Row removal for graded homomorphisms between Specht modules and for graded decomposition numbers

Joint work with Chris Bowman and Matthew Fayers.

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\psi_r \psi_s &= \psi_s \psi_r & & \text{if } |r-s| > 1; \\
y_r \psi_r e(i) &= (\psi_r y_{r+1} - \delta_{i_r,i_{r+1}}) e(i); \\
y_{r+1} \psi_r e(i) &= (\psi_r y_r + \delta_{i_r,i_{r+1}}) e(i);
\end{aligned}$$

$$\psi_r^2 e(i) = \begin{cases} 0 & i_r = i_{r+1}, \\ e(i) & i_{r+1} \neq i_r, i_r \pm 1, \\ (y_{r+1} - y_r)e(i) & i_r = i_{r+1}+1, \\ (y_r - y_{r+1})e(i) & i_r = i_{r+1}-1, \\ (y_{r+1} - y_r)(y_r - y_{r+1})e(i) & i_r = i_{r+1}+1, e = 2; \end{cases}$$

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$$y_1^{\langle \Lambda_\kappa, \alpha_{i_1} \rangle} e(i) = 0;$$

for all admissible r, s, i, j.

Cylotomic KLR algebras and Hecke algebras

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Fact

R_n is $\mathbb{Z}$-graded by setting

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Theorem (Brundan–Kleshchev, '09)

Suppose $e = p$ or $p \nmid e$. Then R_n is isomorphic to the corresponding cyclotomic Hecke algebra.

Multipartitions and tableaux

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$$\mathtt{T} = \begin{array}{l} \begin{array}{|c|c|c|}\hline 1 & 2 & 5 \\ \hline 6 & 8 \\ \cline{1-2} \end{array} \\ \\ \begin{array}{|c|c|}\hline 3 & 7 \\ \hline 4 \\ \cline{1-1} \end{array} \end{array} \quad \in \mathrm{Std}(\lambda).$$

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- *some Garnir relations involving ψ generators*.

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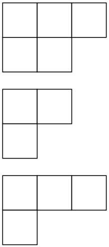

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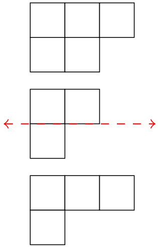
$[\lambda] =$ 

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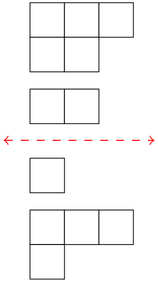
$[\lambda] =$ 

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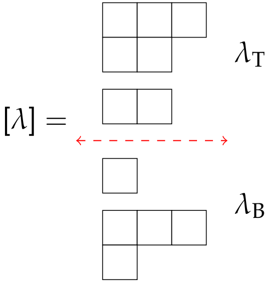

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$$\mathrm{Hom}_{R_n}(\mathrm{S}^{\lambda}, \mathrm{S}^{\mu}) \cong \mathrm{Hom}_{R_{n_{\mathrm{T}}}}(\mathrm{S}^{\lambda_{\mathrm{T}}}, \mathrm{S}^{\mu_{\mathrm{T}}}) \otimes \mathrm{Hom}_{R_{n_{\mathrm{B}}}}(\mathrm{S}^{\lambda_{\mathrm{B}}}, \mathrm{S}^{\mu_{\mathrm{B}}}).$$

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Example: Let $e = 3$, $\kappa = (0, 1, 2)$, $\lambda = ((6, 4, 3^2, 1^3), (5, 4, 2, 1^4),$ $(9^2, 6, 2^2))$ and $\mu = ((10, 4, 3^2, 2^2), (2^5), (13, 9, 6))$.

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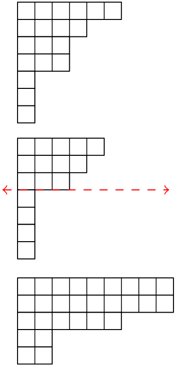
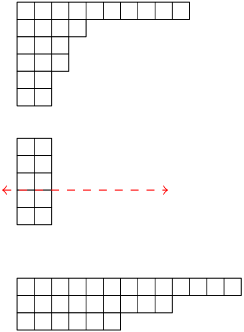
$[\lambda] =$  $[\mu] =$ 

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Thus, we retrieve that $\dim \mathrm{Hom}_{R_n}(\mathrm{S}^{\lambda}_{\kappa}, \mathrm{S}^{\mu}_{\kappa}) = 2v^{13}$.

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Analogously to horizontal cuts, we may talk about vertical cuts, at (c, m) and the left and right pieces it yields.

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With respect to this cut, we redefine the top and bottom pieces to be $\lambda_{\mathrm{T}} = (\lambda_{\mathrm{T}}(r, m), \varnothing, \dots, \varnothing)$ & $\lambda_{\mathrm{B}} = (\varnothing, \dots, \varnothing, (c^r, \lambda_{\mathrm{B}}^{(m)}), \lambda^{(m+1)}, \dots, \lambda^{(l)})$.

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where $d_{\lambda_{\mathrm{T}}\mu_{\mathrm{B}}}$ and $d_{\lambda_{\mathrm{B}}\mu_{\mathrm{B}}}$ are the corresponding graded decomposition numbers in smaller algebras.

Example: $e = 3$, $\kappa = (0, 1)$, $\lambda = ((5^2, 4^2, 3, 2, 1), (9, 6, 4^2, 3, 2^3, 1))$, $\mu = ((5, 4^2, 3^3), (9, 6, 5, 4^2, 2^2, 1^3))$.

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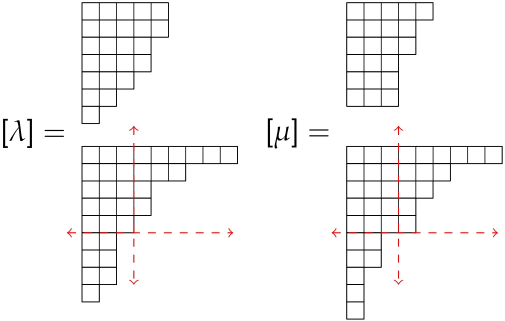
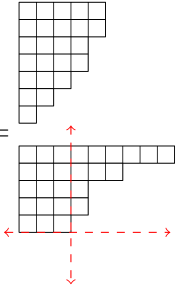
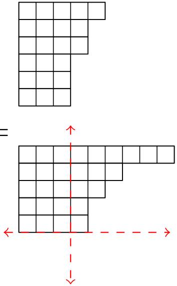
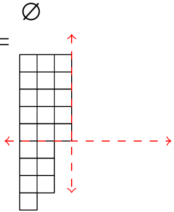
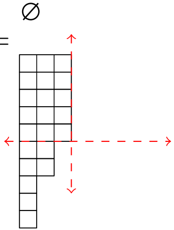


$[\lambda_{\mathrm{T}}] =$ 

$[\mu_{\mathrm{T}}] =$ 

$[\lambda_{\mathrm{B}}] =$ 

$[\mu_{\mathrm{B}}] =$ 

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Thus, we have $d_{\lambda\mu} = v^{12} + 2v^{10} + 2v^{8} + v^{6}$.