Specht modules for the KLR algebras of type C

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Joint work with S. Ariki and E. Park.

Tableaux

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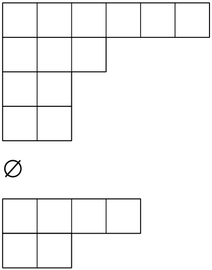

Type C residues

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2	3	4	5	6	7
1	2	3			
0	1				
1	0				

$\varnothing$

1	0	1	2
2	1		

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Let $\ell = 3$, $\kappa = (2, 0, -1)$, and $\lambda = ((6, 3, 2, 2), \varnothing, (4, 2))$.

2	3	2	1	0	1
1	2	3			
0	1				
1	0				

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1	2	3	4	5	6
7	8	9			
10	11				
12	13				

$\varnothing$

14	15	16	17
18	19		

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$$
\begin{array}{|c|c|c|c|c|c|}
\hline
1 & 2 & 3 & 4 & 5 & 6 \\
\hline
7 & 8 & 9 \\
\cline{1-3}
10 & 11 \\
\cline{1-2}
12 & 13 \\
\cline{1-2}
\end{array}
\qquad
\varnothing
\qquad
\begin{array}{|c|c|c|c|}
\hline
14 & 15 & 16 & 17 \\
\hline
18 & 19 \\
\cline{1-2}
\end{array}
\;.
$$

If $\ell = 3$, $\operatorname{res} \mathtt{T}^\lambda = (2,3,2,1,0,1,1,2,3,0,1,1,0,1,0,1,2,2,1)$. If $\ell = \infty$, $\operatorname{res} \mathtt{T}^\lambda = (2,3,4,5,6,7,1,2,3,0,1,1,0,1,0,1,2,2,1)$.

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$$\mathtt{T}_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 & 5 \\ \cline{1-2}\end{array} \quad \mathtt{T}_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 & 5 \\ \cline{1-2}\end{array} \quad \mathtt{T}_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 & 5 \\ \cline{1-2}\end{array}$$

$$\mathtt{T}_4 = \begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 3 & 4 \\ \cline{1-2}\end{array} \quad \mathtt{T}_5 = \begin{array}{|c|c|c|}\hline 1 & 3 & 5 \\ \hline 2 & 4 \\ \cline{1-2}\end{array}$$

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$w^{\mathrm{T}_1} = 1,$

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$$\{e(\nu) \mid \nu \in I^n\} \cup \{x_1, \ldots, x_n\} \cup \{\psi_1, \ldots, \psi_{n-1}\},$$

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For each dominant weight Λ, $R(n)$ has a cyclotomic quotient $R^\Lambda(n)$ which categorifies the corresp. highest weight module $V(\Lambda)$.

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Here we discuss results when $\mathfrak{g}$ is of type C_∞ or $C_\ell^{(1)}$.

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Let $\lambda \in \mathscr{P}^l_n$ and $\kappa \in \mathbb{Z}^l$. We define the graded $R(n)$-module $\mathsf{S}^\lambda_\kappa = \mathsf{S}^\lambda$ to be the module with generator z^λ subject to the relations

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For each $w \in \mathfrak{S}_n$, fix a reduced expression $w = s_{i_1} \dots s_{i_r}$. We define $\psi_w = \psi_{i_1} \dots \psi_{i_r}$.

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$$\mathsf{ch}_q\,\mathsf{S}^\lambda = \sum_{\mathtt{T} \in \mathsf{Std}(\lambda)} q^{\deg \mathtt{T}}\,\mathsf{res}\,\mathtt{T}.$$

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We conjectured that the above result remains true when $\ell < \infty$.

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For $i \in I$ and $k \in \mathbb{Z}_{>0}$, we define $\alpha_{i,n}^\vee = \alpha_i^\vee + \alpha_{i+1}^\vee + \cdots + \alpha_{i+k-1}^\vee$, where we take indices modulo $\ell + 1$ if $\ell < \infty$.

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(SS1) For all $i \in I$, $\langle \Lambda, \alpha_{i,n}^{\vee} \rangle \leq 1$.

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$R^\Lambda(n)$ *is semisimple if and only if conditions (SS1) and (SS2).*