OIST Conference

Sub-Riemannian Analysis and Geometry

January 14-16, 2025

C209 (OIST Center Building) or by Zoom

OIST

OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

沖縄科学技術大学院大学

Aim

Sub-Riemannian analysis and geometry have emerged as vibrant areas of study, with profound implications across various branches of mathematics and beyond. The underlying structures and geometric properties of Carnot-Caratheodory spaces pose intriguing mathematical challenges. This conference serves as a platform for mathematicians and researchers from around the world to converge and explore the rich landscape of mathematical theory alongside many important applications.

Our program encompasses a broad spectrum of topics including various aspects of geometric analysis, partial differential equations, geometric and functional inequalities, control theory, optimal transport, and stochastic processes on sub-Riemannian manifolds. This conference aims to bring together experts, promote collaboration and inspire fresh perspectives on diverse subjects within the realm of sub-Riemannian settings.

Venue

OIST C209 (Center Building)

Online participants can get a Zoom link via the following registration page:

`https://oist.zoom.us/meeting/register/tJYscuChpjovGtYMtVxMQW7Wtp4Ov3paMclX`

Organizers

Qing Liu (OIST, Geometric Partial Differential Equations Unit)
Ye Zhang (OIST, Analysis on Metric Spaces Unit)
Xiaodan Zhou (OIST, Analysis on Metric Spaces Unit)

Schedule (updated)

Tuesday, January 14

10:00-11:00 Maria Gordina (University of Connecticut)

Dimension-independent functional inequalities on sub-Riemannian manifolds

11:20-12:20 Jeremy Tyson (University of Illinois Urbana-Champaign)

The H-type deviation of a step two Carnot group

14:00-15:00 Davide Barilari (University of Padova)

Refined Strichartz Estimates for sub-Laplacians in Heisenberg and H-type groups

15:10-16:10 Andrea Pinamonti (University of Trento)

Some regularity results for balance laws and applications to the Heisenberg group

16:30-17:10 Ye Zhang (Okinawa Institute of Science and Technology)

The heat flow approach to the Prékopa–Leindler inequality

17:20-18:00 Kotaro Hisa (University of Tokyo)

Existence of solutions semilinear parabolic equations with singular initial data in the Heisenberg group

Wednesday, January 15

10:00-11:00 Bianca Stroffolini (University of Naples Federico II)

Natural p-means for the p-Laplacian in the Euclidean space and in the Heisenberg Group

11:20-12:20 Xiao Zhong (University of Helsinki)

Dimer models and Beltrami equation

14:00-15:00 Luca Rizzi (International School for Advanced Studies (SISSA))

Measure contraction properties for sub-Riemannian structures beyond step 2

15:20-16:20 Giovanna Citti (University of Bologna)

A representation formula on the characteristic plane of the Heisenberg group

16:40-17:20 Kenshiro Tashiro (Okinawa Institute of Science and Technology)

RCD condition for spaces with sub-Riemannian type singularities

Thursday, January 16

10:00-11:00 Enrico Le Donne (University of Fribourg)

Variational curve flows in Carnot groups

11:20-12:20 Luca Capogna (Smith College)

Short time existence for curve shortening flows in Carnot groups

Abstracts

Dimension-independent functional inequalities on sub-Riemannian manifolds

Maria Gordina

University of Connecticut

The talk will review recent results on gradient estimates, log Sobolev inequalities, reverse Poincare and log Sobolev inequalities on a class of sub-Riemannian manifolds. As for many of such setting curvature bounds are not available, we use different techniques including tensorization and taking quotients. Joint work with F. Baudoin, L. Luo and R. Sarkar.

The H-type deviation of a step two Carnot group

Jeremy Tyson

University of Illinois Urbana-Champaign

By a classical result of Folland, the fundamental solution for the Laplacian in the sub-Riemannian Heisenberg group is proportional to N^{2-Q} where N denotes Korányi's homogeneous norm and Q is the homogeneous dimension. Capogna, Danielli and Garofalo later observed that an analogous statement is true for the p-Laplacian for every $1 < p < \infty$ and in every H-type group in the sense of Kaplan, namely, the fundamental solution of the p-Laplacian is a multiple of $N^{(p-Q)/(p-1)}$ (or $\log(1/N)$ if $p = Q$). Balogh and Tyson introduced the class of polarizable groups as the largest class of Carnot groups equipped with a similar one-parameter family of fundamental solutions for the p-Laplacians, all defined in terms of a fixed and common homogeneous norm. Such groups can also be characterized as precisely those Carnot groups which admit a polar coordinate decomposition using horizontal radial curves. We conjecture that the only polarizable groups are Kaplan's H-type groups. Towards this end, we seek to recognize the subclass of H-type groups geometrically within the class of all step two Carnot groups (equipped with a coherent Riemannian metric). We introduce and study a numeric measurement, the H-type deviation, which quantifies the degree to which a step two Carnot group fails to be of H-type. As an application, we give several characterizations for H-type groups in terms of analytic identities satisfied by Folland's norm. The H-type deviation of a step two Carnot group turns out to be closely related to the notion of Métivier order, which originates in the study of the Sard problem for abnormal curves. Part of this talk is based on joint work with Luca Nalon (University of Fribourg).

Refined Strichartz Estimates for sub-Laplacians in Heisenberg and H-type groups

Davide Barilari

University of Padova

We obtain refined (non end-point) Strichartz estimates for the sub-Riemannian Schrödinger equation on H-type Carnot groups using Fourier restriction theorems. In particular, we extend the previously known Strichartz estimates obtained for the Heisenberg group also to non radial initial data. Our new argument is based on estimates for the spectral projectors for sub-Laplacians. The same arguments permits to obtain refined Strichartz estimates also for the wave equation on H-type groups. This is a joint work with Steven Flynn.

Some regularity results for balance laws and applications to the Heisenberg group

Andrea Pinamonti

University of Trento

In this talk we prove Hölder regularity of any continuous solution u to a $1d$ scalar balance law, when the source term is bounded and the flux is nonlinear of order $p \in \mathbb{N}$ with $p \geq 2$. Finally, we apply a refinement of the previous result to provide a new proof of the Rademacher theorem for intrinsic Lipschitz functions in the first Heisenberg group. The talk is based on a joint paper with L. Caravenna and E. Marconi.

The heat flow approach to the Prékopa–Leindler inequality

Ye Zhang

Okinawa Institute of Science and Technology

In this talk I will present a new heat equation based proof of the Prékopa–Leindler inequality in the recent paper of K. Ishige, Q. Liu and P. Salani. Using this approach, I will derive an horizontal version of Prékopa–Leindler inequality on the first Heisenberg group.

RCD condition for spaces with sub-Riemannian type singularities

Kenshiro Tashiro

Okinawa Institute of Science and Technology

Sub-Riemannian manifolds are known as typical examples of metric measure spaces that fail to be CD spaces, under some properness assumptions on their sub-Riemannian structures. However, it has been observed that a certain class of singular Riemannian manifolds with sub-Riemannian points, such as Grushin half planes, admits RCD condition. In this talk, we present several examples of metric measure spaces with such sub-Riemannian type singularities that satisfy the RCD condition.

Existence of solutions semilinear parabolic equations with singular initial data in the Heisenberg group

Kotaro Hisa

University of Tokyo

In this talk we obtain necessary conditions and sufficient conditions on the initial data for the solvability of semilinear heat equations with power nonlinearities in the Heisenberg group.

Using these conditions, we can identify the critical exponent of nonlinearities for the global-in-time solvability of the Cauchy problem (so-called the Fujita-exponent), and identify the optimal strength of the singularity of the initial data for the local-in-time solvability.

This talk is based on the joint work with Prof. The Anh Bui (Macquarie University).

Natural p-means for the p-Laplacian in the Euclidean space and in the Heisenberg Group

Bianca Stroffolini

University of Naples Federico II

I will review the notion of asymptotic mean value formulas for the p-Laplacian, whose main contributions are due to J. Manfredi, Julio Rossi, Mikko Parviainen and Felix Del Teso. Next, using the notion of natural p-means introduced by Ishiwata, Magnanini, and Wadade in 2017, I will prove uniform convergence in Lipschitz domains in R^n (and in $C^{1,1}$ domains in the Heisenberg group) of approximations to p-harmonic functions. Finally, I will consider an extended notion, the generalized exterior corkscrew condition, under which the boundary continuity of semidiscrete schemes holds.

This talk is based on some joint works with Juan Manfredi (Pittsburgh), Andràs Domokos and Diego Ricciotti (Sacramento).

Dimer models and Beltrami equation

Xiao Zhong

University of Helsinki

Scaling limits of random tilings present surprising geometric features under suitable boundary conditions and produce deterministic and disordered (also called frozen and liquid) limit configurations with interesting geometric properties. This talk will show how methods from PDEs, in particular, the Beltrami equation, one can understand the delicate geometry of the boundary between the deterministic/frozen and random/liquid phases. It turns out, for instance, that the geometry of the limit shapes within all dimer models is universal, i.e. independent of the specific model. The talk is based on joint work with K. Astala, E. Duse and I. Prause.

Measure contraction properties for sub-Riemannian structures beyond step 2

Luca Rizzi

International School for Advanced Studies (SISSA)

The measure contraction property (MCP in short, introduced by Ohta and Sturm) is the only "classical" synthetic Ricci bound that can be satisfied by sub-Riemannian structures (that are not Riemannian). Thanks to several contributions in the past 15 years, the qualitative picture about the validity of this property is well-understood for self-similar spaces (in particular, Carnot groups) without non-trivial Goh abnormals. We present a work-in-progress, in collaboration with Samuel Borza, showing that MCP can fail in the presence of non-trivial Goh abnormals. In particular we prove that any Carnot group admitting a suitable quotient to the flat Martinet structure does not satisfy any MCP. An explicit algebraic characterization is provided. In particular, the Engel group, the Cartan group (and several other explicit structures of step greater than 3) do not satisfy any MCP. To prove these result, we introduce a weaker version of the essential non-branching assumption that is implied by the validity of the minimizing Sard conjecture, that has independent interest.

A representation formula on the characteristic plane of the Heisenberg group

Giovanna Citti

University of Bologna

The aim of this talk is to study a Laplacian type operator on the characteristic plane in the Heisenberg group H2. Indeed, we introduce a conformal version of the Laplacian on the plane and we prove that the distance induced by the immersion in the ambient space is a good approximation of its fundamental solution. We provide in particular a representation formula for smooth functions in terms of the gradient of the function and the gradient of the approximated fundamental solution. The result has been obtained in collaboration with Baldi and Cupini.

Spectral gaps via small fluctuations

Marco Carfagnini

University of Melbourne

In this talk we will discuss spectral gaps of second order differential operators and their connection to limit laws such as small deviations and Chung's laws of the iterated logarithm. The main focus is on hypoelliptic diffusions such as horizontal Brownian motions on Carnot groups. If time permits, we will discuss spectral properties and existence of spectral gaps on general Dirichlet metric measure spaces.This talk is based on joint works with Maria (Masha) Gordina and Alexander (Sasha) Teplyaev.

Variational curve flows in Carnot groups

Enrico Le Donne
University of Fribourg

Starting with some curve in a Carnot group, we aim to flow the curve to shorten it while keeping its endpoints fixed. We consider Carnot groups equipped with sub-Riemannian distances, and seek to shorten both length and energy. Geodesics will be stationary points of the flow.

As expected, abnormal curves will be obstructions to the theory, and in fact, they will cause the PDE system defining the flow to be discontinuous. However, I will discuss the system's equation and a few properties. These results are in collaboration with L. Capogna (Smith), who will then discuss the short-time existence when one starts with a non-abnormal curve.

Short time existence for curve shortening flows in Carnot groups

Luca Capogna
Smith College

This talk is the follow up of Enrico Le Donne's seminar on variational curve flows in Carnot groups and reports on ongoing joint work with him. We will describe short time existence theorems for the non-local parabolic PDE that describe harmonic map heat flow and for the curvature flow of regular curves in Carnot groups.