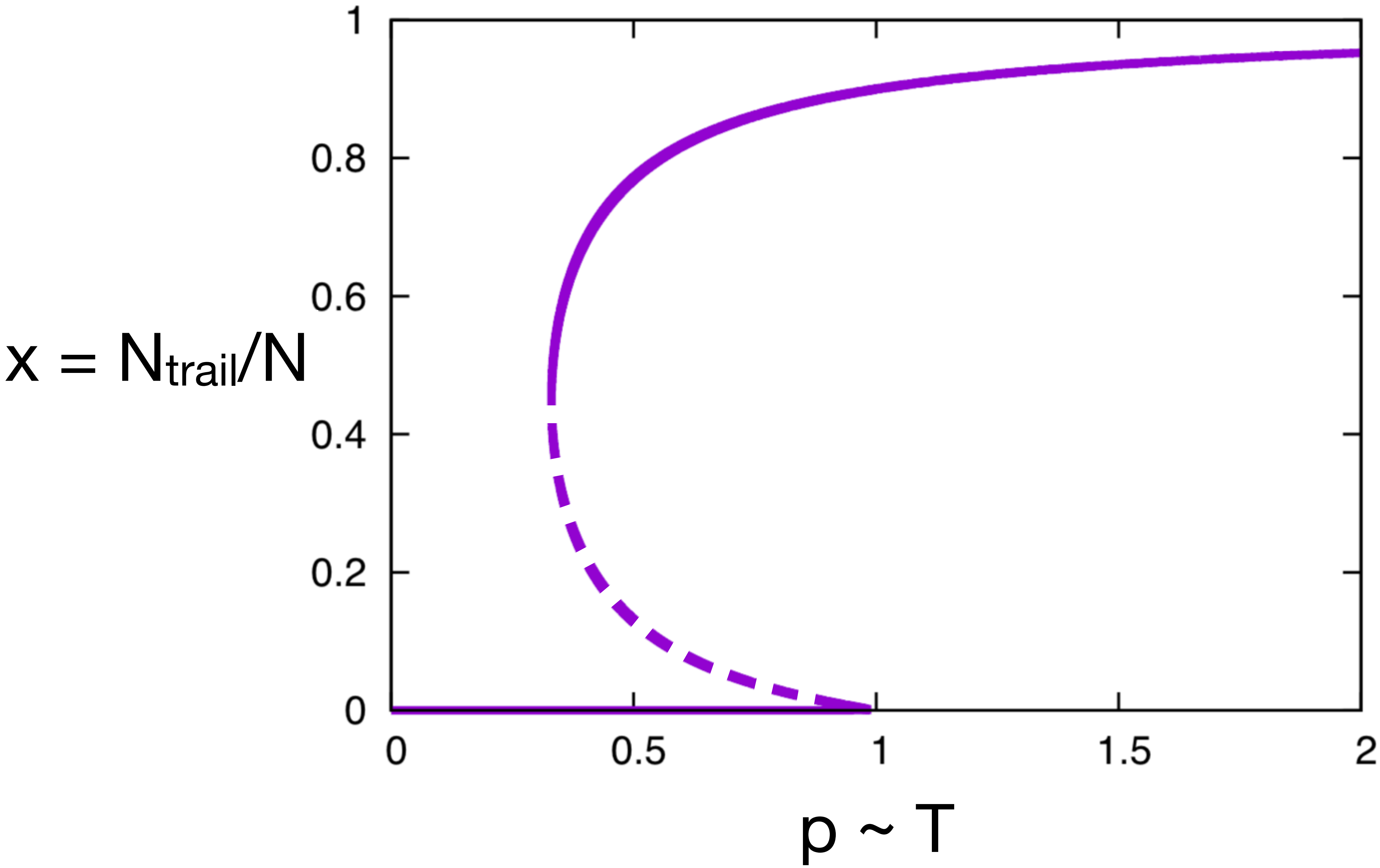
stringy interaction

penalty from Boltzmann factor

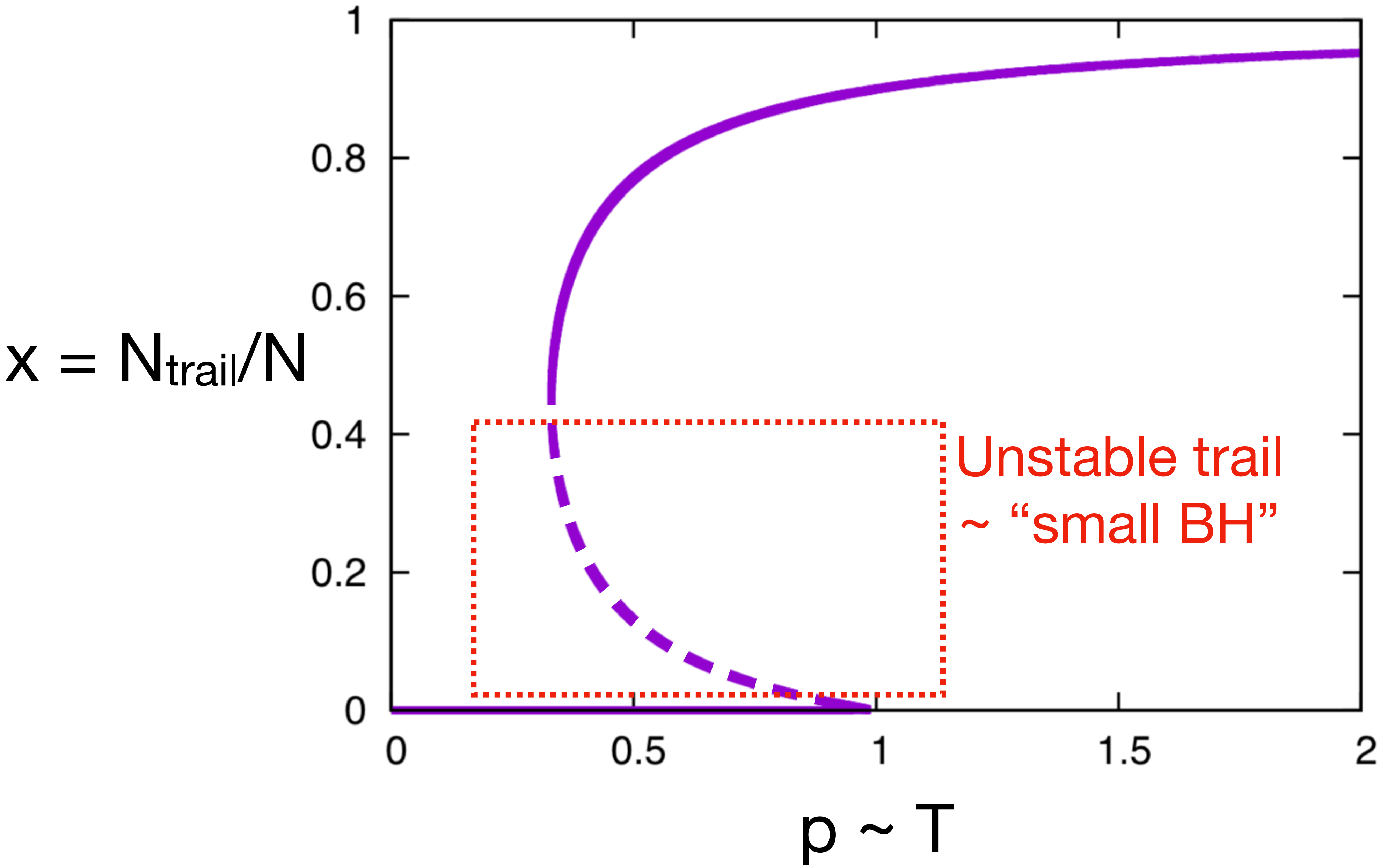




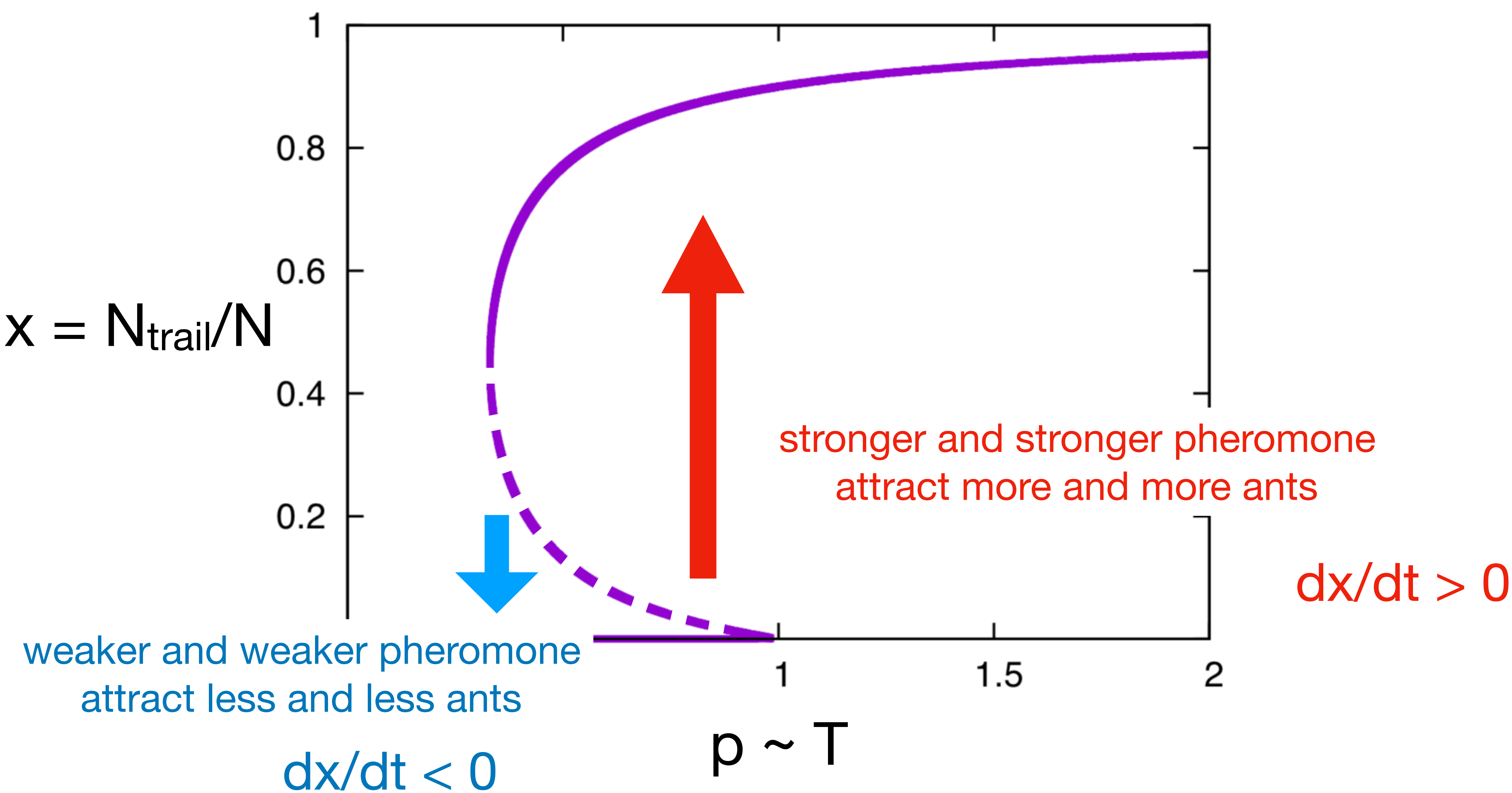




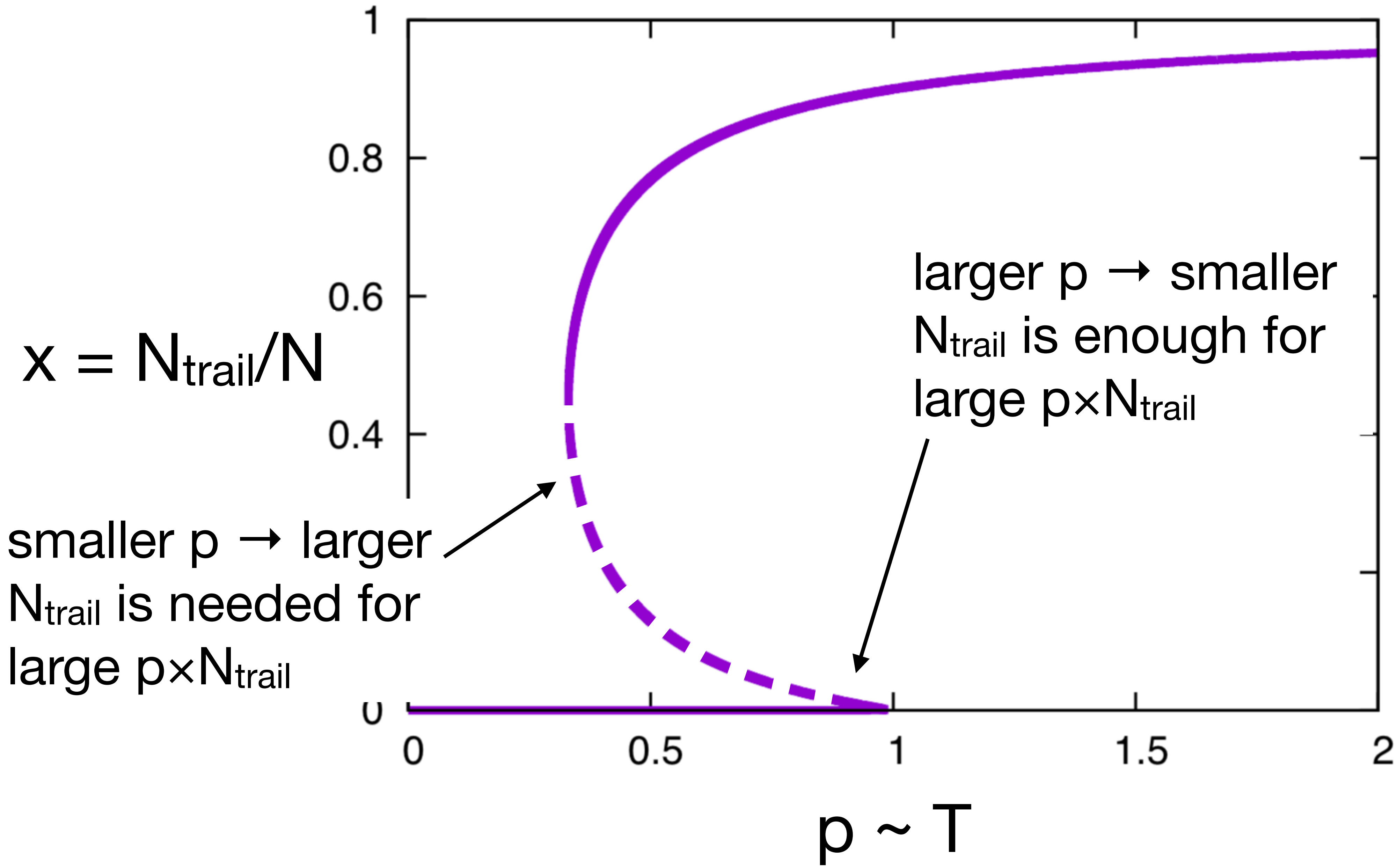
x = Ntrail/N
p ~ T
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2



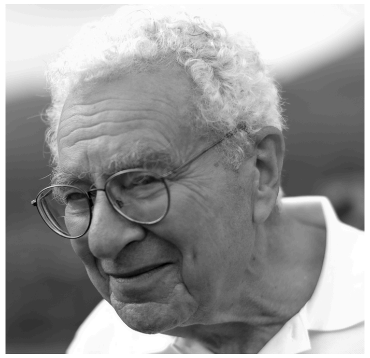
1
0.8
0.6
0.4
0.2
0
x = N_trail/N
Unstable trail
~ “small BH”
0
0.5
1
1.5
2
p ~ T

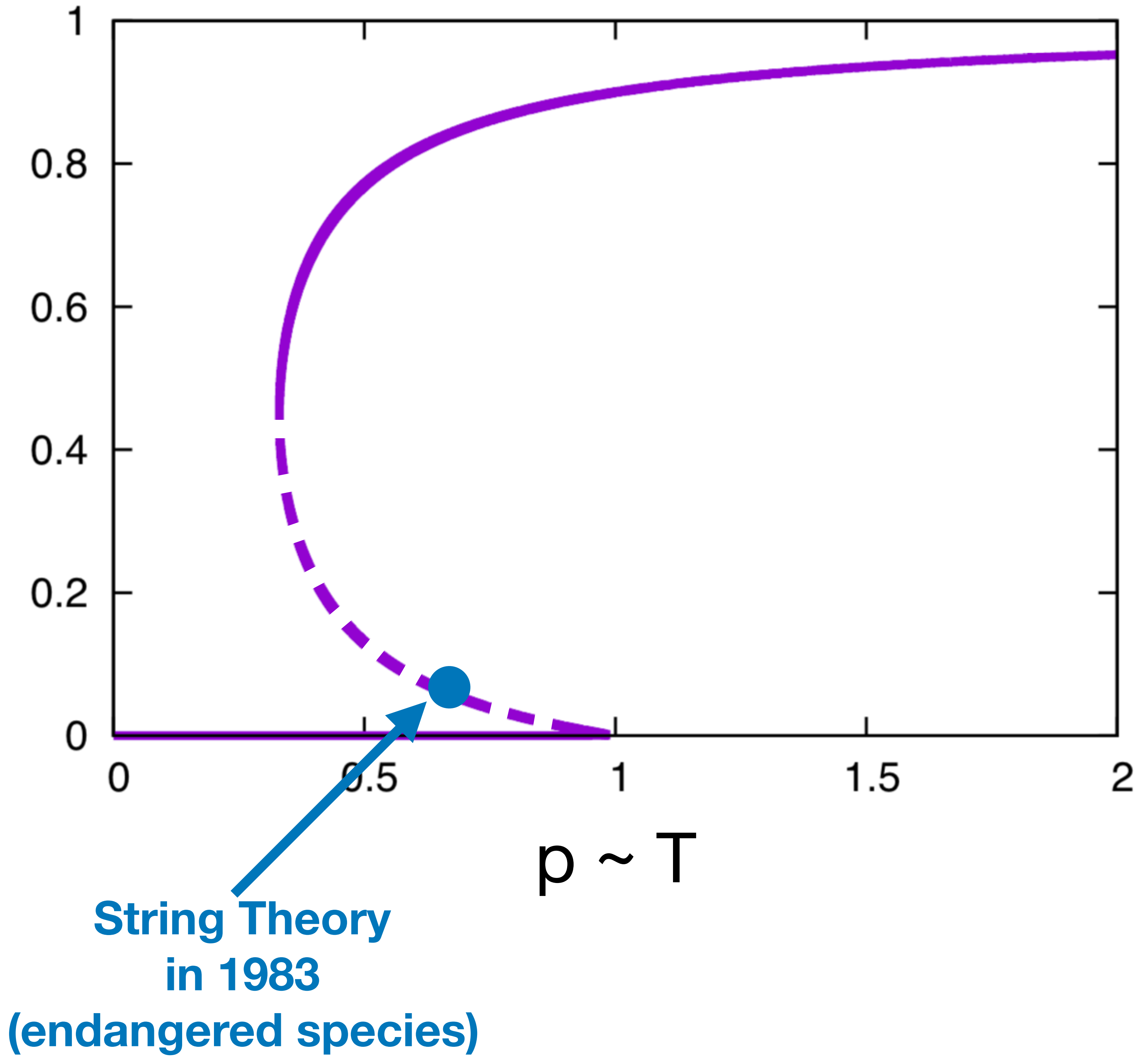


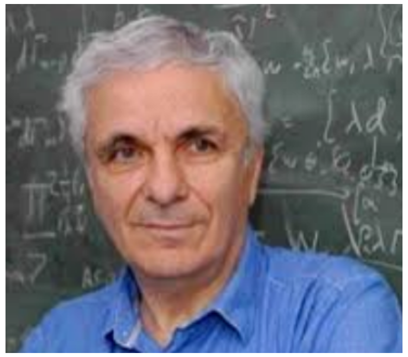
1
0.8
0.6
0.4
0.2
$x = N_{trail}/N$
1
1.5
2
p ~ T
stronger and stronger pheromone
attract more and more ants
dx/dt > 0
weaker and weaker pheromone
attract less and less ants
dx/dt < 0

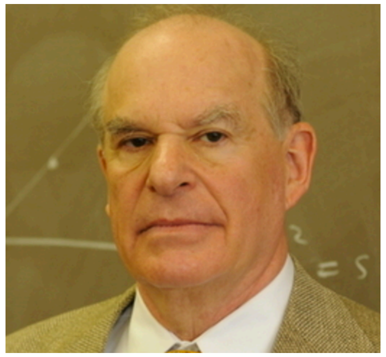


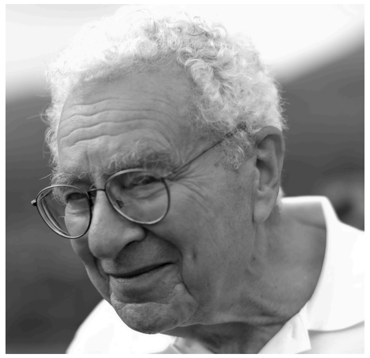
1
0.8
0.6
0.4
0
0
0.5
1
1.5
2
x = Ntrail/N
p ~ T
larger p → smaller Ntrail is enough for large p×Ntrail
smaller p → larger Ntrail is needed for large p×Ntrail

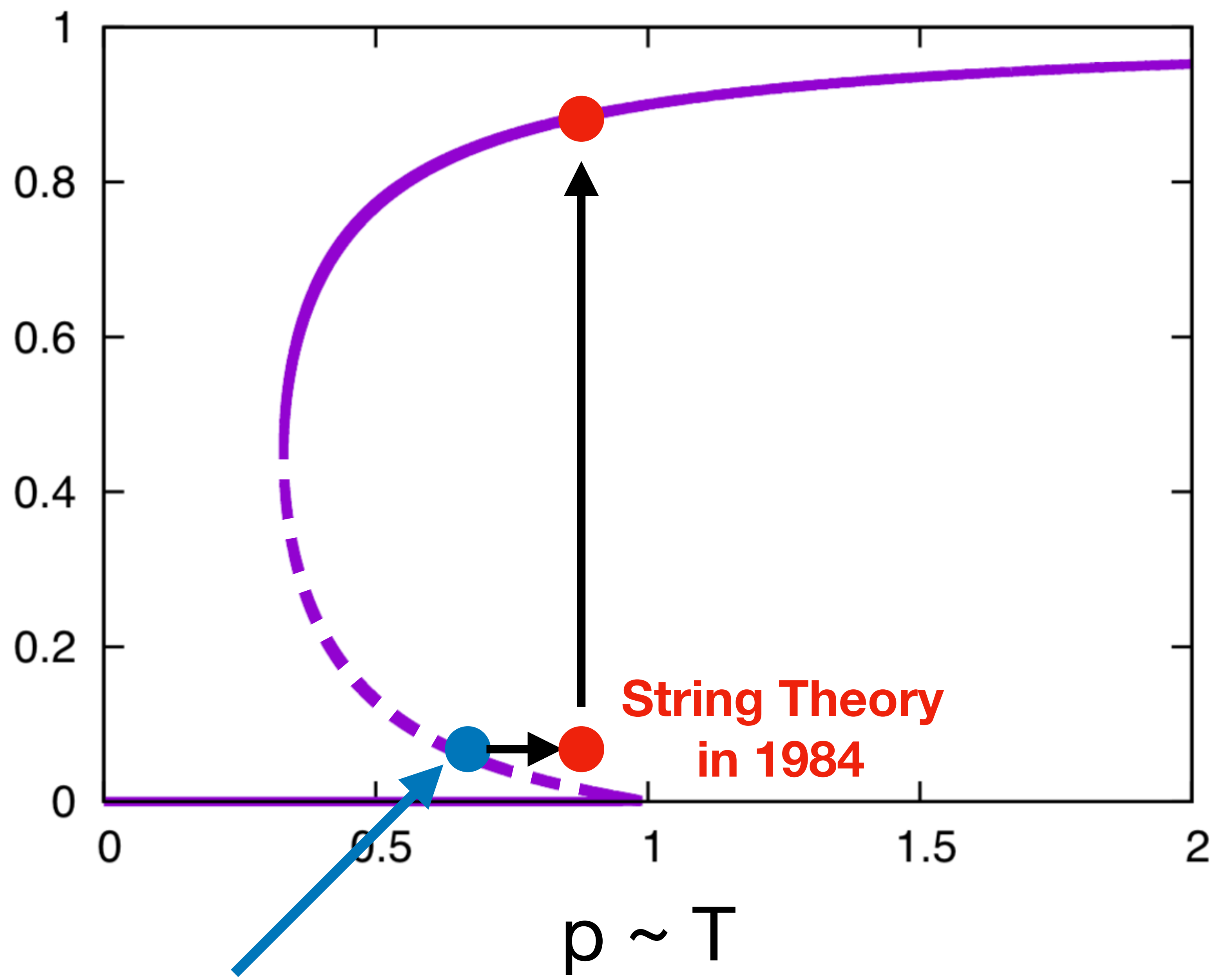


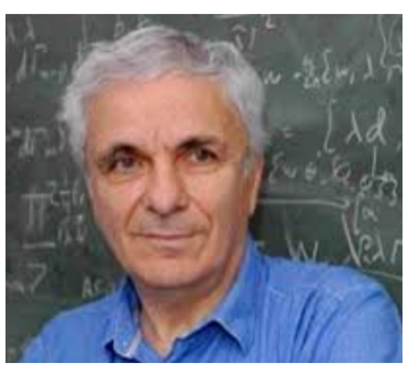

1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
p ~ T
String Theory
in 1983
(endangered species)

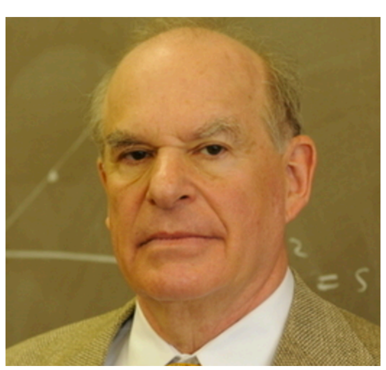






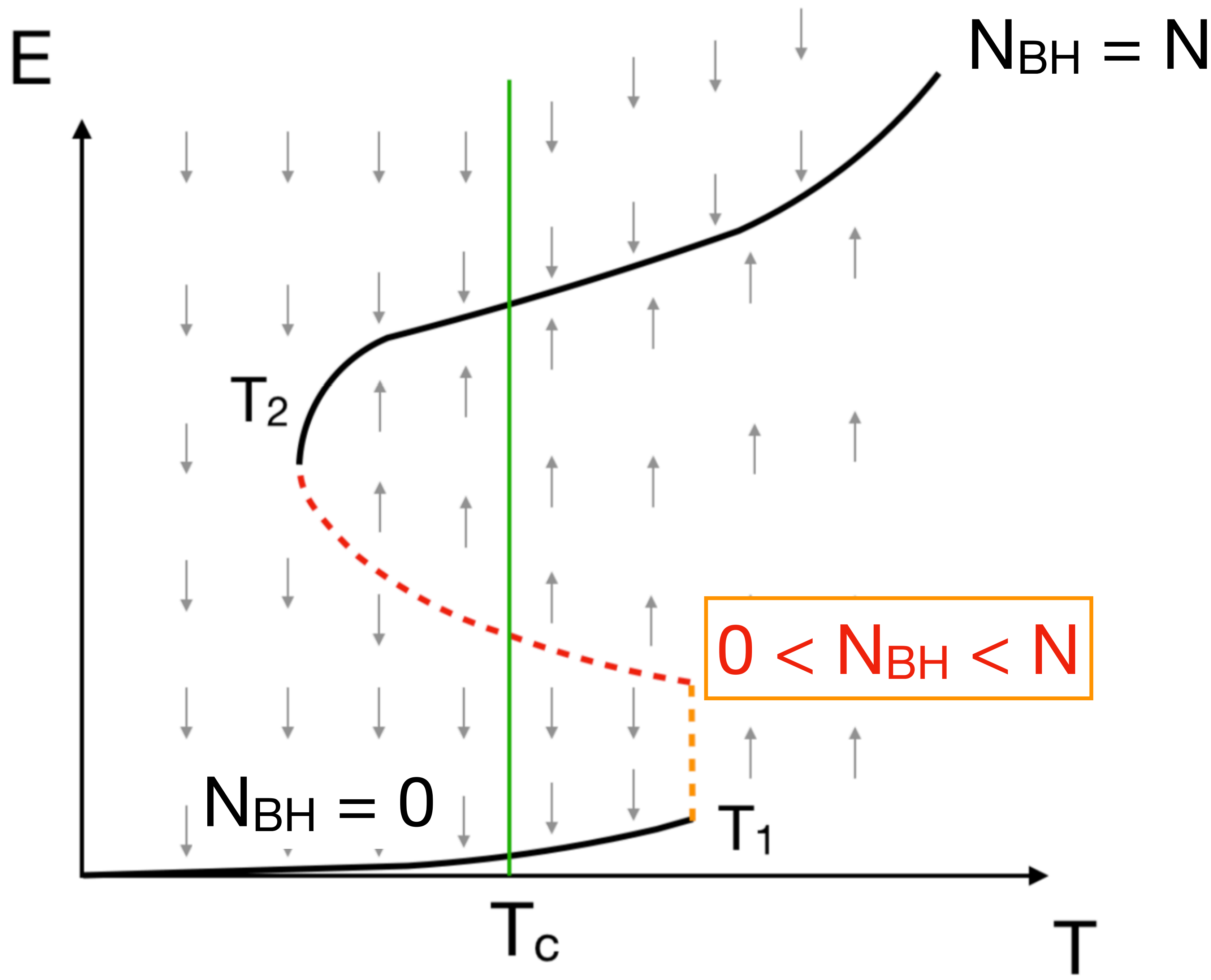






String Theory
in 1983
(endangered species)

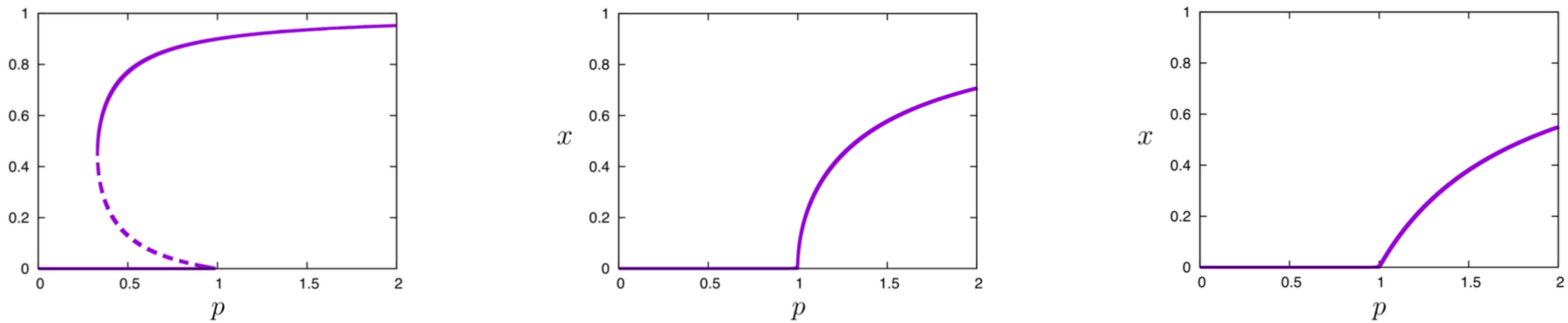
Recent examples:
Deep Learning,
Quantum Computer, ...

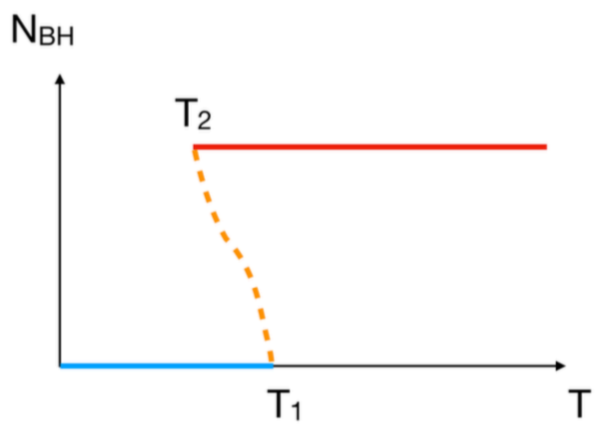


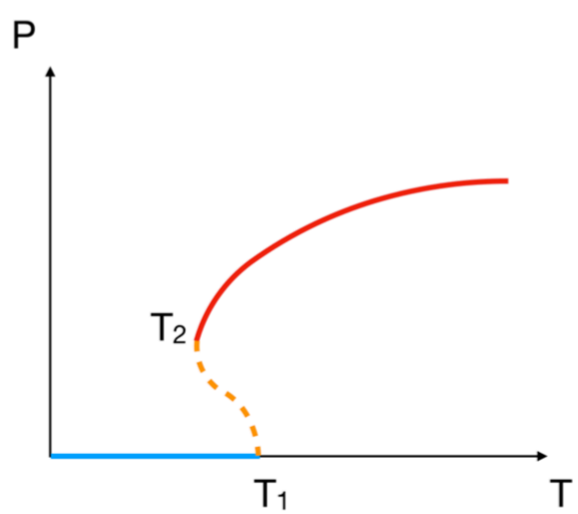
N_{BH} D-branes form the bound state

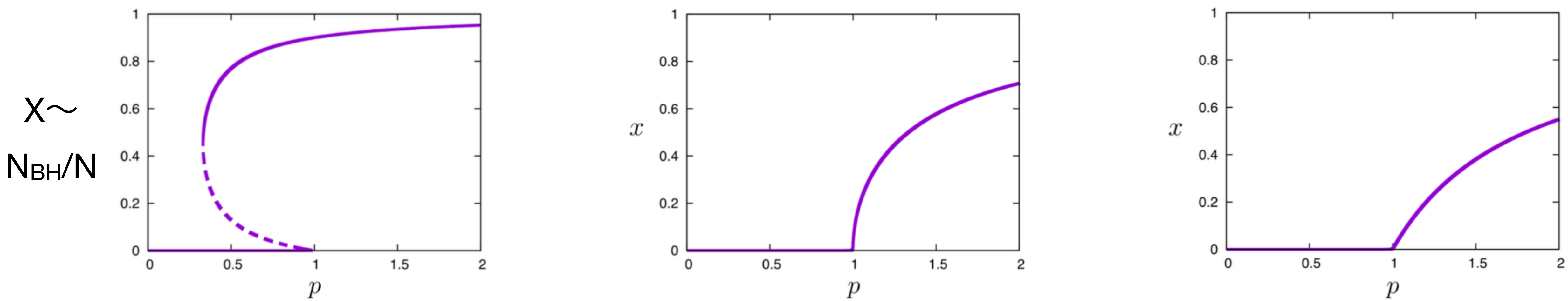
$U(N_{BH})$ is deconfined — ‘partial deconfinement’

$X \sim N_{BH}/N$

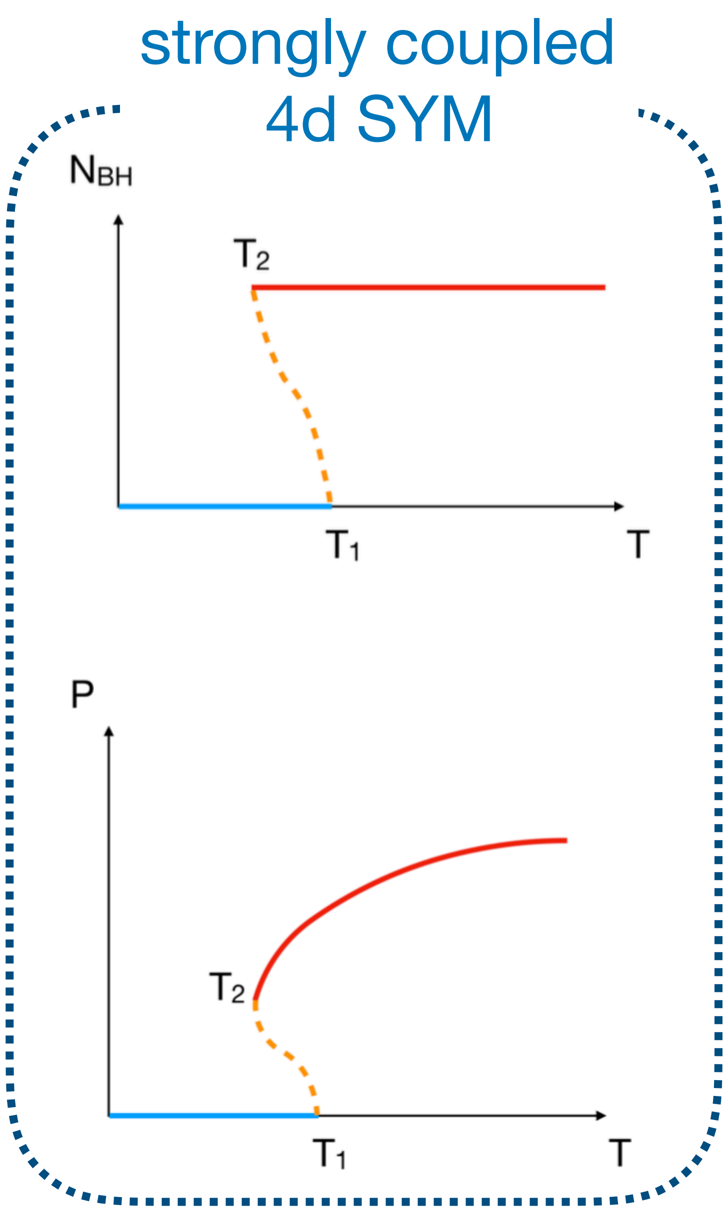




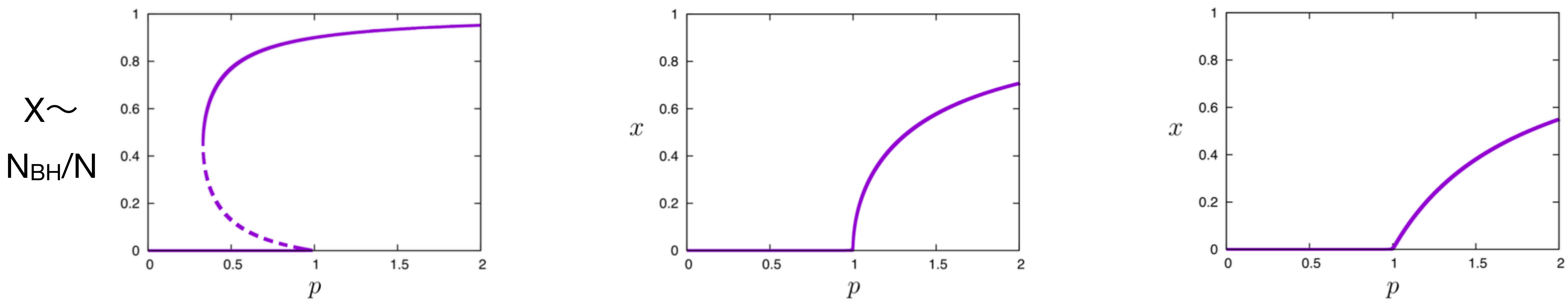




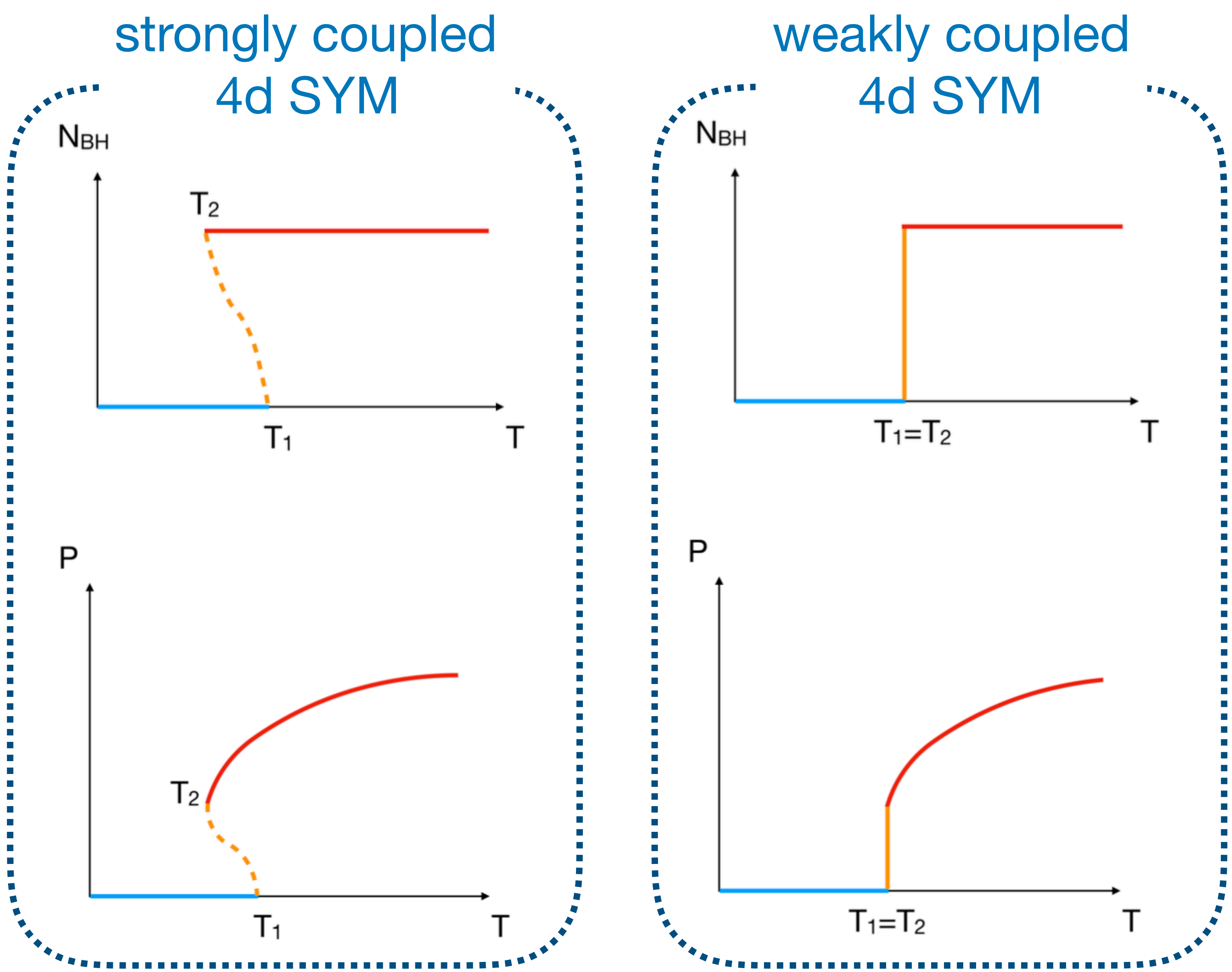
X∼
NBH/N
p
x
p
x
p



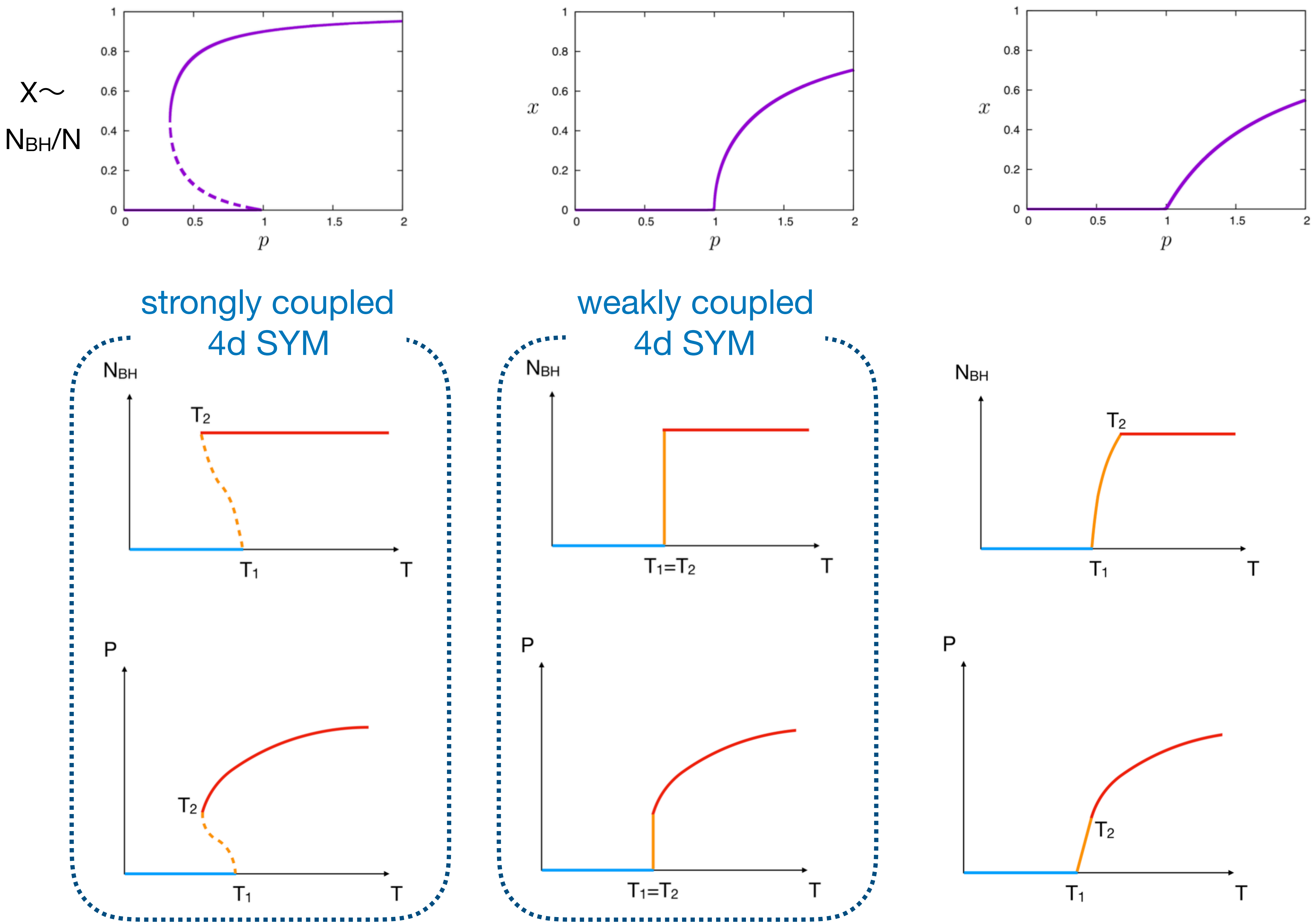
strongly coupled
4d SYM
NBH
T2
T1
T
P
T2
T1
T



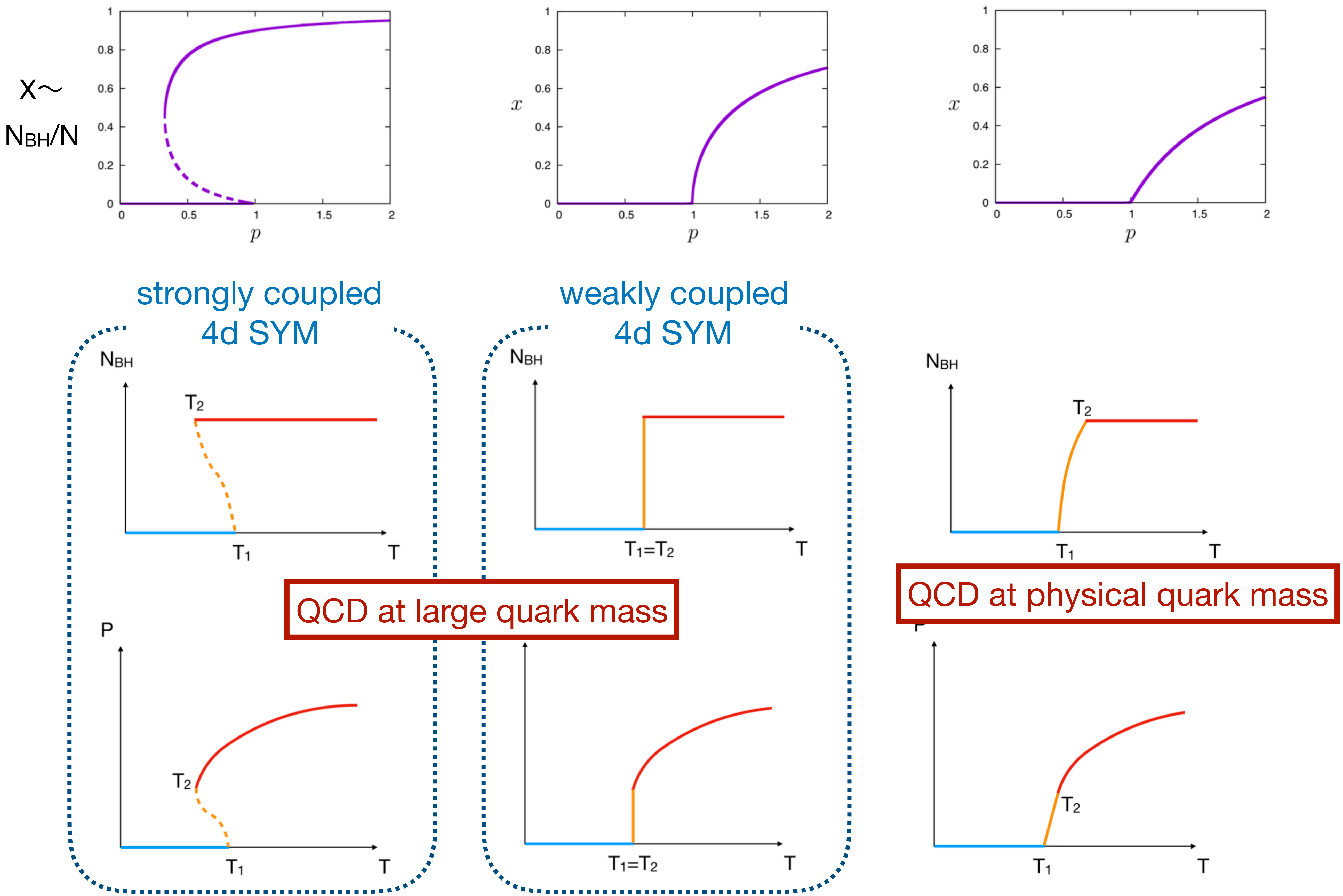
X∼
NBH/N
x
x
p
p
p



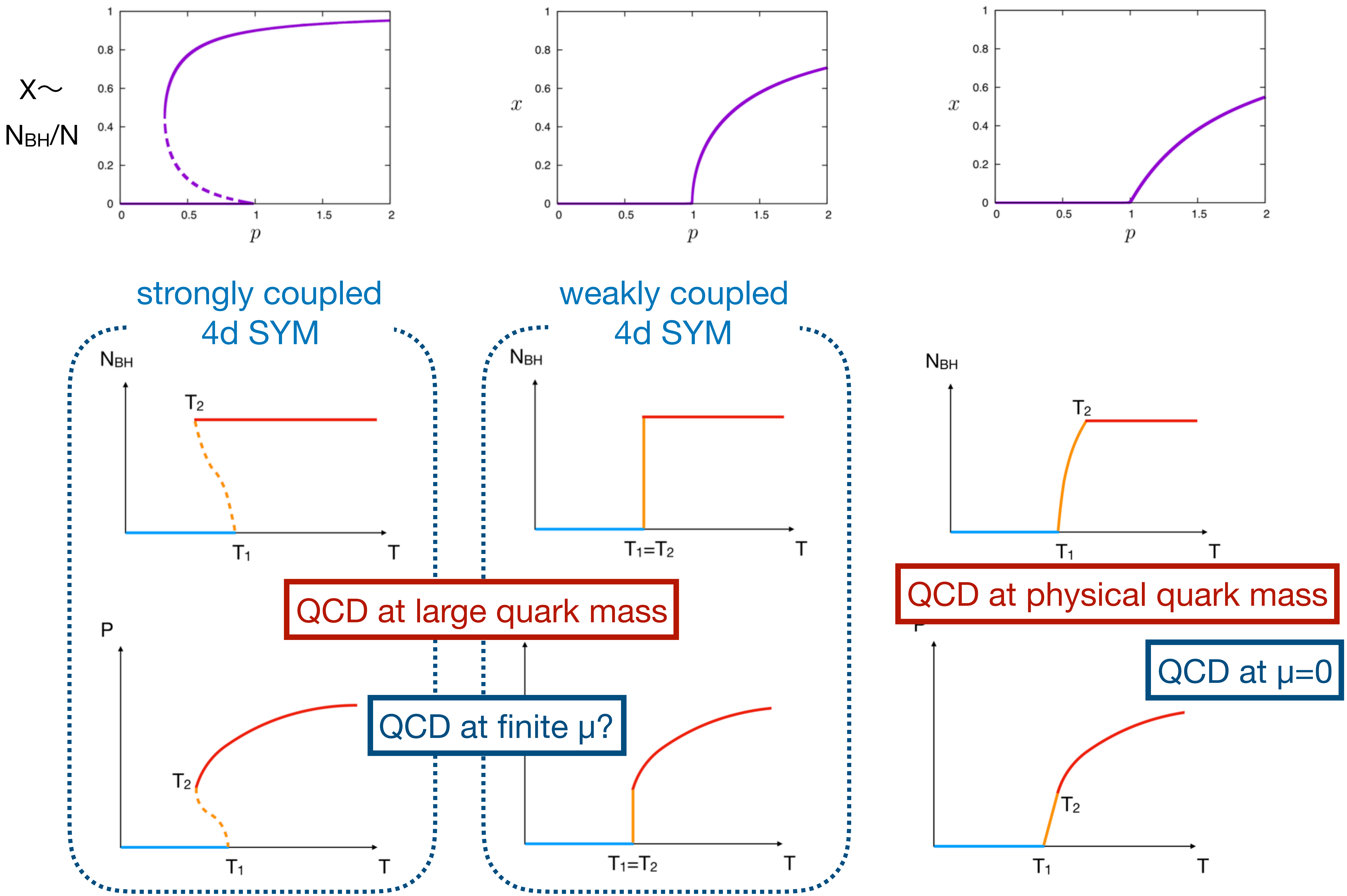
strongly coupled
4d SYM
weakly coupled
4d SYM
NBH
T2
T1
T
P
T2
T1
T
NBH
T1=T2
T
P
T1=T2
T



X∼
NBH/N
x
x
p
p
p
strongly coupled
4d SYM
weakly coupled
4d SYM
NBH
T2
T1
T
P
T2
T1
T
NBH
T1=T2
T
P
T1=T2
T
NBH
T2
T1
T
P
T2
T1
T

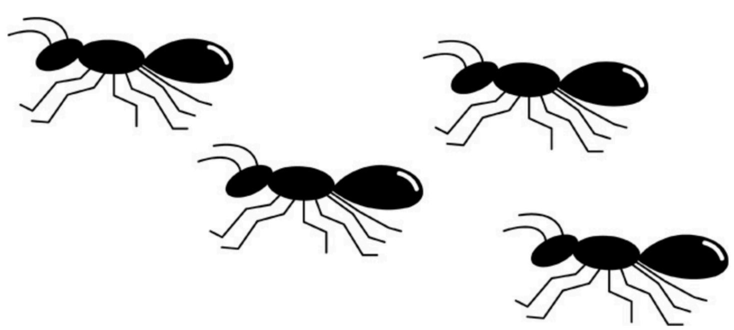


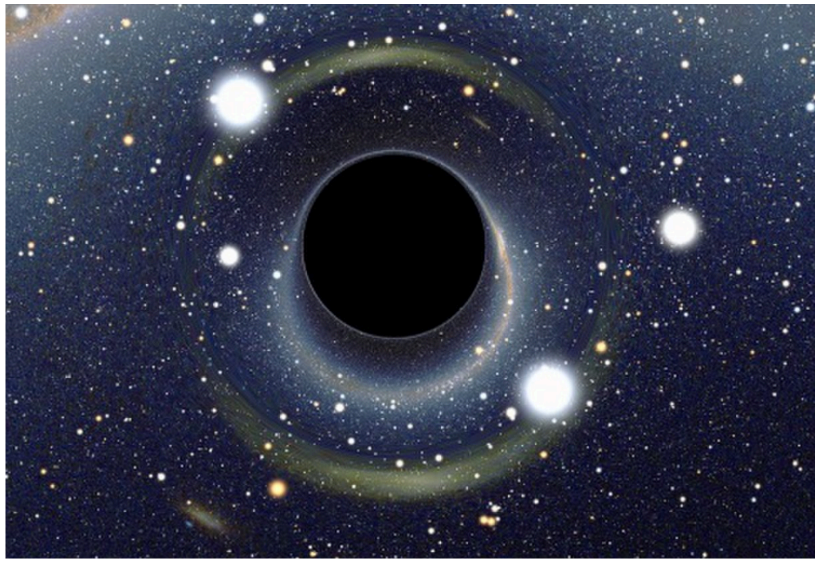
X∼
NBH/N
x
p
strongly coupled
4d SYM
weakly coupled
4d SYM
NBH
T2
T1
T
T1=T2
P
QCD at large quark mass
QCD at physical quark mass



X~
N_{BH}/N
x
p
strongly coupled
4d SYM
weakly coupled
4d SYM
N_{BH}
T_2
T_1
T
$T_1=T_2$
P
QCD at large quark mass
QCD at finite μ?
QCD at physical quark mass
QCD at μ=0

Testing the partial deconfinement





M.H.-Ishiki-Watanabe, 2018, JHEP
M.H.-Jevicki-Peng-Wintergerts, in preparation

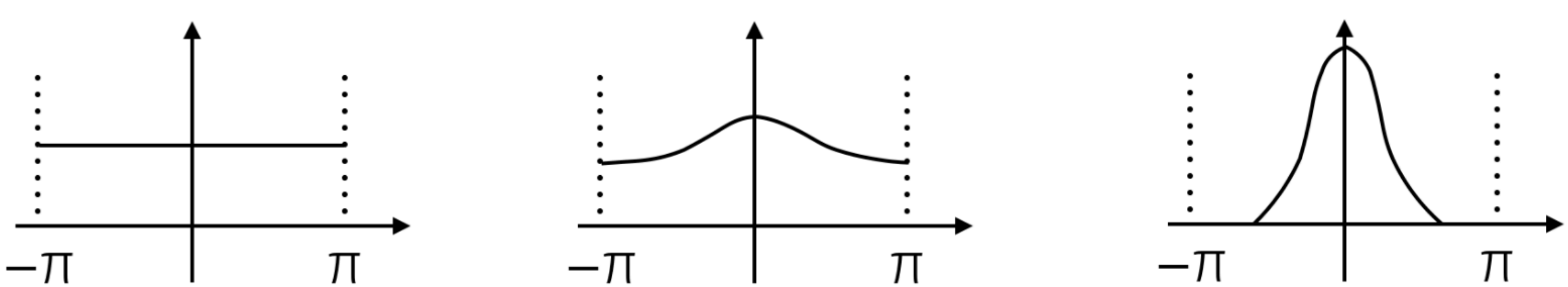
- ‘Polyakov loop’ is a useful
  — even when there is no center symmetry!

$$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:

confined phase
P=0

deconfined phase
P ≠ 0



‘partially’ deconfined **‘completely’ deconfined**

- ‘Polyakov loop’ is a useful
  — even when there is no center symmetry!

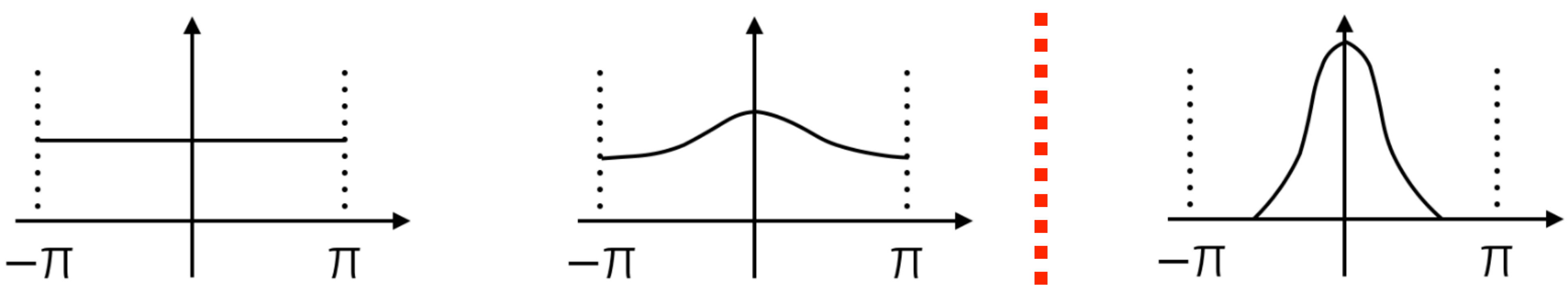
$$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:

Gross-Witten-Wadia transition (GWW)

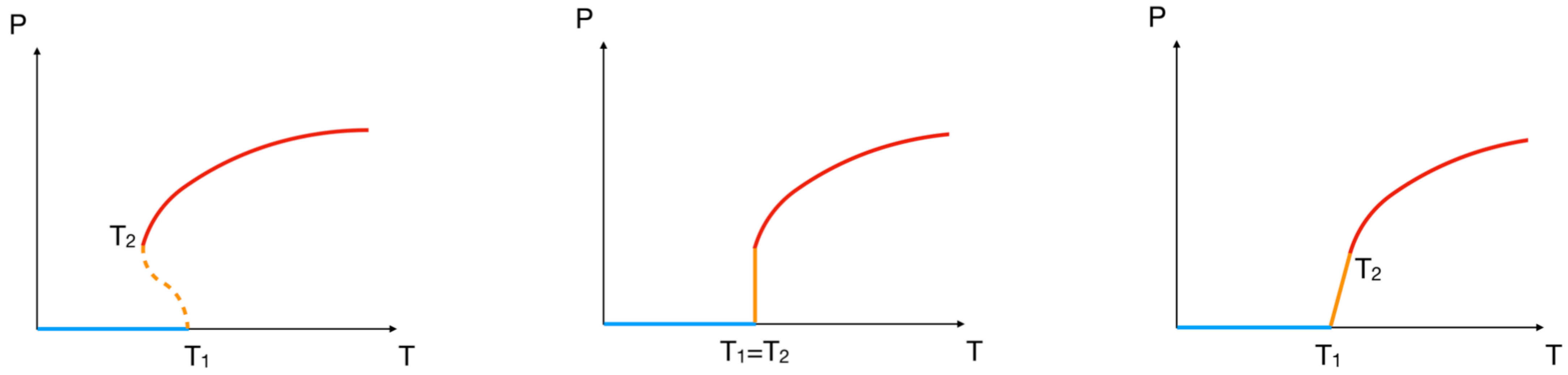
confined phase
P=0

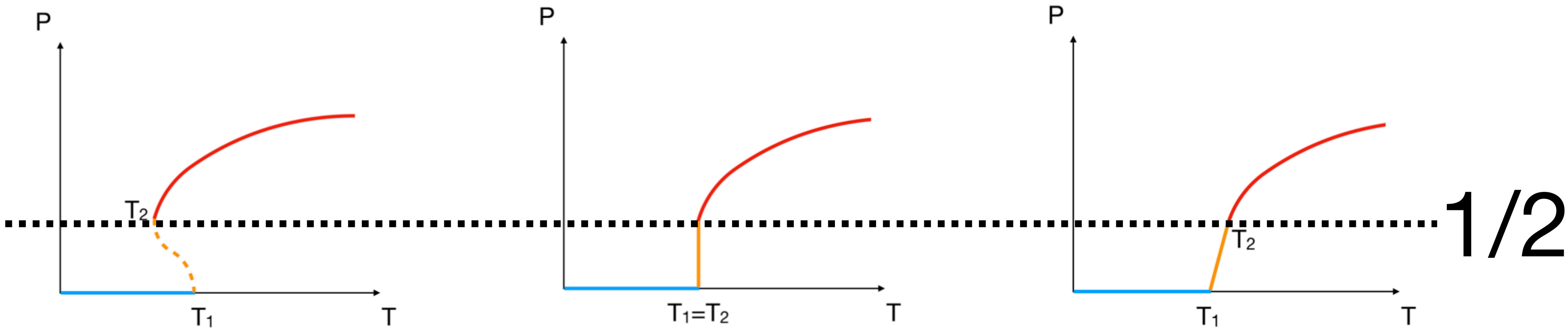
deconfined phase
P ≠ 0



‘partially’ deconfined

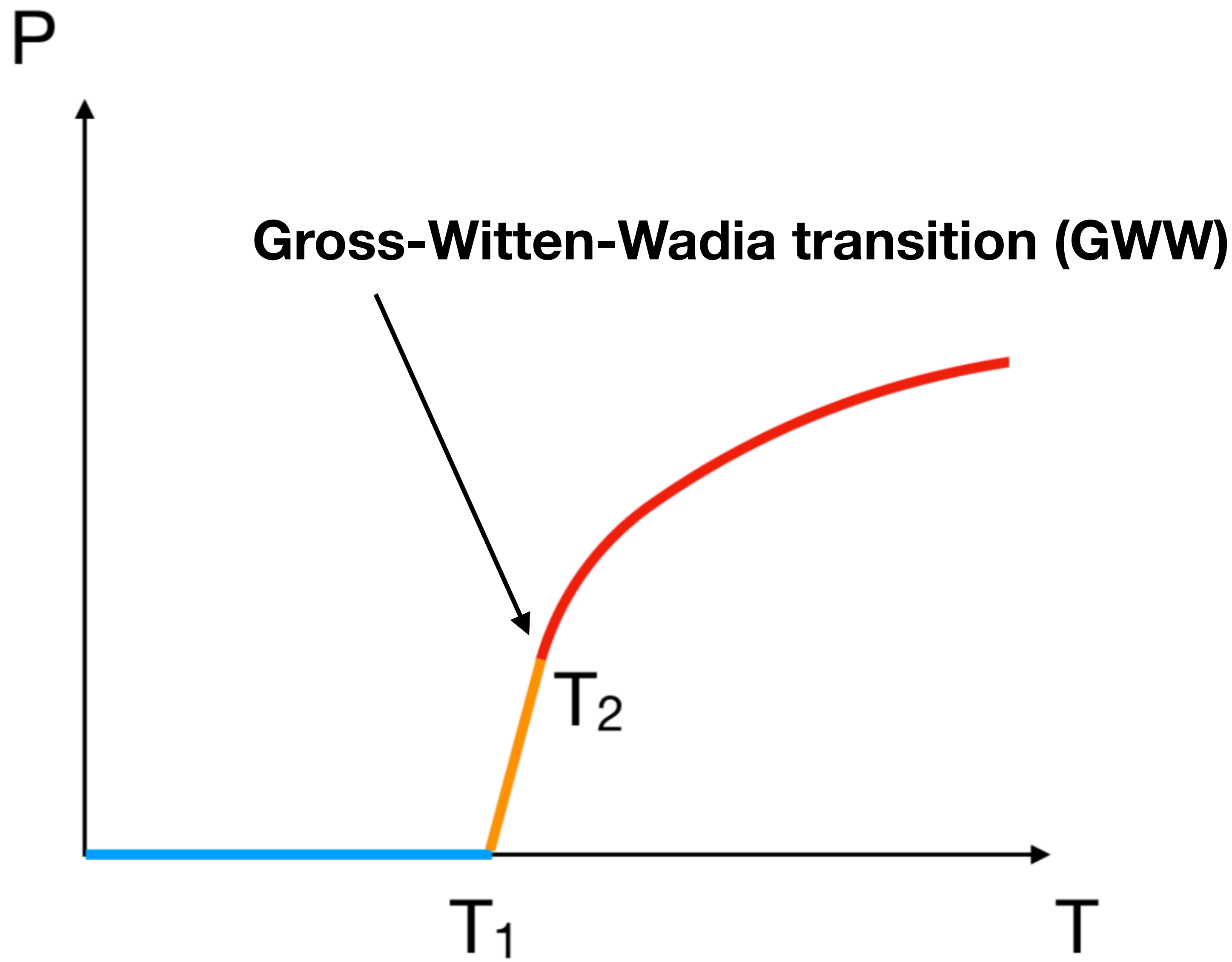
‘completely’ deconfined

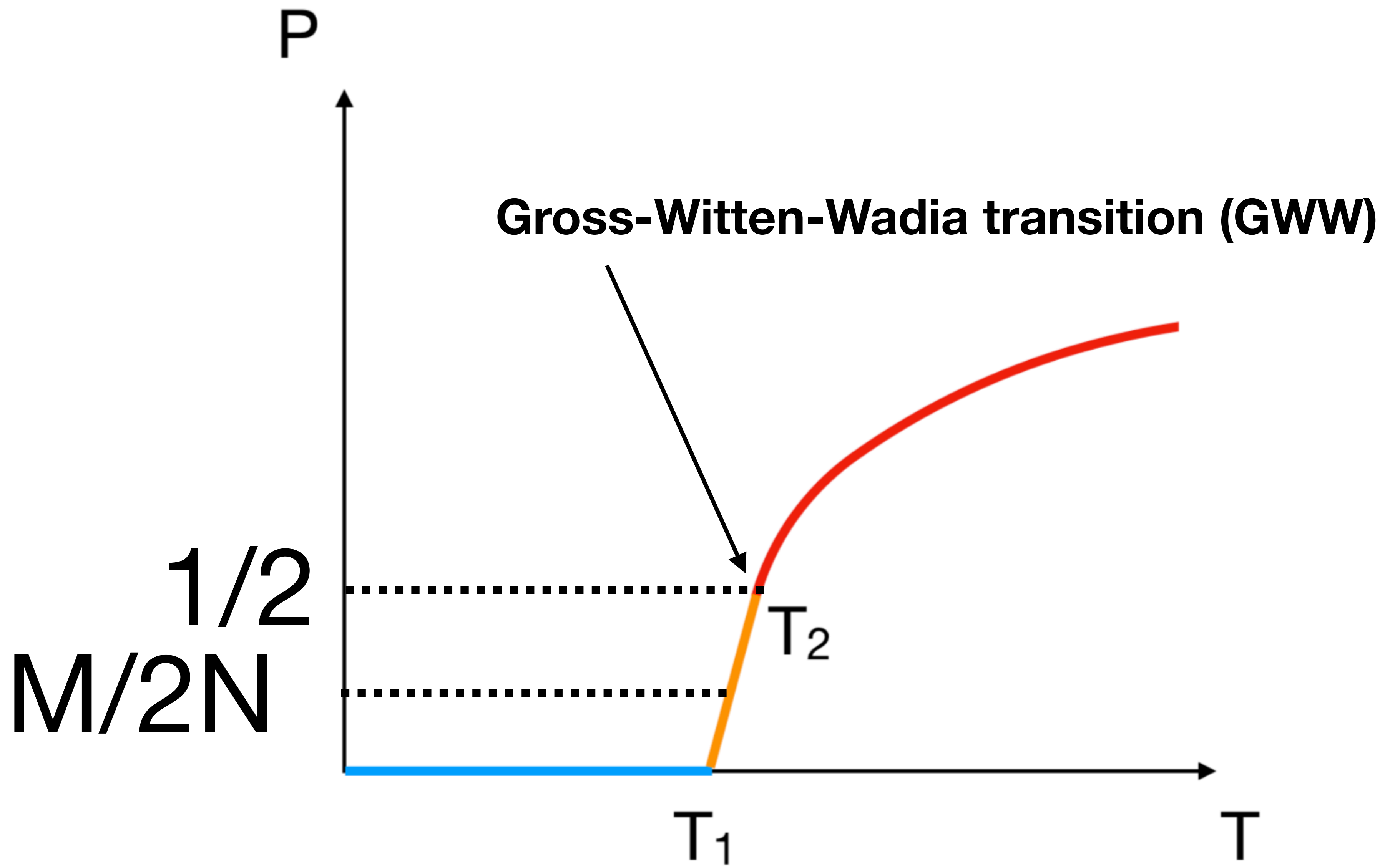

P
T2
T1
T
P
T1=T2
T
P
T2
T1
T

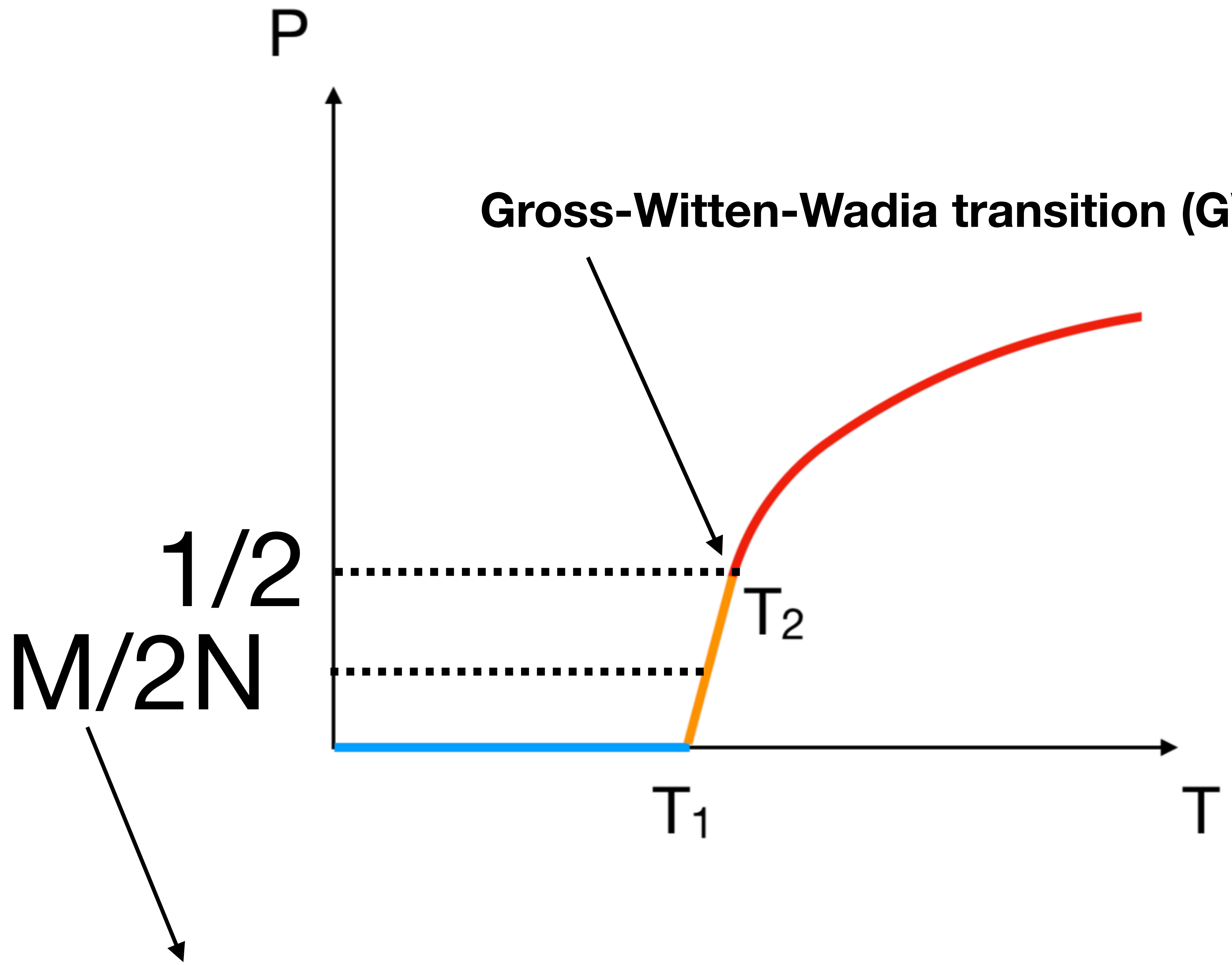


$$\rho(\theta) = \frac{N-M}{N}\rho_{\mathrm{confine}}(\theta) + \frac{M}{N}\rho_{\mathrm{deconfine}}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\mathrm{deconfine}}(\theta)$$

Holds in all examples we have studied.

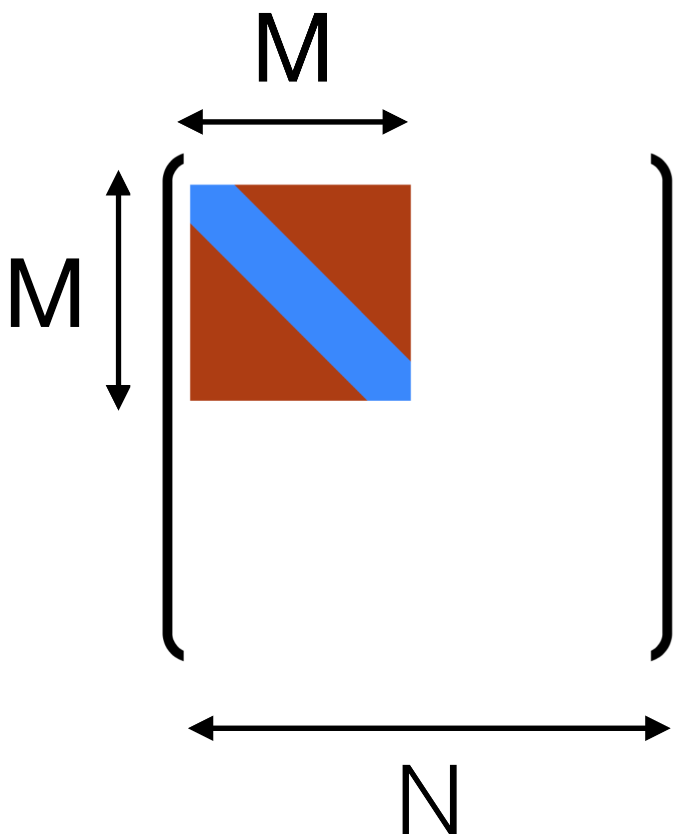

P
Gross-Witten-Wadia transition (GWW)
T_2
T_1
T


P
Gross-Witten-Wadia transition (GWW)
1/2
M/2N
T2
T1
T



$$E = E_{\mathrm{GWW}}(M)$$

$$S = S_{\mathrm{GWW}}(M)$$

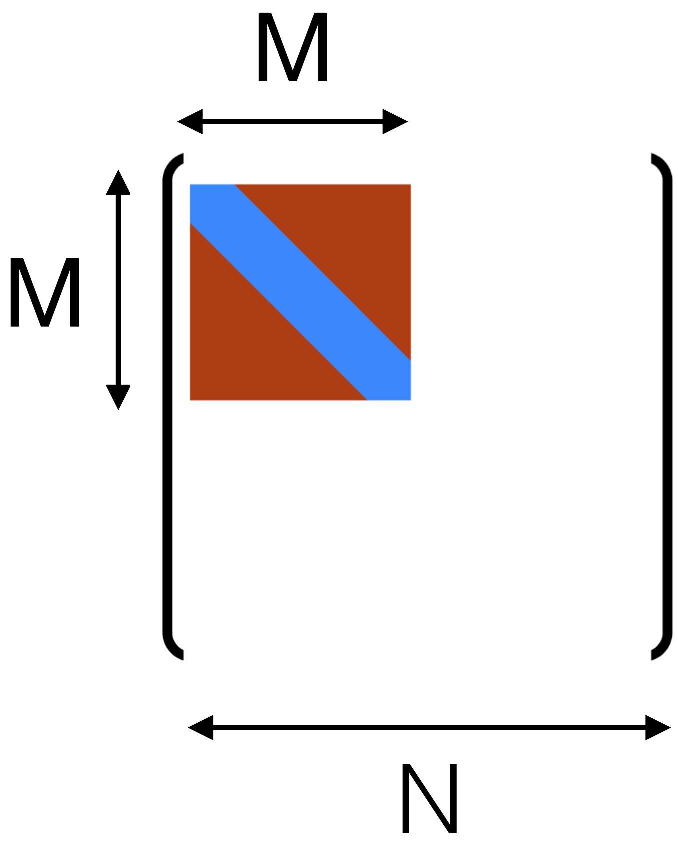


not SU(N)-invariant

$$|E; \mathrm{SU}(M)\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\mathrm{GWW}}(M)$$



not SU(N)-invariant

$$|E;\mathrm{SU}(M)\rangle$$

At weak coupling, this is an energy eigenstate.

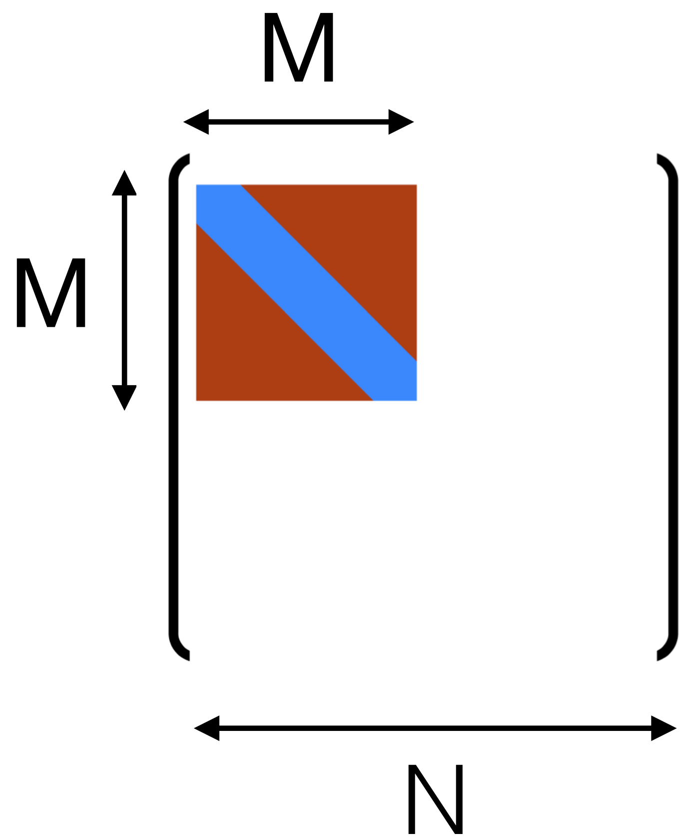
$$S = S_{\mathrm{GWW}}(M)$$

one-to-one correspondence

SU(N)-invariant

This is also an energy eigenstate.

$$|E\rangle_{\mathrm{inv}} \equiv \mathcal{N}^{-1/2}\int dU\,\mathcal{U}\left(|E;\mathrm{SU}(M)\rangle\right)$$



not SU(N)-invariant

$$|E; \mathrm{SU}(M)\rangle$$

At weak coupling, this is an energy eigenstate.

$$S = S_{\mathrm{GWW}}(M)$$

one-to-one correspondence

SU(N)-invariant

This is also an energy eigenstate.

$$|E\rangle_{\mathrm{inv}} \equiv \mathcal{N}^{-1/2} \int dU \, \mathcal{U} \left(|E; \mathrm{SU}(M)\rangle \right)$$

<u>These states explain the entropy precisely.</u>

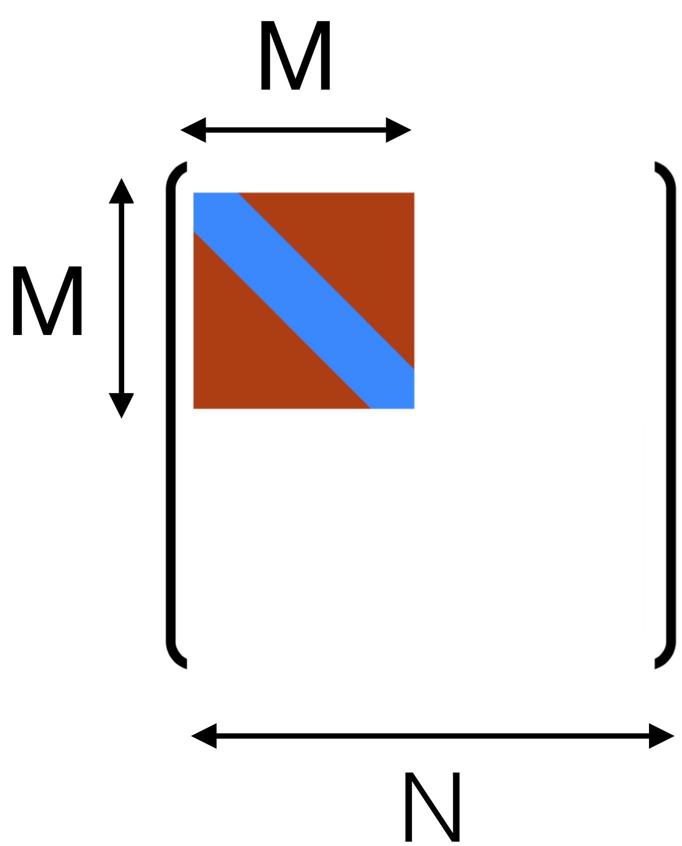
(Analytic calculation doable for weakly coupled QCD on S^3, O(N) vector model, matrix model)

‘Spontaneous gauge symmetry breaking’

M.H.-Jevicki-Peng-Wintergerts, in preparation

SU(N)-invariant

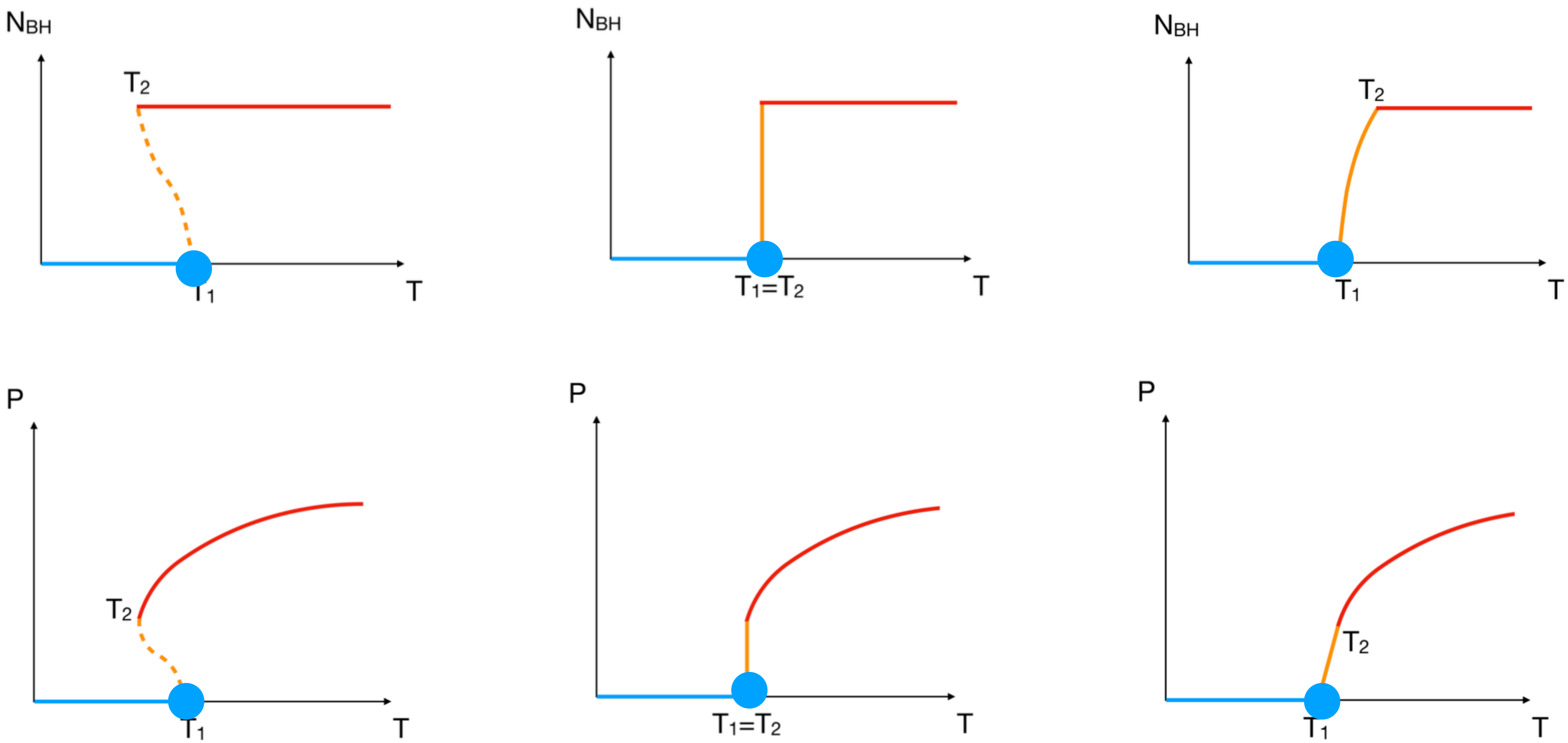
$$|E\rangle_{\rm inv} \equiv \mathcal{N}^{-1/2} \int dU\, \mathcal{U}\left(|E; \mathrm{SU}(M)\rangle\right)$$



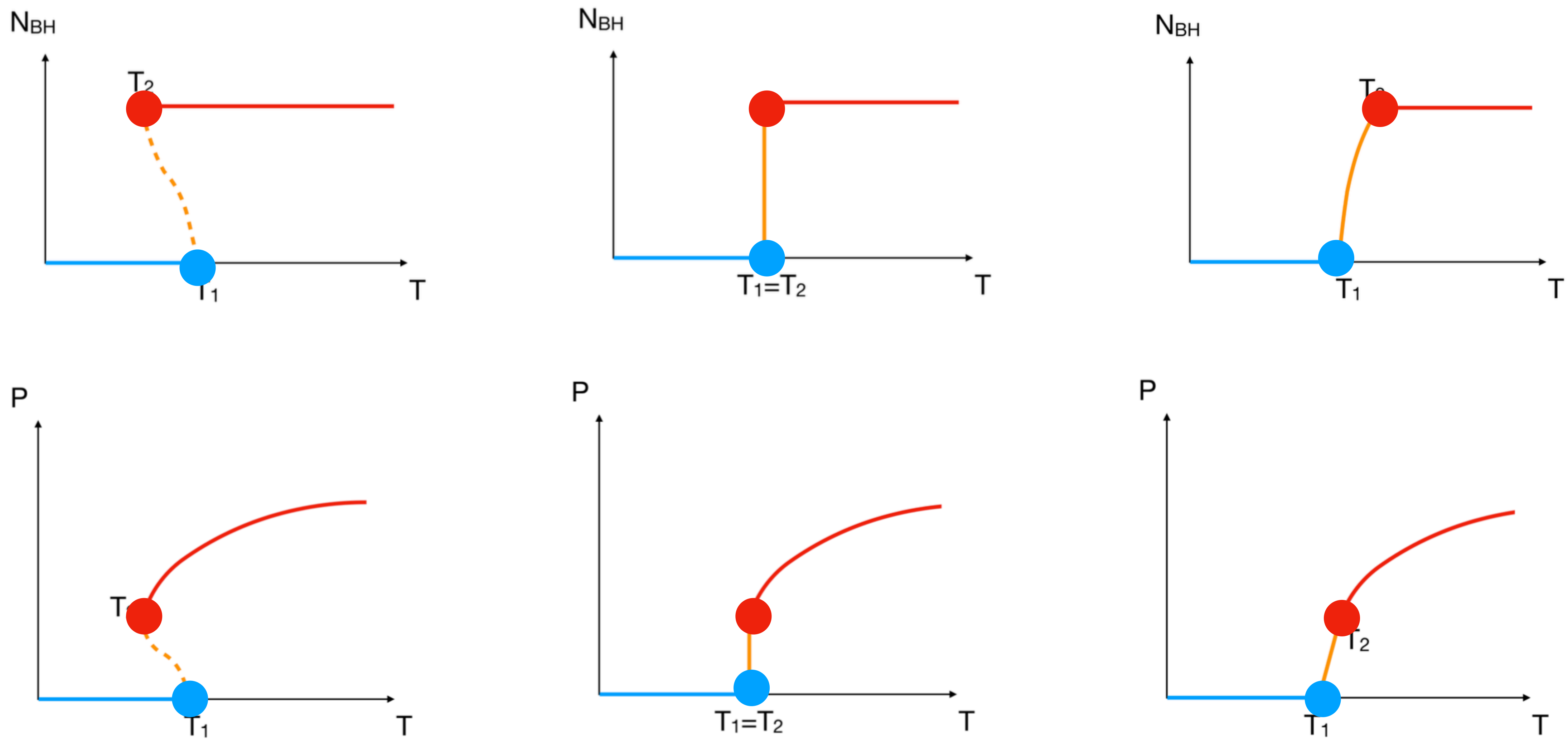
not SU(N)-invariant

$$|E; \mathrm{SU}(M)\rangle$$

- Global part of gauge symmetry breaks spontaneously.
- It is convenient to fix the local part, like usual Higgsing.
- ‘Gauge symmetry breaking’ provides us with a ‘useful fiction’ which makes physics understandable.

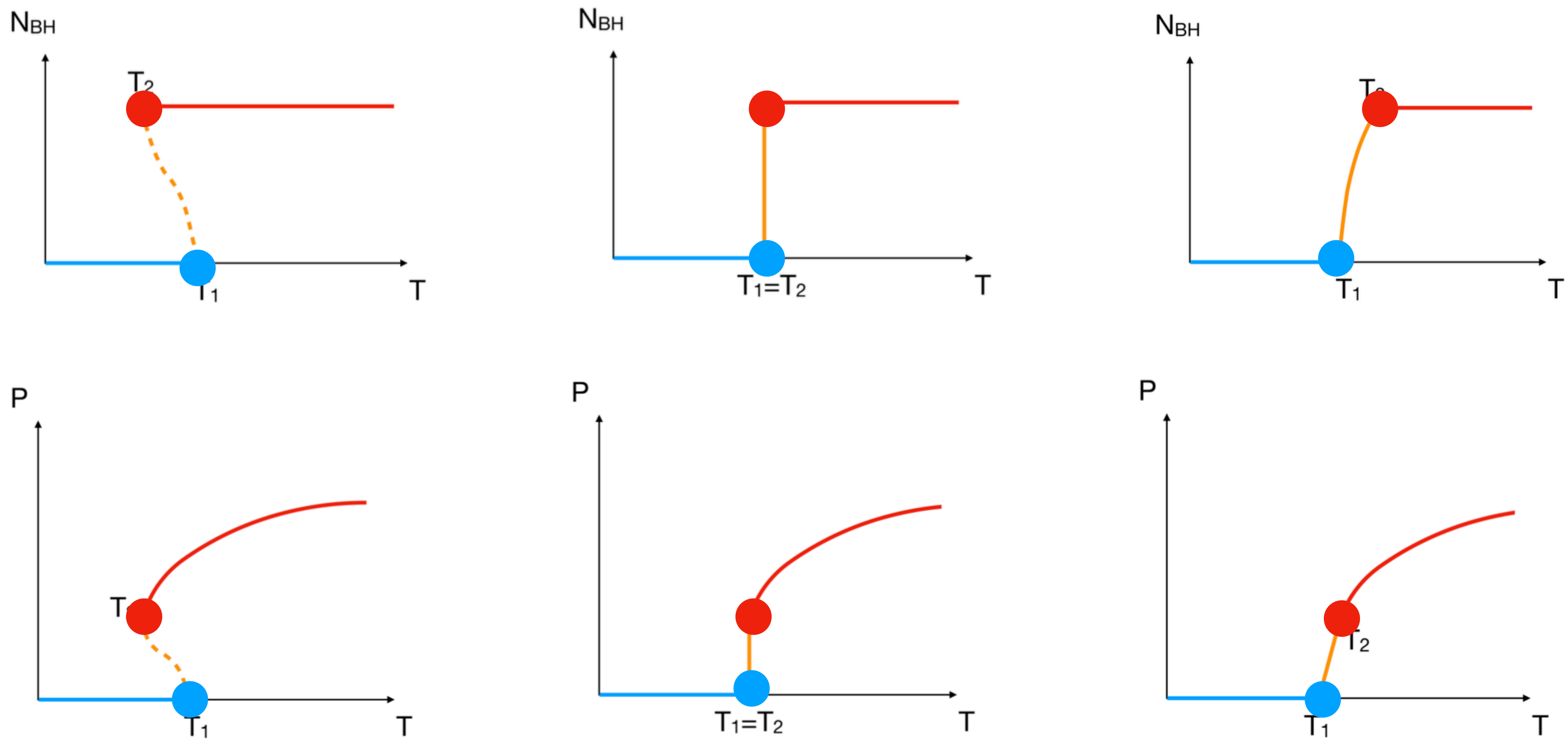


transition 1: confinement to partial deconfinement
(black hole formation begins)



transition 1: confinement to partial deconfinement
(black hole formation begins)

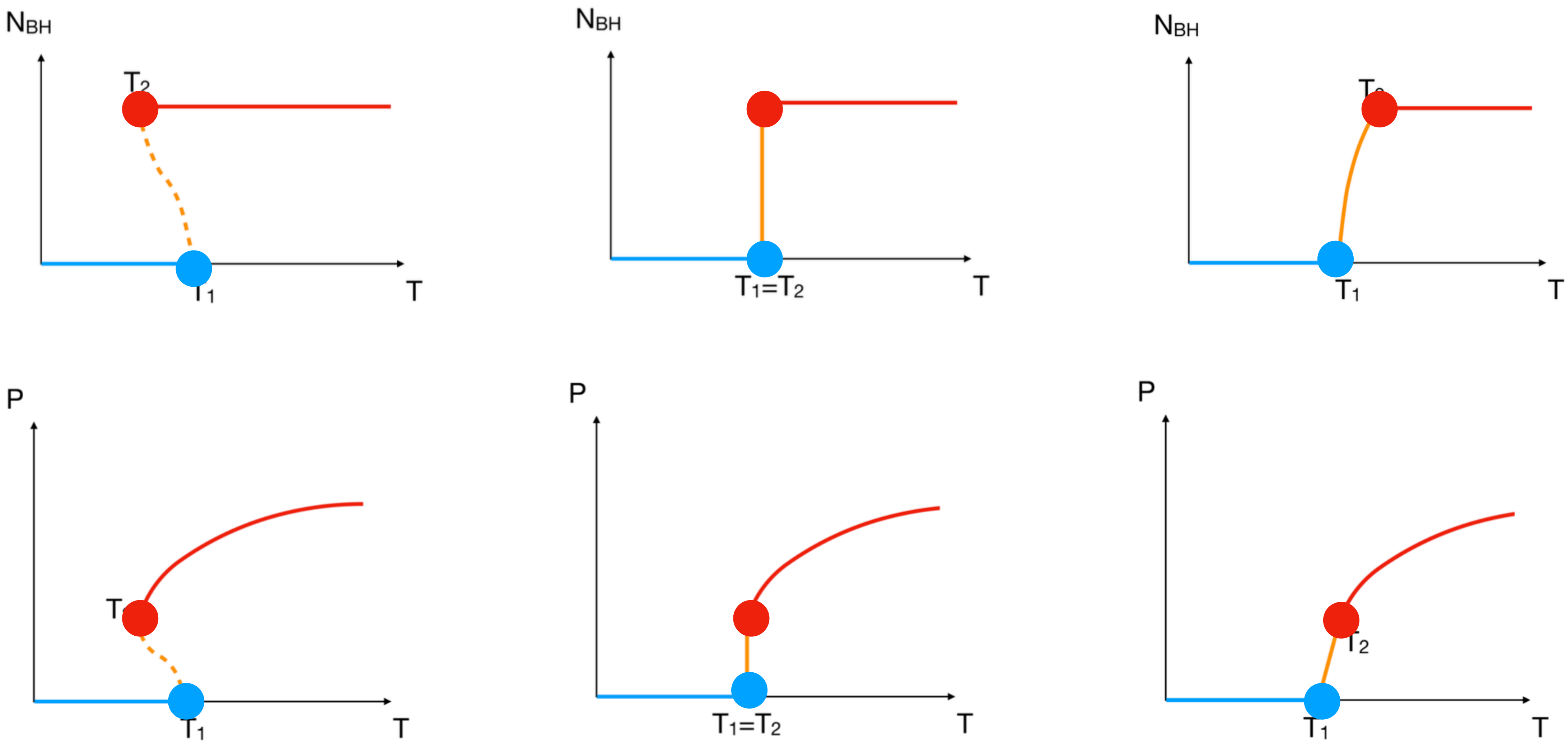
transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)



transition 1: confinement to partial deconfinement
(black hole formation begins)

transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

SU(N) → SU(M)×SU(N−M)×U(1) → SU(N)

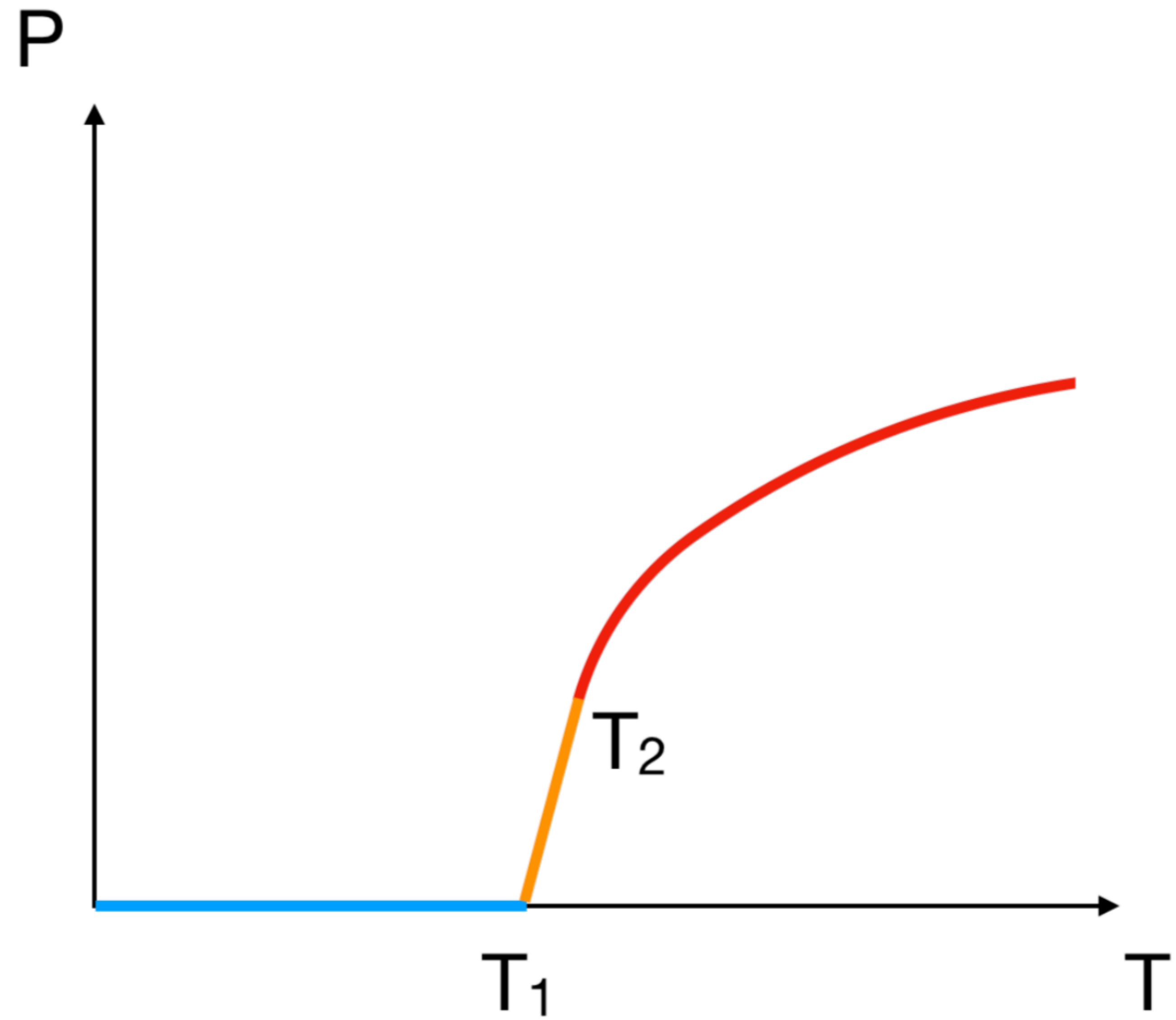


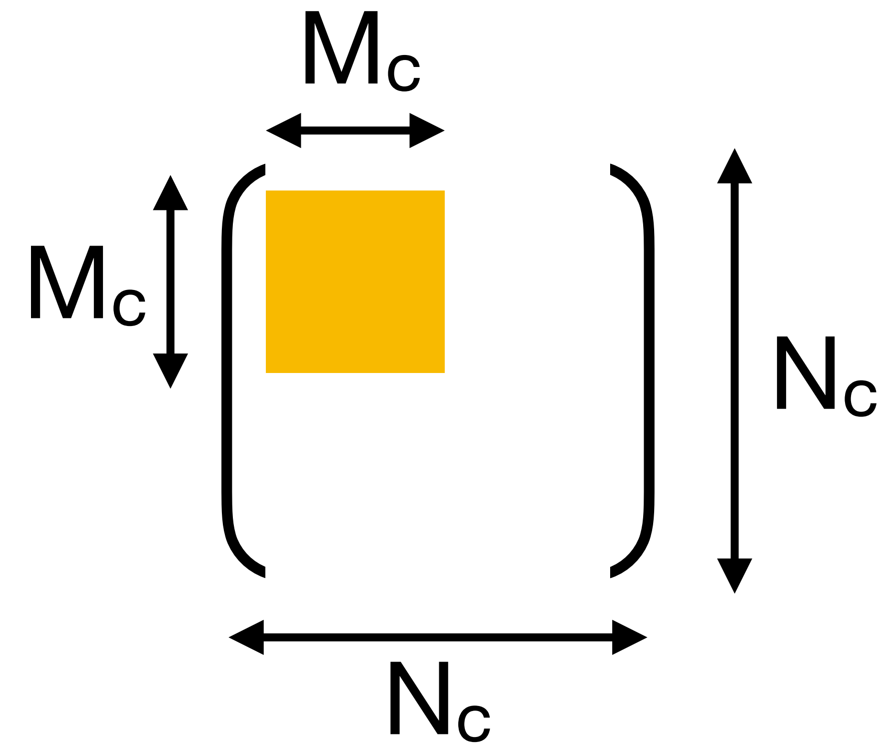
transition 1: confinement to partial deconfinement
(black hole formation begins)

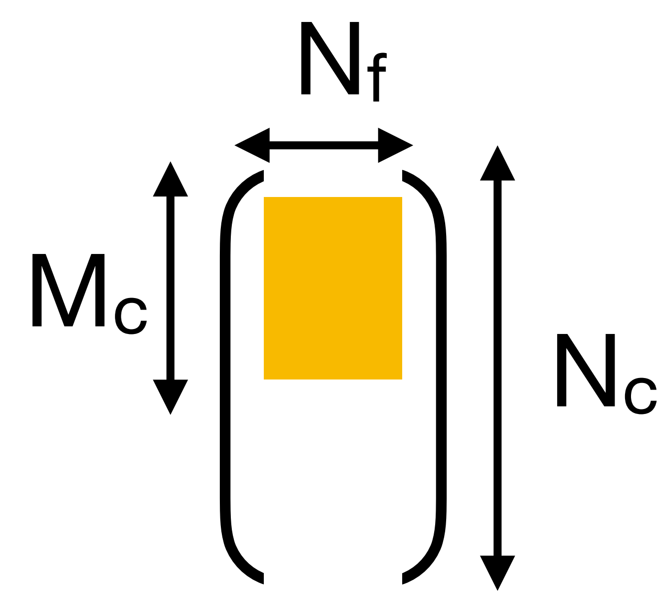
transition 2: partial deconfinement to complete deconfinement
(black hole formation ends)

$$SU(N) \rightarrow SU(M)\times SU(N-M)\times U(1) \rightarrow SU(N)$$

No need for center symmetry. No need for chiral symmetry. Applies to QCD.





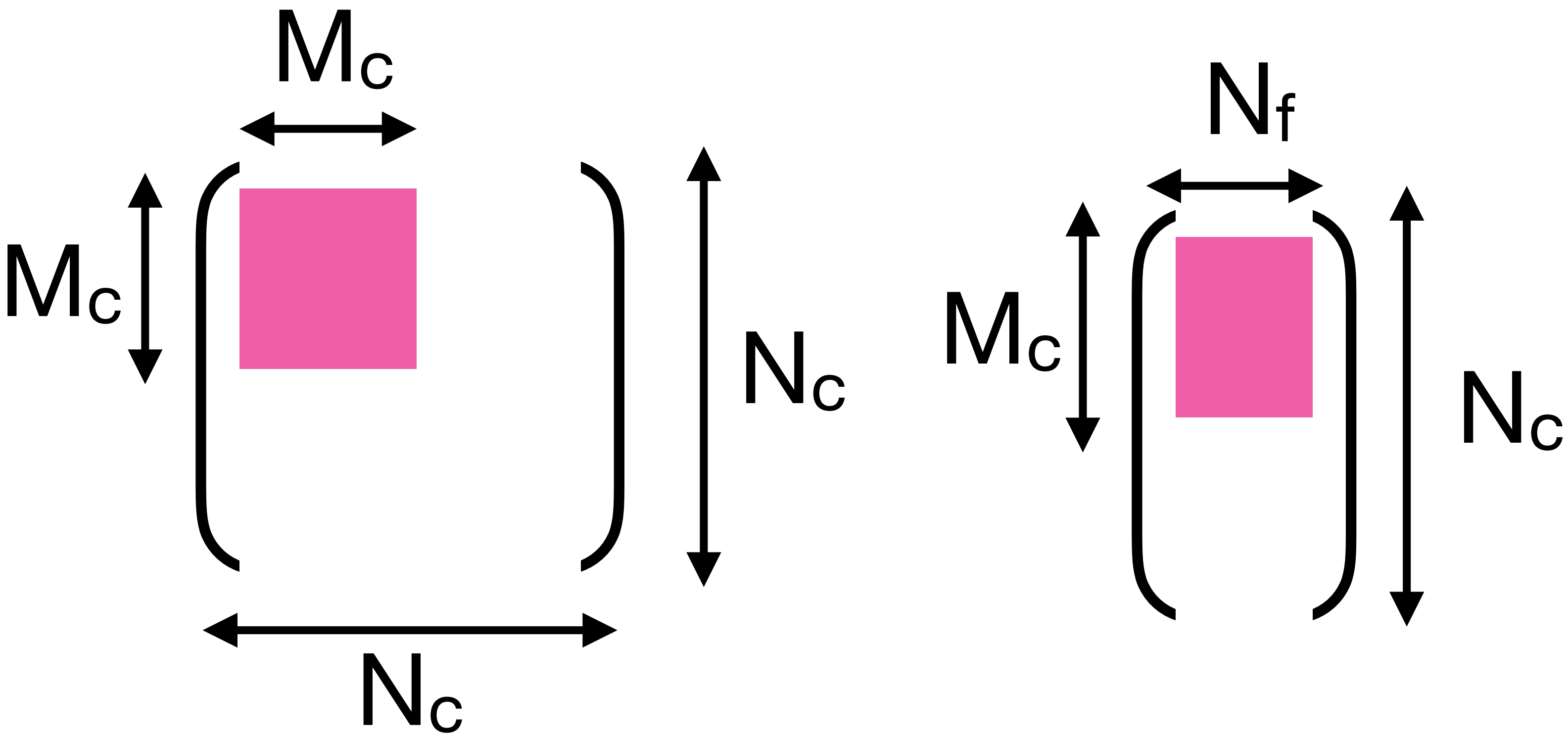


In the chiral limit, the anomaly matching suggests
"confinement ⇄ chiral symmetry breaking"

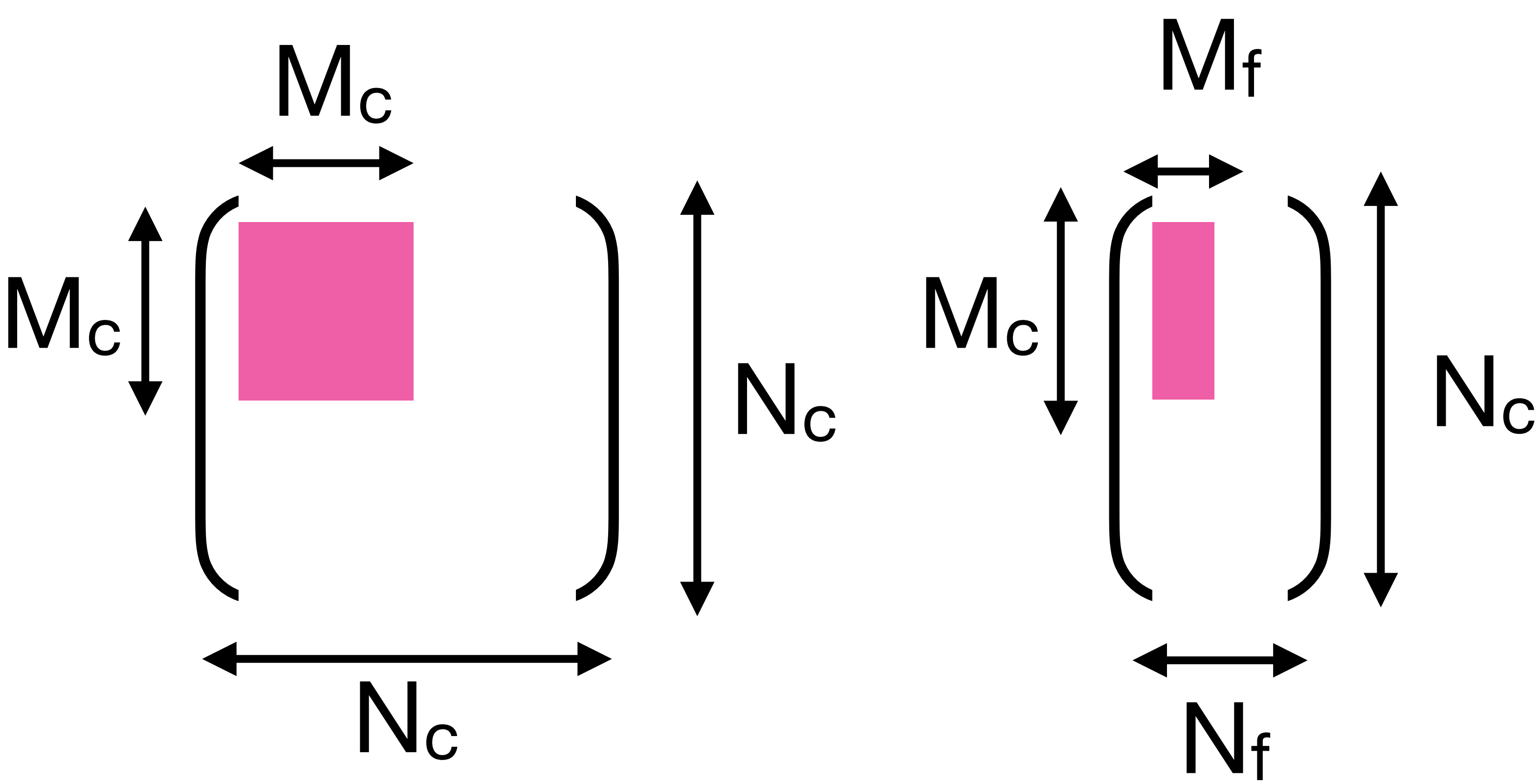
- SU(2) confines/deconfines in QCD → enhanced chiral symmetry?
- Near T_1, free-string-like behavior is expected.

(Still speculative; work in progress)

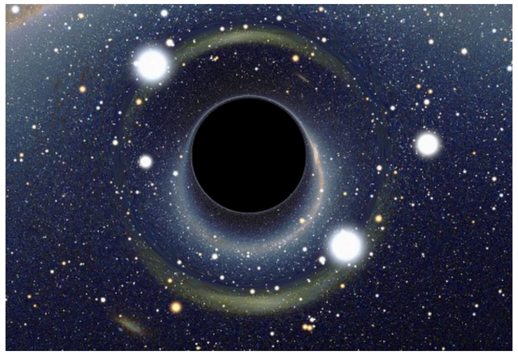
Chiral symmetry breaking?



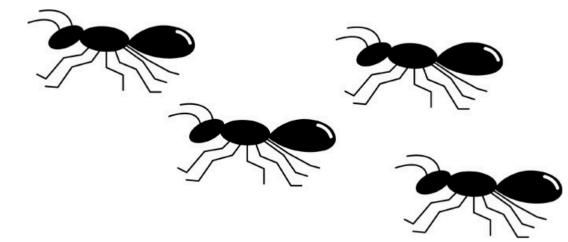
**Natural expectation
at zero chemical potential
and zero mass
due to Vafa-Witten theorem**



**Maybe possible
at finite chemical potential
or $m_u \sim m_d < m_s$**



Conclusion



- ‘Partial deconfinement’ and ‘Schwarzschild Black Hole’ are rather generic in gauge theories.
- Breaking and restoration of gauge symmetry characterize deconfinement. No need for center symmetry.
- Experimental signals in heavy-ion collisions?
- It is important to study gauge theory, in order to understand quantum gravity.