

# Emergent Cosmology from (T)GFT condensates

based on 2008.02774, 2010.09700, 2110.11176 and 2112.12677; with D. Oriti, S. Gielen, A. Polaczek

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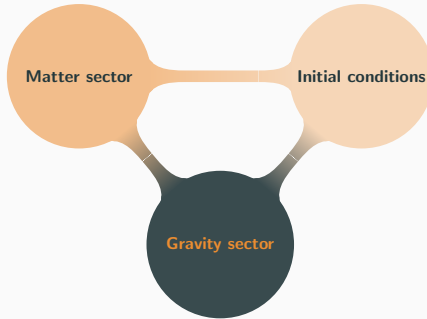
**Luca Marchetti**

QUAST seminar

OIST, 20 September 2022

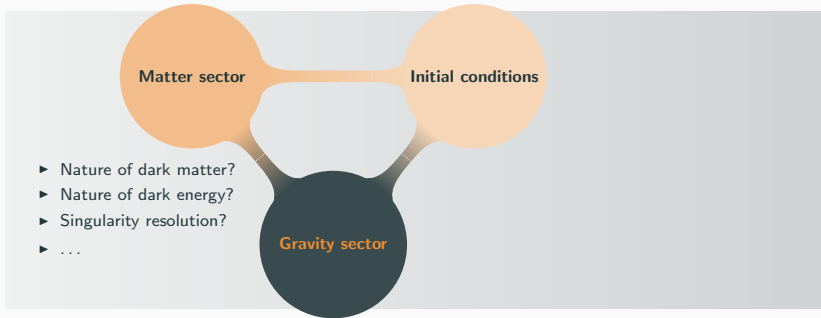
Arnold Sommerfeld Center  
LMU Munich

# The QG perspective on Cosmology

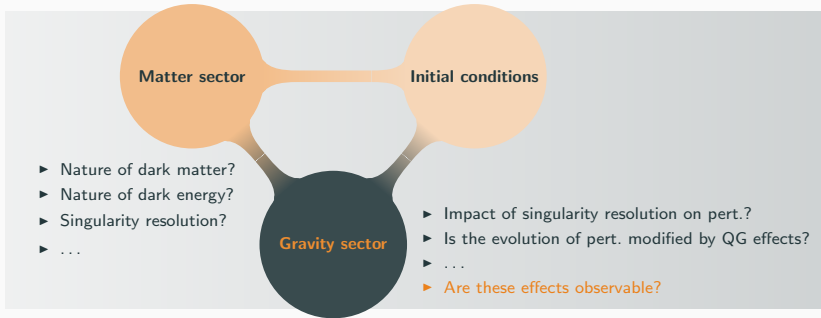


Ashtekar, Kaminski, Lewandowski 0901.0933; Agullo, Ashtekar, Nelson 1302.0254; Gielen, Oriti 1709.01095; Gerhart, Oriti, Wilson-Ewing 1805.03099; ...

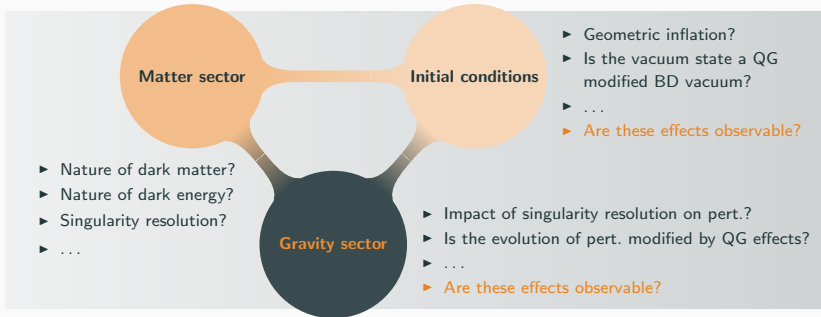
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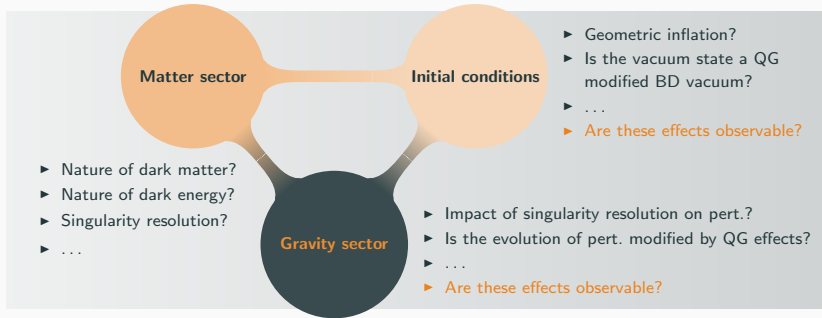
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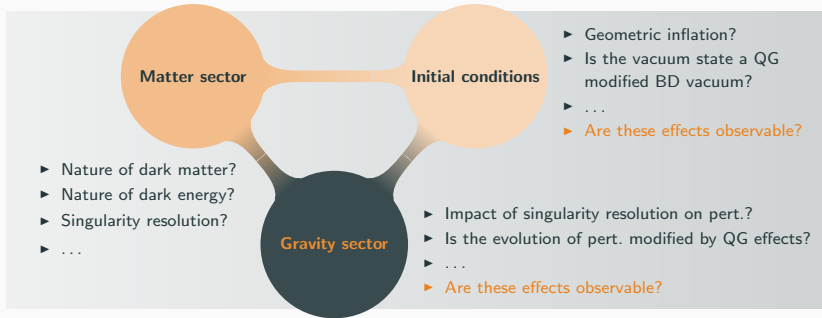
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## Challenges from the QG perspective:

- ▶ How to define (in)homogeneity?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
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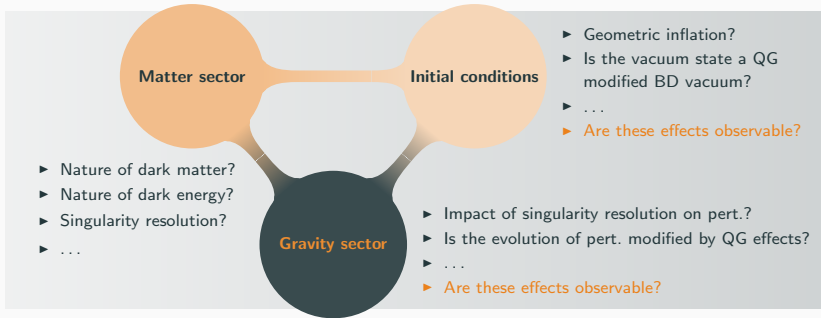


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Relational description

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Relational description

Coarse-graining/  
collective behavior



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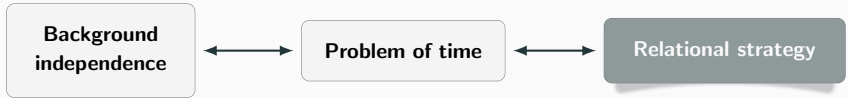
## ● A state-agnostic approach

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# Relational dynamics

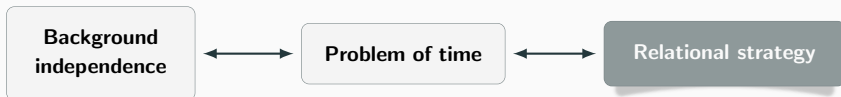
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# Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

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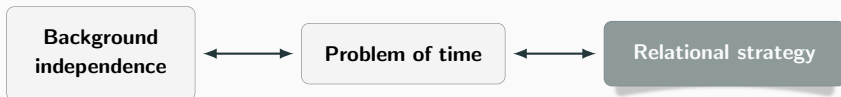
## Classical

Notion of relationality can be classically encoded in

**relational observables:**

- ▶ Take two phase space functions,  $f$  and  $T$  with  $\{T, C_H\} \neq 0$  ( $T$  relational clock).
- ▶ The relational extension  $F_{f,T}(\tau)$  of  $f$  encodes the value of  $f$  when  $T$  reads  $\tau$ .
- ▶ Evolution in  $\tau$  is relational.
- ▶  $F_{f,T}(\tau)$  is a very complicated function, often written in series form.
- ▶ Applications only for (almost) deparametrizable systems, such as GR plus pressureless dust or massless scalar fields.

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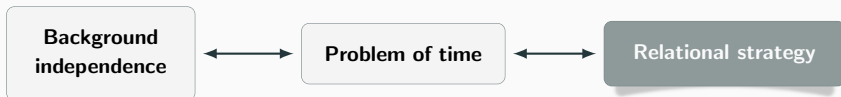
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**Dirac approach**: first quantize, then implement relationality

- ▶ Perspective neutral approach: all variables are treated on the same footing.
- ▶ Poor control of the physical Hilbert space.

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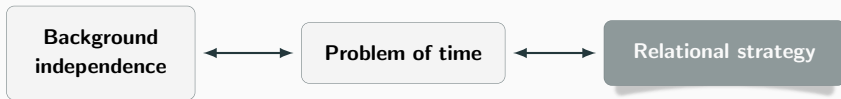
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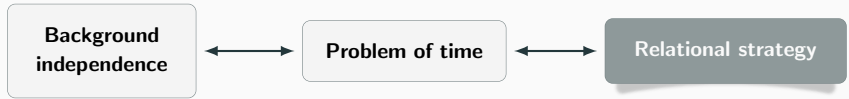
- ▶ Perspective neutral approach: all variables are treated on the same footing.
  - ▶ Poor control of the physical Hilbert space.
- Reduced phase space approach**: first implmt relationality, then quantize
- ▶ No quantum constraint to solve.
  - ▶ Led to quantization of simple deparametrizable models (LQG).
  - ▶ Not perspective neutral. Too complicated to implement in most of the cases.

# Relational strategy and emergent quantum gravity theories

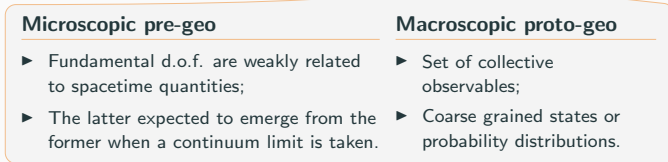


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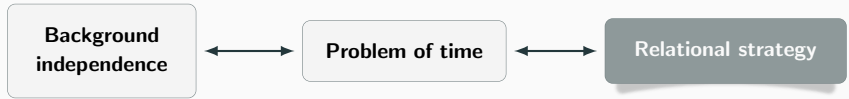
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The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.



# Relational strategy and emergent quantum gravity theories



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## Microscopic pre-geo

- ▶ Fundamental d.o.f. are weakly related to spacetime quantities;
- ▶ The latter expected to emerge from the former when a continuum limit is taken.

## Macroscopic proto-geo

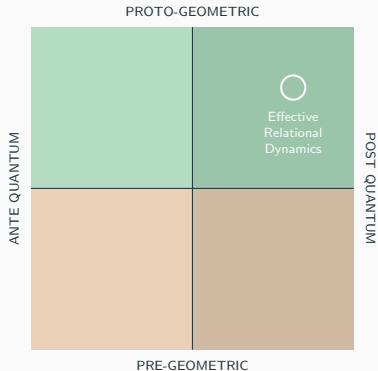
- ▶ Set of collective observables;
- ▶ Coarse grained states or probability distributions.

The quantities whose evolution we want to describe relationally are the result of a coarse-graining of some fundamental d.o.f.

## Effective approaches:

- ▶ Bypass most conceptual and technical difficulties;
- ▶ Relevant for observative purposes.

# Emergent effective relational dynamics

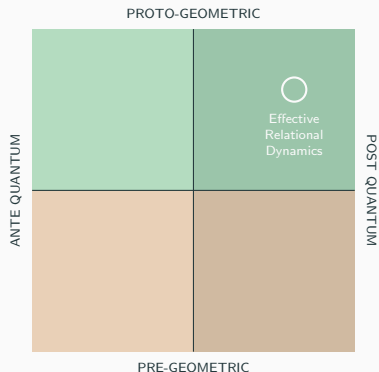


## Basic principles

**Emergence** Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

**Effectiveness** Rel. evolution intended to hold on average. Internal clock not too quantum.

# Emergent effective relational dynamics



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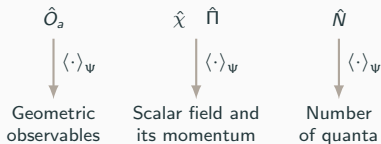
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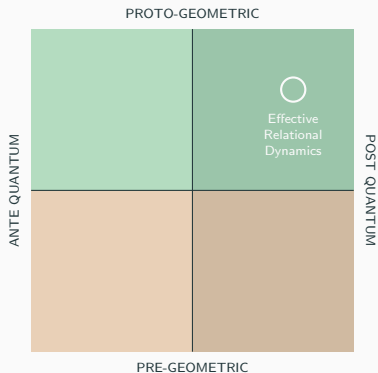
## Concrete example: scalar field clock

### Emergence

- ▶ Identify a class of states  $|\Psi\rangle$  which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- ▶ Identify a set of collective observables:



# Emergent effective relational dynamics



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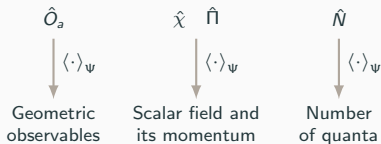
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### Effectiveness

- ▶ It exists a “Hamiltonian”  $\hat{H}$  such that

$$i \frac{d}{d \langle \hat{\chi} \rangle_\Psi} \langle \hat{O}_a \rangle_\Psi = \langle [\hat{H}, \hat{O}_a] \rangle_\Psi,$$

and whose moments coincide with those of  $\hat{\Pi}$ .

- ▶ Relative variance of  $\hat{\chi}$  on  $|\Psi\rangle$  should be  $\ll 1$  and have the characteristic  $\langle \hat{N} \rangle_\Psi^{-1}$  behavior:

$$\sigma_\chi^2 \ll 1, \quad \sigma_\chi^2 \sim \langle \hat{N} \rangle_\Psi^{-1}.$$

# Homogeneous cosmologies from (T)GFT condensates

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# The (T)GFT approach to QG

(Tensorial) Group Field Theories:  
theories of a field  $\varphi : G^d \rightarrow \mathbb{C}$  defined  
on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the “spacetime to be” ( $d = 4$ )  
and  $G$  is the local gauge group of gravity,  
 $G = \text{SL}(2, \mathbb{C})$  or, in many applications,  $G = \text{SU}(2)$ .

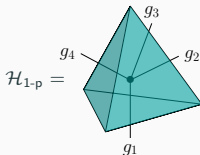
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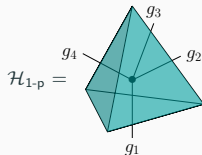
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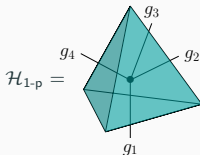
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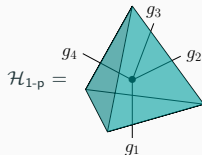
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$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

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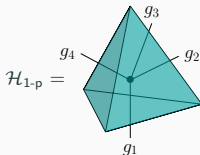
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- ▶ Non-local and combinatorial interactions guarantee the gluing of  $d - 1$ -simplices into  $d$ -simplices.
- ▶  $\Gamma$  are dual to spacetime triangulations.

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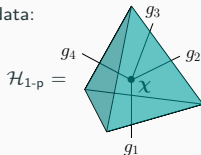
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## Kinematics

Boundary states are  $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.
- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Scalar field discretized on each  $d$ -simplex: each  $d - 1$ -simplex composing it carries values  $\chi \in \mathbb{R}^d$ .



## Dynamics

$Z_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity + matter path integral.

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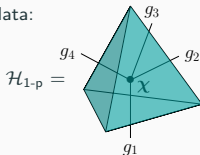
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GFTs are QFTs of atoms of spacetime.

# QG condensates and peaked states

## Extracting continuum physics

- ▶ Identify **coarse-grained states** and **collective observables** with a continuum interpretation.
- ▶ Obtain macroscopic, effective, and **relational** dynamics from the microscopic one.

### (*classical*) (T)GFT condensates

Simplest collective behavior: macroscopic  $\sigma$  dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[ \int d^d \chi \int d\mathbf{g}_I \sigma(\mathbf{g}_I, \chi^\mu) \hat{\varphi}^\dagger(\mathbf{g}_I, \chi^\mu) \right] |0\rangle$$

Collective states

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Relationality

### (clac) Condensate Peaked States

- ▶ If  $\sigma$  is peaked on  $\chi^\mu \simeq x^\mu$ ,  $|\sigma\rangle_x$  encodes relational information about the spatial geometry at  $x^\mu$ .  
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- ▶ Peaking function e.g. Gaussian with non-zero width; reduced wavefunction assumed not to spoil peaking properties.



## Spatial relational homogeneity:

$\sigma$  depends on a single “clock” scalar field  $\chi^0$

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## Observables

**Number**, **volume** (determined e.g. by the mapping with LQG) and **matter** operators (notation:  $(\cdot, \cdot) = \int d\chi^0 d\mathbf{g}_I$ ):

$$\hat{N} = (\hat{\varphi}^\dagger, \hat{\varphi})$$

$$\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$$

$$\hat{X}^0 = (\hat{\varphi}^\dagger, \chi^0 \hat{\varphi})$$

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- ▶  $\langle \hat{O} \rangle_\sigma = O[\tilde{\sigma}]|_{\chi^0=x^0}$ : exp. values of extensive operators are functionals of  $\tilde{\sigma}$  localized at  $x^0$ .

**Spatial relational homogeneity:**  
 $\sigma$  depends on a single “clock” scalar field  $\chi^0$

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## Relationality

- ▶ Averaged evolution wrt  $x^0$  is physical:

$$\langle \hat{\chi} \rangle_\sigma \equiv \langle \hat{X} \rangle_\sigma / \langle \hat{N} \rangle_\sigma \simeq x^0$$

- ▶ ... and satisfies the requirements of effective relational dynamics (in the emergent limit).

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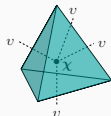
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Wavefunction  
 isotropy



$$\langle \hat{V} \rangle_\sigma = \sum_v V_v |\tilde{\sigma}_v|^2(x^0) \quad v \text{ rep. label}$$

- ▶  $v = j \in \mathbb{N}/2$  for SU(2) (EPRL-like);
- ▶  $v = \rho \in \mathbb{R}$  for SL(2, C) (ext. BC).

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## Mean-field approximation

- ▶ Mesoscopic regime: large  $N$  but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel.
- ▶ Isotropy:  $\tilde{\sigma}_v \equiv \rho_v e^{i\theta_v}$  fundamental variables.

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$$\left( \frac{V'}{3V} \right)^2 \simeq \left( \frac{2 \mathfrak{F}_v V_v \rho_v \operatorname{sgn}(\rho'_v) \sqrt{\mathcal{E}_v - Q_v^2 / \rho_v^2 + \mu_v^2 \rho_v^2}}{3 \mathfrak{F}_v V_v \rho_v^2} \right)^2 \quad \frac{V''}{V} \simeq \frac{2 \mathfrak{F}_v V_v [\mathcal{E}_v + 2\mu_v^2 \rho_v^2]}{\mathfrak{F}_v V_v \rho_v^2}$$



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$$(V'/3V)^2 \simeq 4\pi G/3 \longrightarrow \text{flat FLRW}$$

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### Bounce

- ▶ A **non-zero volume bounce** happens for a large range of initial conditions (at least one  $Q_v \neq 0$  or one  $\mathcal{E}_v < 0$ ).
- ▶ The average singularity resolution may still be spoiled by quantum effects on geometric and clock variables.

# Towards inhomogeneities

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# Scalar perturbations from (T)GFT condensates

Simplest (slightly) relationally inhomogeneous system

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- ▶ 4 MCM **reference** fields  $(\chi^0, \chi^i)$ , with Lorentz/Euclidean invariant  $S_\chi$  in field space.
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- ▶ CPSs around  $\chi^\mu = x^\mu$ , with
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- ▶ Super-horizon GR matching.
- ▶ **No matching** for intermediate modes (because of different coupling with bkg effective metric)!
- ▶ Effective metric signature determined by CPSs.

# A state-agnostic approach

---

# Effective approach for constrained quantum systems

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## Construction of the effective system

### Step 1: definition of the quantum phase space

- ▶ Describe the system with exp. values  $\langle \hat{A}_i \rangle$  and moments:
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- ▶  $\langle \hat{C} \rangle = 0$  and  $\langle (\widehat{\text{pol}} - \langle \widehat{\text{pol}} \rangle) \hat{C} \rangle = 0$  eff. constraints;
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### Step 3: relational rewriting

- ▶ Determine the remaining gauge flow which preserves the gauge conditions.
- ▶ Write evolution of the remaining variables wrt.  $T$  (classical clock).

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- ▶ Interest in a coarse grained system characterized by a small number of macroscopic (1-body) observables.
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- ▶  $\hat{K}$  such that  $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$ .

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- ▶  $\hat{K}$  such that  $[\hat{\Lambda}, \hat{K}] = i\hbar\alpha\hat{K}$ .
- ▶ The procedure can naturally be carried over by choosing as clock variable  $\hat{K}$ .
- ▶ Relational evolution of  $\langle \hat{X} \rangle$  in agreement with classical cosmology.
- ▶ Fluctuations are decoupled from expect. values.
- ▶ If they are small at small  $\langle \hat{K} \rangle$  they stay small even at large  $\langle \hat{K} \rangle$  (probably associated to the constancy of  $\hat{N}$ ).

Results

# Conclusions

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# Results and perspectives

## Relational Dynamics and Emergent QG

### Conclusions

- ✓ Presentation of a scheme to define effective relational dynamics for emergent QG theories.
- ✓ Concrete realization in (T)GFT cosmology.
- ✓ The interplay between quantum effects, emergence and relationality was highlighted.

### Perspectives

- ▶ How to change frame in an effective relational context? (Crucial to study unitarity and frame covariance.)
- ▶ When are QG material frames ideal RFs?
- ▶ Extension of state-agnostic approach to QFT?

## Cosmology from Full QG





### Conclusions

- ✓ **Bkg:** Effective volume dynamics with correct classical limit and possible singularity resolution.
- ✓ **Bkg:** Investigation of the impact of quantum fluctuations.
- ✓ **Bkg:** (almost) state-agnostic extraction of cosmological relational dynamics.
- ✓ **Pert:** First steps towards a relational cosmological perturbation theory **from full QG:**

✓ Super-horizon matching with GR.

✗ No matching with GR at intermediate scales.

### Perspectives

- ▶ **Bkg:** Inclusion of different matter fields, e.g. scalar field with potential. 
- ▶ **Pert:** Impact of bounce on perturbations. 
- ▶ **Pert:** Investigate sub-horizon GR mismatch. 
  - What gravity models do match?
  - Model building? Breakdown of approximations?
- ▶ **Pert:** Study out of condensate perturbations. 
- ▶ **Pert:** Reconstruct an effective metric (produce operators with geometric macro-interpretation).