New constructions for irreducible representations in monoidal categories of type A

Representation Theory Seminar

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Plan

1. Representations of p-adic groups.

2. Quiver Hecke algebras as quantization.

3. Robinson–Schensted–Knuth construction. (w. Erez Lapid)

4. Normal sequences

5. Specht and derived RSK.

1 Representations of p-adic groups

F p-adic field. $G_n = GL_n(F)$

($GL_n(D)$, D/F division algebra fine as well.)

Rep_n = smooth representations

$\varphi: G_n \to GL(V)$ of finite length.

$V/\mathbb{C}$ usually ∞-dim.

These objects are a main focus of the celebrated Langlands program and a prototype for its many developments.

(isomorphism classes of) simple rep's $= \mathrm{Irr}_n \subset \mathrm{Rep}_n$

$$\mathrm{Irr} = \bigcup_{n=0}^{\infty} \mathrm{Irr}_n$$

Bernstein - Zelevinski gave a cornerstone classification.

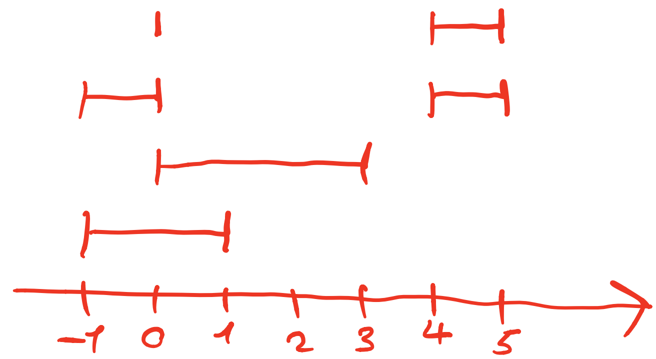
$$\mathrm{Irr} = \text{Zelevinski's multi-segments} \oplus \text{local Langlands reciprocity for super-cuspidal rep's}$$

(Henniart, Taylor-Harris, Scholze)

A <u>segment</u> is a pair of integers $[a,b]$, $a \leq b$.

A <u>multi-segment</u> is a formal finite sum of segments.

e.g. $\mathfrak{m} = [-1,0] + [-1,1] + [0,3] + [4,5] +$
$+ [0,0] + [4,5]$

$\mathfrak{m} =$ 

Given a super-cuspidal $\rho \in \mathrm{Irr}$ and a multi-segment $\mathfrak{m}$, Zelevinski defines a <u>standard</u> rep. $\Sigma(\rho, \mathfrak{m})$ that has a unique irreducible quotient $Z(\rho, \mathfrak{m}) \in \mathrm{Irr}$.

(Up to trivialities,) Z is a bijection.

Reducible rep's are not well-understood, "a step beyond the Langlands program".

Moreover, properties of Irr itself are a source of intensive study (branching laws, appearance in harmonic analysis ...) :

Can we produce "better" models than standard rep's for facilitating that study?

Parabolic induction:

Basic inductive mechanism for producing representations of reductive groups (traces back to Harish-Chandra's philosophy of cusp forms):

An exact functor

$$\mathrm{Rep}_{n_1} \times \mathrm{Rep}_{n_2} \longrightarrow \mathrm{Rep}_{n_1+n_2}$$

$$\pi_1 \boxtimes \pi_2 \longmapsto \pi_1 \times \pi_2$$

aka the Bernstein-Zelevinski product.

Looking at sum of abelian categories

$$\mathrm{Rep} = \bigoplus_{n \in \mathbb{N}} \mathrm{Rep}_n ,$$

the product makes a monoidal structure.

Hard questions:

- For $\pi_1, \ldots, \pi_k \in \mathrm{Irr}$,

  when is $\pi_1 \times \cdots \times \pi_k$ irreducible?
- What are its irr. sub-quotients?
- What is the sub-rep's lattice?

$\underline{2}$ A categorical journey

Pick a supercuspidal $\rho \in \mathrm{Irr}$.

(choice will become irrelevant!)

Let $\mathrm{Irr}_\rho \subset \mathrm{Irr}$ be the set of irr. sub-quotients appearing in all rep'ns of the form

$$(\rho|\det|^{i_1}) \times \cdots \times (\rho|\det|^{i_k})$$

$i_1, \ldots, i_k \in \mathbb{Z}$.

$\mathrm{Rep}_n^\rho \subset \mathrm{Rep}_n$ full sub-category of rep'ns "glued" out of Irr_ρ.

(variation on a Bernstein block)

Quiver Hecke algebras

(Khovanov-Landa-Rouquier algebras)

Those are $\mathbb{Z}$-graded algebras whose rep. theory was designed to categorify quantum groups.

R_n = quiver Hecke algebra of type A_∞,

for weights of height n.

$n \in \mathbb{N}$.

R_n - gmod = graded fin.-dim. R_n-modules

$$M = \bigoplus_{i \in \mathbb{Z}} M_i$$

[shift of grading: gives a new module: $r \in \mathbb{Z}$ $\quad M\langle r \rangle = \bigoplus_i M_{i-r}$]

R_n - mod = (ungraded) fin.-dim. R_n-modules

Categorical equivalences

Bernstein - Bushnell - Kutzko - Heiermann:

Rep^p_n is equivalent to a category $\mathcal{M}_n$ of fin.-dim. modules over an affine Hecke algebra $H_n(q)$.

$(q = q(p)$ prime power$)$

Rouquier: The categories $\mathcal{M}_n$ and R_n-mod are equivalent.

Brundan - Kleshchev showed that cyclotomic quotients of R_n and $H_n(q)$ are in fact isomorphic. More on that later...

$$\begin{tikzcd}
R_n\text{-mod} \arrow[d, phantom, "\wr\ \text{Rouquier}"] & & R_n\text{-gmod} \arrow[ll, "\text{Forget grading}"'] \arrow[ddll, dashed, bend left, "\mathcal{F}", "\text{``grading-forgetting''}"'] \\
\mathcal{M}_n \text{ (affine Hecke alg. modules)} \arrow[d, phantom, "\wr\ \text{Bernstein}"] & & \\
\mathrm{Rep}^{\rho}_n & &
\end{tikzcd}$$

all operations are monoidal:

$$\mathcal{F}(M_1 \circ M_2) \simeq \mathcal{F}(M_1) \times \mathcal{F}(M_2),$$

$$\begin{aligned}
R_{n_1}\text{-gmod} \times R_{n_2}\text{-gmod} &\longrightarrow R_{n_1+n_2}\text{-gmod} \\
M_1 \boxtimes M_2 &\longmapsto M_1 \circ M_2
\end{aligned}$$

is the convolution product.

Monoidal category

$$\mathcal{C} = \bigoplus_n R_n\text{-gmod} \quad \left(\begin{array}{l}\text{categorifying} \\ \text{the quantum} \\ \text{group } U_q(sl_\infty)^+\end{array}\right)$$

becomes a "quantization" of

$$\mathrm{Rep}^\delta = \bigoplus_n \mathrm{Rep}_n^\delta .$$

From the p-adic point of view:

Hidden graded structure uncovered.

Previous hard questions may benefit from developments around monoidal structure of $\mathcal{C}$.

(e.g. cluster algebras, Kang-Kim-Kashiwara-Oh)

$\mathcal{B}$ = Grothendieck ring of Rep^ρ.

When $m = [a,b]$ $a \leq b$ is a single segment, $\Delta(a,b) = Z(\rho, m) \in \mathrm{Irr}_\rho$ are called <u>segment</u> representations.

It is known that

$\mathcal{B} \simeq$ polynomial ring in $\{\Delta(a,b)\}$, $a \leq b \in \mathbb{Z}$.

$$\begin{tikzcd}
\mathcal{C} & & \mathrm{Rep}^\rho \arrow[ll, squiggly, "\text{gradation}"'] \\
U_q(\mathfrak{sl}_\infty)^+_{\mathbb{Z}} & & \mathcal{B} = \text{polynomial ring} \arrow[ll, squiggly, "\text{quantization}"] \arrow[u, squiggly, "\text{categorification}"']
\end{tikzcd}$$

Simple modules:

Given a simple module $L \in \mathcal{C}$,

$\mathcal{F}(L) \in \mathrm{Irr}_\rho$, hence,

$\mathcal{F}(L) = Z(\rho, \mathfrak{m}) = Z(\mathfrak{m})$, for a multisegment $\mathfrak{m}$.

Note, that $\mathcal{F}(L\langle r\rangle) = \mathcal{F}(L)$ for any shift $r \in \mathbb{Z}$.

We set $L_{\mathfrak{m}}$ to be the self-dual (canonical shift of grading) module in $\mathcal{C}$, for which

$\mathcal{F}(L_{\mathfrak{m}}) = Z(\mathfrak{m})$.

Native construction of L_m –

Kleshchev-Ram classification:

Each multisegment m gives rise to a (proper standard) module $KR(m) \in \mathcal{C}$.

This is a (graded) module with a unique irreducible quotient $\cong L_m$.

(simple head)

In fact, $\mathcal{F}(KR(m)) = \Sigma(m)$, quantization of p-adic standard modules.

(K-R construction categorifies PBW bases for the quantum group.)

Homogeneous modules:

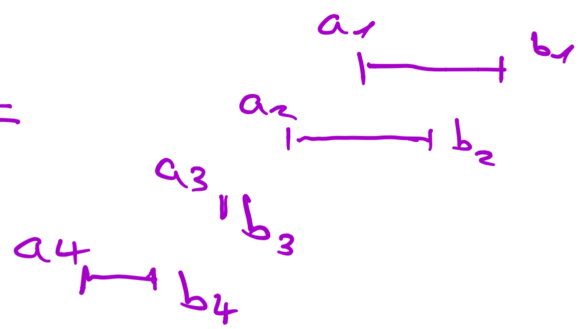
Another work of Kleshchev-Ram identified the simple modules $M \in \mathcal{C}$, for which $M = (M_i)_{i \in \mathbb{Z}}$

$$M_i = \{0\}, \quad i \neq i_0 .$$

It turns out those have a nice description in terms of the Zelevinski / K-R classification:

L_m is homogeneous $\iff$ m is a ladder.

$m =$



$$a_1 > \cdots > a_k$$
$$b_1 > \cdots > b_k$$

Those irr. rep's are well-known in the p-adic literature for their favorable behavior (Lapid-Mínguez, et al.)

3 RSK construction (w. Erez Lapid)

(RSK = Robinson - Schensted - Knuth)

Detour to invariant theory

$$S = \mathbb{C}[x_{i,j}]_{\substack{1 \leq j \leq n \\ 1 \leq i \leq k}} \quad \text{polynomial ring.}$$

For a choice $i_1 > \cdots > i_t \in \{1, \ldots, k\}$

$j_1 > \cdots > j_t \in \{1, \ldots, n\}$

$$(i_1 \cdots i_t \mid j_1 \cdots j_t) = \sum_{\tau \in S_t} \operatorname{sgn}(\tau) \prod_{s=1}^{t} x_{i_s, j_{\tau(s)}} \in S$$

minor

$$= \det(x_{i_s, j_u})_{s,u}$$

$$\begin{array}{c|c} & \begin{array}{cccc} j_1 & j_2 & & j_3 \end{array} \\ \begin{array}{c} i_1 - \\ \\ i_2 - \\ i_3 - \end{array} & \left(\begin{array}{cccc} \cdot & \cdot & & \cdot \\ & & & \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \end{array}\right) k \\ & n \end{array}$$

Now, let us take a <u>bi-tableau</u>

(2 tableaux of same shape)

$$T_1 = \begin{array}{|ccccc|} i_1^1 & i_2^1 & \cdots & i_{t_2}^1 & \cdots \; i_{t_1}^1 \\ i_1^2 & i_2^2 & \iddots & i_{t_2}^2 & \\ \vdots & & & & \\ i_1^s & \cdots & i_{t_3}^s & & \end{array} \qquad T_2 = \begin{array}{|ccccc|} j_1^1 & j_2^1 & \cdots & j_{t_2}^1 & \cdots \; j_{t_1}^1 \\ j_1^2 & j_2^2 & \iddots & j_{t_2}^2 & \\ \vdots & & & & \\ j_1^s & \cdots & j_{t_3}^s & & \end{array}$$

and define a product of minors

$$\tilde{\Lambda}_{T_1,T_2} = \left(i_1^1 \cdots i_{t_1}^1 \,\middle|\, j_1^1 \cdots j_{t_1}^1\right) \cdot \ldots \cdot \left(i_1^s \cdots i_{t_s}^s \,\middle|\, j_1^s \cdots j_{t_s}^s\right)$$

$$\in S \ .$$

<u>Theorem</u>: (Doubilet, Rota, Stein 74'
Dèsarmènien, Kung, Rota 78')

$\{ \tilde{\Lambda}_{T_1, T_2} : T_1, T_2$ are semi-standard$\}$

is a basis for S.

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semi-standard = $i_r^1 \geq i_r^2 \geq \cdots$
$j_r^1 \geq j_r^2 \geq \cdots$, $\forall r$.

In B:

For a s.s. bi-tableau (T_1, T_2),

$\tilde{\Lambda}_{T_1, T_2} \in B$ can be defined by

treating $\underbrace{[\Delta(a,b)]}_{a \leq b}$ as variables $x_{a,b}$.

$$\begin{tikzcd}
\mathcal{C} & & \mathrm{Rep}^{\ell} \arrow[ll, squiggly, "\text{gradation}"'] \\
U_q(\mathfrak{sl}_\infty)^+_{\mathbb{Z}} \arrow[urr, squiggly, "\text{categorification}"'] & & \mathcal{B} \arrow[ll, squiggly, "\text{quantization}"]
\end{tikzcd}$$

$\mathcal{B}$ = polynomial ring

$$\mathcal{B} \supseteq \text{"Rota basis"} = \{ \tilde{\Lambda}_{T_1, T_2} \}$$

What would it mean to pull the Rota basis to the other corners of the square?

Rep-theoretic significance?

For a homogeneous module $L_m \in \mathcal{C}$ given by a ladder multisegment

$$m = [a_1, b_1] + \dots + [a_k, b_k],$$

$$\left[Z(m)\right] = \tilde{\Lambda}_{T_1,T_2} = (a_1 \cdots a_k \mid b_1 \cdots b_k) \in \mathcal{B}.$$

This is the content of the

Lapid–Mínguez–Tadić determinantal identity.

For a general s.s. (T_1, T_2), let

$\ell_1, \dots, \ell_k$ be the ladder multisegments

corresponding to the k rows of the

bi-tableau.

We define a quiver Hecke algebra module

$$\Gamma(T_1, T_2) = L_{\ell_1} \circ \cdots \circ L_{\ell_k} \langle d \rangle \in \mathcal{C}$$

(The shift $d \in \mathbb{Z}$ will be determined later.)

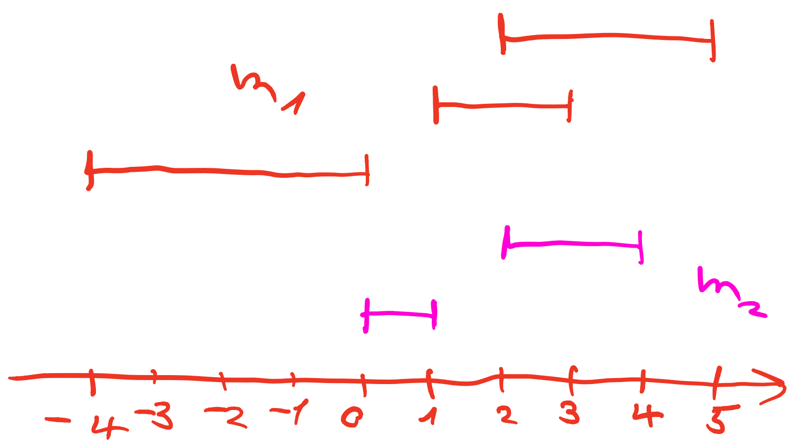
$$[\mathcal{F}(\Gamma(T_1, T_2))] = \tilde{\Lambda}_{T_1, T_2}$$

For example,

$$T_1 \quad \begin{array}{|ccc|} 2 & 1 & -4 \\ 2 & 0 & \end{array} \qquad T_2 \quad \begin{array}{|ccc|} 5 & 3 & 0 \\ 4 & 1 & \end{array}$$

$$l_1 = [2,5] + [1,3] + [-4,0]$$

$$l_2 = [2,4] + [0,1]$$



$$\Gamma(T_1, T_2) = L_{l_1} \circ L_{l_2} \langle d \rangle$$

Robinson - Schensted - Knuth correspondence:

multisets of pairs of integers $\overset{\longleftrightarrow}{\text{RSK}}$ s.s. bi-tableaux on integers

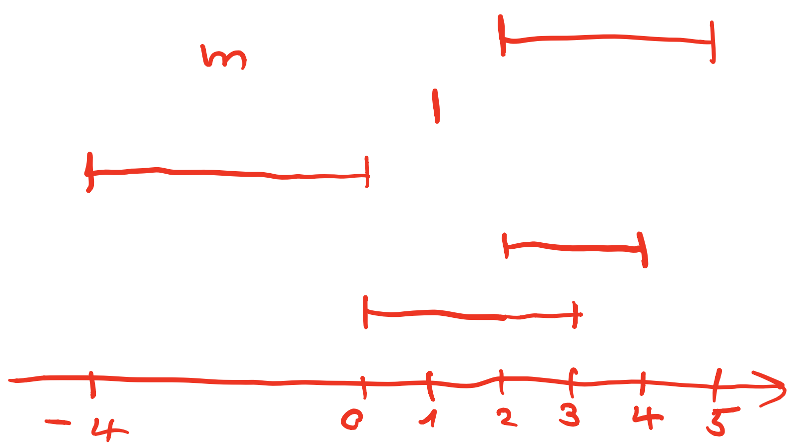
$\cup$|

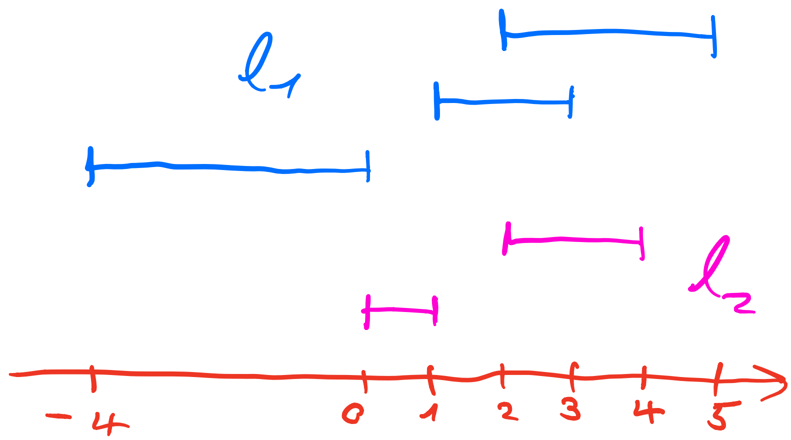
multisegments

$\exists$ bijection with an explicit recursive algorithm.

e.g.

$$\mathrm{RSK}(m) = \begin{array}{|ccc|}\hline 2 & 1 & -4 \\ 2 & 0 & \\ \hline\end{array} \quad \begin{array}{|ccc|}\hline 5 & 3 & 0 \\ 4 & 1 & \\ \hline\end{array}$$





For each multisegment m we define its RSK-standard module as

$$\Gamma(m) := \Gamma(RSK(m)) \in \mathcal{C}$$

G. - Lapid:

(translated to quiver Hecke algebra language)

For a suitable shift $\langle d \rangle$,

L_m appears as a quotient module of $\Gamma(m)$.

Provides an alternative construction of all simple modules, as induced from homogeneous data.

The number of rows in the tableaux of $RSK(m) = \omega(m)$, is the minimal number of

homogeneous modules $L_1, \dots, L_{w(m)}$,
for which L_m can occur as a sub-quotient of $L_1 \circ \dots \circ L_{w(m)}$. (G.)

<u>Conjectures:</u>

- $\{[\Gamma(m)]\}$ are a $\mathbb{Z}[q, q^{-1}]$-basis for $U_q(sl_\infty)^+_{\mathbb{Z}}$.
- Transition matrix between $\{[\Gamma(m)]\}$ and the dual canonical basis unitriangular relative to a natural partial order on bi-tableaux.

4 Normal sequences

Leclerc coined the notion of a real / square-irreducible rep'n: $\pi \in \mathrm{Irr}$, s.t. $\pi \times \pi \in \mathrm{Irr}$.

In the context of quiver Hecke algebras, it was thoroughly studied by Kang-Kashiwara-Kim-Oh.

On the p-adic side the notion was further developed by Lapid-Mínguez.

Given any real simple $L \in \mathcal{C}$,
and any simple $M \in \mathcal{C}$,
$L \circ M$ has a simple head (KKKO)
$H\langle -\tilde{\Lambda}(L,M)\rangle$, where $H \in \mathcal{C}$
is self-dual simple.

A numeric invariant: $\tilde{\Lambda}(L,M) \in \mathbb{Z}$.

Kashiwara-Kim introduced a
concept of a normal sequence
$(L_1, \dots, L_k)$ of real modules:
For a normal sequence,
$L_1 \circ \dots \circ L_k$ has a simple head.

The notion is defined through intertwiner operators (R-matrices).

Inductively: $(L_1, \dots, L_k)$ is a normal sequence, iff, $(L_2, \dots, L_k)$ is a normal sequence with a simple head H, and

$$\tilde{\Lambda}(L_1, H) = \sum_{i=2}^{k} \tilde{\Lambda}(L_1, L_i) \quad \text{holds.}$$

The notion of normal sequences turns out to be useful for questions in the p-adic setting, where the $\tilde{\Lambda}$ invariant was "hidden".

(e.g. G.-Minguez on quotients of standard modules - cyclic rep's)

G.: For any multisegment $\mathfrak{m}$, the ladders $\ell_1, \dots, \ell_{w(\mathfrak{m})}$ read from $RSK(\mathfrak{m})$, give a normal sequence $(L_{\ell_1}, \dots, L_{\ell_{w(\mathfrak{m})}})$.

Corollaries:

1) Each RSK-standard module $\Gamma(\mathfrak{m})$ has a simple head $(\cong L_{\mathfrak{m}})$.

2) $Z(\mathfrak{m}) = F(L_{\mathfrak{m}})$ appears only once in the Jordan–Hölder series of $F(\Gamma(\mathfrak{m}))$.

3) For any $L_{\mathfrak{m}'}\langle r\rangle$, with $\mathfrak{m}' \neq \mathfrak{m}$, appearing in $\Gamma(\mathfrak{m})$, we have $r > 0$.

Also, an explicit formula for the shift $\langle d(m) \rangle$ needed to define $\Gamma(m)$.

5 Derived RSK-standard modules and the Specht construction

Cyclotomic quotients:

(complex alg. q not root of 1)

The affine Hecke algebra $H_n(q)$ (type A) comes with a family of specified finite-dimensional quotients.

$\lambda = (i_1, \dots, i_k)$

(dominant integral weight for sl_∞)

will stand for a choice of a multi-set of integers.

Cyclotomic Hecke algebra : $H_n^\lambda = H_n(q) / (p_\lambda(y_1))$

where $y_1 \in H_n(q)$ is a fixed element,

and $p_\lambda(z) = (z - q^{i_1}) \cdots (z - q^{i_k})$

a fixed polynomial.

Brundan – Kleshchev :

Quotients R_n^λ of R_n can be defined analogously, so that $R_n^\lambda \cong H_n^\lambda$, as algebras. But R_n^λ is graded!

$$H_n(q) = \varprojlim_\lambda H_n^\lambda \quad , \quad R_n = \varprojlim_\lambda R_n^\lambda$$

may be viewed as limits of fin-dim. algebras.

Cyclotomic Hecke algebras,

aka Ariki-Koike algebras, are

of stand-alone interest:

If $\lambda_0 = (i_1)$ (level 1),

$R_n^{\lambda_0} \simeq H_n^{\lambda_0} \simeq$ (finite) Hecke algebra of S_n

$\simeq \mathbb{C}[S_n]$.

i.e. H_n^{λ} stand in between finite

and affine Hecke algebras.

Simple modules:

Let Irr_n^λ be the set of isomorphism classes of simple modules of H_n^λ, or self-dual graded simple modules of R_n^λ. $\mathrm{Irr}^\lambda = \bigcup_n \mathrm{Irr}_n^\lambda$

Using $R_n \twoheadrightarrow R_n^\lambda$, each $M \in \mathrm{Irr}_\lambda$ can be inflated to a simple self-dual module $\mathrm{infl}(M) \in \mathcal{C}$.

We obtain

$$\left(\mathrm{Irr}_\mathcal{C} \text{ is}\right) \text{ self-dual simples in } \mathcal{C} = \bigcup_\lambda \mathrm{Irr}_\lambda$$

What is the relation between the classifications on both sides?

Vazirani - ungraded version,
Kang-Park - graded version,
made comparisons through crystal descriptions of both sides.

R3K generalizes the explicit constructions on both sides!

For level 1 λ_0, the simple modules of $H_n^{\lambda_0}$ are given by partitions $\bar{\mu} = \mu_1 \geq \dots \geq \mu_k$ of the integer n.

Such $D(\bar{\mu}) \in \mathrm{Irr}^{\lambda_0}$,

gives the evaluation module

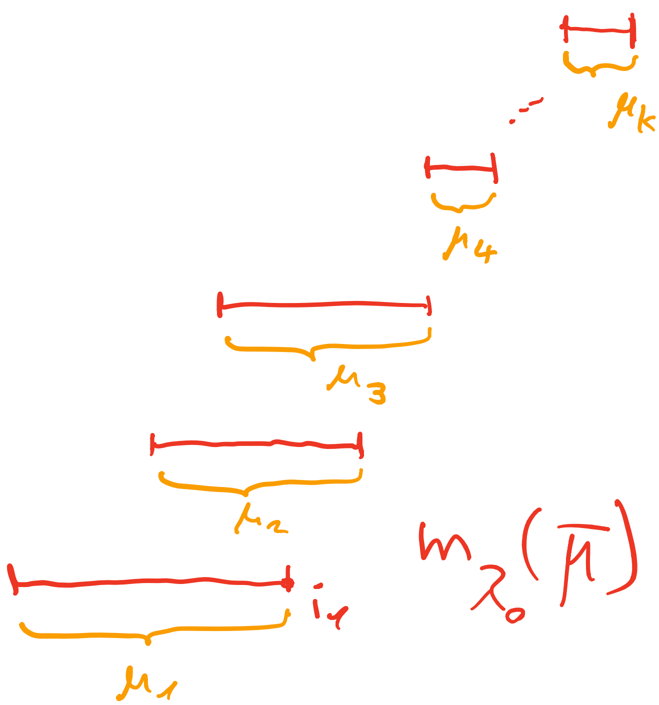
$$ev_{\lambda_0}(\bar{\mu}) := \mathrm{infl}\left(D(\bar{\mu})\right) \in \mathcal{C} .$$

$$\shortparallel$$

$$L_{m_{\lambda_0}(\bar{\mu})}$$

homogeneous module

ladder m.s. of a special type.



For a general λ, Ariki, Grojnowski gave classifications of Irr_n^λ through what amounts to Kleshchev / restricted multi-partitions:

$$D(\overline{\mu}_1, \ldots, \overline{\mu}_k) \in \mathrm{Irr}_n^\lambda$$

$\overline{\mu}_i$ partition of n_i

$n_1 + \ldots + n_k = n$, subject to the "Kleshchev" condinition.

<u>Specht construction:</u>

Ariki's construction of Irr^λ presents simple modules as simple heads of Specht modules.

These generalize the classical combinatorial construction of S_n-module into H_n^λ (Dipper-James-Mathas).

Graded Specht modules were defined by Brundan-Kleshchev-Wang:

$$S(\bar{\mu}_1, \dots, \bar{\mu}_k) \in R_n^\lambda\text{-gmod}$$

$D(\bar{\mu}_1, \dots, \bar{\mu}_k)$ is its simple head.
(Brundan-Kleshchev)

Relation to RSK

On the affine level: $\lambda = \{i_1, \dots, i_k\}$

$$\mathrm{infl}\left(S(\overline{\mu}_1, \dots, \overline{\mu}_k)\right) \simeq ev_{i_1}(\overline{\mu}_1) \circ \dots \circ ev_{i_k}(\overline{\mu}_k)$$

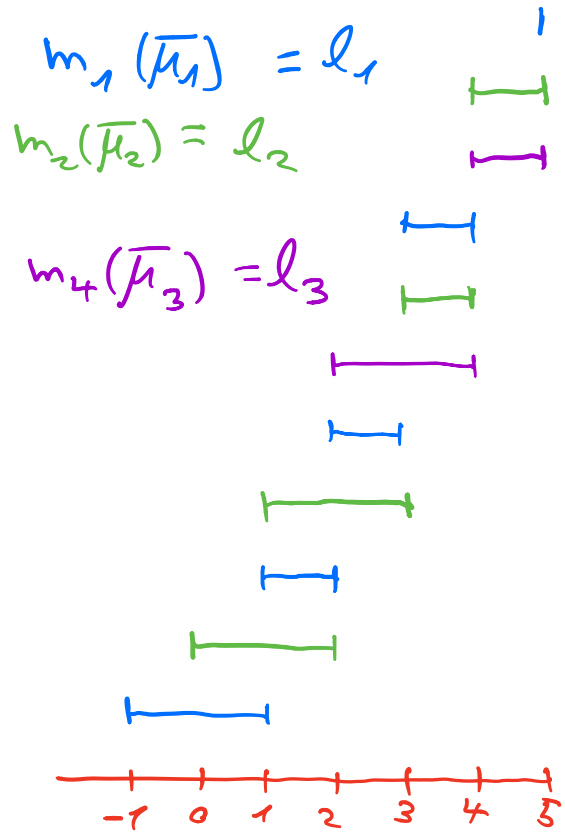
up to a shift of grading. $\in \mathcal{C}$

Product of homogeneous modules, reminds of the RSK construction.

RSK (semi-standard tableaux) and the restricted multipartition conditions are related:

T_1

$$\begin{array}{|ccccc|}
\hline
5 & 3 & 2 & 1 & -1 \\
4 & 3 & 1 & 0 & \\
4 & 2 & & & \\
\hline
\end{array}$$



T_2

$$\begin{array}{|ccccc|}
\hline
5 & 4 & 3 & 2 & \boxed{1} \\
5 & 4 & 3 & \boxed{2} & \\
5 & \boxed{4} & & & \\
\hline
\end{array}$$

$\lambda_0 = (1,2,4)$

$$L_{\ell_1} = ev_1(3,2,2,2,1) = ev_1(\overline{\mu_1})$$

$$L_{\ell_1} = ev_2(3,3,2,2) = ev_2(\overline{\mu_2})$$

$$L_{\ell_3} = ev_4(3,2) = ev_4(\overline{\mu_3})$$

In fact,

$(T_1, T_2) = RSK(m)$, for $m = \ell_1 + \ell_2 + \ell_3$.

(special case)

$$\mathrm{infl}(S(\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3)) \simeq L_{\ell_1} \circ L_{\ell_2} \circ L_{\ell_3} \langle -d \rangle$$
$$\simeq \Gamma(m)$$

In fact, $\Gamma(m)$ is a quotient

$KR(m)$.

Specht, RSK, KR (Zelevinsky) all meet together!

In general, Specht modules may lift, in terms of the bi-tableau construction, to a s.s. skew bi-tableau, with T_2 "trivial":

$$\begin{array}{|c|c|c|c|c|} \hline \; & & 2 & 1 & -1 \\ \hline & 4 & 2 & 0 & \\ \hline 5 & 3 & & & \\ \hline \end{array} \qquad \begin{array}{|c|c|c|c|c|} \hline \; & & 3 & 2 & 1 \\ \hline & 4 & 3 & 2 & \\ \hline 5 & 4 & & & \\ \hline \end{array}$$

Derived RSK-standard modules

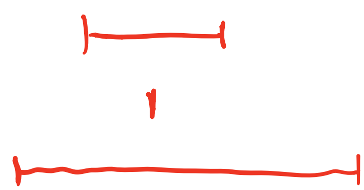
Given a multisegment

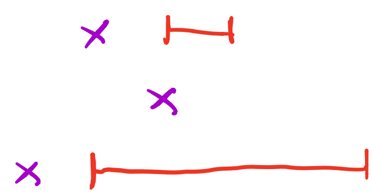
$$m = [a_1, b_1] + \dots + [a_t, b_t]$$

consider the derived m.s.

$$m' = [a_1+1, b_1] + \dots + [a_t+1, b_t],$$

where $[b+1, b]$ is taken as empty.

$m =$ 

$m' =$ 

In the p-adic setting,

the operation $\mathrm{Irr} \to \mathrm{Irr}$

$$Z(m) \mapsto Z(m')$$

expresses the

"highest Bernstein-Zelevinski derivative".

(tangential relation to crystal operators)

G. (in preparation) (ungraded version w. Lapid)

For a RSK-standard module

$$\Gamma(m) = L_{\ell_1} \circ \cdots \circ L_{\ell_w} \langle -d \rangle ,$$

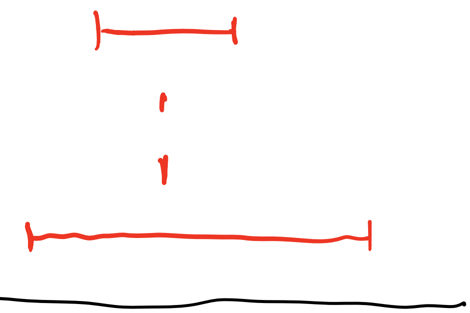
$$\Gamma'(m) := L_{\ell_1'} \circ \cdots \circ L_{\ell_w'} \langle -d' \rangle$$

has a simple head $\simeq L_{m'}$.

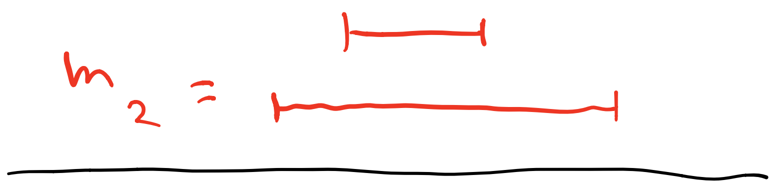
(Note, that maybe $\Gamma'(m) \not\cong \Gamma(m')$)

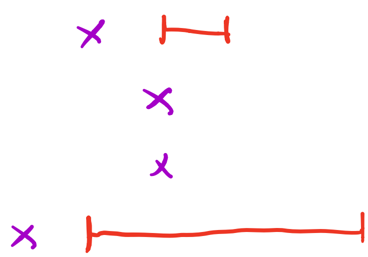
There are many "pre-derivatives":

$m_1 =$



$m_2 =$





$n = m_1' = m_2'$

Hence,

both $\Gamma'(m_1)$ and $\Gamma'(m_2) = \Gamma(n)$

admit L_n as a simple head.

Derived RSK-standard now give

a big family of modules.

- Every proper standard $KR(m)$ is a derived RSK-standard.

-

$$\left\{\begin{array}{c}\text{inflations of}\\ \text{Specht modules}\end{array}\right\}$$

$$\parallel$$

$$\left\{\begin{array}{c}\text{derived}\\ \text{RSK-std}\\ \text{modules}\end{array}\right\} \cap \left\{\begin{array}{c}\text{quotients}\\ \text{of}\\ \text{proper}\\ \text{standard}\\ \text{modules}\end{array}\right\}$$

Can the theory of Specht modules be extended to contain (derived) RSK modules?

Is there a RSK construction in the modular setting?

ご清聴
ありがとう
ございました