Combining Self-critical dynamics and Hebbian learning to explain utility of bursty dynamics in neural networks

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Summary

- Human brain is a self-critical system. In such systems events of any size are could occur. Large scale bursts of neural activity have been observed, yet their utility is unclear and in artificial networks, they are often suppressed as detrimental.
- In our work we demonstrate how such bursts could, in fact, enhance networks' performance when combined with Hebbian learning.
- We based our work on a previously developed algorithm of self-optimization, which combined random disruptions to systems' dynamics with Hebbian learning to allow the dynamical system to "optimize" its own attractors.

Sandpiles and self-criticality in the brain

Activity of neural populations both *in vitro* and *in vivo* is characterized by bursty dynamics. Bursts are almost simultaneous firings of many neurons, which are followed by a quiescent period [2]. These patterns have been observed in a variety of experimental conditions and reproduced in artificial networks. Such spatiotemporal dynamics are not unique to the brain. Distribution in which large-scale events are interspersed with periods of inactivity and minuscule disturbances is described with a power-law frequency spectrum. In such case events of all scales, curtailed only by the size of the system are probable. Such frequency spectra imply that the system is in a critical state. Some systems, the human brain included, are known to tune themselves to the critical state. This phenomenon is known as self-criticality. The simplest computational model of the self-critical system is the Abelian Sandpile model [1]. The dynamics of this cellular automata are governed by a few simple rules:

1. Each site (x_i, y_i) of a finite two-dimensional grid has an associated slope value z_i.
2. If slop exceeds a threshold value site collapses, transferring the slope to the adjacent sites.
3. Slope is incremented by adding grains of "sand" to randomly chosen sites, thus incrementing their slope by 1.

Once the system reaches a stable state (Fig. 2 C) a single grain could cause an avalanche – a cascade of sand movements, which causes significant disruption to slope distribution (Fig. 2 B).
In our model (Fig. 2 A) we use a combination of the Abelian Sandpile model and Hopfield neural network, to solve an optimization problem.

Self-optimization algorithm

Self-optimization algorithm endows a dynamical system with the ability to optimize its own connections [3]. Consider a Hopfield network with N binary neurons. Such network could be used to solve optimization problems, for example, MaxSat problem. Problem constraints are embedded into the connection matrix, and binary vector of neuron states **S,** describes a possible solution to the initial problem. This vector is initialized randomly and iteratively updated (I) until the stable state (fixed point attractor) is reached. Each state has an associated energy value (II), which describes the quality of the solution (Fig. 1 A and D). During self-optimization (Fig. 1 B and C) network is initialized with a random input vector and allowed to converge for a fixed number of timesteps. Then new random state is used to "reset" the model. Weights are augmented at each timestep using Hebbian learning (III). Original weights are saved for testing purposes, as they represent the optimization problem. After self-optimization, only two symmetrical attractors remain in the system, and they correspond to one of the best solutions to the original problem.
In the original work resets are random. However, as it is biologically implausible, in our model we use bursty dynamics of self-critical sandpile to induce system resets.

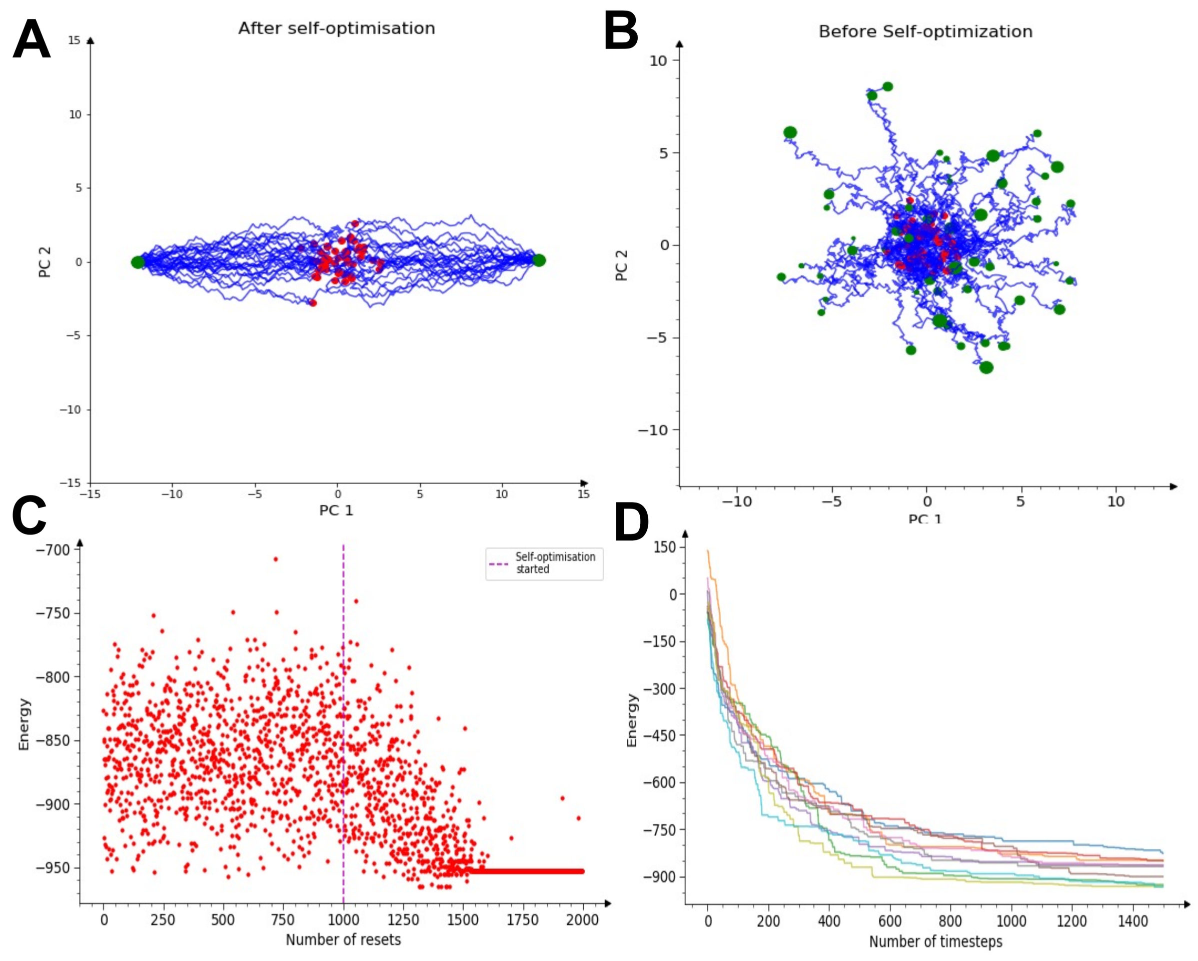

Figure 1. *(top)* 50 illustrative trajectories of the system in primary component space before (**A**) and after (**B**) self-optimization. Green dots represent attractors reached after convergence, and their size is proportional to attractors' energy.
(bottom) Attractors reached by the system before and after self-optimization are marked by red dots (**C**). 10 relaxations from random initial conditions (**D**).

$$s_i(t+1) = \theta\left[\sum_j^N w_{ij}\, s_j(t)\right] \quad \text{(I)}$$

$$Energy = -\frac{1}{2}\sum_{ij}^N w_{ij}\, s_i(t) s_j(t) \quad \text{(II)}$$

$$w_{ij}(t+1) = w_{ij} + \alpha\left[s_i(t) s_j(t)\right] \quad \text{(III)}$$

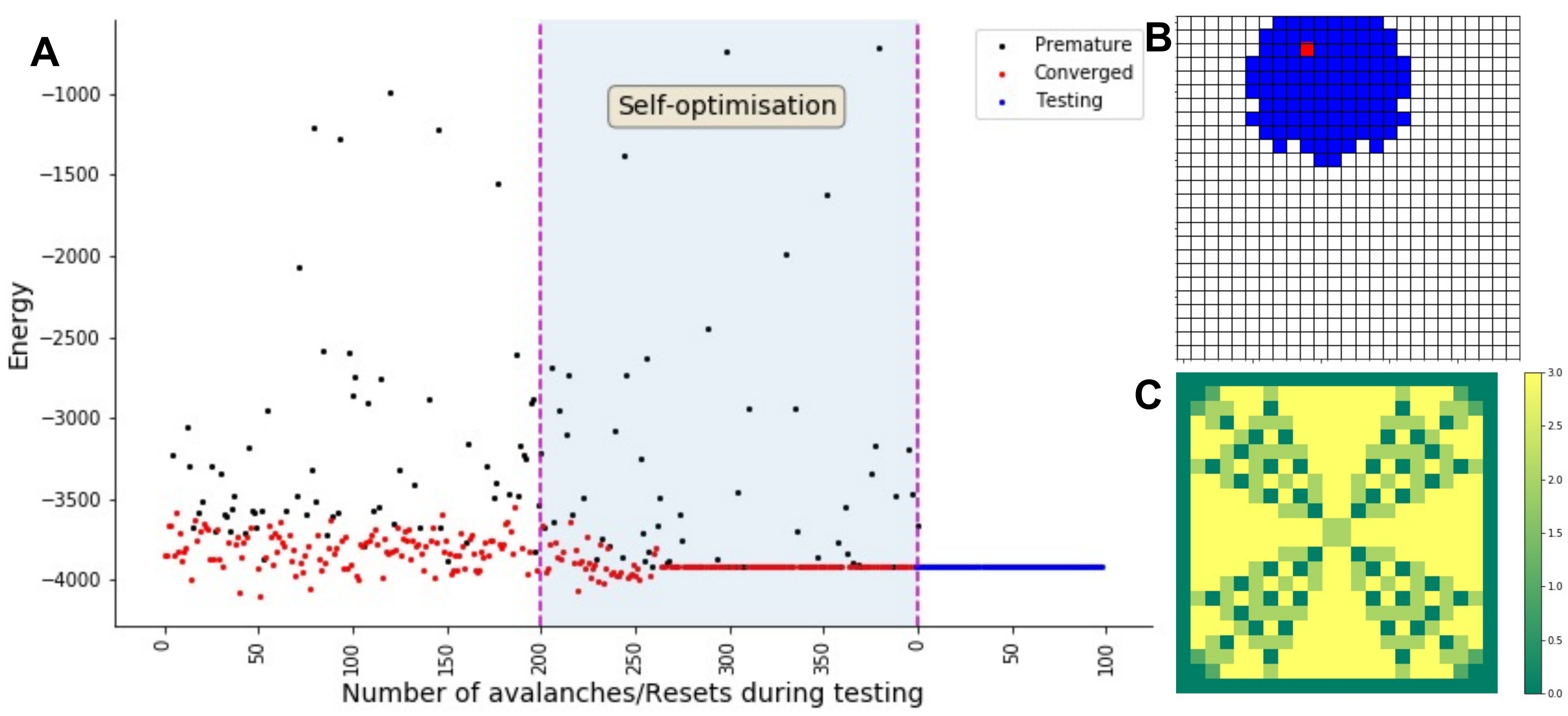

Figure 2 (**A**) Energy of attractors reached by the self-critical system, before and during the self-optimization as well as during testing with fixed weights. (**B**) Single avalanche in the Abelian sandpile caused by adding a grain of sand to the site marked in red. The blue color indicated sites affected by the avalanche. (**C**) The stable state of the sandpile model, color indicates slope.

Results and Futher work

Our model is exceedingly simplistic and is not meant to be a faithful representation of a cortical network. In creating it our main goal was to demonstrate how seemingly detrimental burst of activity could be in fact beneficial to the performance of the system. The next step is to replicate our result in a spiking neural network (SNN), with continuous dynamics. The principal possibility of using self-optimization in SNN model was demonstrated by previous work [4].

Key references:

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