Full runner removal theorem for Ariki-Koike algebras OIST Workshop: Representation Theory of Hecke Algebras and Categorification

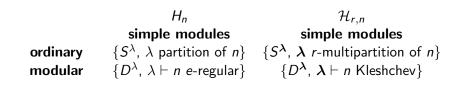
Alice Dell'Arciprete

University of East Anglia

9 June 2023



Iwahori-Hecke and Ariki-Koike algebras



Iwahori-Hecke and Ariki-Koike algebras

$$\begin{array}{ccc} H_n & \mathcal{H}_{r,n} \\ \textbf{simple modules} \\ \textbf{ordinary} \\ \textbf{modular} & \{S^{\lambda}, \lambda \text{ partition of } n\} \\ \{D^{\lambda}, \lambda \vdash n \text{ e-regular}\} & \{S^{\lambda}, \lambda \vdash n \text{ Kleshchev}\} \\ \textbf{QUESTION} & [S^{\lambda}:D^{\mu}] = \textbf{?} & [S^{\lambda}:D^{\mu}] = \textbf{?} \end{array}$$

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Iwahori-Hecke and Ariki-Koike algebras

$$\begin{array}{c|c} H_n & \mathcal{H}_{r,n} \\ \textbf{simple modules} \\ \textbf{ordinary modular} & \{S^{\lambda}, \lambda \text{ partition of } n\} \\ \{D^{\lambda}, \lambda \vdash n \text{ e-regular}\} & \{S^{\lambda}, \lambda \text{ r-multipartition of } n\} \\ \{D^{\lambda}, \lambda \vdash n \text{ e-regular}\} & \{D^{\lambda}, \lambda \vdash n \text{ Kleshchev}\} \end{array}$$

$$\begin{array}{c|c} \textbf{QUESTION} & [S^{\lambda} : D^{\mu}] = \textbf{?} \\ \textbf{char 0} & \textbf{LLT algorithm} & \textbf{LLT-type algorithm} \end{array}$$

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LLT algorithm computes the decomposition numbers for H_n

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LLT algorithm computes the decomposition numbers for H_n

HOW?

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LLT algorithm computes the decomposition numbers for H_n

HOW?

• computation of the **canonical basis elements** of \mathcal{F} :

$$egin{aligned} \mathcal{G}(\mu) = \mu + \sum_{\lambda eq \mu} d^{e}_{\lambda \mu}(q) \lambda, \qquad d^{e}_{\lambda \mu}(q) \in q \mathbb{N}[q] \end{aligned}$$

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LLT algorithm computes the decomposition numbers for H_n

HOW?

• computation of the **canonical basis elements** of \mathcal{F} :

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q-decomposition number



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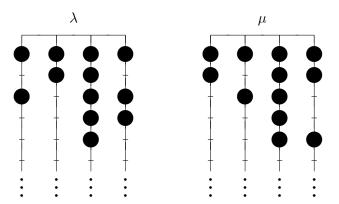
q-decomposition number

• Theorem (Ariki).
$$[S^{\lambda}:D^{\mu}] = d^{e}_{\lambda\mu}(1).$$

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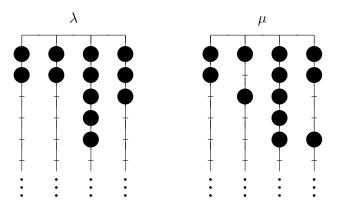
Suppose e = 4. Consider the following abacus displays:



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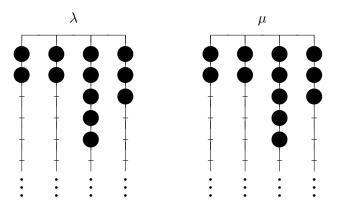
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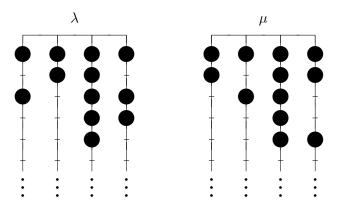
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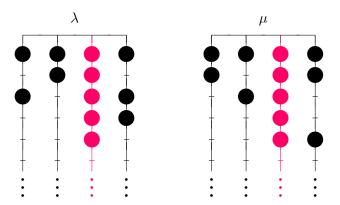
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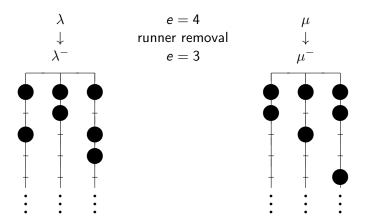
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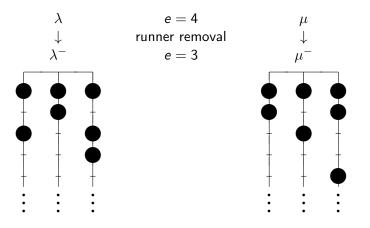
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 $d^4_{\lambda\mu}(q)=d^3_{\lambda^-\mu^-}(q)$

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Full Runner Removal - level 1

Theorem (Fayers)

Let $e \ge 3$. Let λ , $\mu \vdash n$ be in the same block with a "long enough" runner i. Let λ^- , μ^- as before.

Then

$$d^e_{\lambda\mu}(q)=d^{e-1}_{\lambda^-\mu^-}(q).$$

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Theorem (D.)

Let λ, μ be r-multipartitions of n in the same block with μ e-multiregular.

Let $s \in (\mathbb{Z}/e\mathbb{Z})^r$ be a multicharge. Let $k \in \mathbb{Z}_{\geq 0}^r$. Denote by ${}^{+k}$ the addition of a runner full of beads in each component.

If the new inserted runners are "long enough", then

$$d^{e+1}_{\lambda^{+k}\mu^{+k}}(q) = d^e_{\lambda\mu}(q).$$

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Ingredients:

• Addition of a full runner for $\mathcal{H}_{r,n}$

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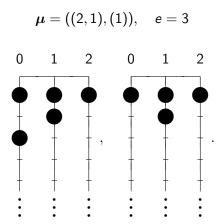
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Ingredients:

- Addition of a full runner for $\mathcal{H}_{r,n}$
- LLT-type algorithm for $\mathcal{H}_{r,n}$

Addition of a full runner - level 2



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Addition of a full runner - level 2

$$\mu^{+(6,10)} = ((3,2,1^2), (7,4,1^2)), \quad e = 4$$

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Proof

Sketch of the proof

r= 2, $\pmb{\mu}=(\mu^{(1)},\mu)$ e-multiregular, $k^{(1)},k^{(2)}\in\mathbb{Z}_{\geq0}$ such that

$$k^{(2)} - k^{(1)} \ge \mu_1 + e - 1.$$

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Sketch of the proof

 $\begin{aligned} r &= 2, \ \mu = (\mu^{(1)}, \mu) \text{ e-multiregular, } k^{(1)}, k^{(2)} \in \mathbb{Z}_{\geq 0} \text{ such that} \\ k^{(2)} - k^{(1)} &\geq \mu_1 + e - 1. \end{aligned}$ $\begin{aligned} \text{STEP 1. } G_{e+1}(\mu^{+k^{(2)}}) &= G_e(\mu)^{+k^{(2)}} \\ &\Longrightarrow G_{e+1}(\mu^{+k^{(2)}}) = \mu^{+k^{(2)}} + \sum_{\mu \triangleright \lambda} d^e_{\lambda\mu}(q) \lambda^{+k^{(2)}} \end{aligned}$

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Proof

STEP 2. $G_{e+1}^{s^+}((\varnothing,\mu^{+k^{(2)}})) = (\varnothing,\mu^{+k^{(2)}}) + \sum_{\mu \triangleright \lambda} d_{\lambda\mu}^e(q)(\varnothing,\lambda^{+k^{(2)}})$

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Sketch of the proof

 $r = 2, \ \mu = (\mu^{(1)}, \mu) \ e$ -multiregular, $k^{(1)}, k^{(2)} \in \mathbb{Z}_{>0}$ such that $k^{(2)} - k^{(1)} > \mu_1 + e - 1.$ **STEP 1.** $G_{e+1}(\mu^{+k^{(2)}}) = G_e(\mu)^{+k^{(2)}}$ $\implies G_{e+1}(\mu^{+k^{(2)}}) = \mu^{+k^{(2)}} + \sum_{\mu \triangleright \lambda} d^{e}_{\lambda\mu}(q) \lambda^{+k^{(2)}}$

Proof

STEP 2. $G^{\boldsymbol{s}^+}_{\boldsymbol{e}+1}((\varnothing,\mu^{+k^{(2)}}))=(\varnothing,\mu^{+k^{(2)}})+\sum_{\mu\rhd\lambda}d^{\boldsymbol{e}}_{\lambda\mu}(q)(\varnothing,\lambda^{+k^{(2)}})$

STEP 3. $G_{e+1}^{s^+}((\varnothing^{+k^{(1)}},\mu^{+k^{(2)}})) = (\varnothing^{+k^{(1)}},\mu^{+k^{(2)}}) + \sum_{\mu \in \mathcal{N}} d_{\lambda\mu}^e(q)(\varnothing^{+k^{(1)}},\lambda^{+k^{(2)}})$

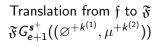
Proof

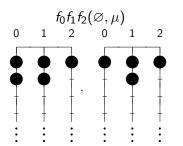
Sketch of the proof

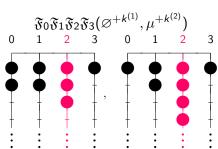
STEP 4.

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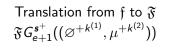
Proof

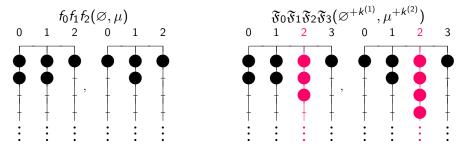
Sketch of the proof

STEP 4.

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$$G_{e}^{s}(\mu)^{+k} = G_{e+1}^{s^{+}}(\mu^{+k})$$

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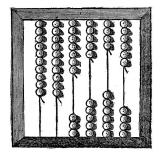
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Thank you for your time!



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