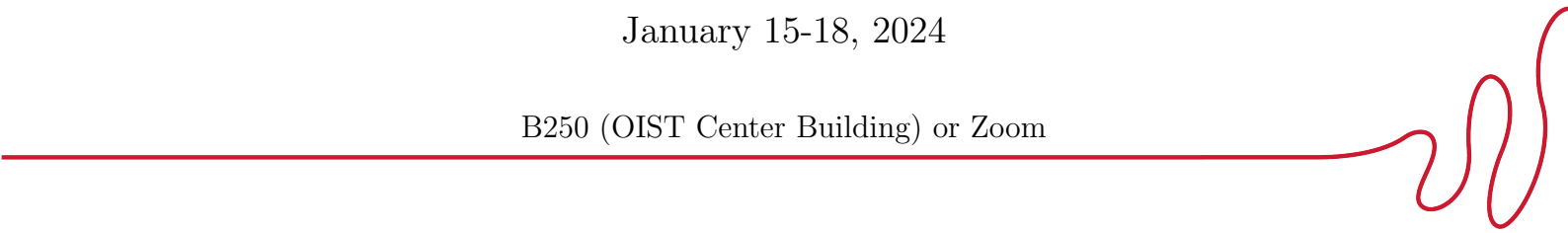
OIST Workshop

Geometric Aspects of Partial Differential Equations

January 15-18, 2024

B250 (OIST Center Building) or Zoom



OIST

OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

沖縄科学技術大学院大学

Aim

Nowadays an increasingly close connection has been developed between partial differential equations and geometry. The past years have witnessed applications of PDEs to a wide variety of geometry related problems arising in both pure mathematics and applied sciences such as isoperimetric problems, surface evolution, control theory, image processing, and optimal transport. On the other hand, geometric analysis also provides many novel techniques to help us understand fundamental properties of PDEs including convex structure, spectral information and asymptotic behavior.

The purpose of this workshop is to present recent progress and new trends in PDEs from geometric perspectives. It aims to bring together the world's leading experts and provide an international forum for researchers who are interested in this field. We plan to discuss various topics on PDEs with emphasis on geometric aspects including convexity of solutions, shape analysis, asymptotic behavior, eigenvalue problems, overdetermined problems, geometric functional inequalities, etc. With an interdisciplinary scope, this workshop also pays attention to numerous applications of recent PDE results to other related areas.

Venue

OIST B250 (Center Building)

Online participants can get a Zoom link via the following registration page:

`https://oist.zoom.us/meeting/register/tJcsf-urrzIjGtxJp830EpFRCd5PyNDn7S8Y`

Orgaizers

Kazuhiro Ishige (University of Tokyo, Graduate School of Mathematical Sciences)
Qing Liu (OIST, Geometric Partial Differential Equations Unit)
Paolo Salani (University of Florence, Department of Mathematics)

Schedule

(* online talks)

Monday, January 15

10:00-11:00 Rolando Magnanini (University of Florence)

New integral identities for torsional creep functions and applications

11:30-12:30 Megumi Sano (Hiroshima University)

Weighted Trudinger-Moser inequalities in the subcritical Sobolev spaces and their applications

14:00-15:00 Ben Weinkove (Northwestern University)

Evolution equations and convexity

15:30-16:30 Norihisa Ikoma (Keio University)

The mountain pass theorem for nonsmooth functionals and its application

17:00-18:00 Katie Gittins (Durham University)*

The heat content of polygonal domains

Tuesday, January 16

10:00-11:00 Pengfei Guan (McGill University)*

Constrained mean curvature flow and isoperimetric problem

11:30-12:30 Shinya Okabe (Tohoku University)

Dynamical approach to a generalized isoperimetric inequality

14:00-15:00 Cristina Trombetti (University of Napels Federico II)

Some isoperimetric estimates for Robin eigenvalues

15:30-16:30 Ryuichi Sato (Fukuoka University)

Existence of global-in-time solutions to a system of fully nonlinear parabolic equations

17:00-18:00 Karoly Boroczky (Alfréd Rényi Institute of Mathematics)*

Uniqueness when the L_p curvature is close to be a constant for $p \in [0, 1)$

Wednesday, January 17

10:00-11:00 Shigeru Sakaguchi (Tohoku University)

Symmetry in overdetermined obstacle problems

11:30-12:30 Xinan Ma (University of Science and Techonology of China)

Jerison-Lee identity semi-linear subelliptic equation on CR manifold

14:00-15:00 Hiroshi Iriyeh (Ibaraki University)*

A sharp estimate of the volume product of convex bodies by means of equipartition

15:30-16:30 Carlo Nitsch (University of Napels Federico II)

On the gradient rearrangement of a function: the BV case

17:00-18:00 Alessio Figalli (ETH Zurich)*

Generic regularity in obstacle problems

Thursday, January 18

9:30-10:30 Daniel Hauer (University of Sydney)

An extension problem for the logarithmic Laplacian

10:45-11:45 Xiaodan Zhou (OIST)

Discontinuous eikonal equations in metric measure spaces

Abstracts

New integral identities for torsional creep functions and applications

Rolando Magnanini

University of Florence

I shall present two general integral identities, which involve the Hessian of two solutions of the Poisson equation with constant source term and their Dirichlet and Neumann boundary values. Applications of the identities are: threshold optimal inequalities for the Serrin problem with locally constant boundary data; new rigidity theorems for the Serrin problem in simply and multiply connected domains; optimal geometric inequalities entailing quermass integrals; quantitative symmetry estimates for the reverse Serrin problem.

Weighted Trudinger-Moser inequalities in the subcritical Sobolev spaces and their applications

Megumi Sano

Hiroshima University

We study the boundedness, the optimality and the attainability of Trudinger-Moser type maximization problems in the subcritical radial Sobolev spaces. Our inequality converges to the original Trudinger-Moser inequality including the optimal exponent and the concentration limit in some sense. Finally, we consider applications of our inequality to elliptic and parabolic problems with exponential nonlinearity. This is a joint work with Masahiro Ikeda(Keio Univ./RIKEN) and Koichi Taniguchi(Tohoku Univ.).

Evolution equations and convexity

Ben Weinkove

Northwestern University

I will describe examples where convexity and concavity are not preserved in time, for the following evolution equations: the Stefan problem, the porous medium equation and the quasi-static droplet model. This is joint work with Albert Chau.

The mountain pass theorem for nonsmooth functionals and its application

Norihisa Ikoma

Keio University

The (symmetric) mountain pass theorem due to Ambrosetti and Rabinowitz is a powerful tool to show the existence of critical points of smooth functionals. Szulkin generalized the results for nonsmooth functionals under the Palais-Smale condition like Ambrosetti and Rabinowitz. On the other hand, when we apply the theorem in order to get a solution to nonlinear elliptic equations, it is sometimes difficult to verify that the corresponding functionals satisfy the Palais-Smale condition. To overcome this difficulty, Struwe exploited a monotone property of minimax values for a family of smooth functionals and proved the existence of Palais-Smale sequences with additional properties. This monotonicity trick was developed further by Jeanjean in the smooth setting. In this talk, we extend the monotonicity trick due to Struwe (and Jeanjean) to nonsmooth functionals as in Szulkin, and show the existence of bounded Palais-Smale sequence at the mountain pass level. As an application of this abstract result, we may obtain nontrivial solutions to the Born-Infeld equation with nonlinear terms. This talk is based on joint work with Jaeyoung Byeon (Korea Advanced Institute of Science and Technology), Andrea Malchiodi (Scuola Normale Superiore di Pisa) and Luciano Mari (University of Turin).

The heat content of polygonal domains

Katie Gittins

Durham University

Let $D \subset \mathbb{R}^2$ be a bounded set with polygonal boundary ∂D. We impose an initial temperature condition on $\mathbb{R}^2 \setminus \partial D$ and can also impose boundary conditions on the edges of ∂D, such as a Dirichlet (cooling) boundary condition.

In such a setting, it is natural to ask: how much heat is left inside D at time t? This quantity is the heat content of D. The small-time asymptotic expansions for the heat content of D encode information about the geometry of D and ∂D. Our goal is to explore how these expansions depend upon the geometry and on various combinations of initial and boundary conditions.

We first review some of the previously known results for the small-time asymptotic expansions for the heat content of D with certain initial and boundary conditions. We then present recent results for the case where D is contained in a larger set with polygonal boundary on which a Neumann (insulating) boundary condition is imposed. The latter is based on joint work with Sam Farrington (Durham University). Time-permitting, we may also discuss some geometric applications of these asymptotic expansions.

Constrained mean curvature flow and isoperimetric problem

Pengfei Guan

McGill University

A constrained mean curvature flow was introduced by Guan-Li in space forms as an effective path to the solution of isoperimetric problem in these spaces in 2015. This flow was extended to general warped product spaces by Guan-Li-Wang afterwards. Very recently, the flow was further generalized to Riemannian manifolds admitting conformal Killing fields by Li-Pan and Flynn-Reznikov. We will discuss the recent results and open problems associated to the constrained mean curvature flow.

Dynamical approach to a generalized isoperimetric inequality

Shinya Okabe

Tohoku University

We devote this talk to a dynamical approach to a generalized isoperimetric inequality on curves. Classically, the isoperimetric inequality states that the lower bound of the isoperimetric ratio $I(X)$ of the curve X, is 1, and the minimizer is a standard round circle. In this talk, we consider the following question: Fix $n > 1$. What is the largest class of maps in H^1 such that $I(X) \geq n$ for all X in the class? In this talk we give a new dynamical approach to the classical and the generalized isoperimetric inequality and give a partial answer to the above question.

Some isoperimetric estimates for Robin eigenvalues

Cristina Trombetti

University of Napels Federico II

In this seminar, I discuss the application of the web functions technique to estimate the first Robin eigenvalue with a negative parameter β. As is well known, in this scenario, a Faber-Krahn-type inequality does not exist. However, with some additional restrictions, a weaker form of Faber-Krahn still holds true, and here is where web functions play a role. It is intriguing to explore whether such techniques can be extended to a "non-flat" ambient space. In particular, in the seminar I will set up and solve the case of a sphere.

Existence of global-in-time solutions to a system of fully nonlinear parabolic equations

Ryuichi Sato

Fukuoka University

In this talk, we consider a system of nonlinear parabolic equations and give a sufficient condition to obtain the existence of global-in-time solutions. Moreover, we discuss the relation between ellipticity and global existence of global-in-time solutions. This talk is based on a joint work with Takahiro Kosugi.

Uniqueness when the L_p curvature is close to be a constant for $p \in [0, 1)$

Karoly Boroczky

Alfréd Rényi Institute of Mathematics

For $p = 0$, it was Firey who initiated the study the uniqueness of solution of the L_p-Minkowski problem on S^n, $n \geq 2$ in 1974, and the scope of the study was extended to all real p by Lutwak in 1990s where the $p = 1$ case is the classical one. For $p > 1$, the uniqueness of the solution was established by Chou and Wang, and also by Hug, Lutwak, Yang and Zhang around 2006. On the one hand, uniqueness of the solution does not hold if $p < 1$ in general as it was shown by Chen, Li, Zhu, and on the other hand, verifying Firey's conjecture, the solution is unique (corresponding to a ball) if the L_p curvature is constant and $-n - 1 < p < 1$ according to the celebrated result by Brendle, Choi, Daskalopoulos.

The natural question is whether the solution is unique if the L_p curvature is close to be a constant and $-n-1 < p < 1$; namely, if the L_p curvature is C^α close to be a constant for some $\alpha \in (0, 1)$. In the case $p = 0$ and $n = 2$, this is verified by the beautiful paper by Chen, Feng, Liu. The talk discusses the novel uniqueness results in the cases $p = 0$, $n = 3$, and $p \in (0, 1)$, $n \geq 2$.

There are various side results of our investigations that might be of independent interest. One of them says that if the L_p curvature is between λ^{-1} and λ for $\lambda > 1$, then the solution is bounded by $C(\lambda, p, n)$ in the cases $p = 0$, $n = 3$ and $p \in (0, 1)$, $n \geq 2$. Another related problem is the characterization of the solution of the L_p-Minkowski problem on S^n if the solution is concentrated to a lower dimensional subspace.

This is joint work with Christos Saroglou.

Symmetry in overdetermined obstacle problems

Shigeru Sakaguchi
Tohoku University

This talk is based on a joint work with Nicola De Nitti. We prove symmetry results for two types of overdetermined obstacle problems: one is Serrin-type and the other concerns some two-phase overdetermined problems where the interface is a level surface of the solutions. The former relies only on the comparison principle and it gives also stability results. The latter relies on Serrin's method of moving planes where the transmission conditions on the interface play a key role.

Jerison-Lee identity semi-linear subelliptic equation on CR manifold

Xinan Ma
University of Science and Techonology of China

D. Jerison and J. M. Lee (JAMS 1988) found a three-dimensional family of differential identities for critical exponent equation on Heisenberg group H^n by using computer. They also care about whether there exists a theoretical framework that would predict the existence and the structure of such formulae. In this talk we answer the question with the help of dimensional conservation and invariant tensors. Then we generalize the Jerison-Lee identities in Cauchy-Riemann(CR) manifold on subelliptic equations. Several new types of identities on CR manifold are found and these identities are used to get the rigidity result for a class of CR Lane-Emden equation in subcritical case, rigidity means that the subelliptic equation has no other solution than some constant at least when a parameter is in a certain range. The rigidity result also deduces the sharp Folland-Stein inequality on closed CR manifold. This is the joint work with Qianzhong OU, AND Tian WU.

A sharp estimate of the volume product of convex bodies by means of equipartition

Hiroshi Iriyeh
Ibaraki University

Mahler's conjecture is one of the classical open problem in the area of convex geometry. It states that for a centrally symmetric convex body K in the n-dimensional Euclidean space, the product of the volume of K and that of the polar body is greater than or equal to $4^n/n!$. It was solved for the two dimensional case in 1938 and the three dimensional case in 2020. In this talk, we will explain the idea of their proof. One of the key ideas is "equipartition" of the convex body K. We also discuss, by using that method, partial results of the non-symmetric version of Mahler's conjecture in the three dimensional. The talk is based on a joint work with Masataka Shibata.

On the gradient rearrangement of a function: the BV case

Carlo Nitsch

University of Napels Federico II

Although the essential features of gradient rearrangement have been known for several decades, they remain relatively unexplored and underestimated. Applications include classic Faber-Krahn inequality for the p-Laplacian eigenvalue and comparison results for solutions to the Hamilton-Jacobi problem. This seminar extends the technique to functions with bounded variation, showing a wider range of applicability. In particular, gradient rearrangement goes beyond Polya Szego limitation of being effective only for functions vanishing at the boundary, paving the way for new developments.

Generic regularity in obstacle problems

Alessio Figalli

ETH Zürich

The classical obstacle problem involves determining the equilibrium state of an elastic membrane that is restricted to remain above a predetermined obstacle. By classical results of Caffarelli, the 'free boundary'—the demarcation line where the obstacle and membrane meet— is smooth outside a set of singular points. Also, explicit examples show that the singular set could be, in general, as large as the regular set. This talk aims to introduce this beautiful problem and describe some classical and recent results on the regularity of the free boundary.

An extension problem for the logarithmic Laplacian

Daniel Hauer

University of Sydney

Motivated by the fact that for positive s tending to zero the fractional Laplacian converges to the identity and for s tending to 1 to the local Laplacian, Chen and Weth [Comm. PDE 44 (11), 2019] introduced the logarithmic Laplacian as the first variation of the fractional Laplacian at $s = 0$. In particular, they showed that the logarithmic Laplacian admits an integral representation and can, alternatively, be defined via the Fourier-transform with a logarithmic symbol. The logarithmic Laplacian turned out to be an important tool in various mathematical problems; for instance, to determine the asymptotic behavior as the order s tends to zero of the eigenvalues of the fractional Laplacian equipped with Dirichlet boundary conditions (see, e.g., [Feulefack, Jarohs, Weth, J. Fourier Anal. Appl. 28(2), no. 18, 2022]), in the study of the logarithmic Sobolev inequality on the unit sphere [Frank, König, Tang, Adv. Math. 375, 2020], or in the geometric context of the 0-fractional perimeter, see [De Luca, Novaga, Ponsiglione, ANN SCUOLA NORM-SCI 22(4), 2021]. Caffarelli and Silvestre [Comm. Part. Diff. Eq. 32(7-9), (2007)] showed that for every sufficiently regular u, the values of the fractional Laplacian at u can be obtained by the co-normal derivative of an s-harmonic function w_u on the half-space (by adding one more space dimension) with Dirichlet boundary data u. This extensionproblem represents the

important link between an integro-differential operator (the nonlocal fractional Laplacian) and a local 2nd-order differential operator. This property has been used frequently in the past in many problems governed by the fractional Laplacian.

In this talk, I will present an extension problem for the logarithmic Laplacian, which shows that this nonlocal integro-differential operator can be linked with a local Poisson problem on the (upper) half-space, or alternatively (after reflection) in a space of one more dimension. As an application of this extension property, I show that the logarithmic Laplacian admits a unique continuous property.

The results presented here were obtained in joint work with Huyuan Chen (Jiangxi Normal University, China) and Tobias Weth (Goethe-Universität Frankfurt, Germany)

Discontinuous eikonal equations in metric measure spaces

Xiaodan Zhou

Okinawa Institute of Science and Technology

In this talk, we focus on the eikonal equation in metric measure spaces, where the inhomogeneous term is allowed to be discontinuous, unbounded and merely p–integrable in the domain with a finite p. Exploiting the notion of Monge solutions in our setting, we establish uniqueness and existence results for the associated Dirichlet boundary problem. The key in our approach is to adopt a new metric, based on the optimal control interpretation, which integrates the discontinuous term and converts the eikonal equation to a standard continuous form. We also discuss the Hölder continuity of the unique solution with respect to the original metric under regularity assumptions on the space and the inhomogeneous term.