First order mean field games on networks

Claudio Marchi

University of Padova

This talk is devoted to finite horizon deterministic mean field games in which the state space is a network. The agents control their velocity, and when they occupy a vertex, they can enter into any incident edge. The running and terminal costs are assumed to be continuous in each edge but not necessarily globally continuous on the network. A Lagrangian formulation of the game is studied. It leads to relaxed equilibria consisting of probability measures on admissible trajectories. We establish the existence of such relaxed equilibria. To any relaxed equilibrium corresponds a mild solution of the mean field game, i.e. a pair (u, m) made of the value function u of a related optimal control problem, and a family $m = (m(t))_t$ of probability measures on the network. Given m, the value function u is characterized by a Hamilton-Jacobi problem on the network. Regularity properties of u and a weak form of a Fokker-Planck equation satisfied by m are investigated.