Note

There have been many Schur alg-like talks already (eg. Carl) with categories whose morphisms are indexed by double cosets
The technology I develop applies to all, but I'll focus on some one not yet discussed

Do p2-3 top. Say walks around pictures on the right, but differ in non-reduced words. We'll go thru Demazure operators since they too have appeared in many talks.

To p5: $R = \mathbb{Q}[x_1, x_2, x_3]$ $\quad s=(12) \quad t=(23)$

$R^s = \mathbb{Q}[x_1+x_2, x_1x_2, x_3]$ $\quad R^t$

$R^{st} = \mathbb{Q}[e_1, e_2, e_3]$

∂_s, i_s, i_t, ∂_t, ∂_{st}, ∂^s_{st}, ∂^t_{st}, i^s_{st}

add ∂ later

Draw upside down

Thm (D): these are all Frob extensions

For $I \subset J \subset S = \{s,t\}$

$\exists$ $i^I_J : R^J \hookrightarrow R^I$

$\partial^I_J : R^I \longrightarrow R^J$, R^J-linear

- R^I free over R^J, finite rank
- $\exists$ (homog) dual bases $\{a_i\}$ $\{b_i\}$

$\partial^I_J(a_i b_j) = \delta_{ij}$.

Ex: $\partial_s(f) = \dfrac{f - sf}{x_1 - x_2}$

$\partial_s(x_1^3) = \dfrac{x_1^3 - x_2^3}{x_1 - x_2} = x_1^2 + x_1x_2 + x_2^2 \in R^s$.

Dual bases $\{1, x_1\}$ $\{-x_2, 1\}$

Ex: $\partial_{st} = \dfrac{\sum_{w \in S_3} \operatorname{sgn}(w)\, w(f)}{\prod_{i<j} (x_i - x_j)}$ $\quad$ Dual bases: Schubert polys.

Geom picture: ~~[illegible]~~

as p5, mention Poincaré Duality.

$p6^{top}$: Do H, define $NC(S_n)$ (K&LP)

Qn: Which compositions in $NC(S_n)$ are nonzero? i.e. rexes?

Presentation? Basis?

~~Longest left double coset rep~~

Thm: $\{\partial_p\}_{p}$ is basis for $\mathrm{Hom}_{NC}(I,J)$

So given basis, what is a rex for p?

Language for composition: p4

Now p6 second half.

Now p7 rephrased.

$\partial_p := D_{\bar{p} w_J^{-1}} : R^J \to R^I$

$$\begin{array}{c} R \\ \downarrow \partial_{w_J} \\ R^J \end{array}$$

$\partial_p \partial_{w_J} = \partial_{\bar{p}}$ I is right desc.

$J \cap \partial_{\bar p} \subset R^I$

$\Rightarrow \mathrm{Im}\ \partial_p|_{R^J} \subset R^I$.

Ex: $\mathrm{Idcop} \in W_I \backslash W / W_J$ when $I \subset J$

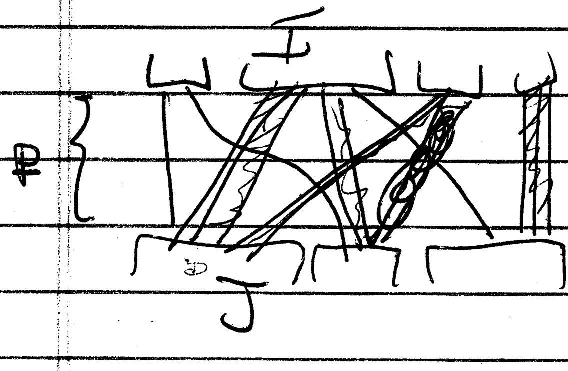
$\bar{p} = w_J$ $\partial_p = D_{id} = id$ gives

$i : R^J \hookrightarrow R^I$

Ex: $\mathrm{Idcop} \in W_J \backslash W / W_I$

$\partial_p = D_{w_J w_I^{-1}} = \partial^I_J$.

Combo of rexes: (It's easy to find them in type A)



$\bar{p} \neq w_I\, p\, w_J$ b/c redundancy

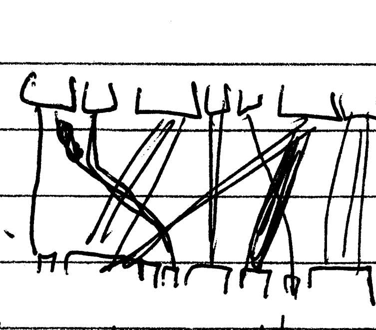
coker red. $K = \mathrm{leftred}(p) = I \cap p J p^{-1}$

$\bar{p} = (w_I w_K^{-1})\, p\, w_J$ | $L = p^{-1} K p = $ right red. $\subset J$

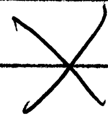
rex for p of form

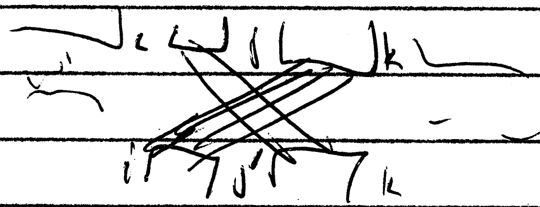
$$[I \supset K] \circ \text{rex for } p^{core} \circ [L \subset J]$$

$p^{core} \in W_K \backslash W / W_L$ $\quad \underline{p}^{core} = \underline{p}$.



think: |X|X| ordering permutation

For each cosy in ordinary rex 

$[\hat{i}\hat{j}\hat{k}, \hat{i}\hat{k}, \hat{i}\hat{j}'\hat{k}]$ produces 

<u>Core</u> cosets are reduced compositions of <u>atomic cosets</u> I_{st}

$$[I_s, I_{st}, I_t] \quad \text{where} \quad w_{I_{st}}\, s\, w_{I_{st}} = t$$

interpr A: Combs of atomic cosets $\hookrightarrow$ combs of ordinary rexes.