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The Dirichlet problem and boundary regularity for nonlinear parabolic equations

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The p-parabolic equation

$$\partial_t u = \Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u)$$

is a nonlinear cousin of the classical heat equation. As such, it offers both difficulties and advantages compared with the heat equation. In the talk, we consider the Perron method for solving the Dirichlet problem for the p-parabolic equation in general bounded domains in R^{n+1}. Compared to space-time cylinders, such domains allow the space domain to change in time. Of particular interest will be boundary regularity for such domains, i.e. whether solutions attain their boundary data in a continuous way. Relations between regular boundary points and barriers will be discussed, as well as some peculiar examples and surprising phenomena related to boundary regularity.
Towards the end I will discuss the same type of questions for two other nonlinear cousins of the heat equation, the porous medium equation

$$\partial_t u = \Delta(u^m)$$

and the so-called normalized p-parabolic equation

$$\partial_t u = |\nabla u|^{2-p}\Delta_p u.$$

The talk is based on collaborations with Jana Björn (Linköping), Ugo Gianazza (Pavia), Mikko Parviainen (Jyväskylä) and Juhana Siljander (Jyväskylä).