Background independent exact renormalisation

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Background dependence

- General relativity is a theory without a fixed background geometry
- Continuum approaches to quantum gravity utilise a background metric

$$\Gamma = \Gamma[g_{\mu\nu}, \bar{g}_{\mu\nu}]$$

This typically breaks diffeomorphism invariance

Background dependence

- A background metric is typically introduced to a) fix the gauge b) To regularise the theory
- Consequence: <u>Infinite</u> avatars of Newton's constant.
- Couplings are related by modified Ward identities.
- Can we have a single metric effective action?

Background independence

- Here I will discuss an approach to quantum gravity that is manifestly background independent and works without fixing the gauge.
- Can be applied to: Gravity, Gauge theory, ...

Background independent exact renormalisation

- Regularise the path integral.
- Derive an exact RG flow equation for the Wilsonian effective action.

- First step is to regularise the path integral in a diffeomorphism invariant manner.
- Regulator: Non-local action + Pauli-Villars determinants (a version of Slavnov's gauge invariant regularisation '74)

QFT requires a <u>UV regulator</u>

$$P(p^2) = \frac{1}{p^2} \to \frac{1}{p^2} e^{-p^2/\Lambda^2}$$

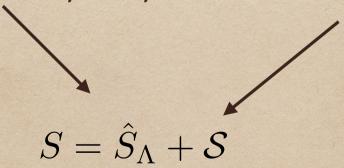
 This can be achieved by modifying the action to depend explicitly on the cutoff scale

$$\frac{1}{2} \int_{x} \partial_{\mu} \phi \partial^{\mu} \phi \to \frac{1}{2} \int_{x} \partial_{\mu} \phi \, e^{-\partial^{2}/\Lambda^{2}} \partial^{\mu} \phi$$

 For gravity we can regularise the propagator by adding non-local terms to the Einstein Hilbert action

$$-\int d^x \sqrt{g} R \to \int \sqrt{g} \left[-R + G^{\mu\nu} \frac{1}{-\nabla^2} (e^{-\nabla^2/\Lambda^2} - 1) R_{\mu\nu} \right]$$

· Action: regularised part plus an 'interaction part'



- Modifying the action with infinite order derivatives only regulates multi-loop diagrams
- One loop diagrams are still divergent (Slavnov '73)
- Simple example: A scalar field in curved spacetime

$$\frac{1}{2}\operatorname{Tr}\log(-\nabla^2) \to \frac{1}{2}\operatorname{Tr}\log(-\nabla^2\mathrm{e}^{-\nabla^2/\Lambda^2})$$

 To regularise the path integral we must also modify the measure to include Pauli-Villars determinants e.g.

$$\int d\phi \to \int d\phi \sqrt{\det \left[-\nabla^2 e^{-\nabla^2/\Lambda^2} + \Lambda^2 \right]}$$

Regularised one-loop trace e.g.

$$\frac{1}{2}\operatorname{Tr}\log(-\nabla^2) \to \frac{1}{2}\operatorname{Tr}\log\left(\frac{-\nabla^2 e^{-\nabla^2/\Lambda^2}}{-\nabla^2 e^{-\nabla^2/\Lambda^2} + \Lambda^2}\right)$$

Regularised integration over geometries

$$\int \prod_{x,\mu\nu} dg_{\mu\nu}(x) \frac{\sqrt{\det G_{\Lambda}[g]}}{\int \prod_{x,\mu} d\xi^{\mu}(x) \sqrt{\det H_{\Lambda}[g]}}$$

- The metrics of the space of spacetime metrics and the space of diffeomorphisms involve Pauli-Villars determinants.
- Choices of Pauli-Villars determinants can fully regularise the path integral

- The exact renormalisation group is a method that ensures that physics is invariant under a change in the cutoff scale.
- The couplings of the theory should depend on the cutoff scale

$$\lambda \to \lambda(\Lambda)$$
 such that $\frac{\partial}{\partial \Lambda} \mathcal{Z} = 0$

 Observables extracted from the path integral independent of the cutoff

A change in the cutoff is accompanied by a field transformation

$$-\int d\phi \,\Lambda \frac{\partial}{\partial \Lambda} e^{-S_{\Lambda}[\phi]} = \int d\phi \int_{x} \frac{\delta}{\delta \phi^{x}} \left(\Psi^{x}[\phi] e^{-S_{\Lambda}[\phi]} \right) = 0$$

 Where we can understand the field transformation as an averaging of the bare fields

$$e^{-S_{\Lambda}[\phi]} = \int d\varphi \, \delta[\phi - b_{\Lambda}[\varphi]] e^{-S_{\Lambda_0}[\varphi]}$$

Then the averaging vector is given by

$$\Psi[b_{\Lambda}] = \Lambda \partial_{\Lambda} b_{\Lambda}[\varphi]$$

The ERGE or <u>flow equation</u> for the Wilsonian action is given by

$$\Lambda \partial_{\Lambda} S + \int_{x} \Psi^{x} \frac{\partial S}{\partial \phi^{x}} = -\int_{x} \frac{\partial \Psi^{x}}{\partial \phi^{x}}$$

To have a well defined flow we should demand that:

$$\Psi^x = -\frac{1}{2} \int_{\mathcal{Y}} \mathcal{K}^{xy} \frac{\delta \mathcal{S}}{\delta \phi^y} + \psi^x$$

The flow then has the form of a heat equation

$$(\mathcal{F} + \Lambda \partial_{\Lambda})e^{-\mathcal{S}} = 0$$

- Requirements for the averaging vector:
- 1. Solutions to the ERGE should lead to regulated path integrals.
- 2. The flow should be quasi-local meaning that it has a well behaved derivative expansion.

Flow as a diffeomorphism on the space of geometries

• Geometric flow:

$$\Lambda D_{\Lambda} \, \tilde{\mu}[g] e^{-S[g]} = 0$$

$$\tilde{\mu}[g] = \frac{\sqrt{\det G_{\Lambda}[g]}}{\sqrt{\det H_{\Lambda}[g]}}$$

$$\Lambda D_{\Lambda} \equiv \Lambda \partial_{\Lambda} + \mathcal{L}_{\Psi}$$

$$\Lambda D_{\Lambda} S = \frac{1}{2} \text{Tr}[G^{-1} \Lambda D_{\Lambda} G - H^{-1} \Lambda D_{\Lambda} H]$$

DeWitt Metric

 Since we work on the space of metrics it is convenient to utilise a DeWitt metric

$$\gamma_{\mu\nu\rho\sigma}(x,y) = \frac{\Lambda^2}{32\pi G_N} \frac{1}{2} \sqrt{g} \left(g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\mu\nu} g^{\rho\sigma} \right) \delta(x,y)$$

- Indices are contracted using the DeWitt metric and functional derivatives are covariant with the Levi-Civita connection —> Field covariant
- Line element:

$$ds^2 = \delta g \cdot \gamma \cdot \delta g$$

What doesn't work

 In analogy to regulating a scalar in curved spacetime one might try:

$$G = \hat{S}^{(2)} + \gamma$$

 This doesn't regularise the theory because the PV vertices diverge too fast for high momentum.

Two levels of regularisation

Instead use that

$$\log(C^2) = 2\log(C)$$

$$\hat{S} = \hat{s} + \frac{1}{2}\hat{s}^{(1)} \cdot \gamma^{-1} \cdot \hat{s}^{(1)}$$

 Regularised metric on geometries (same structure as Bakeyev, Slavnov '96)

$$G = (1 + \gamma^{-1} \cdot \hat{s}^{(2)}) \cdot \gamma \cdot (1 + \gamma^{-1} \cdot \hat{s}^{(2)})$$

• Extra longitudinal terms included in the Hessian which are regularised by the metric on the space of diffeos.

Two levels of regularisation

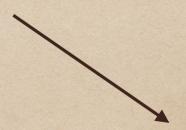
The regularisation would be spoiled if

$$\mathcal{S}^{(n)} \sim \hat{S}^{(n)}$$

Regularisation is not spoilt for

$$S^{(n)} \sim \hat{s}^{(n)}$$

• Note:
$$\hat{S}^{(1)} = (1 + \gamma^{-1} \cdot \hat{s}^{(2)}) \cdot \hat{s}^{(1)}$$



• UV cutoff in the flow equation:
$$C = \frac{1}{1 + \gamma^{-1} \cdot \hat{s}^{(2)}}$$

'classical flow'

The lhs of the flow is given by

$$\Lambda \partial_{\Lambda} (\hat{s} + \frac{1}{2} \hat{s}^{(1)} \cdot \gamma^{-1} \cdot \hat{s}^{(1)} + \mathcal{S}) + (\hat{s}^{(1)} \cdot C^{-1} + \mathcal{S}^{(1)}) \cdot \psi - \frac{1}{2} (\hat{s}^{(1)} \cdot C^{-1} + \mathcal{S}^{(1)}) \cdot \mathcal{K} \cdot \mathcal{S}^{(1)} = \frac{1}{2} \text{Tr}[...]$$

• Fix ψ by

$$\Lambda \partial_{\Lambda} \frac{1}{2} \hat{s}^{(1)} \cdot \gamma^{-1} \cdot \hat{s}^{(1)} + \hat{s}^{(1)} \cdot C^{-1} \cdot \psi = 0$$

$$\psi = -\frac{1}{2}C \cdot (\Lambda \partial_{\Lambda} \gamma^{-1} \cdot \hat{s}^{(1)} + \gamma^{-1} \cdot \Lambda \partial_{\Lambda} \hat{s}^{(1)})$$

• Take $\mathcal{K} = C \cdot \kappa$

The flow

The lhs of the flow is given by

$$\Lambda \partial_{\Lambda}(\hat{s} + \mathcal{S}) + \mathcal{S}^{(1)} \cdot \psi - \frac{1}{2}(\hat{s}^{(1)} + \mathcal{S}^{(1)} \cdot C) \cdot \kappa \cdot \mathcal{S}^{(1)} = \frac{1}{2} \text{Tr}[...]$$

- ullet One can further pick a convenient choice for κ
- The flow generates solutions which preserve the flow. One can show that the trace is UV finite as a consequence.

Conclusions and summary

- New diffeomorphism invariant flow equation
- No background, no gauge fixing, no ghosts
- First test: One-loop divergences of GR are reproduced
- Applications: Quantum gravity and gauge theories (e.g. QCD)