Ultimate-precision quantum clocks: *from mathematical characterization towards light-matter implementations*

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Mischa Woods

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- [M.W. Quantum 5, 381 (2021)] *What is a clock?*
- [M.W., Ralph Silva, Gilles Pütz, Sandra Stupar, Renato Renner, PRX Quantum (2019)] *What is the most accurate clock?*
- [Arman Dost and M.W., arXiv:2303.10029 (2023)] *Can we realise it?*

What is a clock?

[M.W. Quantum 5, 381 (2021)]

$$\mathcal{M}^{t}_{\mathrm{CR_T}\rightarrow\mathrm{CR_T}}(\,\cdot\,)$$

$$\mathcal{H}_d \qquad \mathcal{H}_{N_T}$$

$$\rho_{\mathrm{C}}(t)\otimes|0\rangle\langle 0|_{\mathrm{R_T}}$$

What is a clock? [M.W. Quantum 5, 381 (2021)]

$$\mathcal{M}^{t}_{\mathrm{CR_T}\to\mathrm{CR_T}}(\,\cdot\,)$$

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$$\rho_{\mathrm{C}}(t)\otimes|1\rangle\langle 1|_{\mathrm{R_T}}$$

What is a clock?

[M.W. Quantum 5, 381 (2021)]

$$\mathcal{M}^t_{\mathrm{CR_T}\to\mathrm{CR_T}}(\,\cdot\,)$$

$$\mathcal{H}_d \qquad \mathcal{H}_{N_T}$$

$$\rho_{\mathrm{C}}(t)\otimes|2\rangle\langle 2|_{\mathrm{R_T}}$$

What is a clock? [M.W. Quantum 5, 381 (2021)]

$$\mathcal{M}^t_{\mathrm{CR_T}\to\mathrm{CR_T}}(\,\cdot\,)$$

$$\mathcal{H}_d \qquad \mathcal{H}_{N_T}$$

$$\rho_{\mathrm{C}}(t)\otimes|3\rangle\langle 3|_{\mathrm{R_T}}$$

What is a clock? [M.W. Quantum 5, 381 (2021)]

$$\mathcal{M}^t_{\mathrm{CR_T}\to\mathrm{CR_T}}(\,\cdot\,)$$

$$\mathcal{H}_d \qquad \mathcal{H}_{N_T}$$

$$\rho_{\mathrm{CR_T}}(t) = \sum_{j=0}^{N_T} p_j(t)\,\rho_{\mathrm{C}_j}(t) \otimes |j\rangle\langle j|_{\mathrm{R_T}}$$

What is a clock? [M.W. Quantum 5, 381 (2021)]

1) Self-timing condition:

$$\mathcal{M}^{t_1+t_2}_{\mathrm{CR}\to\mathrm{CR}} = \mathcal{M}^{t_2}_{\mathrm{CR}\to\mathrm{CR}} \circ \mathcal{M}^{t_1}_{\mathrm{CR}\to\mathrm{CR}}$$

2) Continuity:

$$\lim_{t\to 0^+} \|\mathcal{M}^{t}_{\mathrm{CR}\to\mathrm{CR}} - \mathcal{I}_{\mathrm{CR}}\| = 0$$

$$p(t,l|k) := \mathrm{tr}\left[\mathcal{M}^{t}_{\mathrm{CR}\to\mathrm{CR}}(\rho_{\mathrm{C}} \otimes |k\rangle\langle k|_{\mathrm{R}})\, |l\rangle\langle l|_{\mathrm{R}}\right]$$

3) No tick-skipping:

$$\lim_{t\to 0^+} \frac{\sum_{\substack{l=0 \\ l\notin\{k,k+1\}}}^{N_T} p(t,l|k)}{p(t,k+1|k)} = 0$$

4) Clockwork translation invariance:

$\mathrm{tr}_{\mathrm{R}}\left[\mathcal{M}^{t}_{\mathrm{CR}\to\mathrm{CR}}(\rho_{\mathrm{C}} \otimes |k\rangle\langle k|_{\mathrm{R}})\, |k+l\rangle\langle k+l|_{\mathrm{R}}\right]$ is k independent for all k, l, t

What is a clock? [M.W. Quantum 5, 381 (2021)]

Dynamical semi-group representation:

The pair $\rho^0_{\mathrm{C}}, (\mathcal{M}^t_{\mathrm{CR}\to\mathrm{CR}})_{t\geq 0}$ *satisfy 1) to 4), if and only if there exists a Hermitian operator* H *and linear operators* $(L_j)_{j=1}^{N_L}$ *and* $(J_j)_{j=1}^{N_L}$*; which are all time independent, such that for all* $t \geq 0$*:*

$$\mathcal{M}^t_{\mathrm{CR}\to\mathrm{CR}}(\cdot) = \mathrm{e}^{t\mathcal{L}_{\mathrm{CR}}}(\cdot),$$

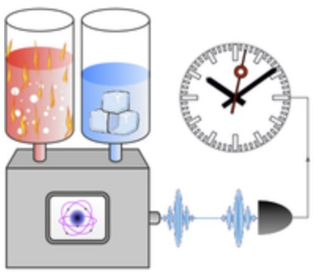
$$\mathcal{L}_{\mathrm{CR}}(\cdot) = -\mathrm{i}[\tilde{H}, (\cdot)] + \sum_{j=1}^{N_L} \tilde{L}_j(\cdot)\tilde{L}_j^\dagger - \frac{1}{2}\{\tilde{L}_j^\dagger \tilde{L}_j, (\cdot)\} + \sum_{j=1}^{N_L} \tilde{J}_j(\cdot)\tilde{J}_j^\dagger - \frac{1}{2}\{\tilde{J}_j^\dagger \tilde{J}_j, (\cdot)\},$$

where $\tilde{H} = H \otimes \mathbb{1}_{\mathrm{R}}$, $\tilde{L}_j = L_j \otimes \mathbb{1}_{\mathrm{R}}$,
$\tilde{J}_j = J_j \otimes O_{\mathrm{R}}$

$$O_{\mathrm{R}} := |1\rangle\langle 0|_{\mathrm{R}} + |2\rangle\langle 1|_{\mathrm{R}} + |3\rangle\langle 2|_{\mathrm{R}} + \ldots + |N_T\rangle\langle N_T - 1|_{\mathrm{R}} + |0\rangle\langle N_T|_{\mathrm{R}}.$$

What is a clock?

Examples:



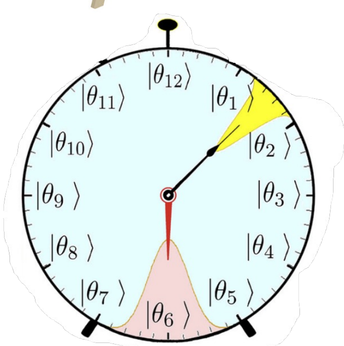
1) Thermodynamic "heat-engine" clock

[P. Erker et. al. PRX (2017)]

$$L_1 = \sqrt{\gamma_h}\sigma_h, \qquad L_2 = \sqrt{\gamma_h \mathrm{e}^{-\beta_h E_h}}\sigma_h^\dagger,$$
$$L_3 = \sqrt{\gamma_c}\sigma_c, \qquad L_4 = \sqrt{\gamma_c \mathrm{e}^{-\beta_c E_c}}\sigma_c^\dagger,$$
$$J_1 = \sqrt{\Gamma}\,|0\rangle\langle d-1|_w\,, \qquad J_2 = J_3 = J_4 = 0,$$

2) Ladder clock [S. Stupar et.al, arXiv:1806.08812 (2018)]

$$L_j = |c_{j+1}\rangle\langle c_j|\,, \quad J_j = 0, \qquad j = 0,1,2,...,d-1$$
$$L_d = 0, \qquad J_d = |c_1\rangle\langle c_d|\,,$$



3) Quasi-Ideal clock [M.W. et al, PRX Quantum (2019)]

$$L_j = 0, \quad J_j = \sqrt{2V_j}\,|\psi_{\mathrm{C}}\rangle\langle t_j|\,,$$
$$j = 0,1,2,...,d$$

[A. Peres, AJP (1980)]

[M.W. et al, Annales Henri Poincaré (2018)]

How well can a clock measure time? [P. Erker et. al. PRX (2017)]

- Delay function of 1st tick $P_{\text{tick}}^{(1)}(t)$:

$P_{\text{tick}}^{(1)}(t)dt$:= *probability of not ticking in interval* $[0, t)$ *followed by ticking in time interval* $[t, t+dt]$

$P_{\text{tick}}^{(k)}(t)$

- Accuracy of kth tick: $R_k := \frac{\mu_k^2}{\sigma_k^2}$ For reset clocks: $R_k := kR_1$

Complete classification of classical clocks

- Definition: Classical clock: a quantum clock without coherence.
  - Classical clocks obey stochastic dynamics

- Theorem (Optimal classical clock) [M.W. et al, PRX Quantum (2019)]

All d dimensional classical clocks satisfy

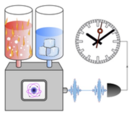
$$R_k \leq kd$$

-- Ladder clock is classical and

$$R_k = kd$$

-- Thermodynamic “heat-engine” clock is quantum but

$$R_k \propto kd$$



Quantum clock Advantage

[M.W. et al, PRX Quantum (2019)]

[M.W. et al, Annales Henri Poincaré (2018)]

-Theorem:

There exists parameters for the quasi-ideal clock such that

$$R_k = kd^2 \qquad \text{as} \qquad d \to \infty$$

-Theorem:

[Y. Yang et. al. ArXiv: 2004.07857 (2020)]

All quantum clocks have accuracy

$$R_1 \leq 2e\pi d^2$$

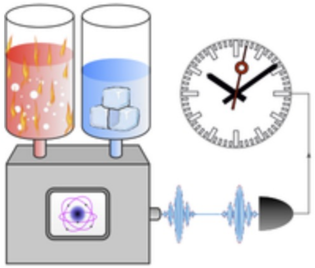
Paper with Arman PT Dost:

What would be needed for an experimental realization?

[Arman Dost and M.W., arXiv:2303.10029 (2023)]

Why this matters:

- Is it physical? Are we cheating?
- What is the thermodynamic cost?



1) *Thermodynamic "heat-engine" clock*

[P. Erker et. al. PRX (2017)]

What does Wall-E propose?

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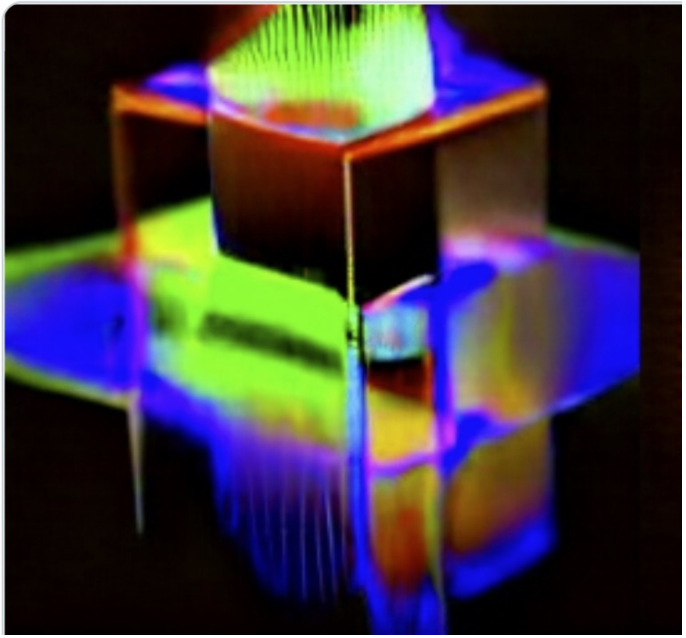
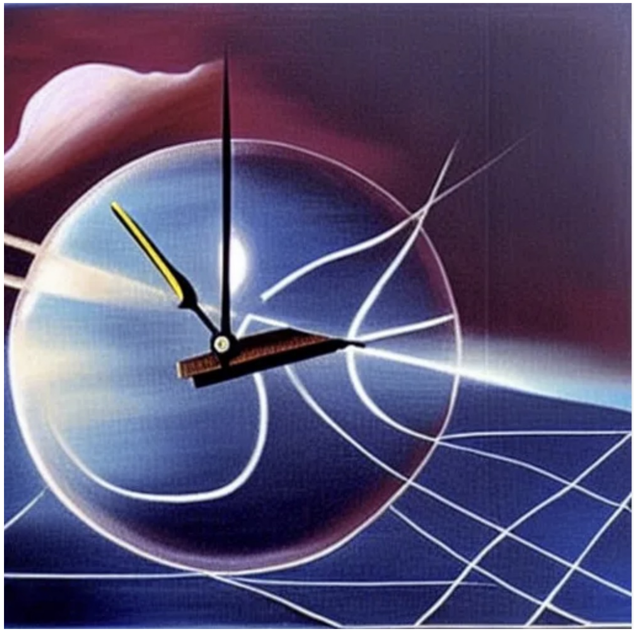


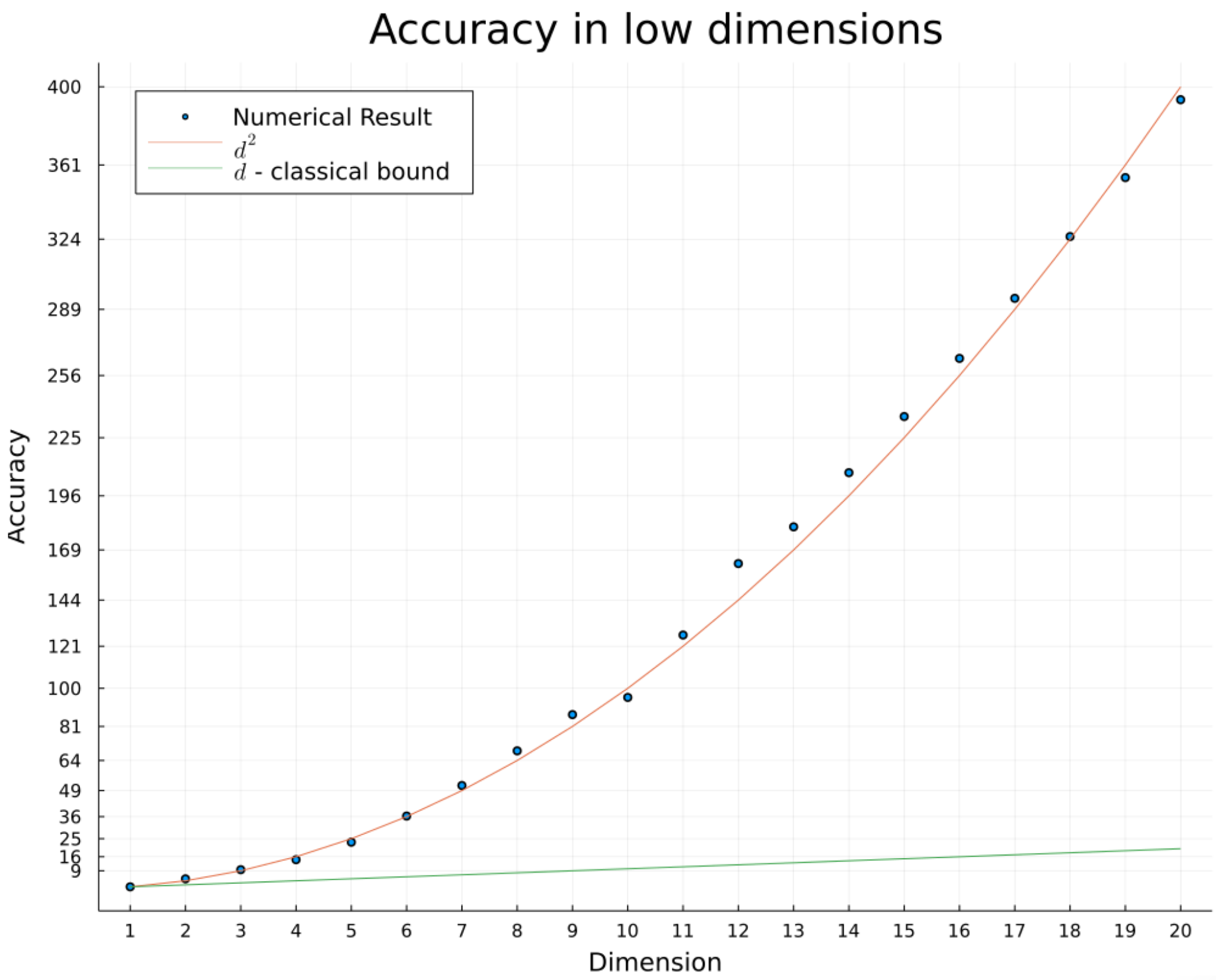
A quantum photonic clock



A quantum clock emitting a photon into the void

Is the quantum advantage achievable in low dimensions?

[Arman Dost and M.W., arXiv:2303.10029 (2023)]



What we need

$$\mathcal{M}^t_{\mathrm{CR}\to\mathrm{CR}}(\cdot) = \mathrm{e}^{t\mathcal{L}_{\mathrm{CR}}}(\cdot),$$

$$\mathcal{L}_{\mathrm{CR}}(\cdot) = -\mathrm{i}[\tilde{H},(\cdot)] + \sum_{j=1}^{N_L} \tilde{L}_j(\cdot)\tilde{L}_j^\dagger - \frac{1}{2}\{\tilde{L}_j^\dagger\tilde{L}_j,(\cdot)\}$$
$$+\sum_{j=1}^{N_L} \tilde{J}_j(\cdot)\tilde{J}_j^\dagger - \frac{1}{2}\{\tilde{J}_j^\dagger\tilde{J}_j,(\cdot)\},$$

$$\tilde{H} = H\otimes \mathbb{1}_{\mathrm{R}} \qquad \tilde{L}_j = L_j \otimes \mathbb{1}_{\mathrm{R}}$$
$$\tilde{J}_j = J_j \otimes O_{\mathrm{R}}$$

- Recall Quasi-ideal clock:

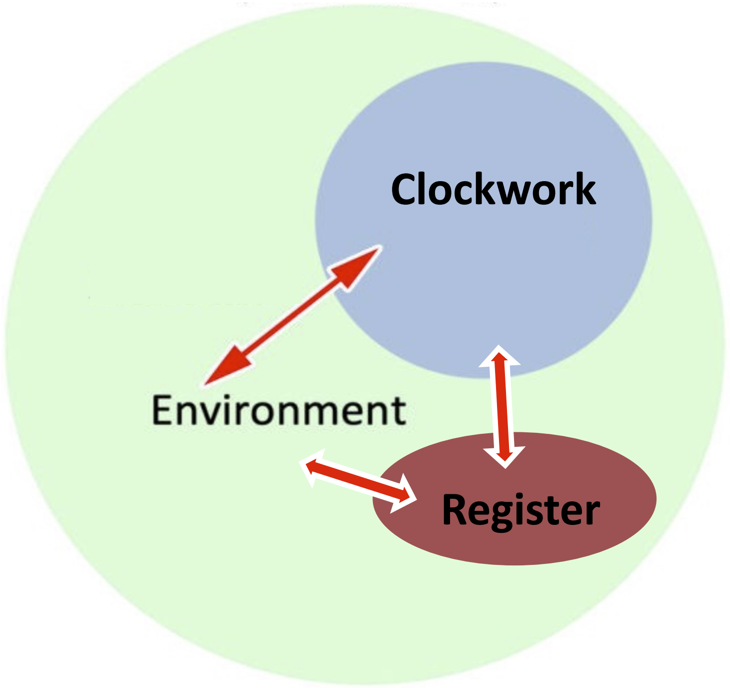
$$L_j = 0, \quad J_j = \sqrt{2V_j}\,|\psi_{\mathrm{C}}\rangle\langle t_j|\,,$$
$$j = 0,1,2,\ldots,d$$

Initial clockwork state
(Reset clock)

$$|t_j\rangle = \frac{1}{\sqrt{d}}\sum_{n=0}^{d-1}\mathrm{e}^{-\mathrm{i}2\pi nk/d}\,|E_n\rangle$$

$$\hat{H} = \sum_{n=0}^{d-1}\omega_0 n\,|E_n\rangle\langle E_n|$$

How to build a Quantum Clock with Quantum advantage?



$$H_{\text{total}} = H_{\text{C}} + H_{\text{E}} + H_{\text{CER}}$$

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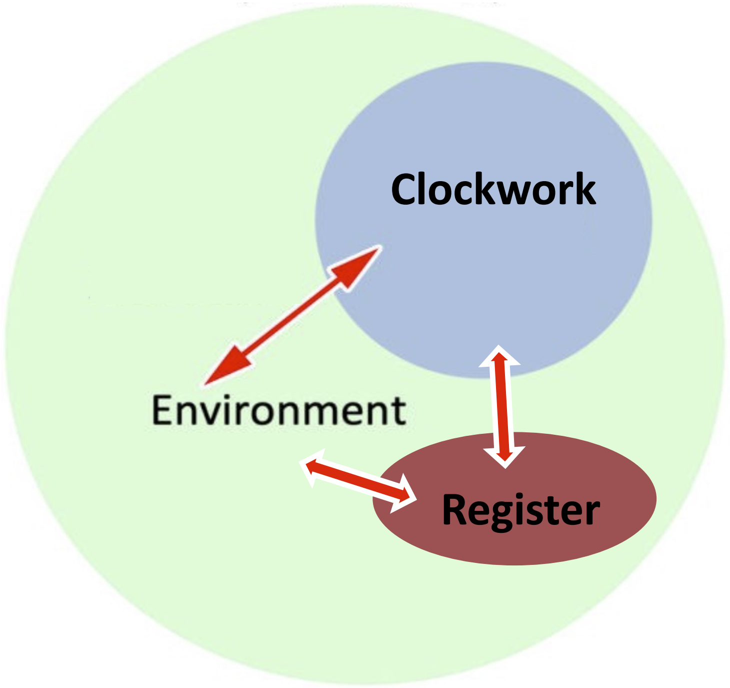


$$H_{\mathrm{total}} = H_{\mathrm{C}} + H_{\mathrm{E}} + H_{\mathrm{CE}}$$

Matter　Light (bath)　$-\vec{D} \cdot \vec{E}$

- Weak Coupling
  - Born-Markov approximation $\tau_B \ll \tau_R$:

$$\frac{d}{dt}\rho_{\mathrm{C}}(t) =$$

$$\sum_{\omega,\omega'} \mathrm{e}^{\mathrm{i}t(\omega'-\omega)}\Gamma(\omega)\Big(D(\omega)\rho_{\mathrm{C}}(t)D^\dagger(\omega') - D^\dagger(\omega')D(\omega)\rho_{\mathrm{C}}(t)\Big)$$

$$+\mathrm{h.c.},$$

$$|E_n\rangle \rightarrow |E_{n+\omega}\rangle$$

- R.W.A. $\tau_S \ll \tau_R$:

$$\frac{d}{dt}\rho_{\mathrm{C}}(t) =$$

$$\sum_{\omega} \Gamma(\omega)\Big(D(\omega)\rho_{\mathrm{C}}(t)D^\dagger(\omega) - D^\dagger(\omega)D(\omega)\rho_{\mathrm{C}}(t)\Big) + \mathrm{h.c.},$$

$$|\psi_{\mathrm{C}}\rangle\langle\psi_{\mathrm{C}}| \, \langle t_j|\rho_{\mathrm{C}}(t)|t_j\rangle - \{V_j \, |t_j\rangle\langle t_j| \, , \rho_{\mathrm{C}}(t)\}$$

$$|t_j\rangle = \frac{1}{\sqrt{d}}\sum_{n=0}^{d-1} \mathrm{e}^{-\mathrm{i}2\pi nk/d} \, |E_n\rangle$$

How to build a Quantum Clock with Quantum advantage?

- Born-Markov approximation $\tau_B \ll \tau_R$

- R.W.A.

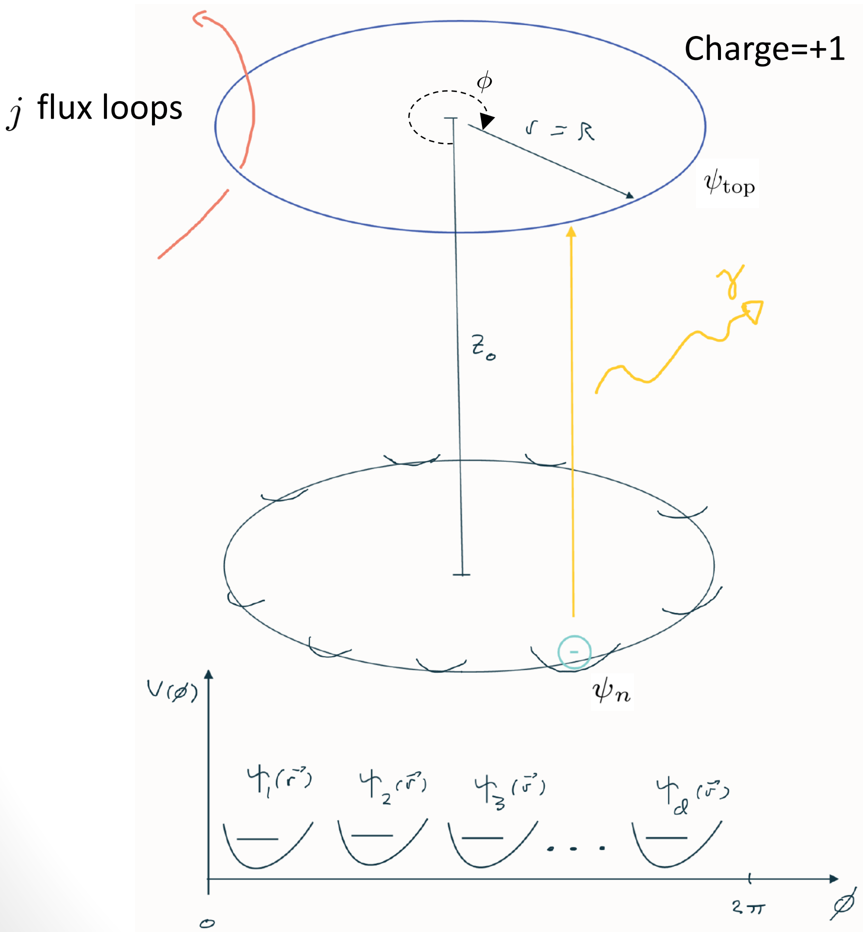
- Phase engineering?

$$|t_j\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} \mathrm{e}^{-\mathrm{i}2\pi nk/d} |E_n\rangle$$

How to build a Quantum Clock with Quantum advantage?

[Arman Dost and M.W., arXiv:2303.10029 (2023)]

The matter Hamiltonian:



- Top ring state:

$$\psi_{\text{top}}(z,r,\phi) = \frac{1}{\sqrt{2\pi R}}\, e^{-i\,\phi j}$$

- Bottom ring states:
  - Spatial localization:

$$\psi_n(z,r,\phi)\psi_k(z',r',\phi') \approx 0$$

  - Rotational symmetry:

$$\psi_1(z,r,\phi - n2\pi/d) = \psi_n(z,r,\phi)$$

$$[D_z]_{\psi_n,\psi_{\text{top}}} =$$

$$q\int dz \int r dr \int d\phi\, \psi_n(z,r,\phi)\, z\, \psi^*_{\text{top}}(z,r,\phi)$$

$$= q\int dz \int r dr \int d\phi\, \psi_1(z,r,\phi - n2\pi/d)\, z\, \frac{e^{-i\,\phi j}}{\sqrt{2\pi R}}$$

$$= e^{-i2\pi nj/d}\, [D_z]_{\psi_1,\psi_{\text{top}}}$$

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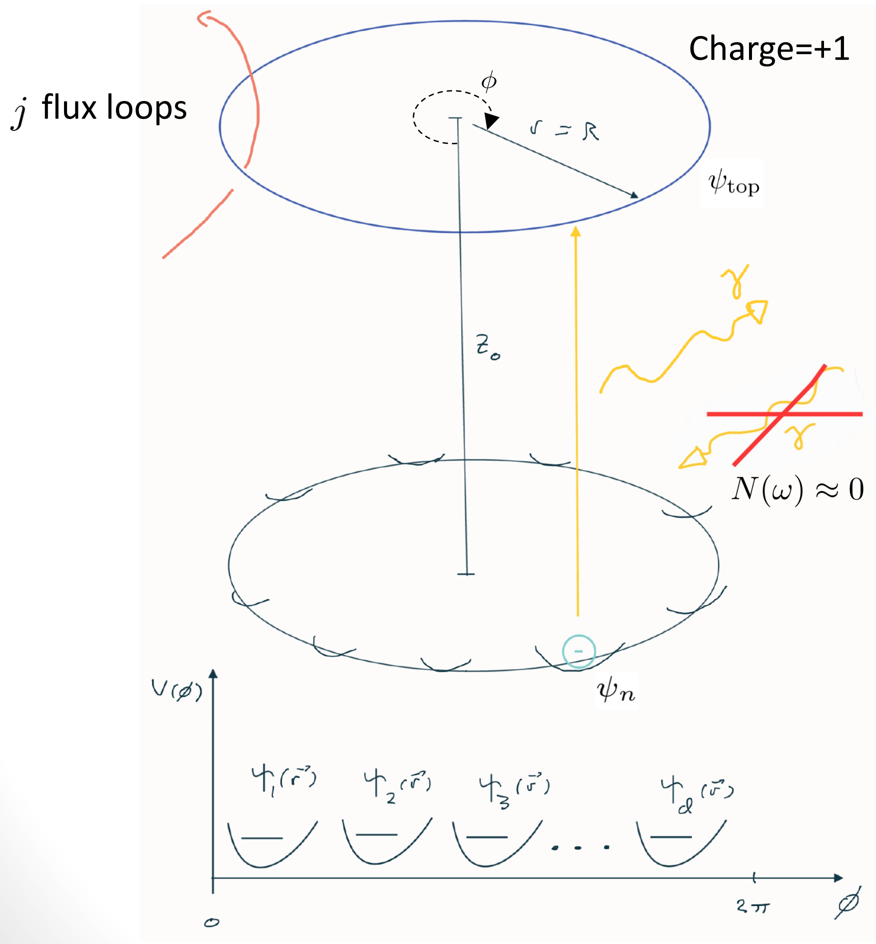

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$$= \mathrm{e}^{-\mathrm{i}2\pi n j/d}\, [D_z]_{\psi_1,\psi_{\text{top}}}$$

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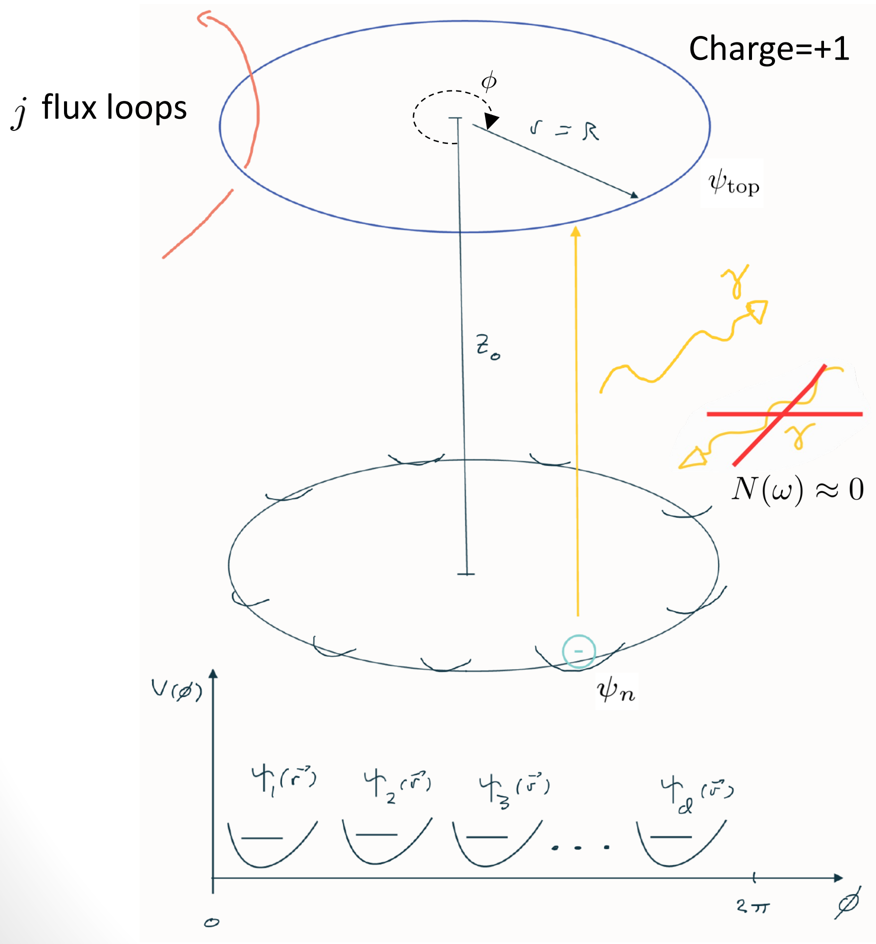

- What we get:

$$H_{\text{total}} = H_{\text{C}} + H_{\text{E}} - D_z\, E_z$$

$$\frac{d}{dt}\rho_{\text{C}}(t) = -\text{i}[H_{\text{C}}, \rho_{\text{C}}(t)]$$
$$+2V_j\, |\psi_{\text{C}}\rangle\langle\psi_{\text{C}}|\, \langle t_j|\rho_{\text{C}}(t)|t_j\rangle - \{V_j\, |t_j\rangle\langle t_j|\,, \rho_{\text{C}}(t)\}$$

$$|t_j\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} \text{e}^{-\text{i}2\pi nj/d}\, |E_n\rangle$$

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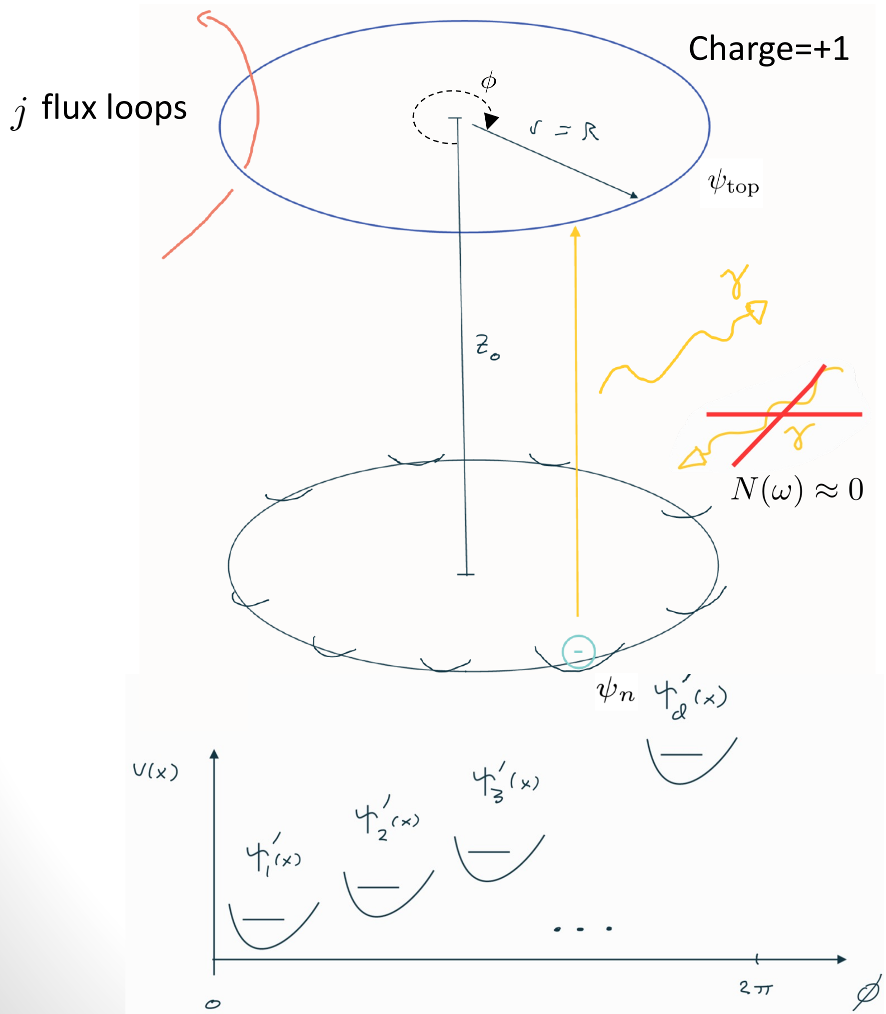

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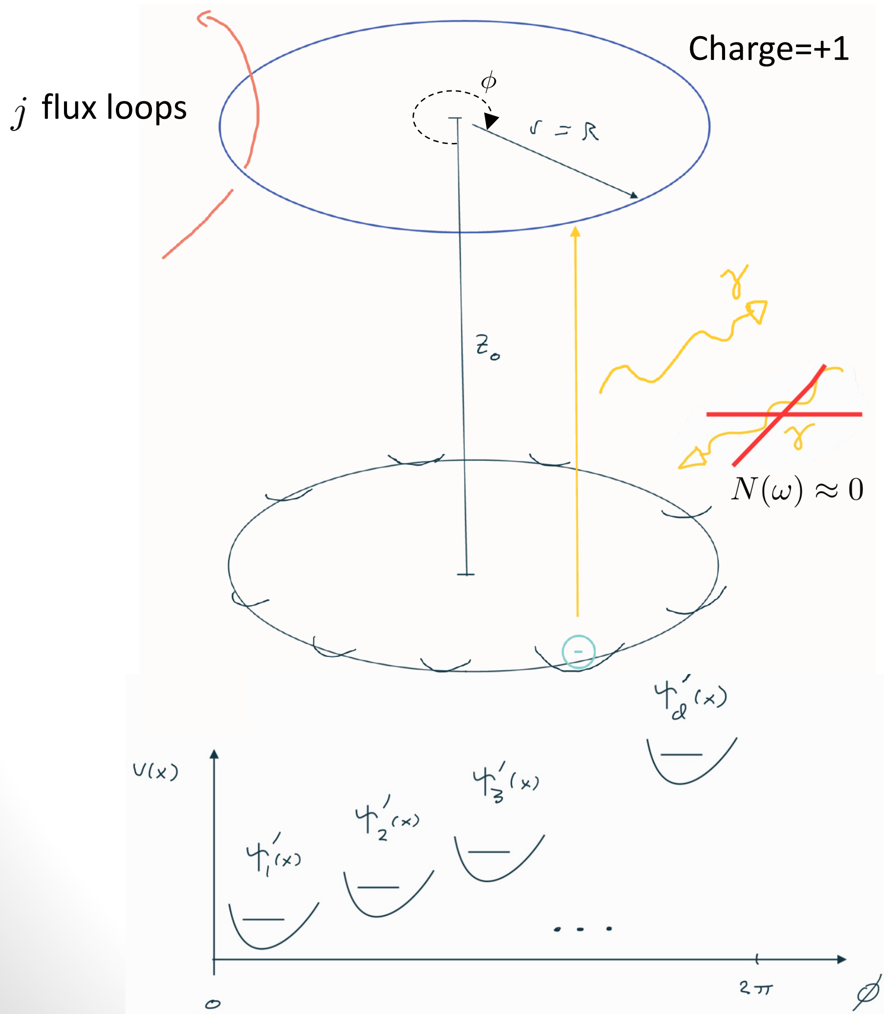

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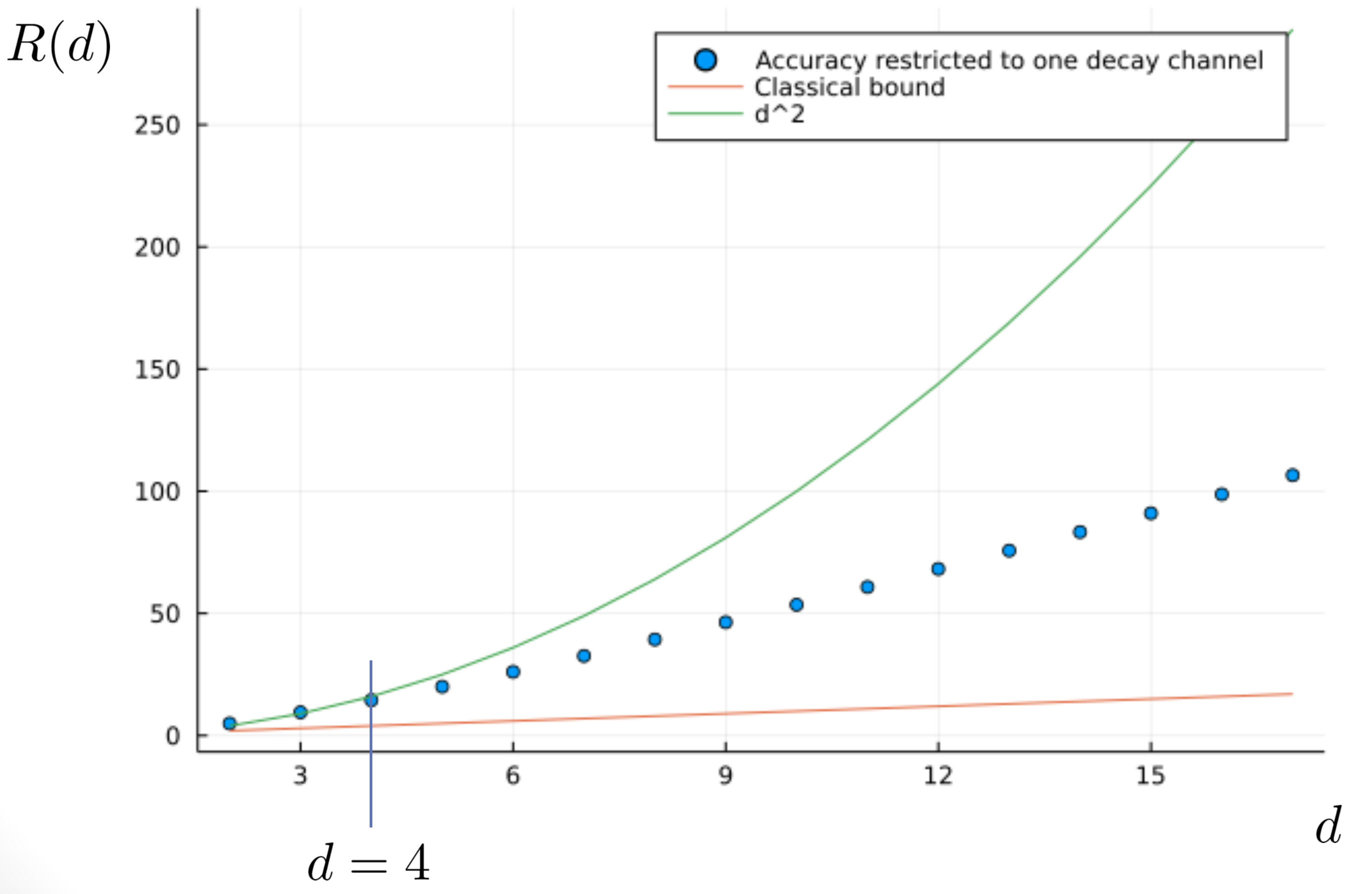
$$\left|t_j\right\rangle = \frac{1}{\sqrt{d}} \sum_{n=0}^{d-1} \mathrm{e}^{-\mathrm{i}2\pi n j/d} \left|E_n\right\rangle$$

- C.f. what we want:

$$\frac{d}{dt}\rho_{\text{C}}(0) = -\mathrm{i}[H_{\text{C}}, \rho_{\text{C}}(t)]$$
$$+\sum_{j=0}^{d-1} 2V_j \left|\psi_{\text{C}}\right\rangle\!\left\langle\psi_{\text{C}}\right| \left\langle t_j\right|\rho_{\text{C}}(t)\left|t_j\right\rangle - \{V_j \left|t_j\right\rangle\!\left\langle t_j\right|, \rho_{\text{C}}(t)\}$$

How to build a Quantum Clock with Quantum advantage?



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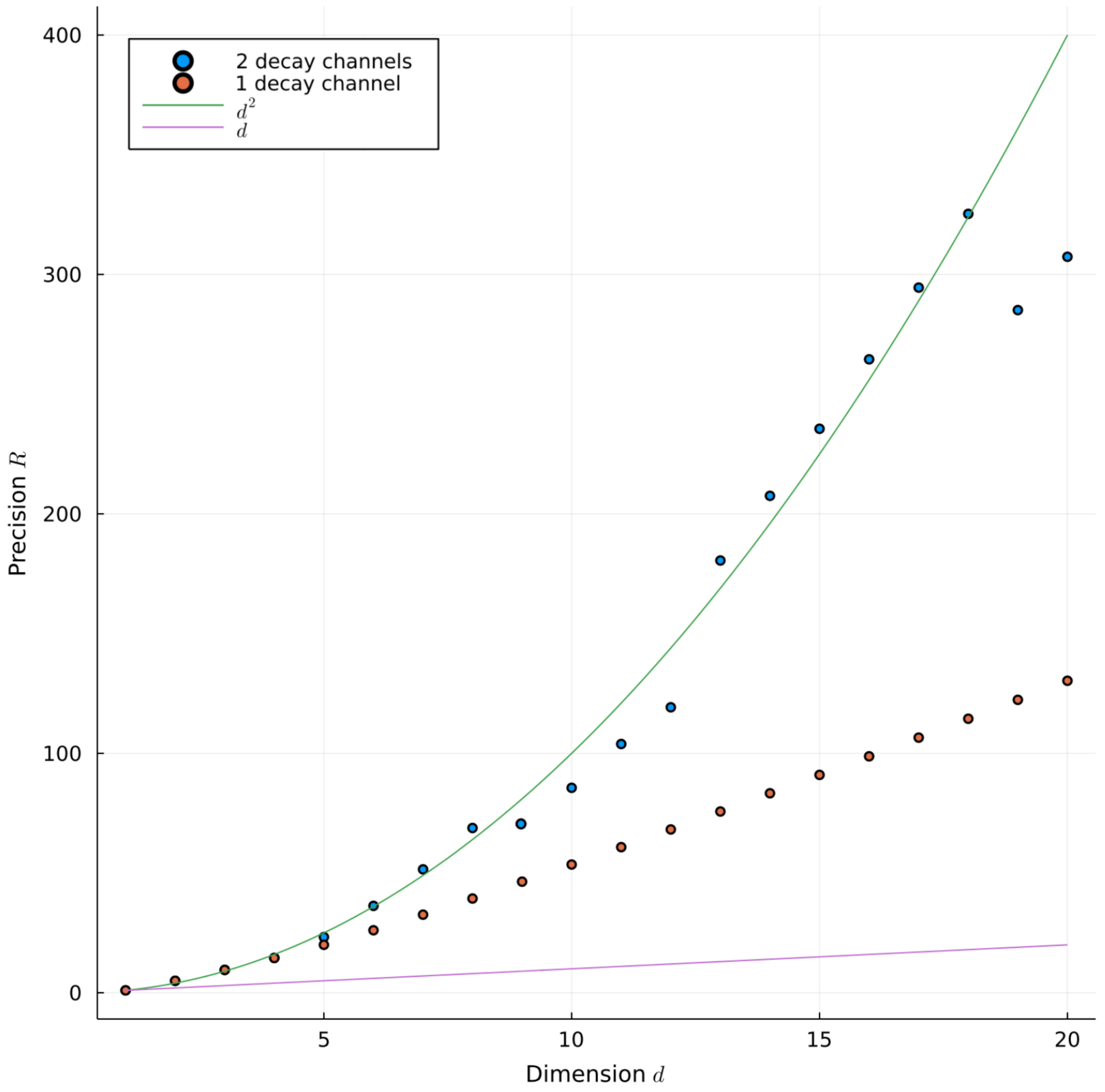
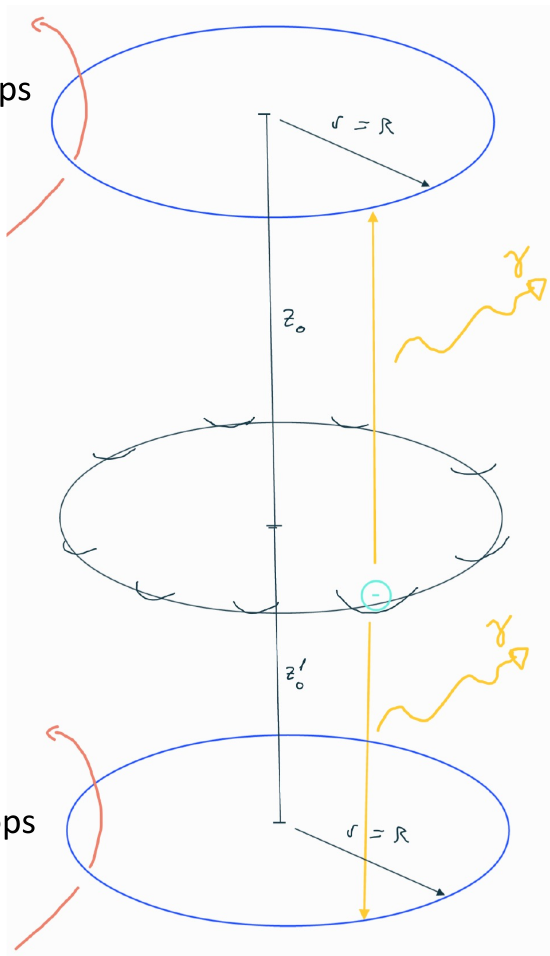


How to build a Quantum Clock with Quantum advantage?

How to construct 2 decay channels:

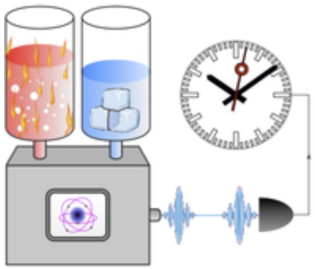
j_1 flux loops



j_2 flux loops

$$\frac{d}{dt}\rho_{\mathrm{C}}(0) = -\mathrm{i}[H_{\mathrm{C}}, \rho_{\mathrm{C}}(t)] + \sum_{j=\{j_1, j_2\}} 2V_j \left|\psi_{\mathrm{C}}\right\rangle\!\left\langle\psi_{\mathrm{C}}\right| \left\langle t_j\right|\rho_{\mathrm{C}}(t)\left|t_j\right\rangle - \left\{V_j \left|t_j\right\rangle\!\left\langle t_j\right|, \rho_{\mathrm{C}}(t)\right\}$$

Entropy production per tick?



1) *Thermodynamic "heat-engine" clock*

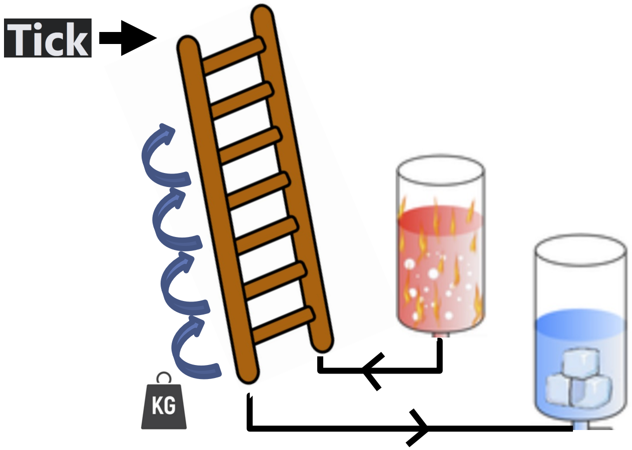
   [P. Erker et. al. PRX (2017)]

Entropy production per tick?

$$\Delta S_{\text{tick}} \approx \beta_v (Q_h - Q_c)$$

$$Q_h := (d-1)E_h$$

$$Q_c := (d-1)E_c$$



Entropy production per tick?

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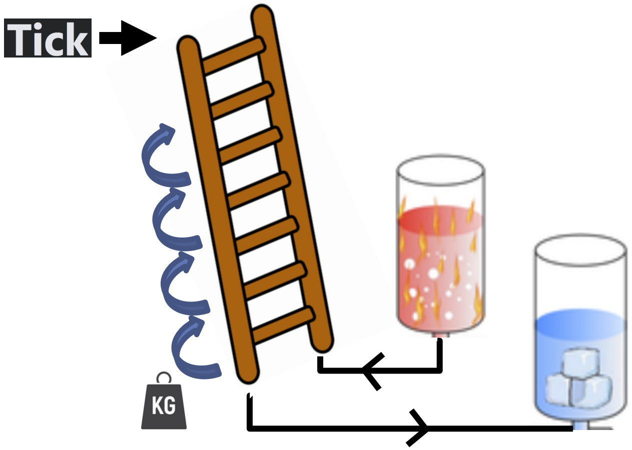

$$R = \frac{\Delta S_{\text{tick}}}{2}$$

Entropy production ~~per tick~~ ?

$$\mathcal{L}(\cdot) = [H, \cdot] + \mathcal{D}(\cdot)$$

$$\sigma(\rho(t)) := -\frac{d}{dt} S(\rho(t)||\tau_\beta) = \frac{d}{dt} S(\rho(t)) + J(\rho(t))$$

$$J(\rho) := \beta \, \mathrm{tr}[H \mathcal{D} \rho]$$
$$= \mathrm{tr}[\mathcal{L}(\rho) \ln(\tau_\beta)]$$

Entropy production ~~per tick~~ ?

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Entropy production per tick?

$$\mathrm{tr}[\mathcal{L}_{\mathrm{C}}^{\mathrm{nt}} \left(\rho_{\mathrm{C}}^{\mathrm{nt}}(\tilde{t}\,)\right) \ln(\tau_\beta)]$$

Entropy production ~~per tick~~ ?

$$\mathcal{L}(\cdot) = [H, \cdot] + \mathcal{D}(\cdot)$$

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$$\begin{aligned} J(\rho) &:= \beta \, \mathrm{tr}[H \mathcal{D} \rho] \\ &= \mathrm{tr}[\mathcal{L}(\rho) \ln(\tau_\beta)] \end{aligned}$$

Entropy production per tick?

$$\int_0^t \mathrm{d}\tilde{t} \; \mathrm{tr}[\mathcal{L}_\mathrm{C}^\mathrm{nt}\left(\rho_\mathrm{C}^\mathrm{nt}(\tilde{t}\,)\right) \ln(\tau_\beta)]$$

Entropy production ~~per tick~~ ?

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$$\begin{aligned} J(\rho) &:= \beta \, \mathrm{tr}[H \mathcal{D} \rho] \\ &= \mathrm{tr}[\mathcal{L}(\rho) \ln(\tau_\beta)] \end{aligned}$$

Entropy production per tick?

$$P_{\mathrm{tick}}(t) \int_0^t \mathrm{d}\tilde{t} \; \mathrm{tr}[\mathcal{L}_{\mathrm{C}}^{\mathrm{nt}} \big(\rho_{\mathrm{C}}^{\mathrm{nt}}(\tilde{t}\,)\big) \ln(\tau_\beta)]$$

Entropy production ~~per tick~~ ?

$$\mathcal{L}(\cdot) = [H, \cdot] + \mathcal{D}(\cdot)$$

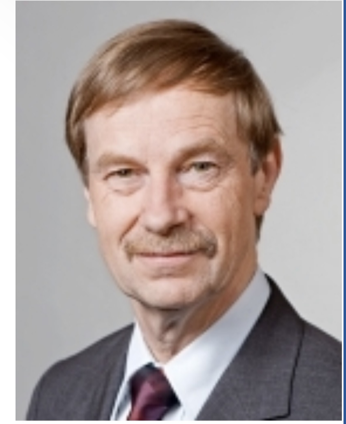
$$\sigma(\rho(t)) := -\frac{d}{dt} S(\rho(t)||\tau_\beta) = \frac{d}{dt} S(\rho(t)) + J(\rho(t))$$

$$\begin{aligned} J(\rho) &:= \beta \, \mathrm{tr}[H \mathcal{D} \rho] \\ &= \mathrm{tr}[\mathcal{L}(\rho) \ln(\tau_\beta)] \end{aligned}$$

Entropy production per tick?

$$\Delta S_{\mathrm{tick}} := \int_0^\infty \mathrm{d}t \, P_{\mathrm{tick}}(t) \int_0^t \mathrm{d}\tilde{t} \; \mathrm{tr}[\mathcal{L}_{\mathrm{C}}^{\mathrm{nt}} \big(\rho_{\mathrm{C}}^{\mathrm{nt}}(\tilde{t}\,)\big) \ln(\tau_\beta)]$$

Entropy production ~~per tick~~ ?



$$\mathcal{L}(\cdot) = [H, \cdot] + \mathcal{D}(\cdot)$$

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Entropy production per tick?

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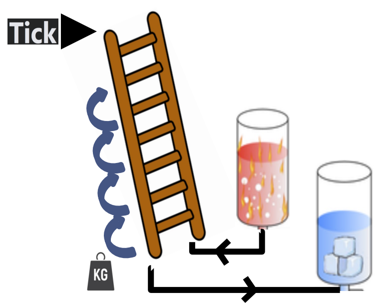
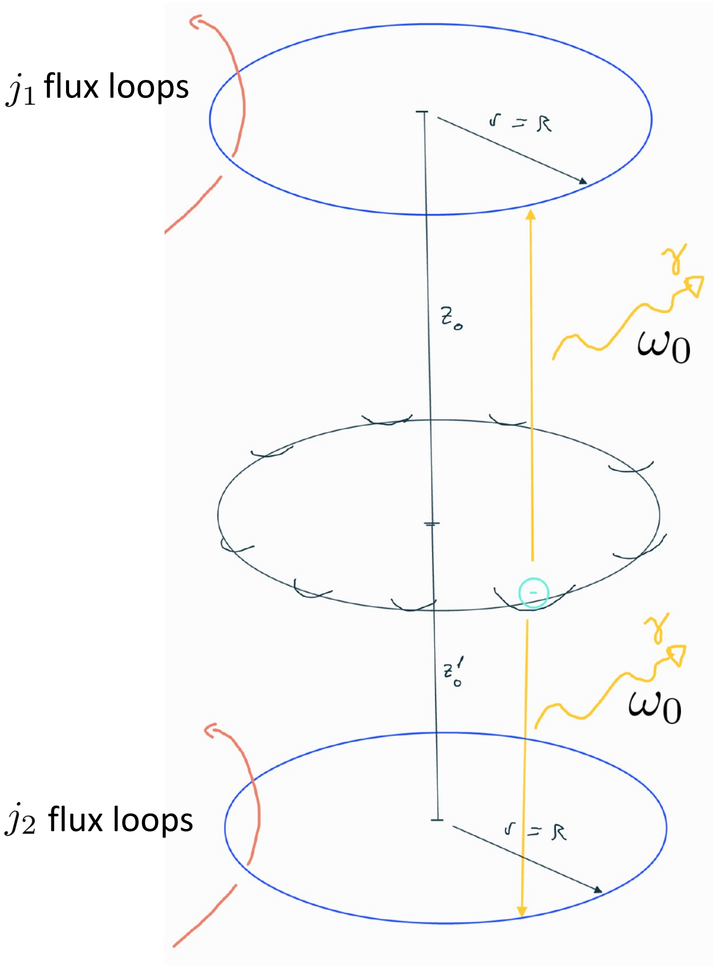
$$R = \frac{\Delta S_{\mathrm{tick}}}{2}$$



Entropy production per tick ?



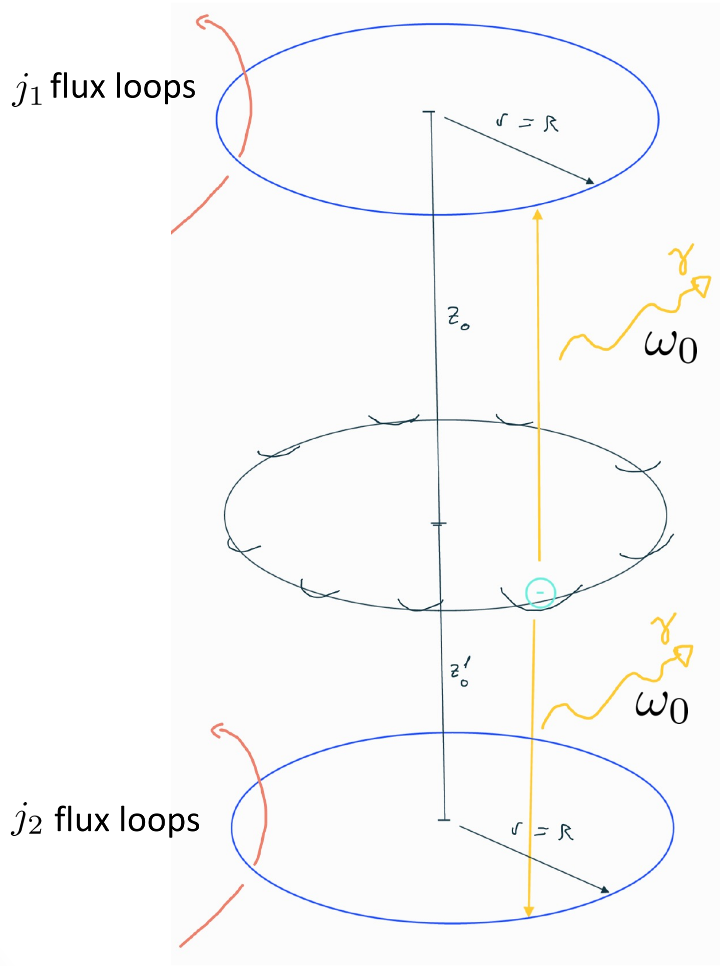
$$\Delta S_{\text{tick}} \approx \beta(\omega_0 + \omega d/2)$$

No

$$\Delta S_{\text{tick}} \approx \beta_v(Q_h - Q_c)$$

$$R \approx \left(\frac{\Delta S_{\text{tick}}}{\beta} - \omega_0\right)^2 \frac{4}{\omega^2}$$

Entropy production per tick ?



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Minimal entropy per tick quadratically improved!

- Is the most accurate quantum clock realizable?
  - Weak coupling limit sufficient for quantum advantage
  - Up to $d = 8$ only require 2 decay channels
  - Discreate Fourier transform realizable with flux loops
  - Quantum entropy advantage too
  - Experimental implementation via superconducting qubits?

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QInfo Inria team

Location: ENS Lyon and Uni. Grenoble

Faculty members

- Alastair Abbott
- Guillaume Aubrun
- Omar Fawzi
- Daniel Stilck França
- Mischa Woods

Postdoctoral researchers

- Cyril Elouard
- Mizanur Rahaman
- Ala Shayeghi

PhD students

- Emily Beatty
- Paul Fermé
- Aadil Oufkir
- Hoang-Duy Ta

Multiple PhD and postdoc positions