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1 Introduction

My research is focused on proof theory, a subfield of mathematical logic. I investigate ordinal representation systems, which are the main tools in the ordinal analysis of mathematical theories, and which play an essential role in gauging the proof strength of axiom systems or particular mathematical theorems the comparative strength of which might be of interest. Besides contributing to the deeper understanding of classical ordinal representation systems, my main area of research is the investigation of representation systems derived from so-called elementary patterns of resemblance, which are of interest in their own right as they present combinatorial structures the properties of which give rise to mathematical theorems of great proof strength.

2 Activities and Findings

During the first year as an STA I have further developed the theory of ordinal arithmetical tools I have been using to analyze patterns of resemblance. As a result I was able to prove so-called Bachmann property of fundamental sequences for the corresponding classical ordinal notations. This property is crucial when relating ordinal limits of rising complexity, approximated by fundamental sequences, to hierarchies of fast growing number theoretic functions. Such hierarchies of fast growing functions (cf. Hardy hierarchy) are an essential tool when determining to which degree a mathematical theory is able to recognize computable number theoretic functions as (provably) computable. The findings just described will be published as accepted paper and presented at the conference *Computability in Europe 2024 – Twenty years of Theoretical and Practical Synergies* – to take place in Amsterdam. An extended version of the paper will shortly be submitted for journal publication.

3 Collaborations

Plans for collaboration with researchers from Ghent University (Belgium) and the University of Barcelona (Spain) will be discussed in detail this summer, since the findings described above enable the formulation of Goodstein principles of higher complexity.

4 Publications and other output

G. Wilken: *Fundamental sequences based on localization.* LNCS Proceedings of the 20th Conference on Computability in Europe (CiE): Twenty Years of Theoretical and Practical Synergies, Amsterdam 2024. To appear.