OIST Conference

Geometric PDEs and Applications

January 16-18, 2023

C209 (OIST Center Building) or Zoom

OIST

OKINAWA INSTITUTE OF SCIENCE AND TECHNOLOGY GRADUATE UNIVERSITY

沖縄科学技術大学院大学

Aim

The past years have witnessed cutting-edge applications of geometry-related partial differential equations in material sciences, image processing, control theory, optimal transport and many other fields. Motivated by these new challenges, novel techniques have been developed for solving various nonlinear PDEs and understanding their analytic and geometric properties.

This conference aims to bring together top mathematicians who actively contribute to research on related topics including fully nonlinear equations, calculus of variations, convex analysis, shape analysis, asymptotic behavior, geometric functional inequalities and so on. It will provide an opportunity to broaden our knowledge on a large variety of subjects in analysis and geometry as well as many new applications.

In addition to the OIST funds, this conference is also partially supported by KAKENHI from the Japan Society for the Promotion of Science (JSPS). The support from JSPS is gratefully acknowledged.

Venue

OIST C209 (Center Building)

Online participants can get a Zoom link via the following registration page:

`https://oist.zoom.us/meeting/register/tJ0tc-GrqDkjEtUnb4OGVQjxkTSuHs1Ip3Np`

Orgaizers

Qing Liu (OIST, Geometric Partial Differential Equations Unit)
Hiroyoshi Mitake (University of Tokyo, Graduate School of Mathematical Sciences)
Xiaodan Zhou (OIST, Analysis on Metric Spaces Unit)

Schedule

Monday, January 16

10:00-11:00 Hung Tran (University of Wisconsin, Madison)

Periodic homogenization of Hamilton-Jacobi equations: some recent progress

11:30-12:30 Yifeng Yu (University of California, Irvine)

Existence of effective burning velocity in cellular flow for curvature G-equation

14:00-15:00 Goro Akagi (Tohoku University)

Rate of convergence to asymptotic profiles for fast diffusion on domains via energy methods

15:10-16:10 Tianling Jin (Hong Kong University of Science and Technology)

Optimal regularity and fine asymptotics for the porous medium equation in bounded domains

16:30-17:30 Shuntaro Tsubouchi (University of Tokyo)

Continuous differentiability across a facet; gradient regularity results for very singular elliptic problems

Tuesday, January 17

10:00-11:00 Inwon Kim (University of California, Los Angeles)

Tumor growth with nutrients: regularity of interface

11:30-12:30 Adam Oberman (McGill University)

Mathematical aspects of deep learning

14:00-15:00 Olivier Ley (IRMAR, INSA-Rennes)

Some uniqueness results for the motion by mean curvature of entire graphs

15:10-16:10 Norbert Pozar (Kanazawa University)

Continuum limit of dislocations with annihilation in one dimension

16:30-17:30 Shuhei Kitano (Waseda University)

$W^{\sigma,p}$ estimates for fully nonlinear integral equations

Wednesday, January 18

9:00-10:00 Julio Rossi (University of Buenos Aires)

Asymptotic mean value properties for nonlinear partial differential equations

10:10-11:10 Jiwoong Jang (University of Wisconsin, Madison)

Level-set forced mean curvature flow with the Neumann boundary condition

11:30-12:30 Erbol Zhanpeisov (Okinawa Institute of Science and Technology)

Existence of solutions for fractional semilinear parabolic equations in Besov-Morrey spaces

Abstracts

Periodic homogenization of Hamilton-Jacobi equations: some recent progress

Hung Tran

University of Wisconsin, Madison

I first give a quick introduction to front propagations, Hamilton-Jacobi equations, and homogenization theory. I will then show that the optimal rate of convergence is $O(\varepsilon)$ in the convex setting and some nonconvex cases. I will also mention finer results on the effective fronts in two dimensions. Connections to stable norms in Riemannian geometry will also be made. Based on various joints work with W. Jing and Y. Yu.

Existence of effective burning velocity in cellular flow for curvature G-equation

Yifeng Yu

University of California, Irvine

G-equation is a popular level set model in turbulent combustion, and becomes an advective mean curvature type evolution equation when the curvature effect is considered:

$$G_t + \left(1 - d\,\mathrm{div}\frac{DG}{|DG|}\right)_+ |DG| + V(x)\cdot DG = 0.$$

In this talk, I will show the existence of effective burning velocity under the above curvature G-equation model when V is a two dimensional cellular flow. The non-coercivity here is one of the major difficulties in homogenization of mean curvature type equations. To overcome that, we introduce a new approach that combines PDE methods with a dynamical analysis of the Kohn-Serfaty deterministic game characterization of the curvature G-equation based on the special structure of the cellular flow. This is a joint with Hongwei Gao, Ziang Long and Jack Xin.

Rate of convergence to asymptotic profiles for fast diffusion on domains via energy methods

Goro Akagi

Tohoku University

This talk is concerned with the Cauchy-Dirichlet problem for fast diffusion equations on bounded domains. It is well known that every (possibly sign-changing) weak solution vanishes in finite time at the unique power rate, and therefore, asymptotic profiles for such vanishing solutions are defined as a limit of rescaled solutions, which solve the Cauchy-Dirichlet problem for a fast diffusion equation with a blow-up reaction. Asymptotic profiles are characterized as nontrivial equilibria of the rescaled problem (see pioneer works of Berryman and Holland in 1980s and subsequent results for qualitative results). Recently, Bonforte and Figalli (CPAM, 2021) established an important quantitative result of the convergence of rescaled solutions to nondegenerate positive asymptotic profiles. More precisely, they proved an exponential convergence of nonnegative rescaled solutions to nondegenerate positive asymptotic profiles in a weighted L^2 space with a *sharp rate* (in view of some linearized analysis) by developing a *nonlinear entropy method*. In this talk, we shall develop a different approach to prove exponential convergence with rates for nondegenerate asymptotic profiles with definite or changing signs. In particular, if we restrict ourselves to nondegenerate positive asymptotic profiles, we shall exhibit an H^1_0 convergence with the sharp rate. Our method of proofs is based on an energy method rather than entropic one, and a key ingredient is a quantitative gradient inequality established based on an eigenvalue problem with weights, which was already introduced by Bonforte and Figalli and also plays a crucial role in our analysis.

Optimal regularity and fine asymptotics for the porous medium equation in bounded domains

Tianling Jin

Hong Kong University of Science and Technology

We prove global Holder gradient estimates for bounded positive weak solutions of porous medium equations and fast diffusion equations in smooth bounded domains with homogeneous Dirichlet boundary condition. This allows us to establish their optimal global regularity and finer asymptotics. This talk is based on the joint works with Xavier Ros-Oton and Jingang Xiong.

Continuous differentiability across a facet; gradient regularity results for very singular elliptic problems

Shuntaro Tsubouchi

University of Tokyo

This talk is concerned with C^1-regularity for weak solutions to very singular elliptic equations or systems involving both one-Laplacian and p-Laplacian. The main difficulty is that these elliptic problems become no longer uniformly elliptic near a facet, a place where a gradient vanishes. Continuity of derivatives even across a facet was first established by the speaker and Prof. Yoshikazu Giga, in a special case where a weak solution is both scalar-valued and convex. Without these assumptions, the speaker recently found it possible to prove continuous differentiability of weak solutions. The aim of this talk is to explain how continuity of a gradient is demonstrated.

Tumor growth with nutrients: regularity of interface

Inwon Kim

University of California, Los Angeles

We will study a tumor growth model with nutrients. The model presents dynamic patch solutions due to the contact inhibition among the tumor cells. We show that when the nutrients do not diffuse and the cells do not die, the tumor density exhibits regularizing dynamics. These results are in sharp contrast to the models either with nutrient diffusion or with death rate in tumor cells.

Mathematical aspects of deep learning

Adam Oberman

McGill University

In this introductory talk, I will give a mathematical perspective on shallow and deep learning, for an audience with a background in PDEs.

Some uniqueness results for the motion by mean curvature of entire graphs

Olivier Ley
IRMAR, INSA-Rennes

I will present a review of old results obtained with G. Barles, S. Biton, M. Bourgoing, P. Cardaliaguet and E. Chasseigne about uniqueness of solutions for the mean curvature equation of entire graphs. The motivation come from an intriguing result of Ecker and Huisken (1991) where existence of a global in time smooth solution of the mean curvature equation of entire graphs was proven for every locally Lipschitz initial data, regardless of the growth of the data. In our works, we use different approaches including geometrical and analytical tools, which bring partial results of uniqueness. However, up to my knowledge, the most general result is still open.

Continuum limit of dislocations with annihilation in one dimension

Norbert Pozar
Kanazawa University

In this talk I discuss the proof of the many-particle limit for a system of particles in one dimension. The particles carry a signed charge and interact via a logarithmic potential. When two particles with opposite charges meet, they annihilate and are removed from the system. This serves as a simplified model of dislocation dynamics in a crystalline lattice.

This system can be equivalently described by a Hamilton-Jacobi-type level set formulation with nonlocal interaction of a lattice of level sets. By carefully analyzing the ODE system near the annihilation points and establishing comparison principle for the both the particle level set formulation and the continuous limit problem, we are able to generalize the convergence proof in the earlier work of Forcadel, Imbert and Monneau (2009) to the singular interactions and annihilation of multiple particles.

This talk is based on joint work with Mark Peletier (TU Eindhoven) and Patrick van Meurs (Kanazawa Uni.).

$W^{\sigma,p}$ estimates for fully nonlinear integral equations

Shuhei Kitano
Waseda University

In this talk, we will consider several regularity results for viscosity solutions of fully nonlinear integral equations. For fully nonlinear second-order partial differential equations, Caffarelli established three regularity estimates: $C^{2,\alpha}$ estimates, $C^{1,\alpha}$ and $W^{2,p}$ in the early 1980s. In the case of integral equations, the analogs of the first two estimates have already been proved by Serra (2015) and Caffarelli-Silvestre (2011) respectively, whereas the $W^{\sigma,p}$ estimate for integral equations is our new result. The key is to analyze Riesz potentials of solutions for integral equations, which are also solutions of certain second-order partial differential equations.

Asymptotic mean value properties for nonlinear partial differential equations

Julio Rossi

University of Buenos Aires

In recent years there has been an increasing interest in whether a mean value property, known to characterize harmonic functions, can be extended in some weak form to solutions of nonlinear equations. This question has been partially motivated by the surprising connection between Random Tug-of-War games and the normalized $p-$Laplacian discovered some years ago, where a nonlinear asymptotic mean value property for solutions of a PDE is related to a dynamic programming principle for an appropriate game.

Our goal in this talk is to show that an asymptotic nonlinear mean value formula characterizes weak solutions (in the viscosity sense) for the wide class of partial differential equations, including eigenvalues of the Hessian and the classical Monge-Ampère equation.

Joint work with: P. Blanc (Jyvaskyla, Finland), F. Charro (Detroit, USA), C. Esteve (Bilbao, Spain) and J.J. Manfredi (Pittsburgh, USA).

Level-set forced mean curvature flow with the Neumann boundary condition

Jiwoong Jang

University of Wisconsin, Madison

In this talk, we discuss a forced mean curvature flow with the right angle condition to the boundary. We show that a sufficient condition on the forcing term can be imposed in terms of the curvature of the boundary to obtain gradient estimates global in time. Also, we obtain gradient estimates local in time without the condition on the forcing term. Next, we prove the large-time convergence and compute the large-time profile in the radially symmetric setting. We also provide examples in which the solution is not globally Lipschitz, implying that the condition on the forcing term is sharp. This is joint work with Dohyun Kwon, Hiroyoshi Mitake and Hung V. Tran.

Existence of solutions for fractional semilinear parabolic equations in Besov-Morrey spaces

Erbol Zhanpeisov

Okinawa Institute of Science and Technology

We consider the local well-posedness of nonlinear parabolic equations including higher-order semilinear equations and viscous Hamilton-Jacobi equations in inhomogeneous Besov-Morrey spaces. Recently, Ishige, Kawakami, and Okabe (2022) obtained sufficient conditions for the existence of solutions to these equations for the initial data in Radon measures. In this talk, we establish the existence of solutions for a large class of initial data including measures other than Radon measures such as the derivative of Radon measures by introducing inhomogeneous Besov-Morrey spaces.