

Spinning geodesic Witten diagrams and Mellin representations

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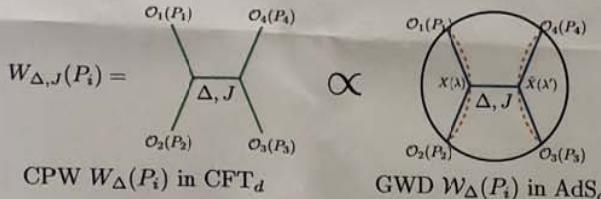
with Heng-Yu Chen and En-Jui Kuo (National Taiwan U.)

Abstract

We revisit the so-called geodesic Witten diagrams (GWD) which is proposed as the holographic dual of conformal partial waves (CPW).

Hijano, Kraus, Perlmutter and Snively [1508.00501]

$$\langle \mathcal{O}_1(P_1)\mathcal{O}_2(P_2)\mathcal{O}_3(P_3)\mathcal{O}_4(P_4) \rangle = \sum_{\mathcal{O}_{\Delta,J}} \lambda_{12}\lambda_{34} W_{\Delta,J}(P_i)$$



CPW $W_{\Delta}(P_i)$ in CFT_d

GWD $\mathcal{W}_{\Delta}(P_i)$ in AdS_{d+1}

1. Embedding Formalism

AdS_{d+1} or R^d coordinates are embedded in $\text{M}^{1,d+1}$. cf. Weinberg [1006.3480]

$$\text{AdS}_{d+1} \subset \text{M}^{1,d+1} \quad \text{Poincaré AdS}_{d+1}; \quad \{y^a, z\}, \quad X^A \in \text{M}^{1,d+1}$$

$$(X^+, X^-, X^a) = \frac{1}{z^2}(1, z^2 + y^2, y^a), \quad X \cdot X = -1$$

$$\mathbb{R}^d \subset \text{M}^{1,d+1}$$

$$\text{Euclid } \mathbb{R}^d; \quad \{y^a\}, \quad P^A \in \text{M}^{1,d+1}$$

$$(P^+, P^-, P^a) = (1, y^2, y^a), \quad P \cdot P = 0$$

We focus on only symmetric transverse traceless (STT) tensors:

$$\text{In AdS, } T(X, W) = W^{A_1} \dots W^{A_l} T_{A_1 \dots A_l}(X) \quad X \cdot W = W \cdot W = 0$$

$$\text{In R, } F(P, Z) = Z^{A_1} \dots Z^{A_l} F_{A_1 \dots A_l}(P) \quad P \cdot Z = Z \cdot Z = 0$$

※ W and Z are polarization vectors.

2. Conformal Partial Waves (CPWs)

◆ Integral representation of CPWs

Dolan and Osborn [1108.6194]
Sleight [1610.01318]

CPWs can be decomposed into two 3-pt. functions.

$$W_{\Delta,J}(P_i) \sim \int_{\partial} dP_0 \int_{-\infty}^{\infty} d\nu \frac{\mathcal{K}_{\Delta_{12},\Delta_{34},J}(\nu)}{\nu^2 + (\Delta - h)^2} \times \langle \mathcal{O}_1(P_1)\mathcal{O}_2(P_2)\mathcal{O}_{h+i\nu,J}(P_0) \rangle \langle \mathcal{O}_{h-i\nu,J}(P_0)\mathcal{O}_3(P_3)\mathcal{O}_4(P_4) \rangle$$

The diagram shows a 3-point function decomposition of a CPW. It consists of two parts: a left part where a central node Δ, J is connected to $\mathcal{O}_1(P_1)$ and $\mathcal{O}_4(P_4)$, and a right part where it is connected to $\mathcal{O}_2(P_2)$ and $\mathcal{O}_3(P_3)$. Below these, a 3-point function is shown with a central node $\mathcal{O}_{h+i\nu,J}(P_0)$ connected to $\mathcal{O}_1(P_1)$ and $\mathcal{O}_4(P_4)$, and another node $\mathcal{O}_{h-i\nu,J}(P_0)$ connected to $\mathcal{O}_2(P_2)$ and $\mathcal{O}_3(P_3)$.

Costa et al [1107.3554], [1109.6321]

◆ Spinning CPWs

CPWs with external spinning fields can be constructed by using differential operators $\mathcal{D}_{\text{Left}}$ and $\mathcal{D}_{\text{Right}}$.

$$W_{\mathcal{O}_{\Delta,J}}^{\{n_{10}, n_{20}, n_{12}\}; \{n_{30}, n_{40}, n_{34}\}}(P_i, Z_i) \equiv \mathcal{D}_{\text{Left}}^{n_{10}, n_{20}, n_{12}} \mathcal{D}_{\text{Right}}^{n_{30}, n_{40}, n_{34}} W_{\mathcal{O}_{\Delta,J}}(P_i)$$

$$\mathcal{D}_{\text{Left}} \equiv \begin{array}{c} \mathcal{O}_{r_1}(P_1) \\ \diagdown \quad \diagup \\ \mathcal{O}_{\Delta_{3,J}}(P_3) = \end{array} \quad \mathcal{D}_{\text{Right}} \equiv \begin{array}{c} \mathcal{O}_{\Delta_{1,J}}(P_1) \\ \diagup \quad \diagdown \\ \mathcal{O}_{\Delta_{3,J}}(P_3) \end{array} \quad \text{where } \mathcal{D}_{\text{Left}} \equiv H_{12}^{n_{10}} D_{12}^{n_{20}} D_{21}^{n_{12}} D_{11}^{m_1} D_{22}^{m_2} \quad \mathcal{D}_{\text{Right}} \equiv H_{34}^{n_{34}} D_{34}^{n_{30}} D_{43}^{n_{40}} D_{33}^{n_{34}} D_{44}^{m_4}$$

D-operators generate tensor structures.

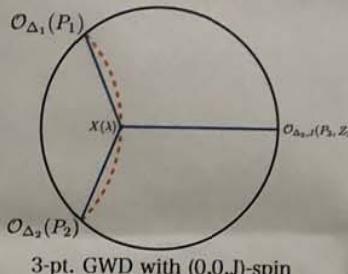
Def. of D-operators

$$D_{11} \equiv ((P_1 \cdot P_2)Z_1^A - (Z_1 \cdot P_2)P_1^A) \frac{\partial}{\partial P_2^A} + ((P_1 \cdot Z_2)Z_1^A - (Z_1 \cdot Z_2)P_1^A) \frac{\partial}{\partial Z_2^A}$$

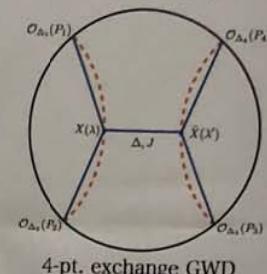
$$D_{12} \equiv \dots$$

$$H_{ij} \equiv 2\{(Z_i \cdot P_j)(Z_j \cdot P_i) - (Z_i \cdot Z_j)(P_i \cdot P_j)\}$$

3. Geodesic Witten Diagrams (GWDs)



3-pt. GWD with $(0,0,J)$ -spin



4-pt. exchange GWD

Ex. 3-pt. GWD $(0,0,J)$ -spin

$$\int_{\gamma_{12}} d\lambda \Pi_{\Delta_1,0}(P_1, X) (K \cdot \nabla)^J \Pi_{\Delta_2,0}(P_2, X) \Pi_{\Delta_3,J}(P_3, X; Z_3, W) \Big|_{X=X(\lambda)} = S_{\Delta_{1,2,3}}^J \beta_{\Delta_{12}, \Delta_3} \begin{bmatrix} \Delta_1 & \Delta_2 & \Delta_3 \\ 0 & 0 & J \\ 0 & 0 & 0 \end{bmatrix} \sim \langle \mathcal{O}_1(P_1)\mathcal{O}_2(P_2)\mathcal{O}_{3,J}(P_3, Z_3) \rangle$$

3-point GWD is proportional to 3-pt. function.

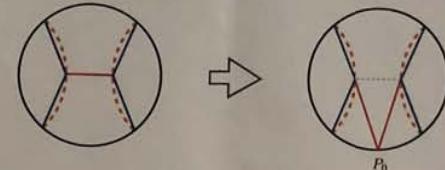
Nishida and Tamaoka [1609.04563]

Ex. 4-pt (Δ, J) -exchange GWD: $\mathcal{W}_{\Delta,J}(P_i)$

$$\mathcal{W}_{\Delta,J}(P_i) = \frac{1}{[J! (\frac{d-1}{2})_J]^2} \int_{\gamma_{12}} \int_{\gamma_{34}} \Pi_{\Delta_1,0}(P_1, X) (K \cdot \nabla_X)^J \Pi_{\Delta_2,0}(P_2, X) \times \Pi_{\Delta_3,0}(P_3, \tilde{X}) (\tilde{K} \cdot \tilde{\nabla}_{\tilde{X}})^J \Pi_{\Delta_4,0}(P_4, \tilde{X}) \Pi_{\Delta,J}(X, \tilde{X}; W, \tilde{W})$$

◆ The split representation of bulk-bulk propagator Costa et al [1404.5625]

$$\Pi_{\Delta,J}(X, \tilde{X}; W, \tilde{W}) = \frac{1}{\pi J!(h-1)_J} \int_{-\infty}^{\infty} d\nu \int_{\partial} dP_0 \frac{\nu^2}{\nu^2 + (\Delta - h)^2} \times \Pi_{h+i\nu,J}(X, P_0; W, D_Z) \Pi_{h-i\nu,J}(\tilde{X}, P_0; \tilde{W}, Z_0)$$



$$\mathcal{W}_{\Delta,J}(P_i) = \int dP_0 \int_{-\infty}^{\infty} d\nu \frac{\nu^2 C_{h+i\nu,J} C_{h-i\nu,J} \beta_{\Delta_{12},h+i\nu,J} \beta_{\Delta_{34},h-i\nu,J}}{\nu^2 + (\Delta - h)^2} \times \begin{bmatrix} \Delta_1 & \Delta_2 & h+i\nu \\ 0 & 0 & J \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \Delta_3 & \Delta_4 & h-i\nu \\ 0 & 0 & J \\ 0 & 0 & 0 \end{bmatrix}$$

This is same as the integral representation of CPW.

$$(\text{CPW}) \quad W_{\Delta,J}(P_i) \sim \mathcal{W}_{\Delta,J}(P_i) \quad (\text{GWD})$$

This relation holds even for spinning CPWs (GWDs).

$$\mathcal{D}_{\text{Left}}^{n_{10}, n_{20}, n_{12}} \mathcal{D}_{\text{Right}}^{n_{30}, n_{40}, n_{34}} W_{\Delta,J}(P_i) \sim \mathcal{D}_{\text{Left}}^{n_{10}, n_{20}, n_{12}} \mathcal{D}_{\text{Right}}^{n_{30}, n_{40}, n_{34}} \mathcal{W}_{\Delta,J}(P_i)$$

4. Mellin representation

A GWD has the following Mellin representation:

$$\mathcal{W}_{\Delta,J}(P_i) = \int_{-i\infty}^{i\infty} \frac{ds dt}{(4\pi i)^2} \int_{-\infty}^{\infty} \frac{1}{\nu^2 + (\Delta - h)^2} \omega_{\nu,J}^{\text{GWD}}(t) P_{\nu,J}(s, t) \prod_{i < j} \Gamma(\delta_{ij}) P_{ij}^{-\delta_{ij}}$$

"Mellin amplitude"

"Kinematic" factors

where

$$\mathcal{M}_{\nu,J}(s, t)$$

$$\omega_{\nu,J}^{\text{GWD}}(t) = \frac{\Gamma(\frac{h+i\nu-J-t}{2})}{8\pi \Gamma(\pm i\nu) \Gamma(\frac{\Delta_1+\Delta_2-t}{2}) \Gamma(\frac{\Delta_3+\Delta_4-t}{2})} \quad \text{no poles of double trace ops.}$$

$P_{\nu,J}(s, t)$: the Mack polynomial