Tensor Representations for The Drinfeld Double of The Taft Algebras

Georgia Benkart[6], Rekha Biswal[7], Ellen Kirkman[8], Van Nguyen[9], Jieru Zhu[10]

WINART 2

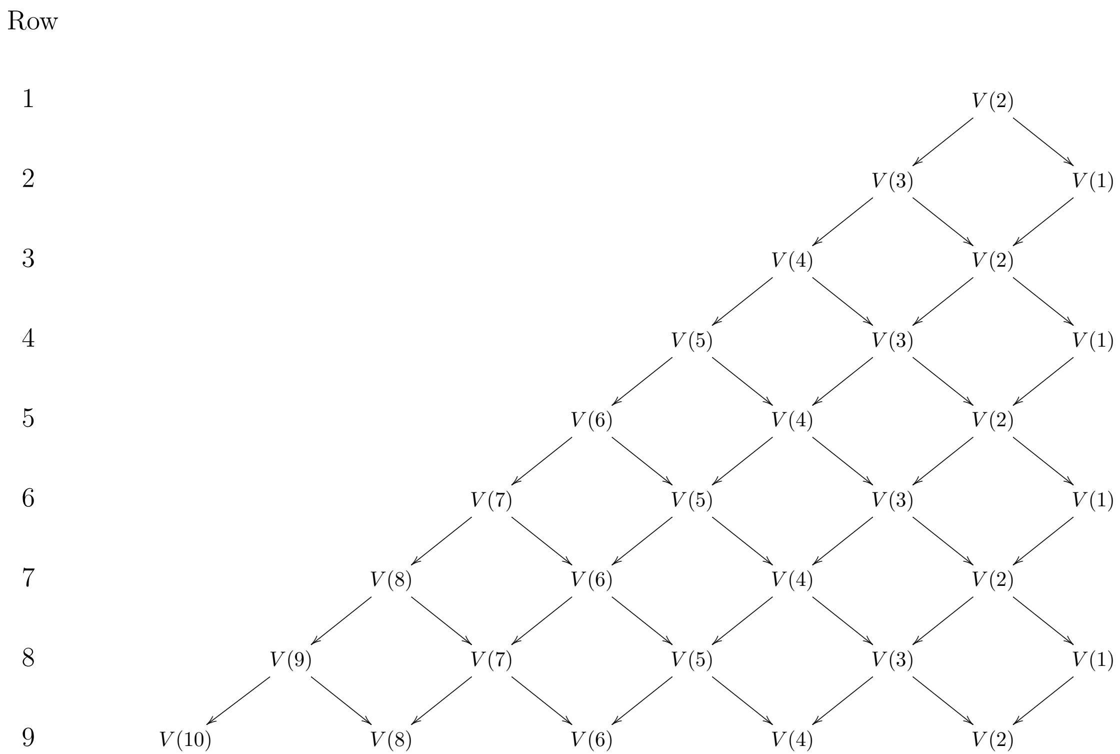
Okinawa Institute of Science and Technology

November 2020

[6]University of Wisconsin-Madison
[7]Max Planck Institute for Math
[8]Wake Forest University
[9]US Naval Academy
[10]Hausdorff Institute of Math

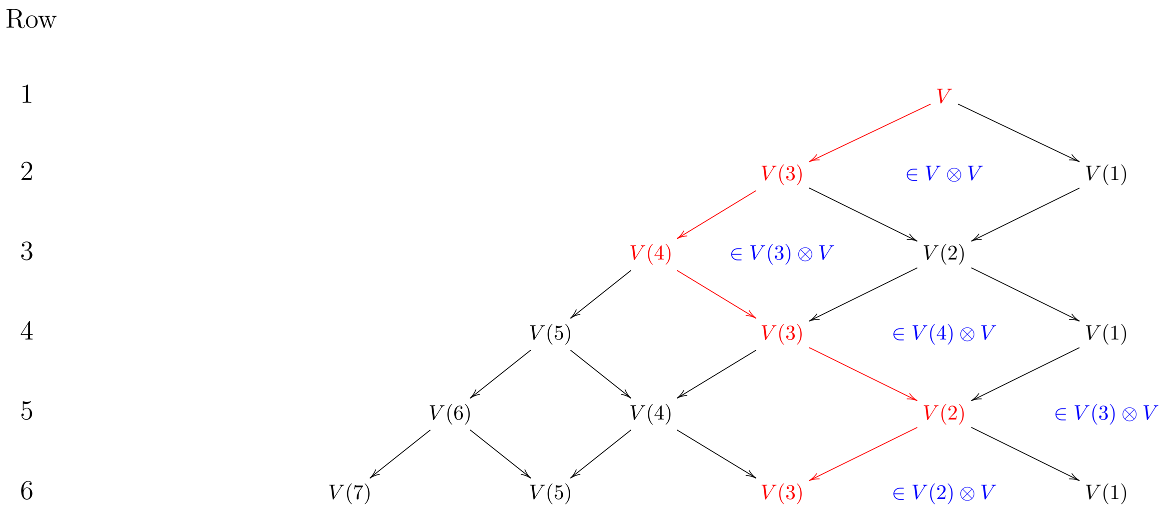
Bratteli Graph

Take $n = 11$



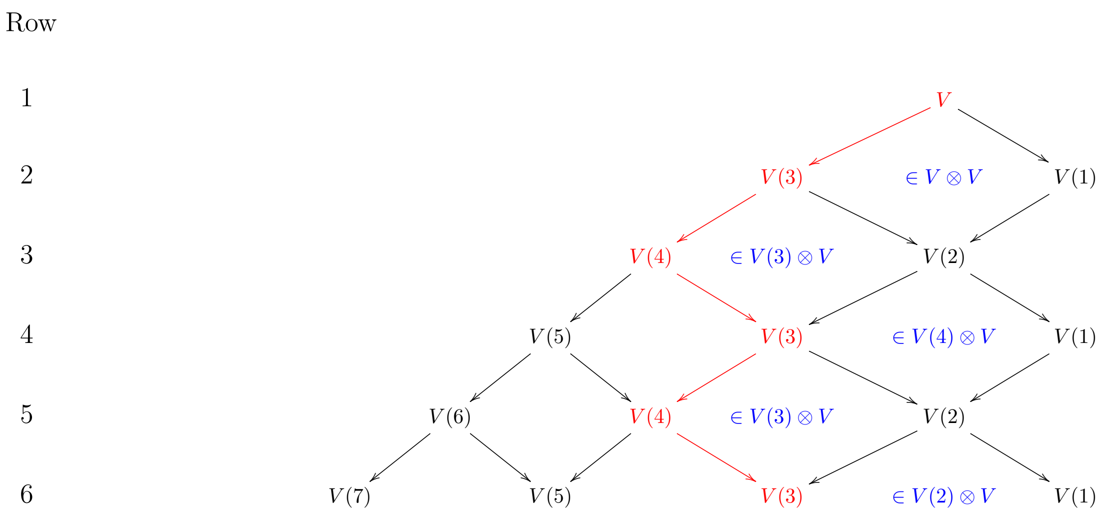
A Directed Path

Here $V = V(2,0)$

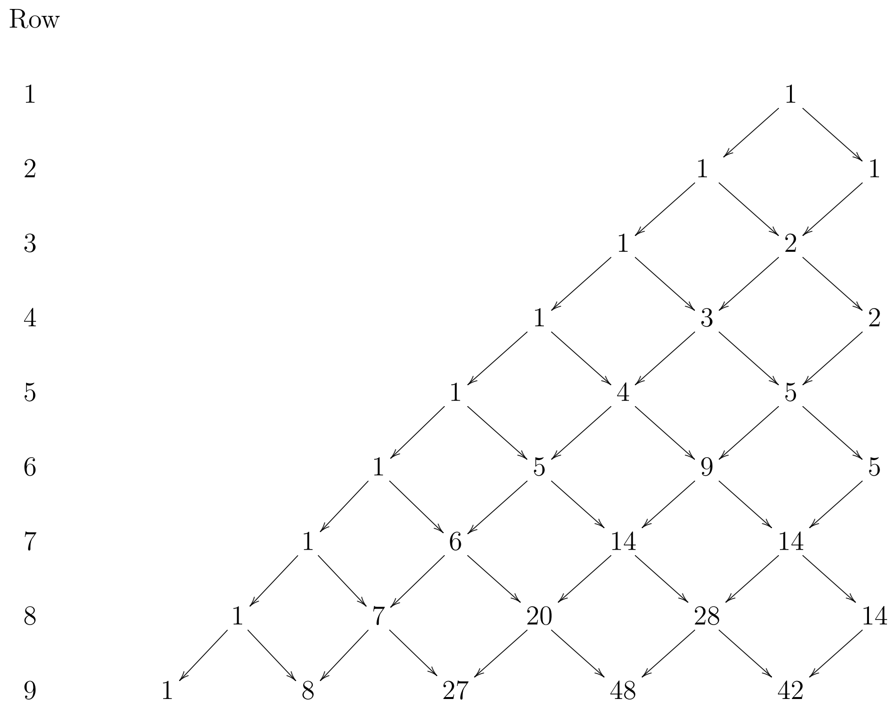


Another Directed Path

Here $V = V(2,0)$



Truncated Pascal's Triangle



Truncated Pascal's Triangle

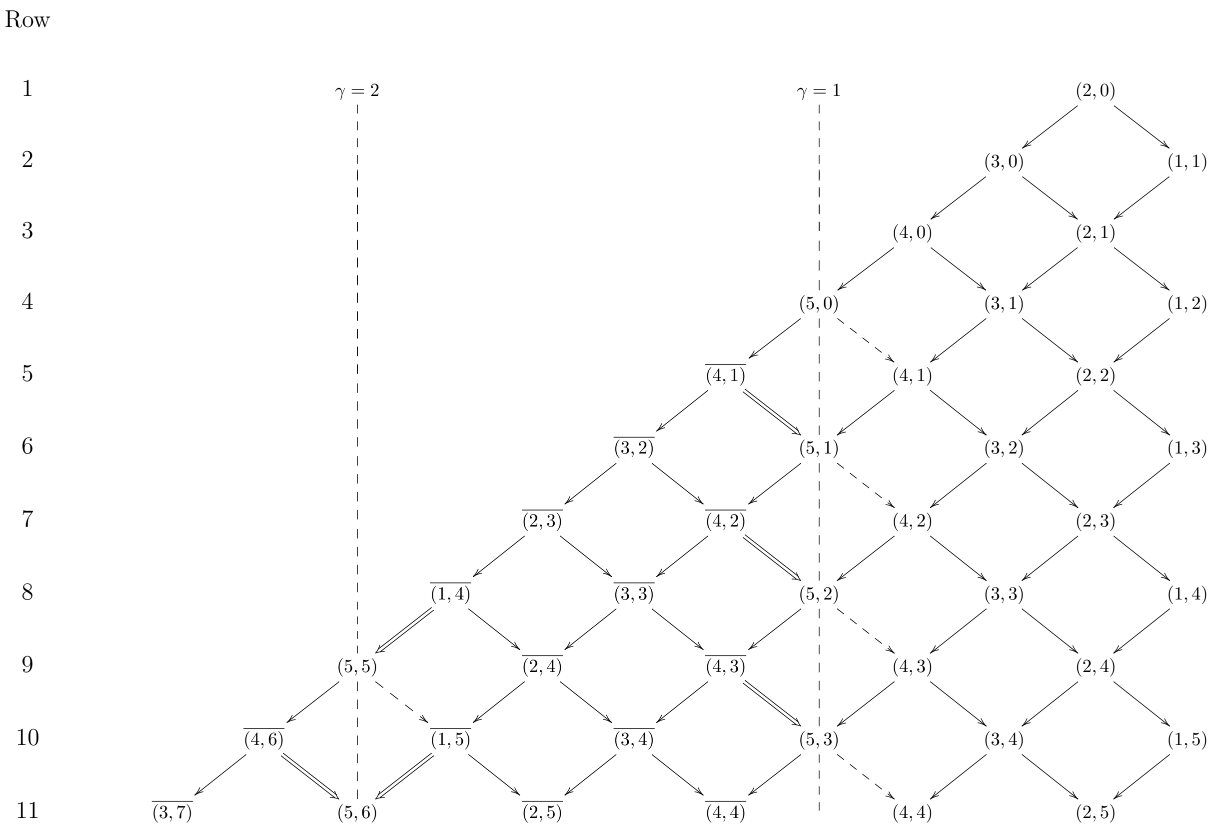
Row (k)											$\dim \mathrm{TL}_k$
1									1		1^2
2								1		1	$1^2 + 1^2$
3							1		2		$1^2 + 2^2$
4						1		3		2	$1^2 + 3^2 + 2^2$
5					1		4		5		$1^2 + 4^2 + 5^2$
6				1		5		9		5	$1^2 + 5^2 + 9^2 + 5^2$
7			1		6		14		14		$1^2 + 6^2 + 14^2 + 14^2$
8		1		7		20		28		14	$1^2 + 7^2 + 20^2 + 28^2 + 14^2$
9	1		8		27		48		42		$1^2 + 8^2 + 27^2 + 48^2 + 42^2$

Dimension of $\mathrm{TL}_k(\xi)$

Theorem (Folklore Theorem)

$$\dim \mathrm{TL}_k(\xi) = \sum_{i:node\ in\ Row\ k} (\#\ directed\ paths\ with\ target\ i)^2$$

Nonsemisimple Case: $n \leq k \leq 2n-2$. (Here $n=5$)



Dimension Count

Theorem (BBKNZ)

When $1 \leq k \leq 2n-2$,

$$\dim \mathrm{TL}_k(\xi) = \dim \mathrm{End}_{D_n}(V^{\otimes k})$$

Dimension Count

Theorem (BBKNZ)

When $1 \leq k \leq 2n-2$,

$$\dim \mathrm{TL}_k(\xi) = \dim \mathrm{End}_{D_n}(V^{\otimes k})$$

Corollary (BBKNZ)

$$\mathrm{TL}_k(\xi) \simeq \mathrm{End}_{D_n}(V^{\otimes k})$$

Comparison to the quantum group

Theorem (Andersen-Stroppel-Tubbenhauer '18)

When $q^n = 1$ *and* $k \in \mathbb{Z}_{\geq 0}$

$$\mathrm{TL}_k(\xi) \simeq \mathrm{End}_{U_q(\mathfrak{sl}_2)}((\mathbf{k}^2)^{\otimes k})$$

However,

$$\mathrm{TL}_k(\xi) \to \mathrm{End}_{D_n}(V^{\otimes k})$$

fails to be surjective when $k > 2n - 2$.