Stochastic tug-of-war type games and partial differential equations

Mikko Parviainen

University of Jyväskylä

The fundamental works of Doob, Hunt, Itô, Kakutani, Kolmogorov, Lévy and many others have shown a profound and powerful connection between the classical linear partial differential equations and probability. A well known example is the interplay between the Laplace equation and the Brownian motion. There is an interplay, discovered more recently, also in the nonlinear case. In this talk, we consider stochastic zero sum games, regularity of value functions, and related nonlinear PDEs. A particular example of such a stochastic game is the tug-of-war with noise. Passing to the limit with the step size in this game gives rise to the normalized or game theoretic p-Laplace and p-parabolic equations. As an application, the connection between the games and PDEs allows us to develop methods to deal with nonlinear PDEs by playing games. For example, we obtain a proof for regularity of p-harmonic functions by playing carefully chosen strategies. We discuss several recent approaches to the regularity in this context.