The Mullineux map

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If $\mathrm{char}(\mathbb{F}) = p$ and λ is p-regular, then S^λ has a unique simple quotient D^λ (the James module). The set

$$\left\{ D^\lambda \;\middle|\; \lambda \text{ a } p\text{-regular partition of } n \right\}$$

is a complete set of simple modules for $\mathfrak{S}_n$ in characteristic p.

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Mullineux problem: Describe the map t_p combinatorially.

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  $$t_3 : (3) \longmapsto (2,1), \qquad (2,1) \longmapsto (3).$$

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So $m_p = k_p = x_p = s_p = t_p$. Write these all as m_p.

Mullineux’s bijection

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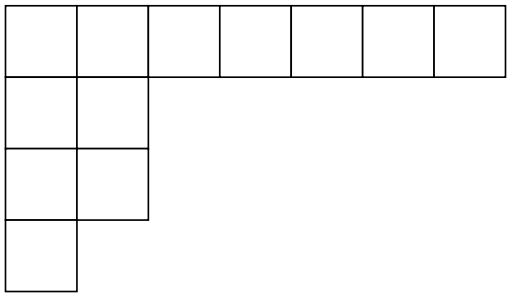
e.g. $\lambda = (7, 2^2, 1)$



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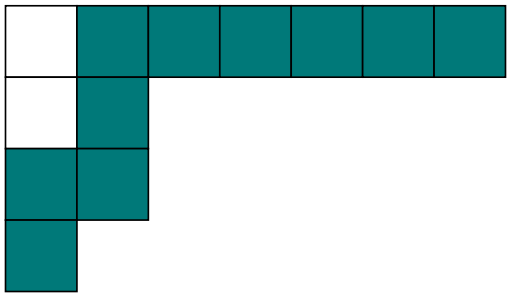

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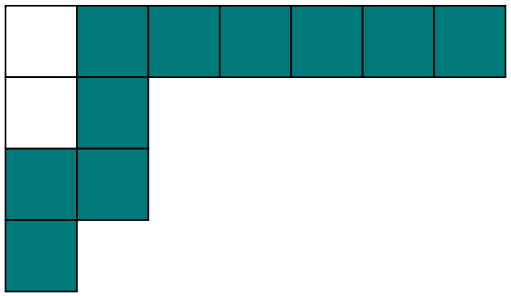

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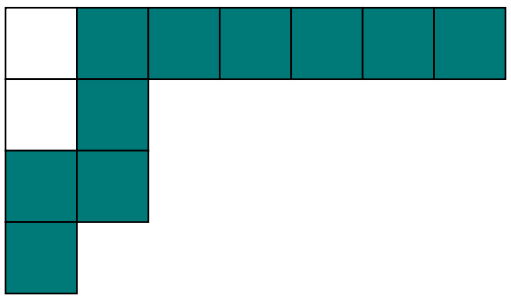

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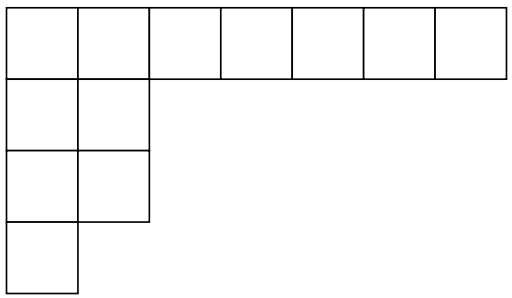

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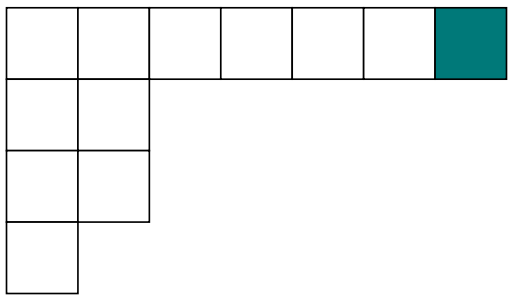

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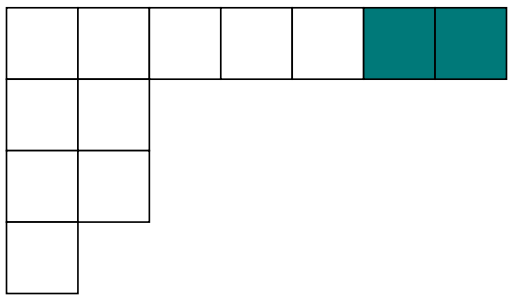

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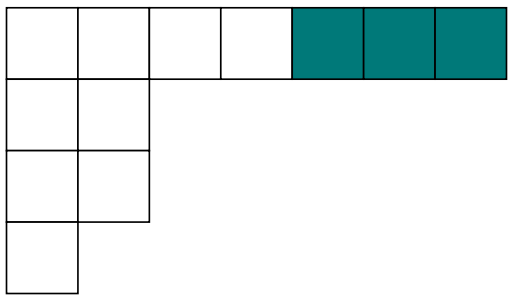

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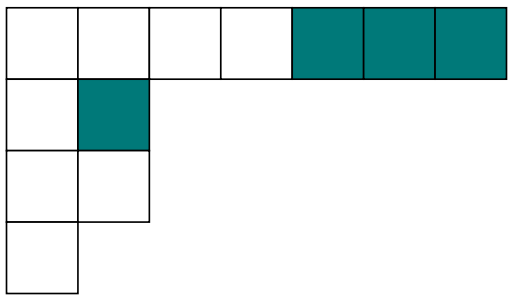

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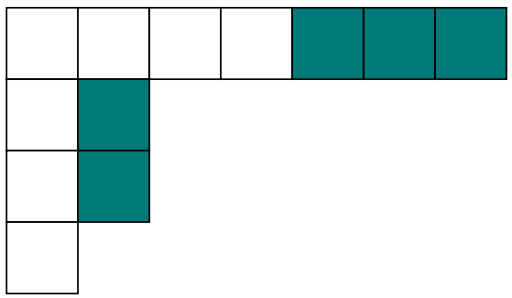

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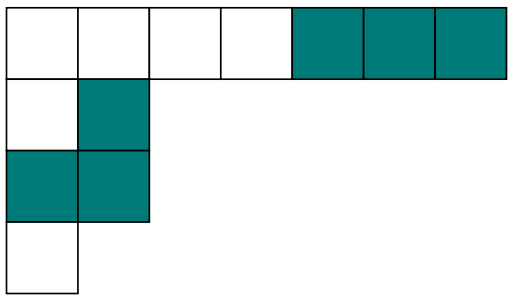

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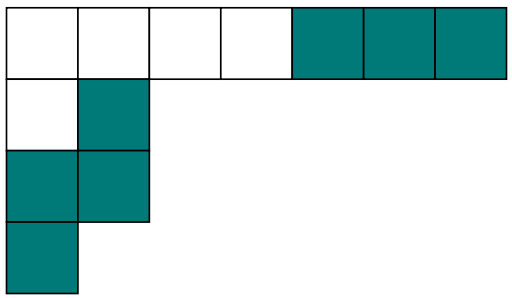

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Mullineux symbol of λ: $\begin{pmatrix} n_1 & n_2 & \dots \\ r_1 & r_2 & \dots \end{pmatrix}$.

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Mullineux bijection: $m_p(\lambda)$ is the p-regular partition whose Mullineux symbol is

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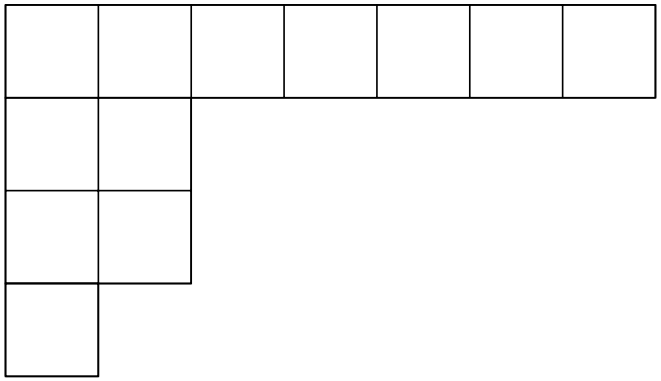
Mullineux's bijection – example



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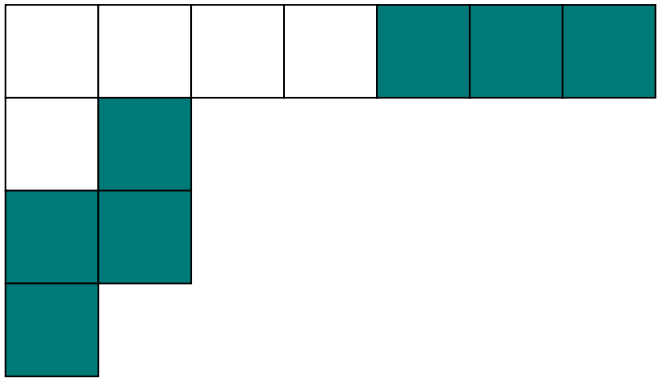

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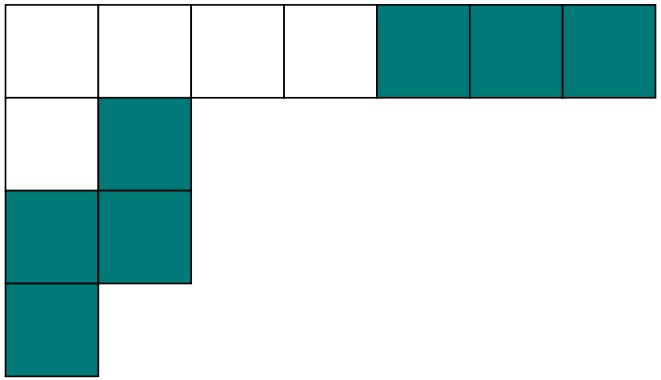

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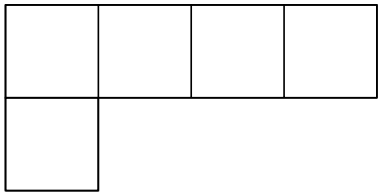

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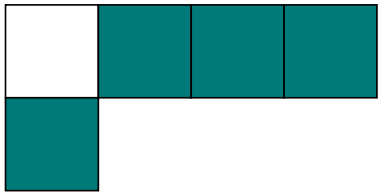

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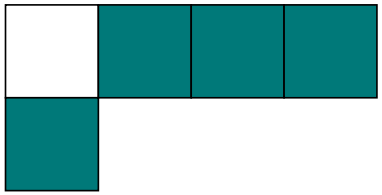

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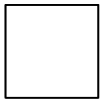

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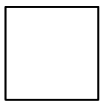

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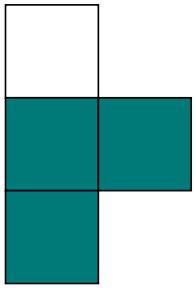

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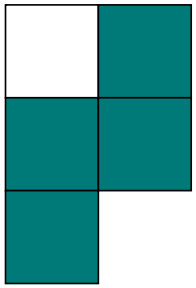

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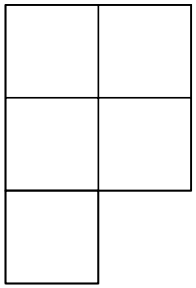

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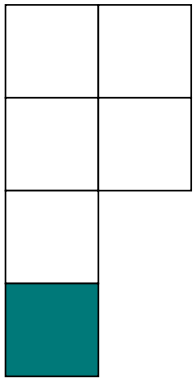
Mullineux's bijection – example



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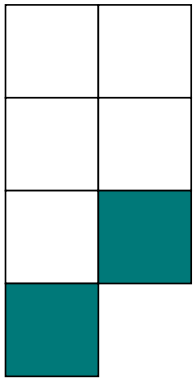

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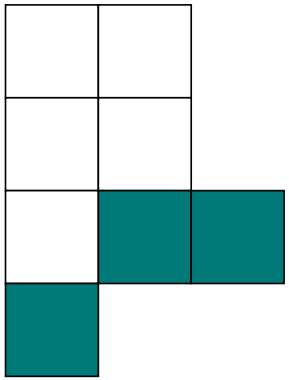

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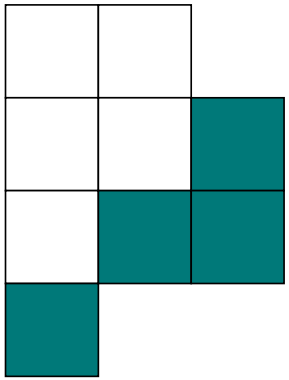

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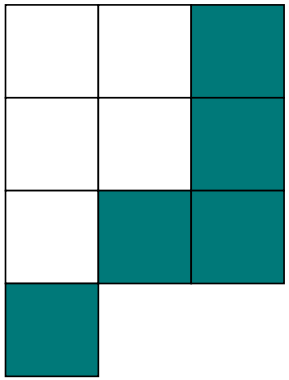

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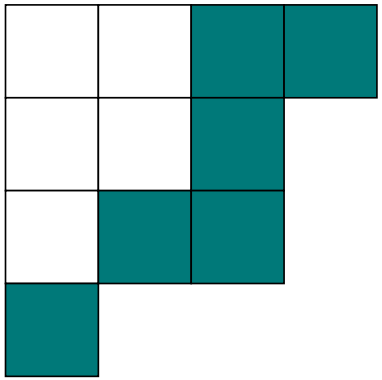

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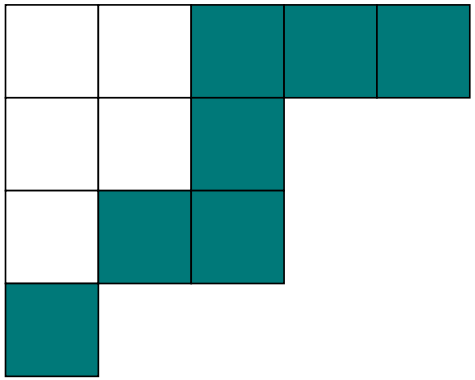

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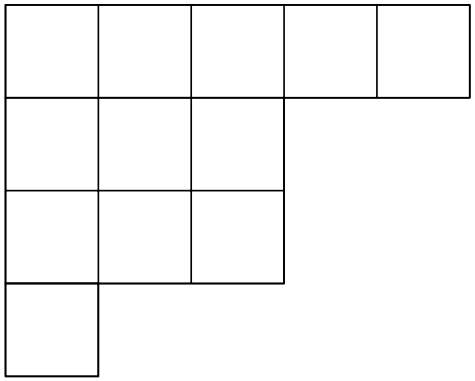

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$$m_3(7, 2^2, 1) = (5, 3^2, 1).$$

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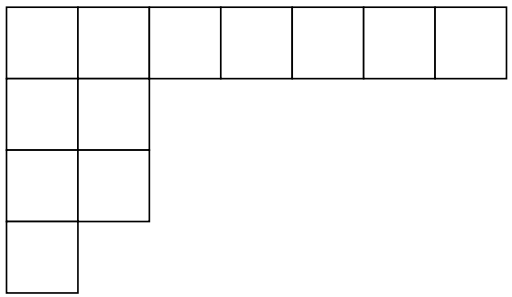

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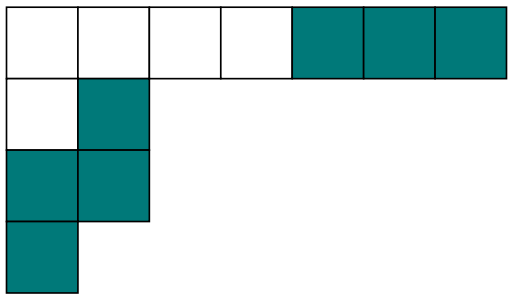

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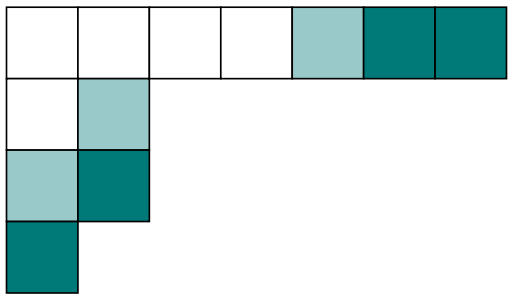

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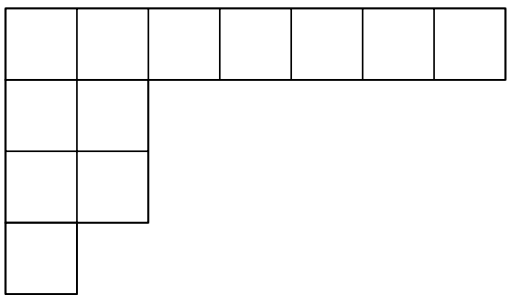

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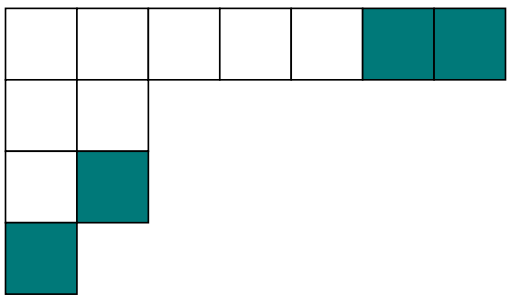

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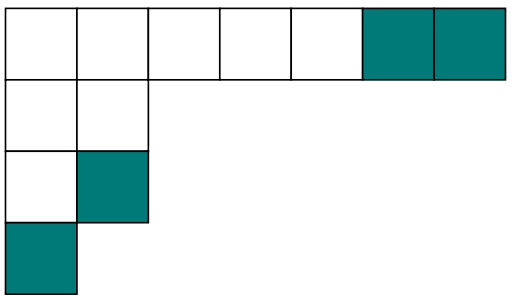

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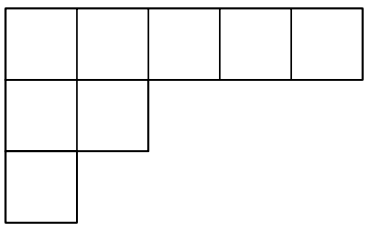

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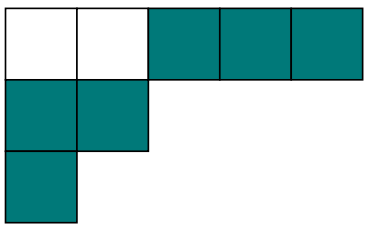

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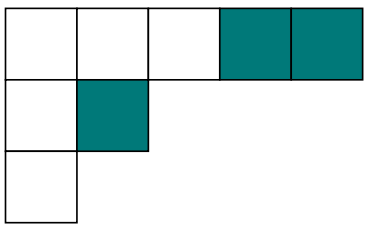

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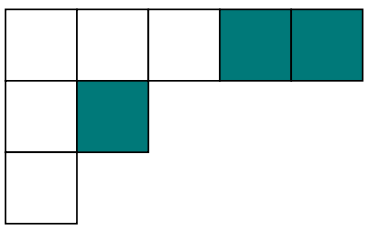

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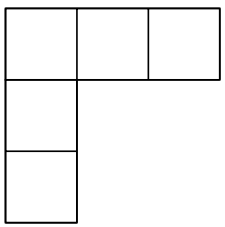

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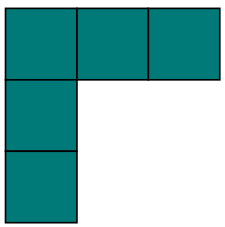

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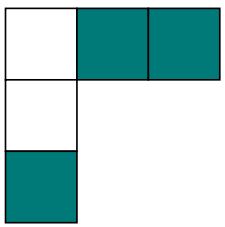

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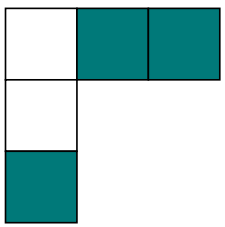

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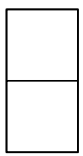

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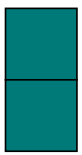

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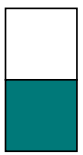

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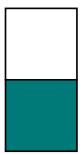

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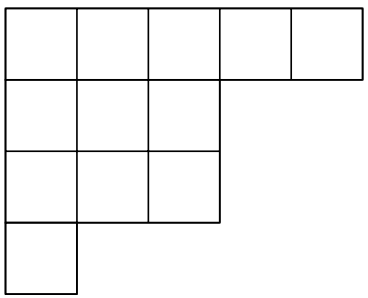

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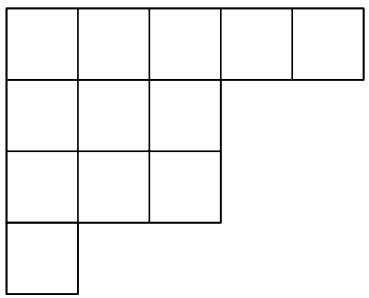

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Kleshchev's modular branching rule:

$$\operatorname{soc}(e_i D^\lambda) = \begin{cases} D^{\tilde{e}_i\lambda} & \text{if } \lambda \text{ has a good } i\text{-node} \\ 0 & \text{otherwise.} \end{cases}$$

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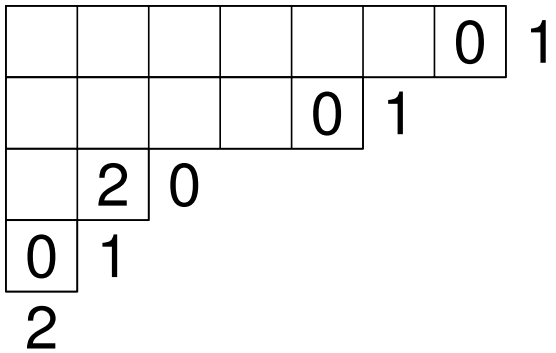

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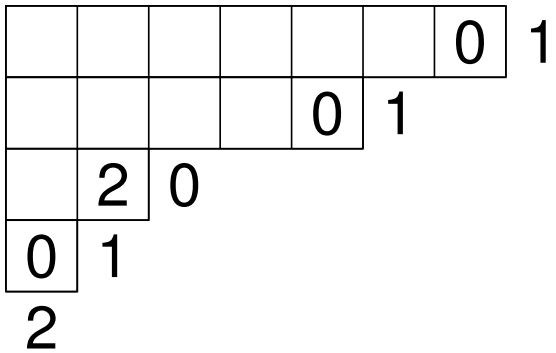

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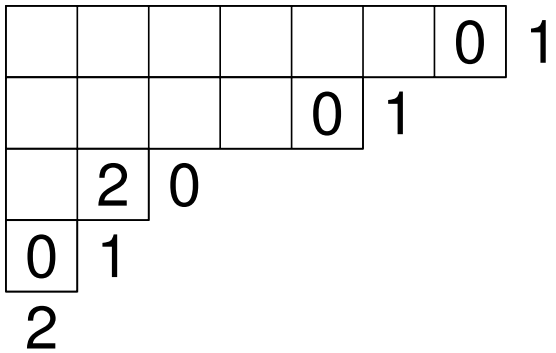

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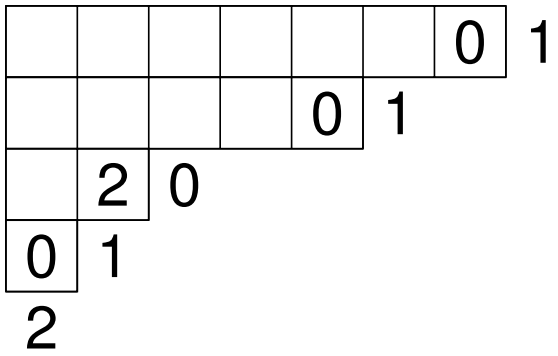

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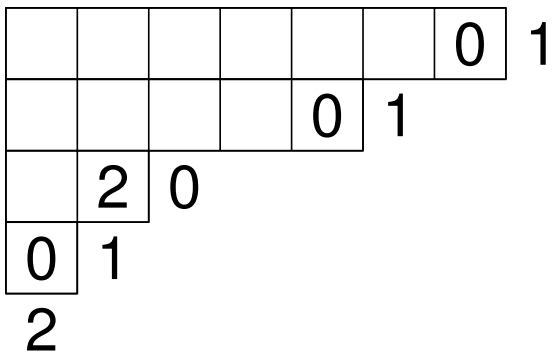

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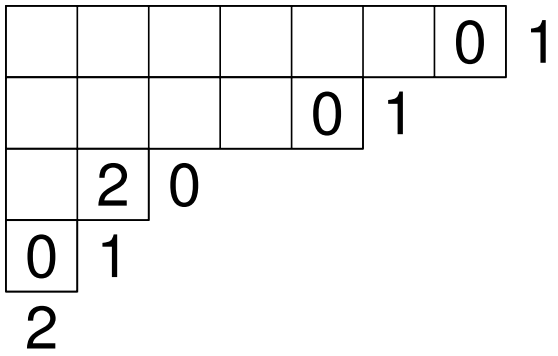

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Then $m_p(\lambda)$ is the partition such that

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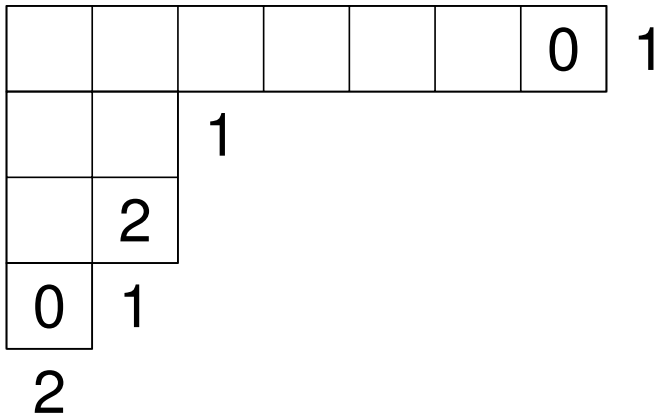

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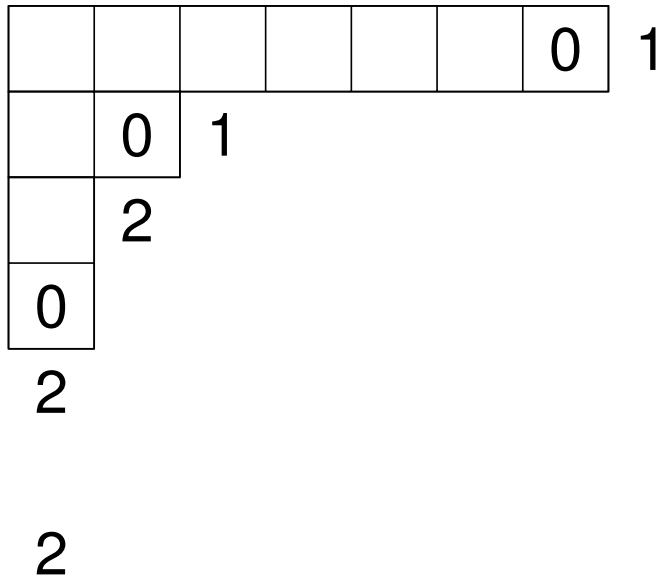

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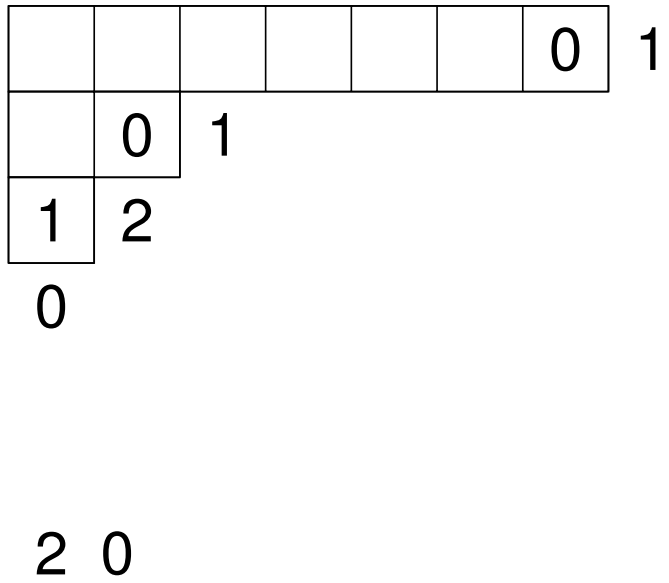

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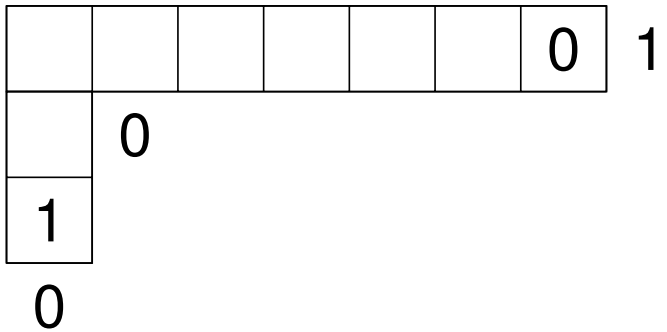

2 0 0

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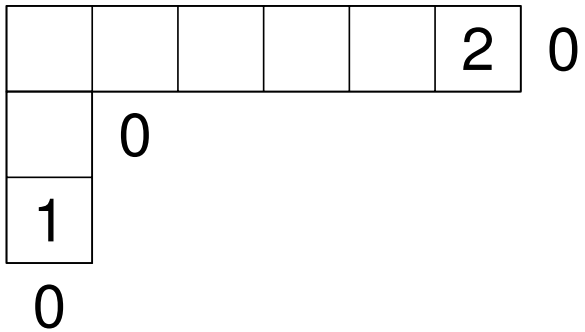

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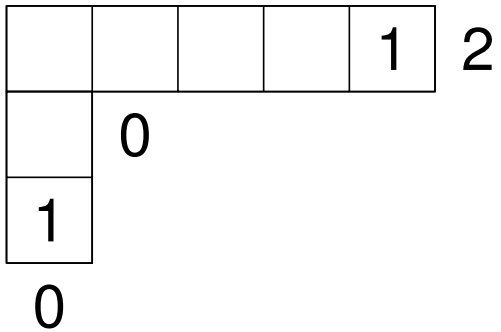

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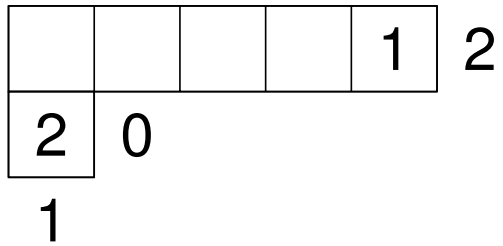

2 0 0 0 2 1

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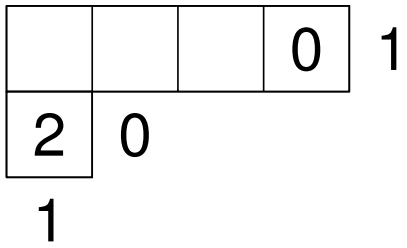

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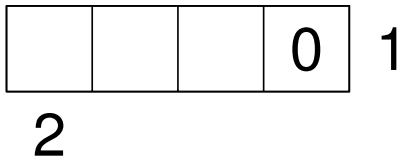

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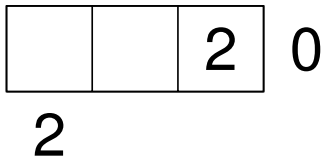

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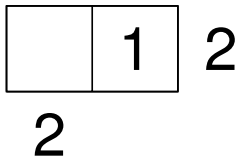

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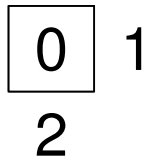

2 0 0 0 2 1 1 2 0 2 1

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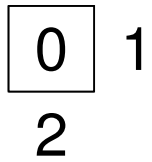

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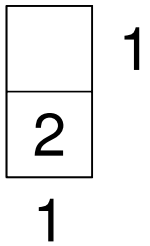

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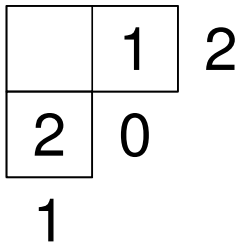

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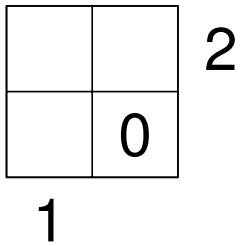

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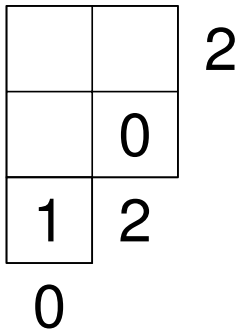

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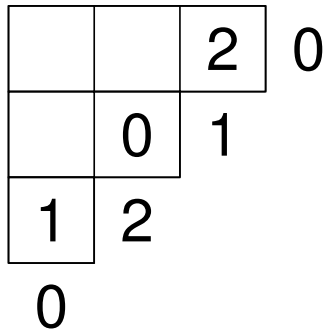

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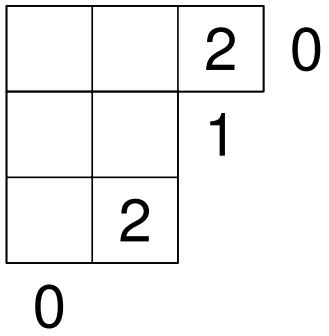

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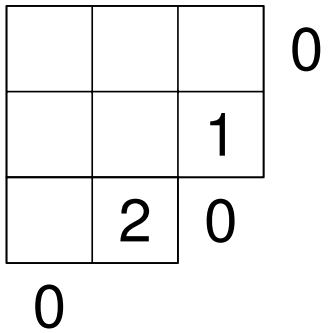

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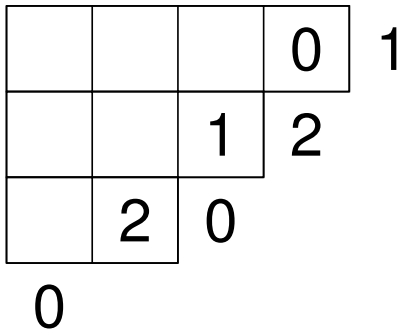

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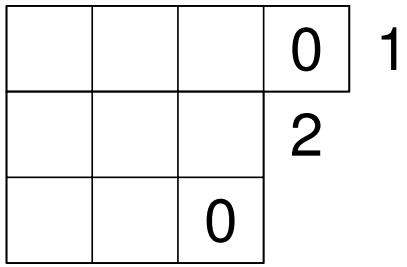

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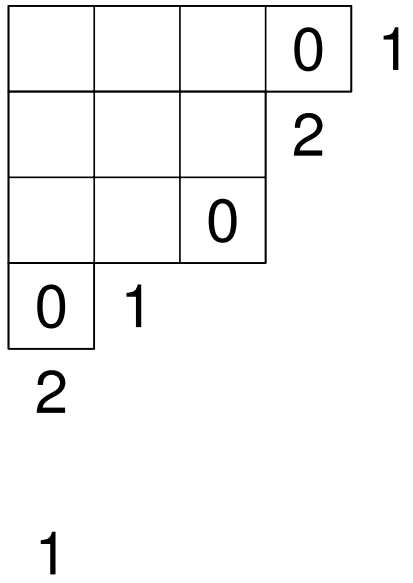

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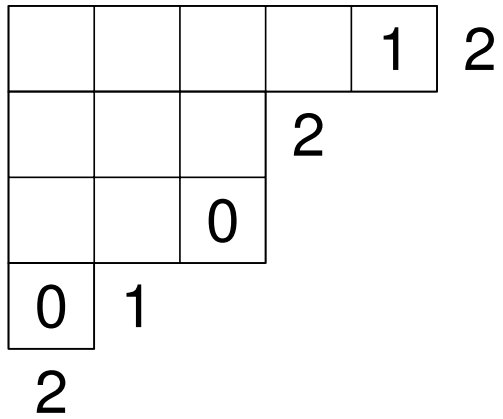

$$m_3(7, 2^2, 1) = (5, 3^2, 1).$$

Brundan–Kujawa–Serganova bijection

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BKS bijection: $m_p(\lambda)'$ is the resolution of $-(\lambda')$.

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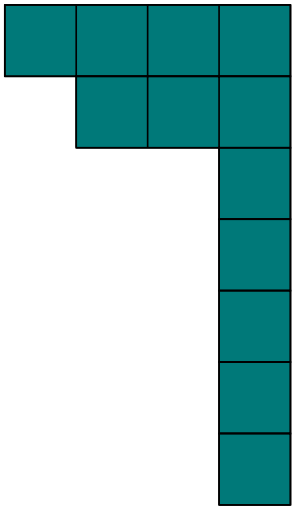

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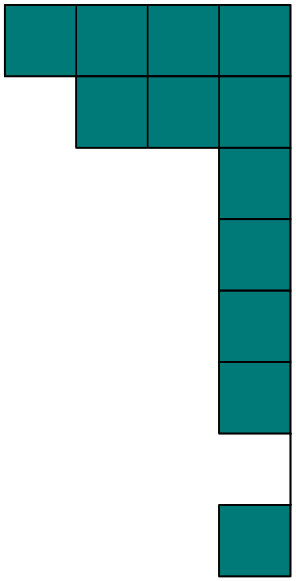

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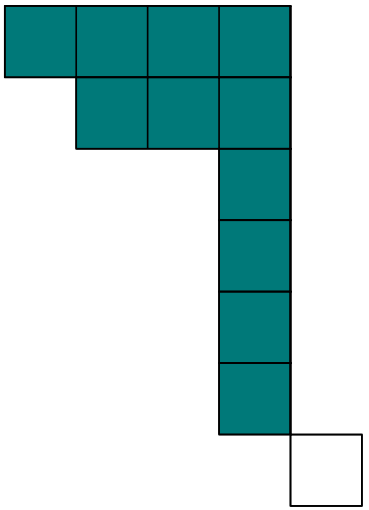

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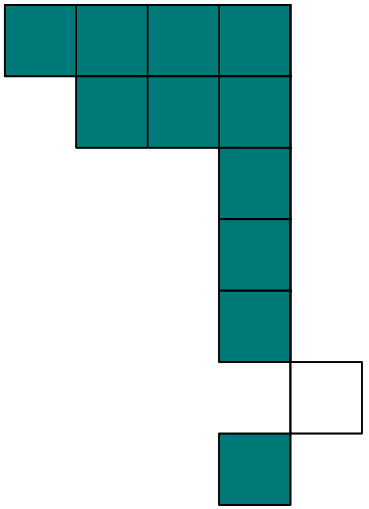

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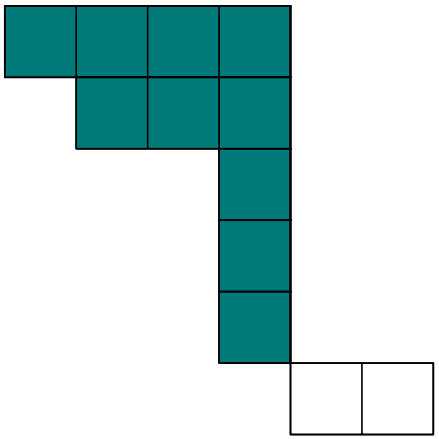

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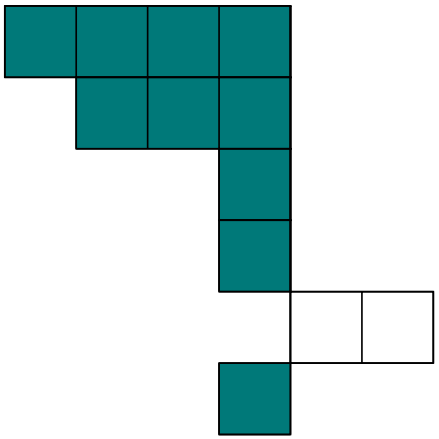

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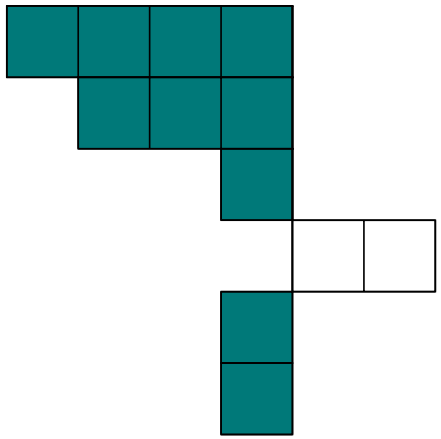

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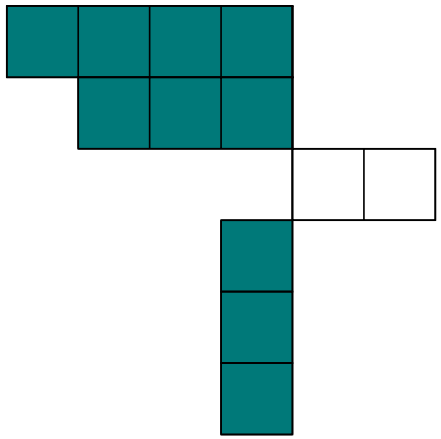

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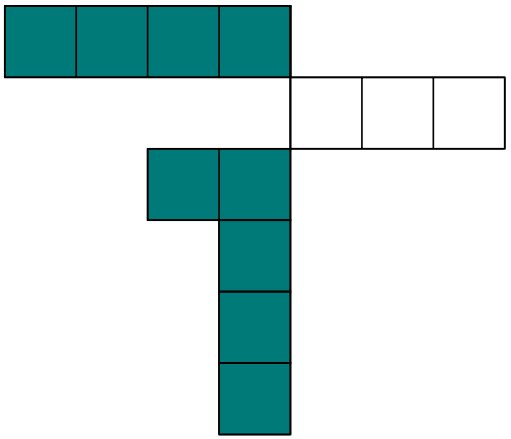

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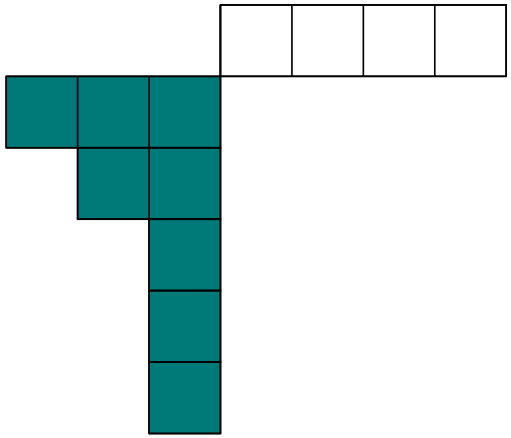

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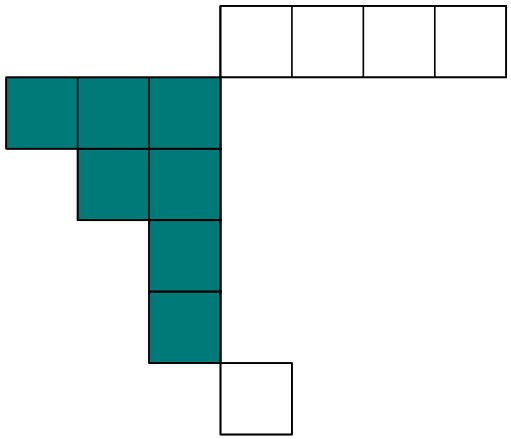

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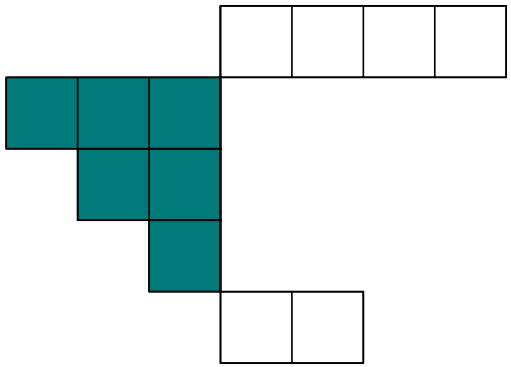

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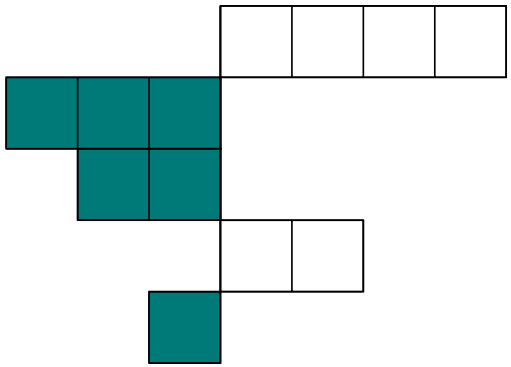

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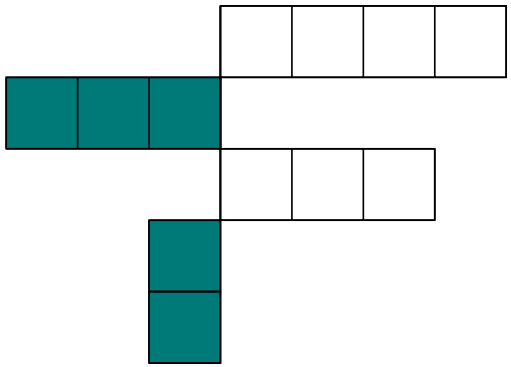

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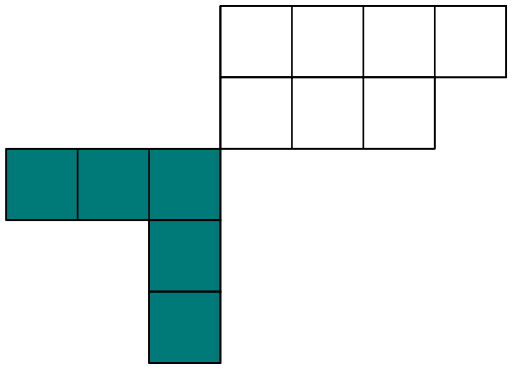

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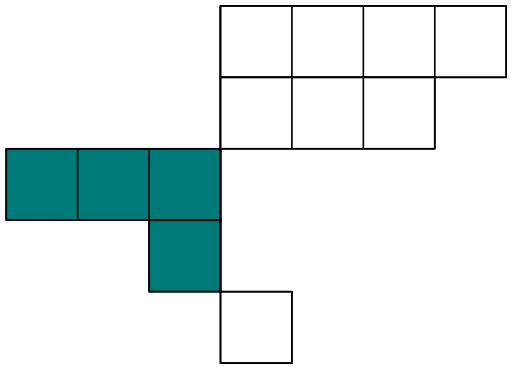

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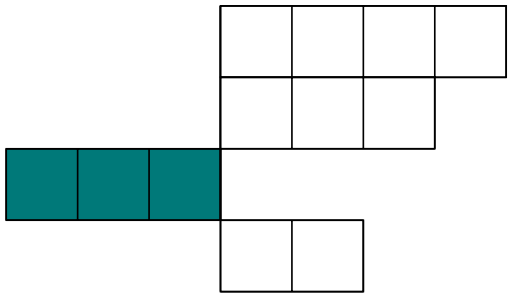

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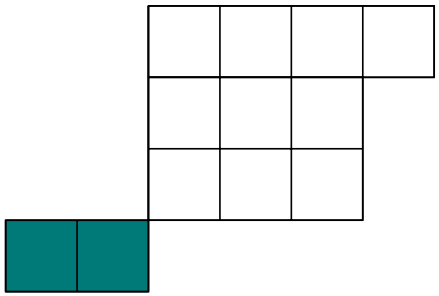

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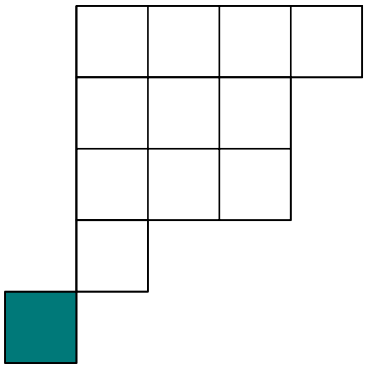

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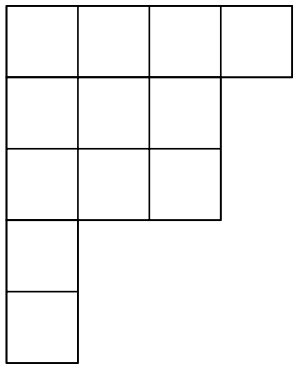

Crystals

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arrows: $\lambda \xrightarrow{i} \mu$ when $\lambda = \tilde{e}_i\mu$, for $i \in \mathbb{Z}/p\mathbb{Z}$.

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arrows: $\lambda \xrightarrow{i} \mu$ when $\lambda = \tilde{e}_i\mu$, for $i \in \mathbb{Z}/p\mathbb{Z}$.

B_{reg} is isomorphic to the crystal graph for the highest-weight crystal $B(\Lambda_0)$ for $\widehat{\mathfrak{sl}}_p$ (Misra–Miwa 1990). Kleshchev's bijection says that m_p is a "signed automorphism" of this graph, i.e.

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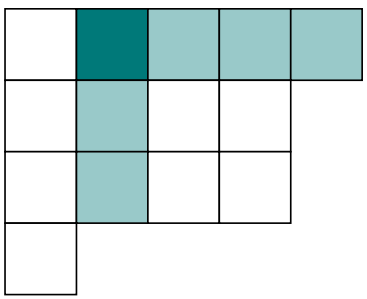

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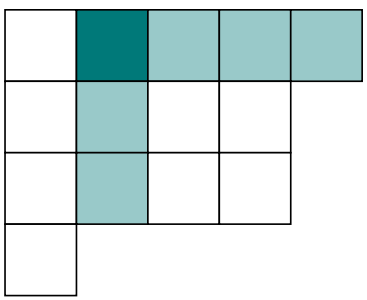

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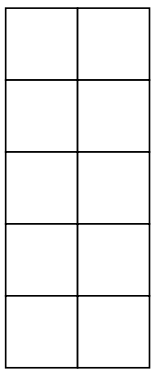

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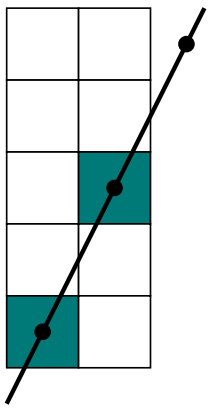

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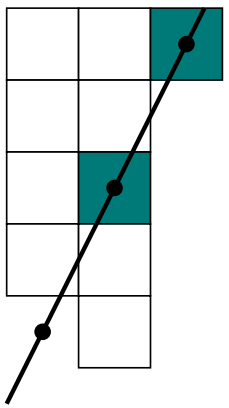

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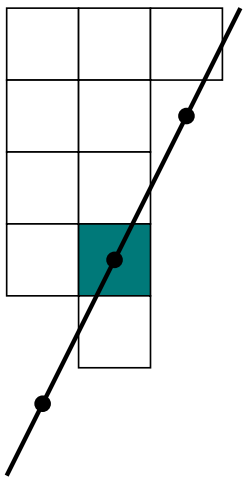

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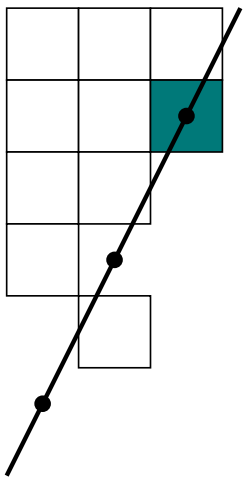

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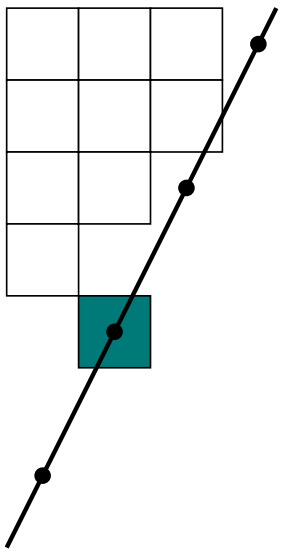

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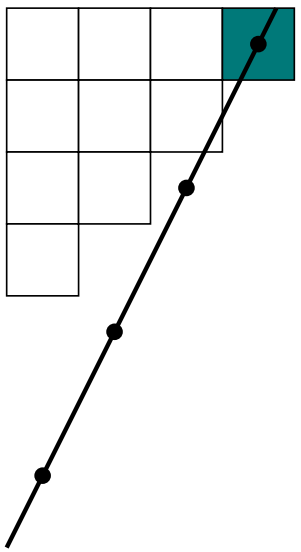

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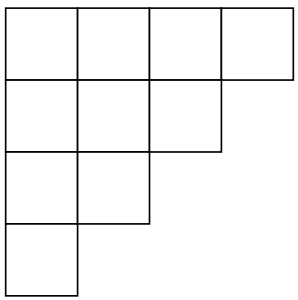

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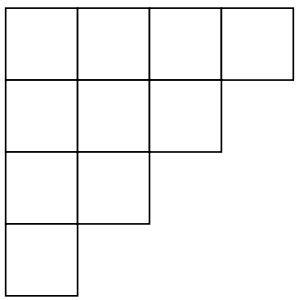

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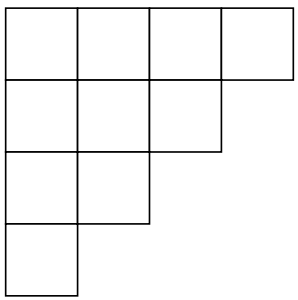

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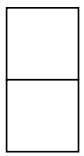

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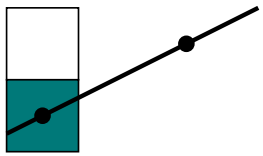

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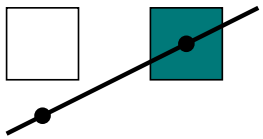

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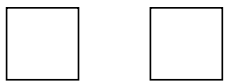

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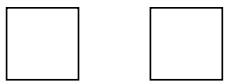

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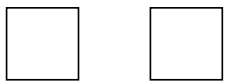

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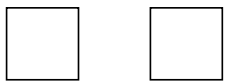

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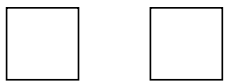

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Theorem (Millan Berdasco 2019): Each (p, c)-equivalence class contains a unique most dominant partition.

(Case $c = p-1$ is due to Berg.)

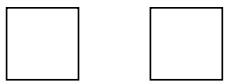
Define the (p, c)-regularisation of λ to be the most dominant partition in the (p, c)-class containing λ.

(p, c)-regularisation

Take $c \in \{1, \ldots, p-1\}$. (p, c)-ladder: line of gradient $\frac{p-c}{c}$.

We want to define (p, c)-regularisation.

e.g. $p = 3$, $c = 2$:



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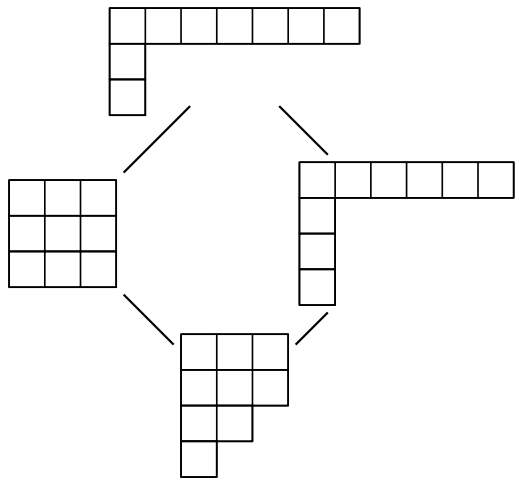

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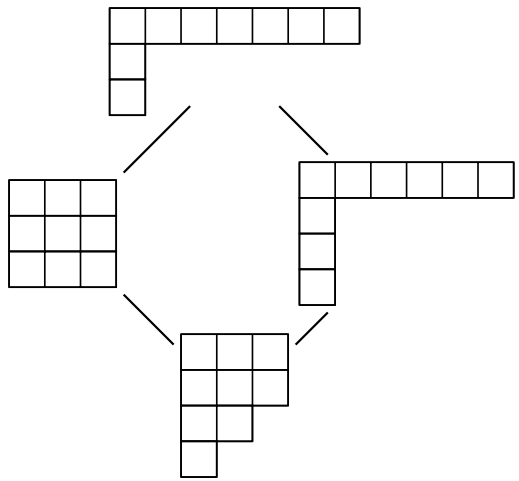

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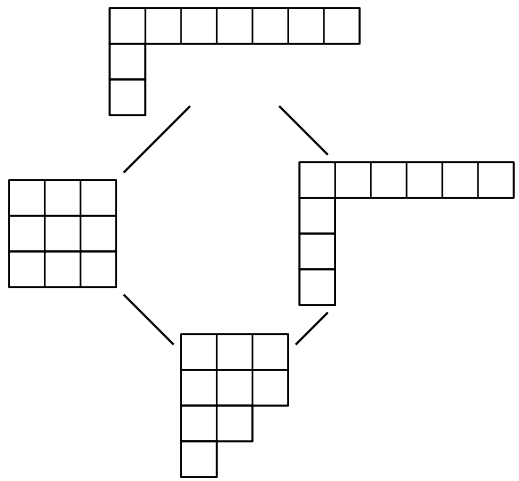

Millan Berdasco gives an algorithm to compute the (p, c)-regularisation using the abacus.

Theorem (Millan Berdasco 2019): Let $A_c^\flat = (c-1, 2c-1, 3c-1, \dots)$. Then the (p, c)-regularisation of λ is the unique $A_c^\flat$-regular partition in the same (p, c)-class as λ.

(p, c)-regularisation and crystals

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More generally, take any rational $c \in [1, p-1]$. Define

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$p = 3$, $c = \frac{3}{2}$: the isomorphism

$$B_{(1,3,4,6,7,9,\dots)} \longrightarrow B_{(1,2,4,5,7,8,\dots)}.$$

is given by $(3, \frac{3}{2})$-regularisation $(= (6, 3)$-regularisation).

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4. Output μ.

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If we discard partitions of size bigger than 11, then there are six crystals B_A:

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\begin{aligned}
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&B_{(1,3,4,\dots)} \\
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Theorem (Bessenrodt–Olsson–Xu): $m_p(\lambda) \trianglerighteq (\lambda')^{\mathrm{reg}}$, with equality if and only if every hook of length divisible by p is shallow.

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New algorithm gives a much easier proof of this.