Schurian-infinite blocks of type A Hecke algebras

Joint work with Susumu Ariki and Sinéad Lyle

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$$\text{representation-finite} \Rightarrow \text{Schurian-finite.}$$

The converse is not true in general – e.g. preprojective algebras of type other than A_n for $1 \leqslant n \leqslant 4$ are representation-infinite, but Schurian-finite.

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We want to determine the Schurian-finiteness of blocks of type A Hecke algebras, using the above proposition.

Hecke algebras

The Iwahori–Hecke algebra of the symmetric group is the unital, associative $\mathbb{F}$-algebra $\mathscr{H}_n$ with generators $T_1, T_2, \dots, T_{n-1}$ and relations

$$\begin{aligned} (T_i - q)(T_i + 1) &= 0 && \text{for all } i, \\ T_i T_j &= T_j T_i && \text{for } |i-j| > 1, \\ T_i T_{i+1} T_i &= T_{i+1} T_i T_{i+1} && \text{for } 1 \leqslant i \leqslant n-2, \end{aligned}$$

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If $e \leqslant n$, the simple modules appear as quotients of the Specht modules: $\{\mathrm{D}^\lambda \mid \lambda \vdash n,\ \lambda \text{ is } e\text{-regular}\}$.

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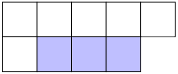
$[\lambda] =$ 

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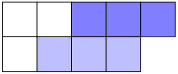
$[\lambda] =$ 

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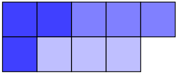
$[\lambda] =$ 

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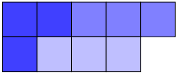
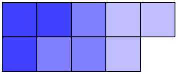
$[\lambda] =$ 



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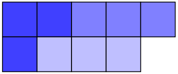
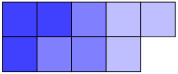
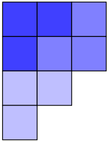
$[\lambda] =$ 



$[\mu] =$ 

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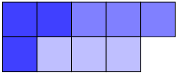
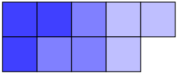
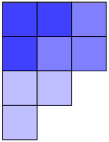
$[\lambda] =$ 



$[\mu] =$ 

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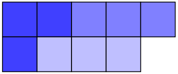
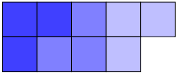
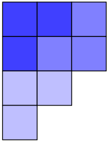
$[\lambda] =$ 



$[\mu] =$ 

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Graded decomposition numbers

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The graded decomposition number $d_{\lambda\mu}^{e,p}(v)$ is defined to be the graded composition multiplicity of D^μ in S^λ. In other words

$$d_{\lambda\mu}^{e,p}(v) = [\mathsf{S}^\lambda : \mathsf{D}^\mu]_v = \sum_{d\in\mathbb{Z}} [\mathsf{S}^\lambda : \mathsf{D}^\mu\langle d\rangle] v^d \in \mathbb{N}[v, v^{-1}].$$

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Combining this with an argument involving idempotent truncation, we're able to obtain our main tool for showing that a given block of $\mathscr{H}_n$ is Schurian-infinite.

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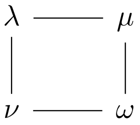

which is $A_3^{(1)} \rightsquigarrow$ the result (in characteristic 0).

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Theorem (Lyle–S.)

Suppose $e \geqslant 3$, and that B is any block of $\mathscr{H}_n$ with weight $\geqslant 4$. Then B is Schurian-infinite in any characteristic.

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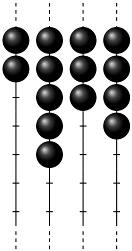

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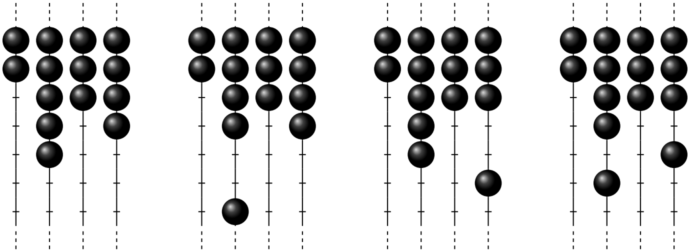
Let $e = 4 \rightsquigarrow$ 4-runner abacus. Let $\rho = (4, 3, 2, 1^3)$ be a core, and $\lambda = (12, 3, 2, 1^3)$, $\mu = (10, 5, 2, 1^3)$ and $\nu = (8, 7, 2, 1^3)$ be three partitions in the weight 2 block with core ρ.



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$$d^{e,0}_{\lambda\mu}(v) = d^{e-1,0}_{\lambda^-\mu^-}(v).$$

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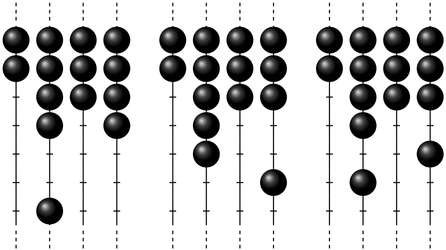

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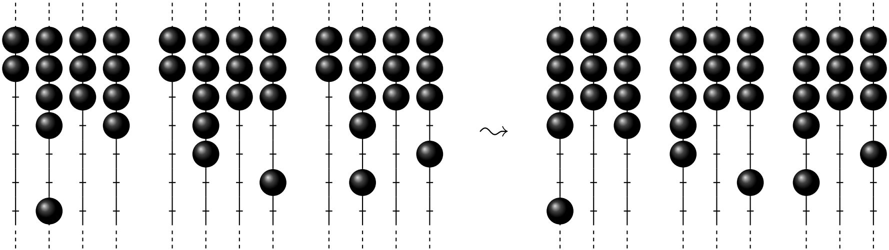

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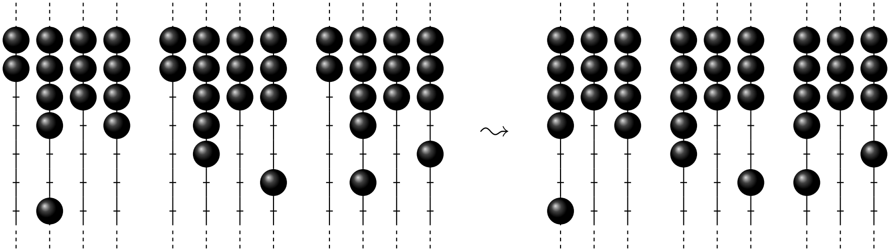

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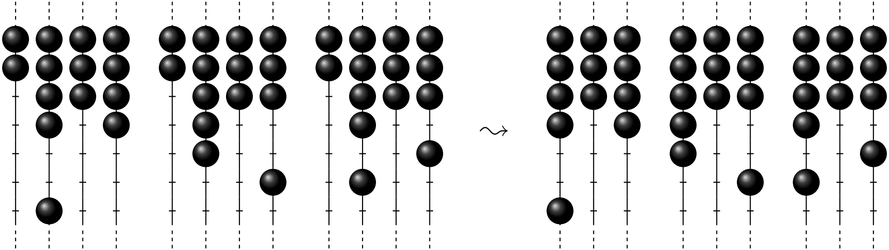

We obtain $\lambda^- = (7, 1)$, $\mu^- = (6, 2)$, and $\nu^- = (4^2)$. In particular, we have

$$d^{e,0}_{\mu\lambda}(v) = d^{e-1,0}_{(6,2)(7,1)}(v) = v \quad \text{and} \quad d^{e,0}_{\nu\lambda}(v) = d^{e-1,0}_{(4^2)(7,1)}(v) = 0.$$

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AIM: In a given block, choose four partitions so that all the ‘action’ happens on the runners with positions p_{e-2}, p_{e-1}, p_e. Then we can remove all but these three runners, using the runner removal result. We want to get (†) or (‡).

Recap

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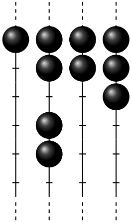

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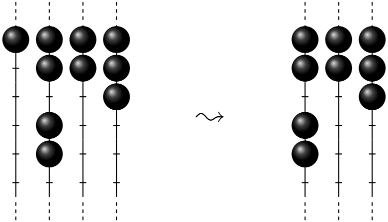

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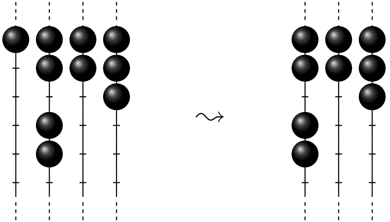

so that $\omega^- = \langle 3^2 \rangle = (4, 2^2)$. Check: these four partitions yield (†).

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If $e = 3$ and $p = 2$, difficulty is caused by the 'RoCK block', which does not have the required submatrix, and other ad hoc methods are required.

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When $e = 3$ and $p = 2$, the RoCK block is once again difficult to handle, and requires ad hoc treatment.

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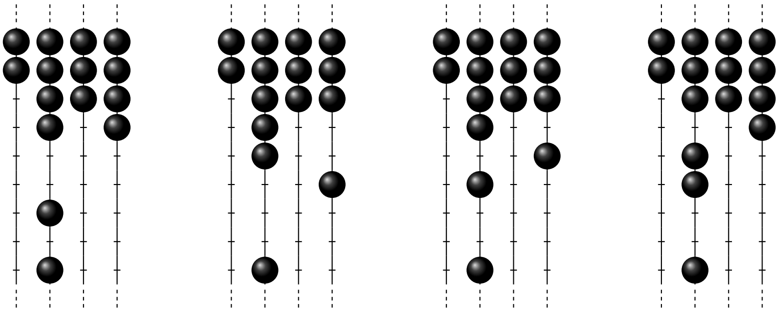
Theorem (Donkin, 1998)

If λ, μ, $\bar{\lambda}$, and $\bar{\mu}$, are as in either case above, then $d^{e,p}_{\lambda\mu}(1) = d^{e,p}_{\bar{\lambda}\bar{\mu}}(1)$.

Example

Let $e = 4$, and take the core $\rho = (7, 4, 3, 2, 1^3)$ (almost as before), and take the weight 5 block with this core. Take partitions $\lambda = (19, 12, 3, 2, 1^3)$, $\mu = (19, 10, 5, 2, 1^3)$, $\nu = (19, 8, 7, 2, 1^3)$, and $\omega = (19, 8, 5, 4, 1^3)$.



The corresponding submatrix may be computed by row-removal, and matches the weight 2 matrix previously found!

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All these come together to yield that all blocks of weight $\geqslant 2$ are Schurian-infinite.