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Critical Sobolev Spaces and Subspaces of BMO

Daniel Spector
National Taiwan Normal University

It is well-known that functions in critical Sobolev spaces embed into the space of functions of bounded mean oscillation (BMO) originating in the work of John and Nirenberg. Less well-known is the fact that they actually embed into BMO on subspaces of every smaller dimension. In this talk we introduce a class of spaces which are finer targets of these critical Sobolev embeddings than BMO that capture this phenomena, which we term beta-dimensional BMO. Interestingly, these spaces also gives an answer to the question of which BMO functions admit restrictions in BMO of subspaces. The key tool is a capacitary analogue of the John-Nirenberg inequality for the Hausdorff content, obtained recently in a joint work with You-Wei Chen.