Noncommutative geometry for bimetric gravity models

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[Hassan-Rosen, 2012; Akrami-Koivisto-Sandstad, 2013]

$$S = -\frac{M_g^2}{2}\int d^4x\sqrt{-\det g}R(g) - \frac{M_f^2}{2}\int d^4x\sqrt{-\det f}R(f) +$$
$$+ m^2 M_g^2 \int d^4x \sqrt{-\det g}\sum_{n=0}^{4}\beta_n e_n +$$
$$+ \int d^4x\sqrt{\det g}\mathcal{L}_m(g,\Phi) + \alpha\int d^4x\sqrt{-\det f}\mathcal{L}_m(f,\Phi)$$

- f, g - metric tensors
- $R(f), R(g)$- Ricci scalars
- M_f, M_g - mass scales

Free parameters

[Akrami-Koivisto-Mota-Sandstad(2013), Akrami-Koivisto-Sandstad(2012), Berg-Buchberger-Enander-Mortsell-Sjors(2011), von Strauss-Schmidt–May-Enander-Mortsell-Hassan(2012)]:

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- The most important free parameters are β_n's and the mass parameter m.
- Different choices $\Rightarrow$ Different cosmological scenarios
- How to fix them using *hidden symmetries* or is it possible to *derive* bimetric gravity models from some *first principles*?

Example - FLRW geometry

Friedmann-Lemaître-Roberston-Walker-type metrics

$$ds_g^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

$$ds_f^2 = -X(t)^2 dt^2 + Y(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta d\phi^2 \right) \right)$$

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- ...they will contain free parameters

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- Introduce coefficients $B_n = \frac{m^2\beta_n}{H_0^2}$, where H_0 is the value of Hubble parameter $H(g)$ at present time
- Derive the evolution equation (under the simplified assumption $k = 0$)

$$\frac{H(g)^2}{H_0^2} = \frac{B_0}{3} + B_1 y + B_2 y^2 + \frac{B_3}{3} y^3 + \Omega_m + \Omega_\gamma,$$

where $ya = Y$ and $\Omega_i = \frac{\rho_i}{3H_0 M_g^2}$.

- Derive quartic equation in y with parameters $B_0, ..., B_4, \Omega_m, \Omega_\gamma$.

- Assumption (for simplification) $\Omega_\gamma^0 \approx 0$.
- Then there are six free parameters $\beta_0, ..., \beta_4, \Omega_m^0$.
- Fixing them, one can get an evolution of y.
- Calculate $H(g)$ as a function of z.
- Compare with the observations.

- There are ranges of initial parameters consistent both with observational data and ΛCDM model
- Even slight change of these parameters can drastically change cosmological scenario.

Issues related with bimetric gravity models

- Unclear geometrical interpretation.
- Large freedom of the choice of parameters - appropriate physical theory should allow for falsification tests!

Our proposal

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- It will produce concrete predictions on cosmological parameters that can be tested in astronomical observations.

- {locc. compact Hausdorff M} $\equiv$ {comm. $C^* -$ algebra $C_0(M)$}
- {$C^* -$ algebra} $\equiv$ {ops. on a Hilbert space}
- differential structure (on closed, oriented Riemannian spin^c manifolds): the Dirac operator D acting on $L^2(M, S)$ determines the geodesic distance on M

[Connes, 2013]

The metric and spin structure of a closed orientable Riemannian spin^c - manifold M can be encoded into a system

$$(C^\infty(M), L^2(M,S), D).$$

For $\dim M$ *even, the are two important operators in the associated Clifford algebra* Cl*:*

- $\mathbb{Z}/2\mathbb{Z}$ *- grading* γ^5,
- *charge conjugation operator* C,

that satisfy few compatibility conditions among themselves an the Dirac operator.

Spectral triple

$$(A, H, D, \gamma, J)$$

- A - $*$ - algebra represented (in a faithful way) on a Hilbert space H
- $\mathbb{Z}/2\mathbb{Z}$ - grading γ on H s.th. $\gamma = \gamma^*$ and $[\gamma, A] = 0$
- antilinear isometry J
- selfadjoint operator D (with a compact resolvent)
- ... see e.g. [Lizzi, 2018]

Why spectral triples?

- Almost-commutative geometry ($C^\infty(M) \otimes A_f$) can explain the origin of the Higgs mass in the Standard Model. Furthermore, from the spectral action one can derive the form of an action for this model, and find relations between appropriate parameters. [Chamseddine-Connes-Marcoli, 2015]
- There are known successes in the derivation of Hilbert-Einstein action in Connes-Lott cosmology [Ackermann, 1996][Kastler,1995][Kalau-Walze, 1995]

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Spectral action method

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- Define $\Omega_D := \text{span}\{a[D, b] \ : a, b \in A\}$
- Consider $D_{\mathcal{A}} = D + \mathcal{A} + J\mathcal{A}J^{-1}$ for $\mathcal{A} \in \Omega_D$

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$$\mathcal{S} = \text{Tr} f\left(\frac{D_{\mathcal{A}}}{\Lambda}\right),$$

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- The first term should give an effective action of the theory

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We have to compute

$$\mathcal{S}(D) = \Lambda^2 \mathrm{Wres}(D_{\mathcal{A}}^{-4}) + \mathrm{Wres}(D_{\mathcal{A}}^{-2}),$$

where the Wodzicki residue for a pseudodifferential operator P is given by

$$\mathrm{Wres}(P) = (2\pi)^{-\dim M} \int_{\|\xi\|=1} \mathrm{tr}(\sigma(P)(\xi))d\xi,$$

where $\sigma(P)$ is the principal symbol of P.

Spectral triple for bimetric gravity?

First candidate

- We propose to use modified almost-commutative approach with appropriate chosen finite spectral triples
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- Our first candidate: $A_F = \mathbb{C} \oplus \mathbb{C}$,
- but on the product geometry we use Dirac-like operator of the form

$$D = \begin{bmatrix} D_f & \gamma^5_M \phi \\ \gamma^5_M \phi^* & D_g \end{bmatrix},$$

where $\phi : M \to \mathbb{C}$.

The main task

Compute the spectral action and compare with bimetric gravity models

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Compute the spectral action and compare with bimetric gravity models

- We need to know how to compute appropriate integrals.
- Required integrals are not known in the literature (rational functions over higher spheres).
- We have to develop tools for computing them analytically and to express the solution in *an invariant way* - express in terms of traces/determinants of metrics f and g, or/and their contractions etc..
- Alternative approach: find relations between them, use symmetries and avoid calculations?
- Develop numerical methods?
- Determine only leading terms?

- Simplified models - toroidal FLRW metrics [Sitarz, 2019] - qualitative discussion of action and EOMs. Expected (and confirmed in some cases) fading oscillation terms between metrics f and g.
- Symmetries can eliminate tedious calculations - at this moment it is more or less clear which terms (and why) have to be zero from symmetry reasons.
- Some class of the rational integrals can be computed analytically (or at least prepared for numerical computations) using generalizations of known results [Othmama, 2011].
- The full result is still not known - it is still work in progress.

- We propose an alternative approach to the bimetric gravity models based on spectral geometry.
- It may provide an explanation of the origin of these models together with their parameters and the values of coefficients can be compared with observational data.
- It is still work in progress.

ご清聴ありがとうございました