Large spin analytic bootstrap via Mellin

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Based on

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Sampling of the results

Anomalous dimension of large spin double trace operators: $\phi\partial_{\mu_1}\cdots\partial_{\mu_\ell}\phi$

All orders in inverse spin J

$$\gamma_\ell \sim \sum_{i=0}^{\infty} \frac{\gamma^i}{J^{2i}} \qquad J^2 = (\ell + \Delta_\phi)(\ell + \Delta_\phi - 1)$$

$$\gamma^i = -\frac{C_m}{J^{\tau_m}} \sum_{q=0}^{i} \sum_{n=0}^{i-q} \sum_{k_1=0}^{i-q-n} (-1)^n \, 2^{1+\ell_m} \, \mathfrak{b}_{k_1}(\Delta_\phi) \, \mathfrak{b}_{i-k_1-n-q}\left(\frac{\tau_m}{2} - \Delta_\phi + q\right)$$
$$\times \frac{(\tau_m + 2\ell_m - 1)\,\Gamma(-h + \ell_m + \tau_m + 1)\,\Gamma^2(2\ell_m + \tau_m - 1)}{n!\,q!\,\Gamma(1 - h + q + \ell_m + \tau_m)\,\Gamma^4\left(\ell_m + \frac{\tau_m}{2}\right)\,\Gamma(\ell_m + \tau_m - 1)}$$
$$\times \frac{\Gamma\left(q + \frac{\tau_m}{2}\right)\Gamma\left(n + q + \frac{\tau_m}{2}\right)\Gamma^2(\Delta_\phi)}{\Gamma^2\left(-n - q + \Delta_\phi - \frac{\tau_m}{2}\right)} P(q + \tau_m/2, 0)$$

h=d/2, d= dimension

Flashing the results for epsilon expansion

$$\phi \partial_{\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_\ell} \phi$$

$$\Delta_\ell = 2 + \ell - \epsilon + \frac{1}{54}\epsilon^2 \left(1 - \frac{6}{\ell\,(\ell+1)}\right) + \delta_\ell^{(3)}\,\epsilon^3 + \delta_\ell^{(4)}\,\epsilon^4 + \delta_\ell^{(5)}\,\epsilon^5 + \cdots$$

$$\delta_\ell^{(5)} \sim \frac{\epsilon^5}{7085880\ell^2}\Big(2430\zeta(3)(144\log(\ell) + 144\gamma_E - 59) - 2332800\zeta(5)$$
$$- 135\log(\ell)\,(24\log(\ell)(12\log(\ell) + 36\gamma_E - 41) + 48\gamma_E(18\gamma_E - 41)$$
$$+162\pi^2 - 31) + 27\left(5\gamma_E(24\gamma_E(41 - 12\gamma_E) + 31) + 216\pi^4 - 810\gamma_E\pi^2\right)$$
$$+ 20385\pi^2 + 33770\Big)$$

Plan of talk

Plan of talk

- Review of large spin bootstrap

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- Bootstrap in Mellin space

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- Epsilon expansion at large spin

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- Summary

Quick Review of conformal bootstrap

Quick Review of conformal bootstrap

s-channel

$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{(x_{12}^2 x_{34}^2)^{\Delta_\phi}} \sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell}(u,v)$$

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If OPE is associative then these two expansion must give the same result

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Obtain all $\Delta, \quad C_{\Delta,\ell}$ You know everything..!!

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This is the conventional bootstrap program

Bootstrap equation

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell}(u,v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \sum_{\Delta,\ell} C_{\Delta,\ell}\, g_{\Delta,\ell}(v,u)$$

Bootstrap equation

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Difficult to solve this equation analytically and get information

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Simplifies in a certain limit...large spin limit and for double trace operators

$\ell \gg 1 \qquad v \ll u \ll 1$

Fitzpatrick-Kaplan-Poland-Simmons-Duffin, Komargodski-Zhiboedov

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$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \left(1 + \sum_{\Delta,\ell} C_{\Delta,\ell}\, v^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v,u)\right)$$

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$$f_{\Delta,\ell}(v) = (1-v)^{\ell}\, {}_2F_1(\beta,\beta,2\beta;1-v) \qquad \beta = \frac{\Delta+\ell}{2}$$

Large spin bootstrap

$$v \ll u \ll 1$$

Leading term

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v) = \left(\frac{u}{v}\right)^{\Delta_\phi}$$

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The r.h.s. diverges as v ~ 0.

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Each term on the l.h.s. goes as $f_{\Delta,\ell}(v) \sim \log v$

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Need to sum over infinite large spin double trace operators

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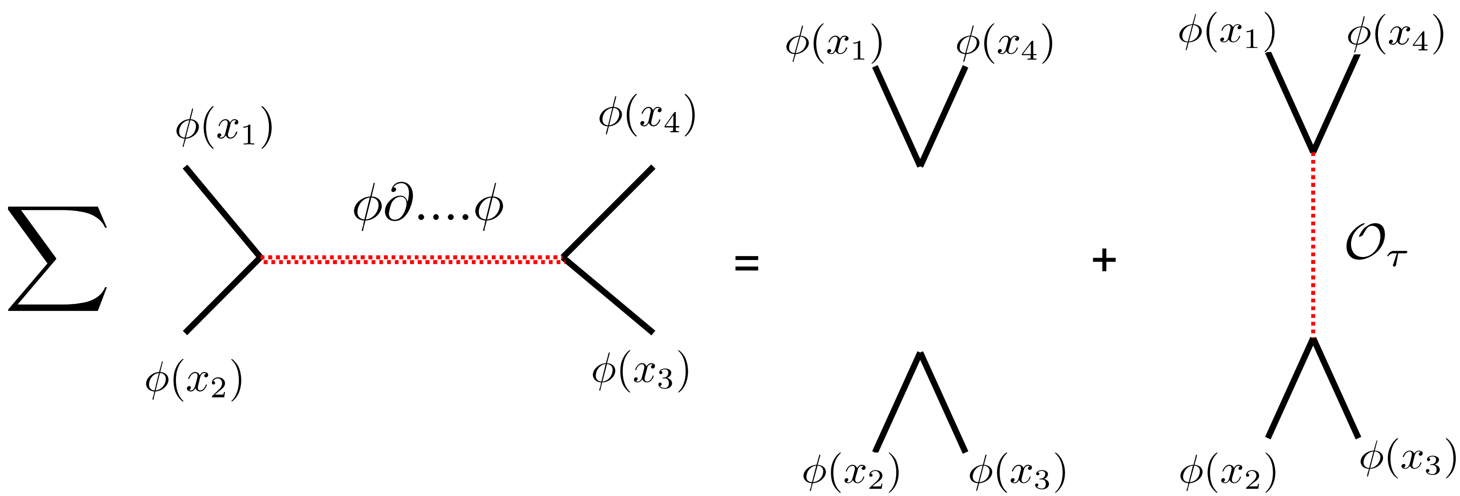
Need to sum over infinite large spin double trace operators on the left to reproduce the divergence on the r.h.s.



$\sum$
$\phi(x_1)$
$\phi(x_2)$
$\phi\partial....\phi$
$\phi(x_4)$
$\phi(x_3)$
=
$\phi(x_1)$
$\phi(x_4)$
$\phi(x_2)$
$\phi(x_3)$
+
$\phi(x_1)$
$\phi(x_4)$
$\mathcal{O}_\tau$
$\phi(x_2)$
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Large spin bootstrap

Double trace operators

$$O \sim \phi \partial_{\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_\ell} \partial^{2n} \phi$$

Large spin bootstrap

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The Unknowns

$$\Delta = 2\Delta_\phi + 2n + \ell + \gamma_{n,\ell}$$

Anomalous dimension

$$\ell \gg 1$$

$$C_{\Delta,\ell} = C_{n,\ell}(1 + \delta C_{n,\ell})$$

OPE coefficient

Large spin bootstrap

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OPE coefficient

$n = 0$ for simplicity

Large spin bootstrap

Subleading term of the bootstrap equation

$$\sum_{\Delta,\ell} C_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v) = \left(\frac{u}{v}\right)^{\Delta_\phi} \left(\sum_{\Delta,\ell} C_{\Delta,\ell}\, v^{\frac{\Delta-\ell}{2}}\, f_{\Delta,\ell}(v,u)\right)$$

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Small v behavior on the rhs : power of v is controlled by the twist $\tau = \Delta - \ell$

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We focus on the minimal twist operators on the rhs

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twist τ_m spin ℓ_m OPE coeff C_m

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twist τ_m spin ℓ_m OPE coeff C_m

$$f^0_\ell(v) = (1-v)^\ell\, {}_2F_1(\Delta_\phi+\ell, \Delta_\phi+\ell, 2\Delta_\phi+2\ell; 1-v)$$

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Subleading term of the bootstrap equation

Coefficient of Log u term on the lhs involves the anomalous dimension

$$u^{\Delta_\phi} \ \log u \sum_\ell C_\ell \, \frac{\gamma_\ell}{2} \, f_\ell^0(v)$$

Subleading term of the bootstrap equation

Coefficient of Log u term on the lhs involves the anomalous dimension

$$u^{\Delta_\phi}\ \log u \sum_{\ell} C_\ell \,\frac{\gamma_\ell}{2}\, f_\ell^0(v)$$

Assume the following expansion of the anomalous dimension in large spin limit

$$\gamma_\ell = \gamma_0 + \frac{\gamma_1}{\ell} + \cdots$$

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Known Mean field OPE coefficient

$$C_\ell = \frac{2\,\Gamma^2(\Delta_\phi + \ell)\,\Gamma(2\Delta_\phi + \ell - 1)}{\ell!\,\Gamma^2(\Delta_\phi)\,\Gamma(2\Delta_\phi + 2\ell - 1)}$$

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Large spin sum on the lhs can be done at the leading order by converting

$$\sum_\ell \to \int dl$$

Subleading term of the bootstrap equation

$$u^{\Delta_\phi}\,\log u \sum_{\ell} C_\ell \,\frac{\gamma_\ell}{2}\, f^0_\ell(v)$$

Summing over $\ell \gg 1$

Subleading term of the bootstrap equation

$$u^{\Delta_\phi} \; \log u \sum_\ell C_\ell \, \frac{\gamma_\ell}{2} \, f_\ell^0(v)$$

Summing over $\ell \gg 1$

We have from the lhs $$u^{\Delta_\phi} \; \log u \, v^{\tau_m/2-\Delta_\phi} \, \gamma_0 \, \frac{\Gamma^2(\Delta_\phi - \frac{\tau_m}{2})}{\Gamma^2(\Delta_\phi)}$$

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Focus on the logu term from the rhs $$\left(\frac{u}{v}\right)^{\Delta_\phi}\left(C_m\,v^{\frac{\tau_m}{2}}\,f_{\tau_m,\ell_m}(v,u)\right)$$

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Compare the log u term to get the anomalous dimension

$$\gamma_0 = -\frac{2\Gamma^2\left(\Delta_\phi\right)\Gamma\left(2\ell_m+\tau_m\right)}{\Gamma^2\left(\ell_m+\frac{\tau_m}{2}\right)\Gamma^2\left(\Delta_\phi-\frac{\tau_m}{2}\right)}\left(\frac{1}{\ell}\right)^{\tau_m}\ C_m,$$

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Similar analysis for non log terms gives the OPE coefficients

Large spin bootstrap

Subleading order corrections can be done following Alday-Zhiboedov (2015)

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Anomalous dimension: asymptotic expansion in inverse J

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Involves two recursion relations that one needs to solve

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Simplifies in Mellin space...gives an all order expression !

Conformal bootstrap in Mellin space

Mellin transform

$$\tilde{f}(s) = \int_0^\infty dx\, x^{s-1}\, f(x)$$

Inverse Mellin transform

$$f(x) = \frac{1}{2\,\pi} \int_{-i\infty}^{i\infty} ds\, x^{-s}\, \tilde{f}(s)$$

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For $\qquad \tilde{f}(s) = \dfrac{1}{s - \Delta}$

$$f(x) = \frac{1}{x^\Delta}$$

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Scaling behavior of CFT correlation function

Mellin transform

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Scaling behavior of CFT correlation function

Mellin representation captures conformal symmetry automatically

The Mellin amplitude

Mack (2009), Penedones, Costa-Goncalves- Penedones ,
Fitzpatrick-Kaplan- Penedones-Raju, van Rees ....

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Mellin transform of the 4-pt function

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Mellin transform of the 4-pt function

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(-t)^2\, \Gamma(s+t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

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Mellin amplitude

Properties of Mellin amplitude

$$\mathcal{A}(u,v) = \int ds\, dt\, u^s\, v^t\, \Gamma(-t)^2\, \Gamma(s+t)^2\, \Gamma(\Delta_\phi - s)^2\, \mathcal{M}(s,t)$$

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- Channel dualities are manifest as s, t exchange

Mellin transform of s channel conformal block

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$$G(u,v) \sim u^{\frac{\Delta-\ell}{2}} (1-v)^{\ell} \, {}_2F_1\left(\frac{\Delta+\ell}{2}, \frac{\Delta+\ell}{2}, \Delta+\ell; 1-v\right)$$

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Expand in small anm dim $\quad \Delta = 2\Delta_\phi + \ell + \gamma$

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$$\frac{1}{s-\frac{\Delta-\ell}{2}} \to \Gamma^2(\Delta_\phi - s)\left(\frac{1}{2}\gamma + (\Delta_\phi - s)(\gamma\gamma_E - 1)\right)$$

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$$G(u,v)=\sum_{\ell}\int\frac{ds}{2\pi i}\frac{dt}{2\pi i}u^{s}\,v^{t}\,\Gamma^{2}(s+t)\,\Gamma^{2}(-t)\,\Gamma^{2}(\Delta_{\phi}-s)\,q_{\ell}^{(s)}(s)\,Q_{\ell,0}^{2s+\ell}(t)$$

Residue at double pole $s=\Delta_{\phi}$

$$u^{\Delta_{\phi}}\,\log u\,(\cdots),\quad u^{\Delta_{\phi}}\,(\cdots)$$

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Play a key role in repackaging the equations in Mellin space

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where

$$\beta_\ell(\Delta_\phi)=\frac{2^{-\ell}\Gamma(2(\ell+\Delta\phi))}{\ell!\,\Gamma(\Delta\phi)^2\Gamma(\ell+\Delta\phi)^2}$$

Strategy in Mellin space

Mellin transform of the t channel can be obtained from the s channel by replacing $s \to t + \Delta_\phi, \quad t \to s - \Delta_\phi$

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Boils down to simple equations in Mellin space

Mellin transform of t channel conformal block

Minimal twist operator $\tau_m = \Delta_m - \ell_m$

$$G^t(u,v) = C_m \int ds\, dt\, u^s\, v^t\, \rho\, \frac{\Gamma(\frac{\Delta_m - \ell_m}{2} - \Delta_\phi - t)\Gamma(\frac{2h - \Delta_m - \ell_m}{2} - \Delta_\phi - t)}{\Gamma^2(-t)}$$
$$\times P(t + \Delta_\phi, s - \Delta_\phi)$$

↓

Mack polynomial, polynomial in s, t

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$$\rho = \Gamma^2(\Delta_\phi - s)\,\Gamma^2(-t)\,\Gamma^2(s+t)$$

Evaluate the residue at $s = \Delta_\phi$

Mellin transform of t channel conformal block

Focus on the log u term

$$G^t(u,v) = C_m\, u^{\Delta_\phi} \log u \int dt\, v^t\, \Gamma^2(\Delta_\phi + t)\Gamma(\frac{\Delta_m - \ell_m}{2} - \Delta_\phi - t)$$
$$\times \Gamma\left(\frac{2h - \Delta_m - \ell_m}{2} - \Delta_\phi - t\right) P(t + \Delta_\phi, 0)$$

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Re express the t dependence in terms of Q(t) polynomials.

$$G^t(u,v) = -\sum_\ell u^{\Delta_\phi} \log u \int dt v^t\, \Gamma^2(\Delta_\phi + t)\, \Gamma^2(-t)\, q^t_\ell\, Q^{2\Delta_\phi + \ell}_\ell(t)$$

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$$q^t_\ell = 2\,\beta_\ell(\Delta_\phi)\, C_m \int dt\, \Gamma(\tau_m/2 - \Delta_\phi - t)\Gamma((2h - \Delta_m - \ell_m)/2 - \Delta_\phi - t)$$
$$\times\, \Gamma^2(\Delta_\phi + t)\, P(t + \Delta_\phi, 0)\, {}_3F_2\left[\begin{matrix} -\ell,\ 2\Delta_\phi + \ell - 1,\ \Delta_\phi + t \\ \Delta_\phi \quad , \quad \Delta_\phi \end{matrix}; 1\right]$$

t integral has pole at $\qquad t = \tau_m/2 - \Delta_\phi + r$

Bootstrap equation in Mellin space

$$\sum_\ell u^{\Delta_\phi} \log u \int dt\, v^t\, \Gamma^2(\Delta_\phi + t)\, \Gamma^2(-t)\, (\mathfrak{q}_\ell^s + \mathfrak{q}_\ell^t)\, Q_\ell^{2\Delta_\phi+\ell}(t) = 0$$

$$\boxed{\mathfrak{q}_\ell^s + \mathfrak{q}_\ell^t = 0}$$

Algebraic eq !!

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$$\mathfrak{q}_\ell^t = \sum_{r=0}^{\infty} 2\beta_\ell(\Delta_\phi)\, C_m\, \frac{(-1)^r}{r!} \Gamma^2(\tau_m/2+r)\Gamma(h-\Delta_m-r)$$

$$\times\, P(\tau_m/2+r,0)\, {}_3F_2\left[\begin{matrix} -\ell,\ 2\Delta_\phi+\ell-1,\ \tau_m/2+r \\ \Delta_\phi\ \ ,\ \ \Delta_\phi \end{matrix};1\right]$$

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Need the large spin behavior of continuous Hahn polynoimal.

Asymptotics of continuous Hahn polynomial

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$$

$$
\sim \frac{(-1)^n \ell! \Gamma(s)^2 \Gamma(\ell-n-1+s-t)\Gamma(n+s+t)}{n!\Gamma(2s+\ell-1)\Gamma(-t-n)^2\Gamma(s+t)\Gamma(1+\ell+n+s+t)}
$$

Asymptotics of continuous Hahn polynomial

$$ {}_3F_2\left[\begin{matrix} -\ell,\ 2s+\ell-1,\ s+t \\ s\quad,\qquad s \end{matrix};1\right] $$

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$$ \frac{\Gamma(\lambda+\alpha)}{\Gamma(\lambda+\beta)} = \sum_{k=0} d(\alpha,\beta,k) J^{\alpha-\beta-2k}, $$

$$ J = \sqrt{\lambda(\lambda+\alpha+\beta-1)} $$

J. L.Fields

Asymptotics of continuous Hahn polynomial

PD, K. Ghosh, A. Sinha

$$ {}_3F_2\left[\begin{matrix}-\ell,\ 2s+\ell-1,\ s+t \\ s\ \ ,\ \ s\end{matrix};1\right] \sim \sum_{n,k_1,k_2=0}^{\infty} \frac{(-1)^n\,\Gamma^2(s)\,(s+t)_n}{n!\Gamma^2(-t-n)}\,\mathfrak{b}_{k_1}(s)\,\mathfrak{b}_{k_2}(t)J^{-\alpha} $$

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Given in terms of generalised Bernoulli polynomial

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$$ {}_3F_2\left[\begin{matrix}-\ell, \; 2s+\ell-1, \; s+t \\ s \quad , \qquad s\end{matrix}; 1\right] \sim \sum_{n,k_1,k_2=0}^{\infty} \frac{(-1)^n\,\Gamma^2(s)\,(s+t)_n}{n!\Gamma^2(-t-n)}\,\mathfrak{b}_{k_1}(s)\,\mathfrak{b}_{k_2}(t) J^{-\alpha} $$

$$ J^2 = (\ell+s)(\ell+s-1) $$

$$ \alpha = 2k_1 + 2k_2 + 2n + 2s + 2t $$

Given in terms of generalised Bernoulli polynomial

Only even powers of J will appear

In the large spin limit $$\gamma_\ell \sim \sum_{i=0}^{\infty} \frac{\gamma^i}{J^{2i}}$$ in s channel

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Comparing powers of J from both sides of the bootstrap equation will fix the anomalous dimension at all orders in inverse J

Result: Anomalous dimension

$$\gamma^i = -\frac{C_m}{J^{\tau_m}} \sum_{q=0}^{i} \sum_{n=0}^{i-q} \sum_{k_1=0}^{i-q-n} (-1)^n \, 2^{1+\ell_m} \, \mathfrak{b}_{k_1}(\Delta_\phi) \, \mathfrak{b}_{i-k_1-n-q}(\frac{\tau_m}{2} - \Delta_\phi + q)$$

$$\times \frac{(\tau_m + 2\ell_m - 1) \, \Gamma(-h + \ell_m + \tau_m + 1) \, \Gamma^2(2\ell_m + \tau_m - 1)}{n! \, q! \, \Gamma(1 - h + q + \ell_m + \tau_m) \, \Gamma^4\left(\ell_m + \frac{\tau_m}{2}\right) \, \Gamma(\ell_m + \tau_m - 1)}$$

$$\times \frac{\Gamma\left(q + \frac{\tau_m}{2}\right) \Gamma\left(n + q + \frac{\tau_m}{2}\right) \Gamma^2(\Delta_\phi)}{\Gamma^2\left(-n - q + \Delta_\phi - \frac{\tau_m}{2}\right)} P(q + \tau_m/2, 0)$$

Expression for anomalous dimension to all orders in 1/J

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Expression for anomalous dimension to all orders in $1/J$

e.g. $i = 0$

$$\gamma_0 = -\frac{2\Gamma^2(\Delta_\phi) \, \Gamma(2\ell_m + \tau_m)}{\Gamma^2\left(\ell_m + \frac{\tau_m}{2}\right) \Gamma^2\left(\Delta_\phi - \frac{\tau_m}{2}\right)} \left(\frac{1}{J}\right)^{\tau_m} C_m,$$

Result: OPE coefficient

Similar expression for OPE coefficient comes from the non log term

$$\delta C_{0,\ell} = \sum_{i=0}^{\infty} \frac{\delta C_{0,\ell}^{(i)}}{J^{2i}}$$

$$\delta C_{0,\ell}^{(i)} = \frac{\text{function}(\tau_{\rm m}, \Delta_\phi, \ell_{\rm m})}{J^{\tau_m}} C_m$$

Given in terms of generalised Bernoulli polynomial and Mack polynomial

Epsilon expansion in the large spin limit

Critical exponents via Epsilon expansion...

$$S = \int d^{4-\epsilon} \left[(\partial_\mu \phi)^2 + \lambda \phi^4 \right]$$

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$$\Delta_\phi = \frac{d-2}{2} + \frac{N+2}{4\,(N+8)^2}\epsilon^2 + \frac{N+2}{4\,(N+8)^2}\left[6\,\frac{3N+14}{(N+8)^2} - \frac{1}{4}\right]\epsilon^3$$

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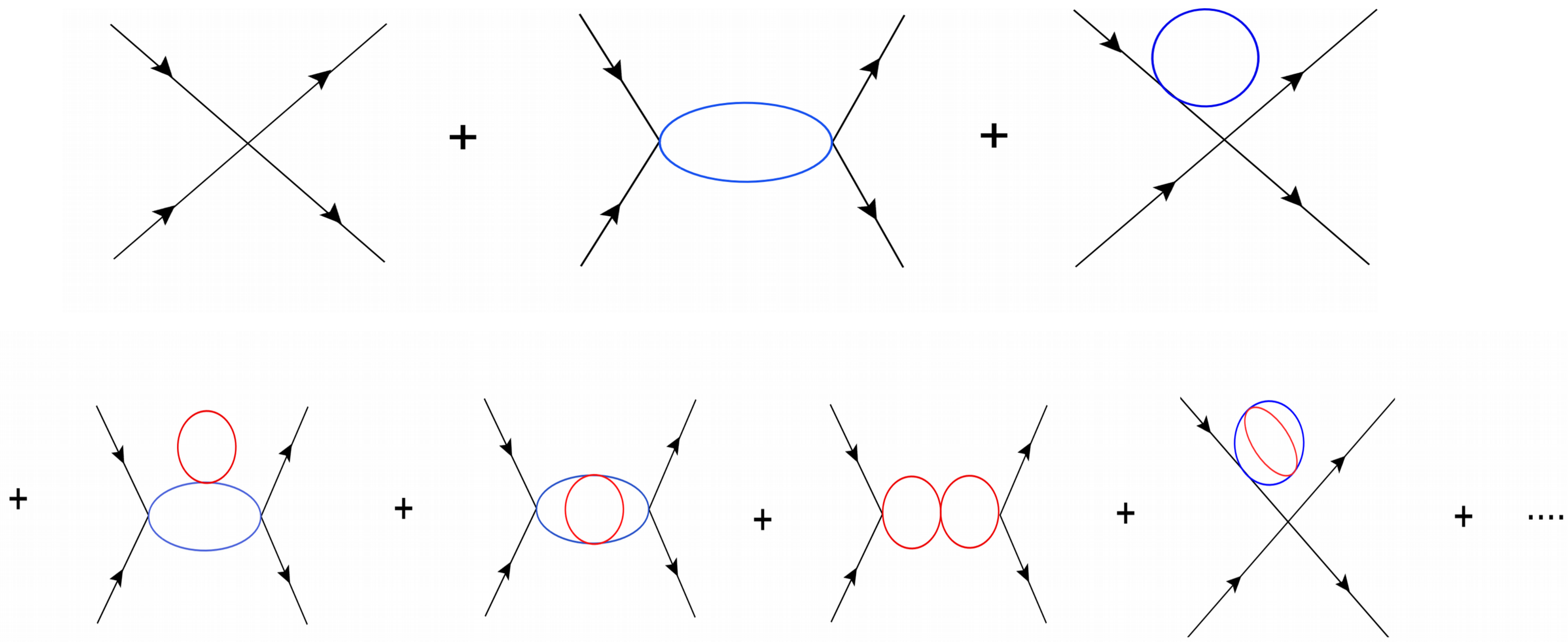
d=3, N=1

Anm. dim.	critical exponents	$\epsilon = 1$
Δ_ϕ	$\eta = 2\Delta_\phi - d + 2$	0.519
Δ_{ϕ^2}	$\alpha = 2 - \frac{d}{d-\Delta_{\phi^2}}$	1.45

Computing Δ for ϕ^2 till 2 loops, in Wilson Fisher fixed point CFT



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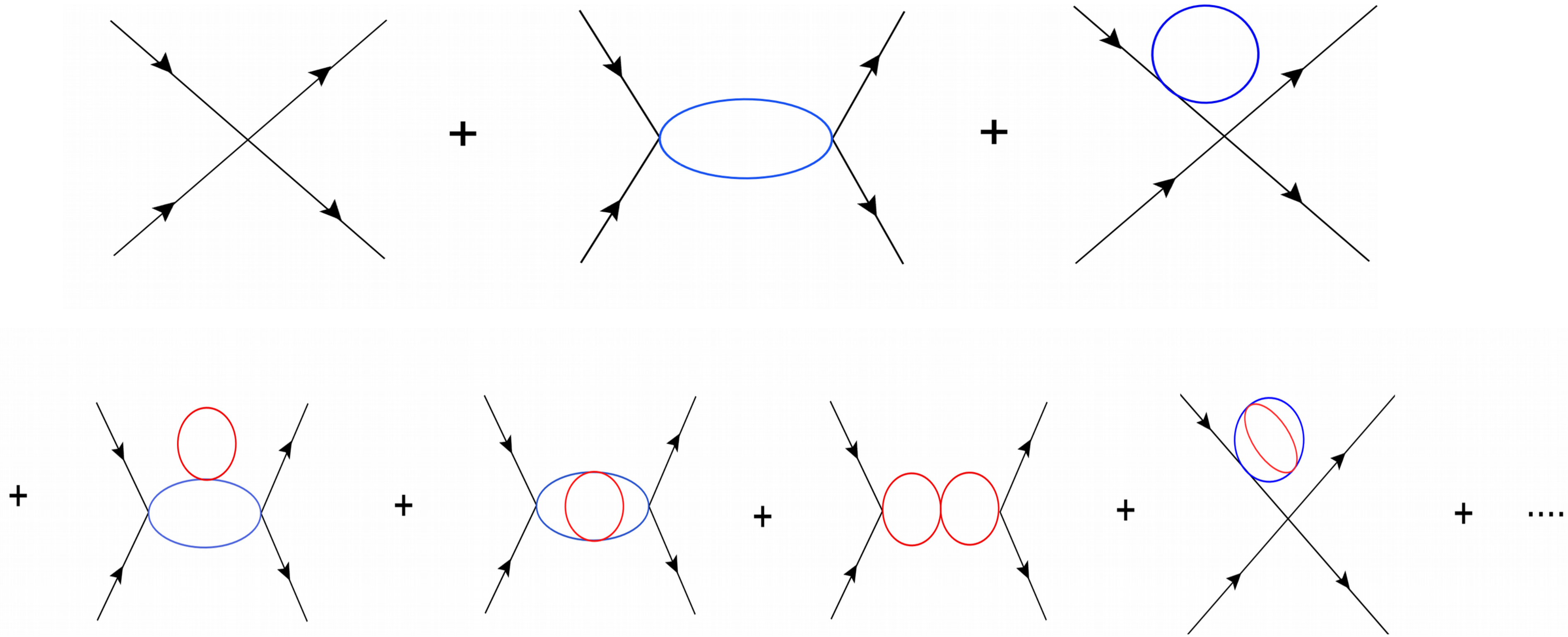

Computing till 3 loops involves many more diagrams

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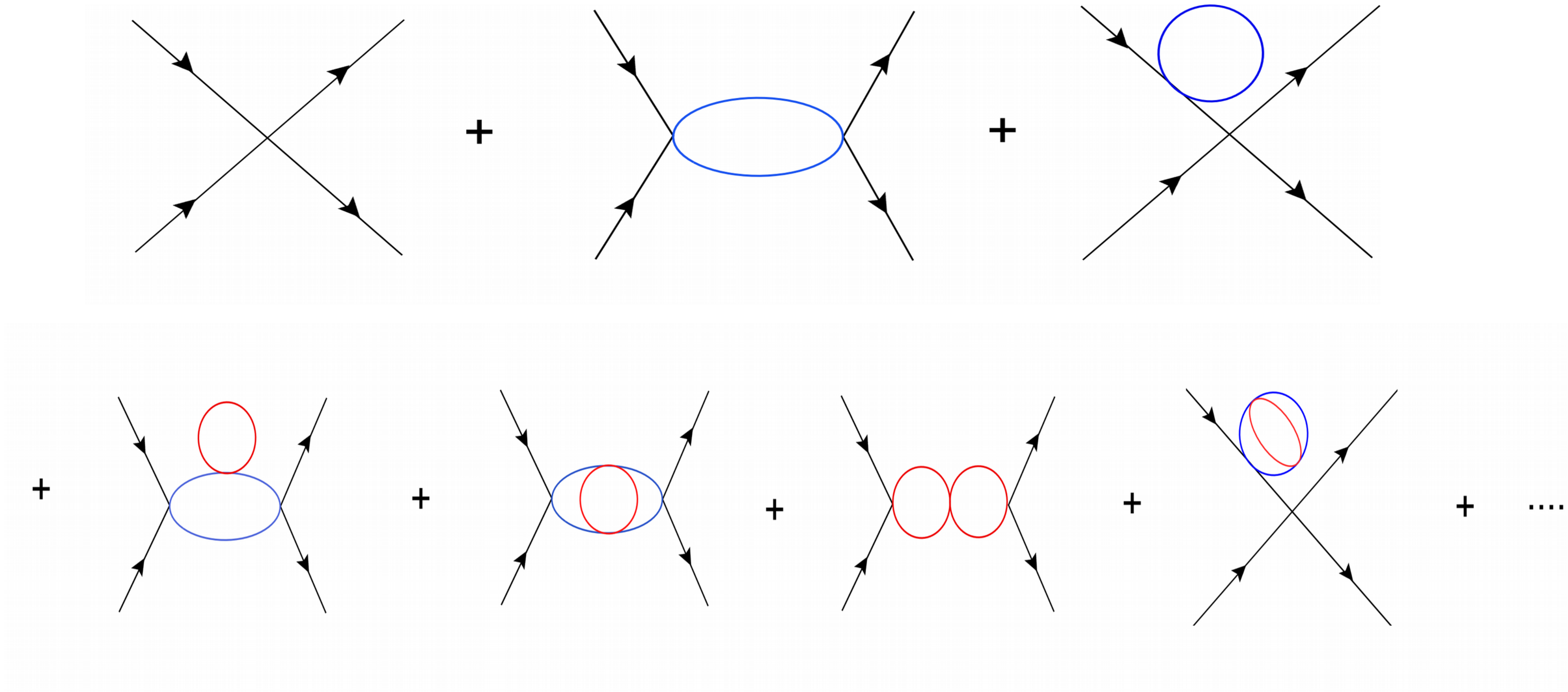

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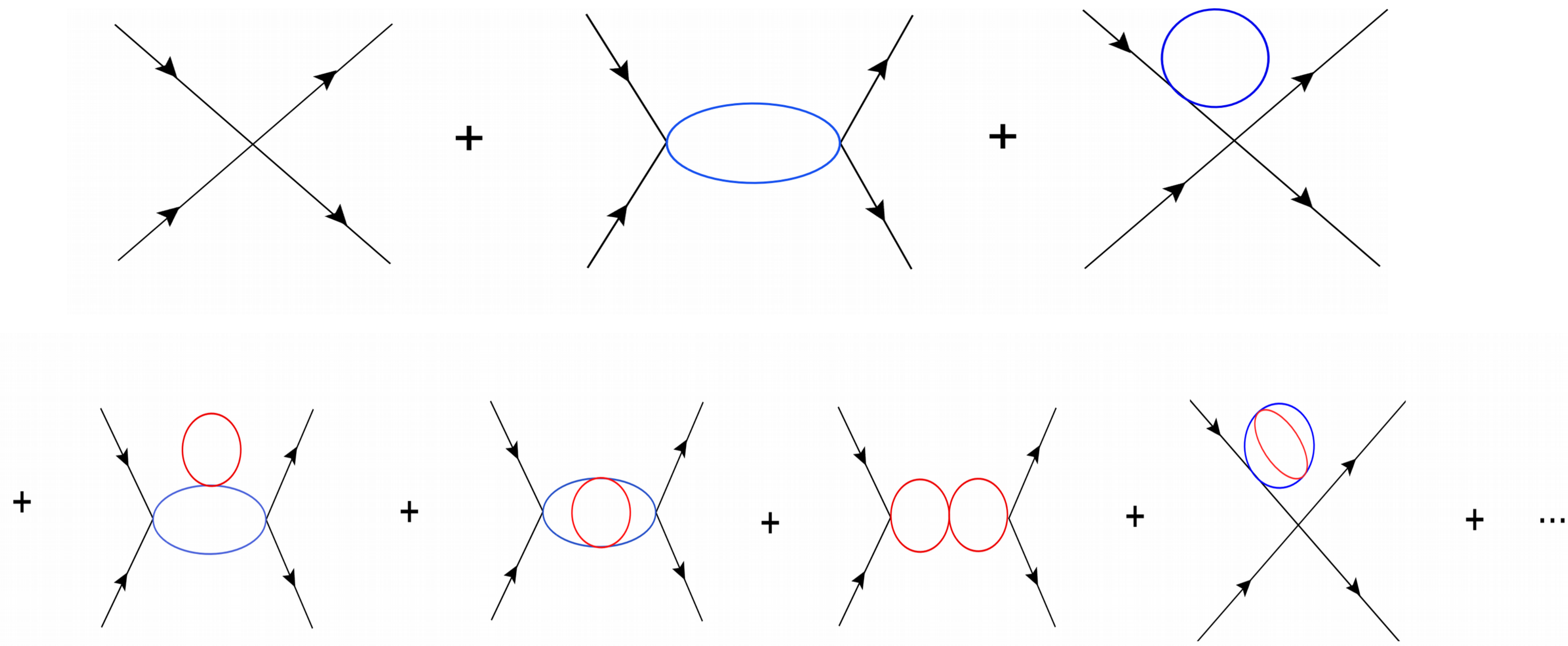

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Results from large spin bootstrap can be used to determine the operator dimension of $\phi\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_\ell}\phi$ in the large spin limit

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Need to sum over ℓ_m

Strategy for epsilon expansion

$$\gamma_0 = \sum_{\ell_m} -\frac{2\Gamma^2\left(\Delta_\phi\right)\Gamma\left(2\ell_m+\tau_m\right)}{\Gamma^2\left(\ell_m+\frac{\tau_m}{2}\right)\Gamma^2\left(\Delta_\phi-\frac{\tau_m}{2}\right)}\left(\frac{1}{\ell}\right)^{\tau_m} C_m,$$

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$$C_0 = 2 - \frac{2\epsilon}{3} - \frac{34\epsilon^2}{81} + C_0^{(3)}\epsilon^3 + C_0^{(4)}\epsilon^4 + C_0^{(5)}\epsilon^5 + O(\epsilon^6)$$

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$$\tau_m = 2 - \epsilon + \frac{1}{54}\epsilon^2\left(1 - \frac{6}{\ell_m\left(\ell_m+1\right)}\right) + \delta_\ell^{(3)}\epsilon^3 + \delta_\ell^{(4)}\epsilon^4 + O(\epsilon^5)$$

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$$\gamma_0 \sim \# \frac{\epsilon^2}{\ell^2} + \# \frac{\epsilon^3}{\ell^2} + \# \frac{\epsilon^4}{\ell^2} + f(\delta_\phi^{(4)}, \delta_0^{(3)}, \delta_0^{(4)}, \delta_{\ell_m}^{(3)}, C_0^{(3)}) \frac{\epsilon^5}{\ell^2}$$

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Feynman diagram	Mellin bootstrap
$\delta_\phi^{(4)}$, $\delta_0^{(3)}$, $\delta_0^{(4)}$, $\delta_{\ell_m}^{(3)}$	$C_0^{(3)}$

Kleinert et al.
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Step 3: Do the sum over ℓ_m

Compute the anomalous dimension of higher spin operators at e^5 in the large spin limit

Sampling of new result: Large spin anm. dimension

PD, A. Kaviraj
JHEP1802(2018)153

$$\phi\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_\ell}\phi$$

$$\Delta_\ell = 2+\ell-\epsilon+\frac{1}{54}\epsilon^2\left(1-\frac{6}{\ell\,(\ell+1)}\right)+\delta_\ell^{(3)}\,\epsilon^3+\delta_\ell^{(4)}\,\epsilon^4+\delta_\ell^{(5)}\,\epsilon^5+\cdots$$

$$\delta_\ell^{(5)} \sim \frac{\epsilon^5}{7085880\ell^2}\Big(2430\zeta(3)(144\log(\ell)+144\gamma_E-59)-2332800\zeta(5)$$
$$-135\log(\ell)\left(24\log(\ell)(12\log(\ell)+36\gamma_E-41)+48\gamma_E(18\gamma_E-41)\right.$$
$$\left.+162\pi^2-31\right)+27\left(5\gamma_E(24\gamma_E(41-12\gamma_E)+31)+216\pi^4-810\gamma_E\pi^2\right)$$
$$+20385\pi^2+33770\Big)$$

Subsequent order of anomalous dimension can also be computed using

$$\gamma_1 = \sum_{\ell_m} C_{\ell_m} \frac{(2\Delta_\phi - 1)\,\tau_m\,(\Delta_\phi)\;2^{\tau_m+2\ell_m-1}\Gamma^2\Gamma\left(\ell_m+\frac{\tau_m}{2}+\frac{1}{2}\right)}{\sqrt{\pi}\Gamma\left(\ell_m+\frac{\tau_m}{2}\right)\Gamma^2\left(\Delta_\phi-\frac{\tau_m}{2}\right)}\left(\frac{1}{\ell}\right)^{\tau_m}$$

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This gives the anomalous dimension at the order $\quad (\cdots)\,\frac{\epsilon^5}{\ell^3}$

Subsequent order of anomalous dimension can also be computed using

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Relation between the usual bootstrap and Mellin bootstrap

Thank you