## LECTURE 13. SPONTANEOUS SUSY BREAKING. NON-RENORMALIZATION THEOREM

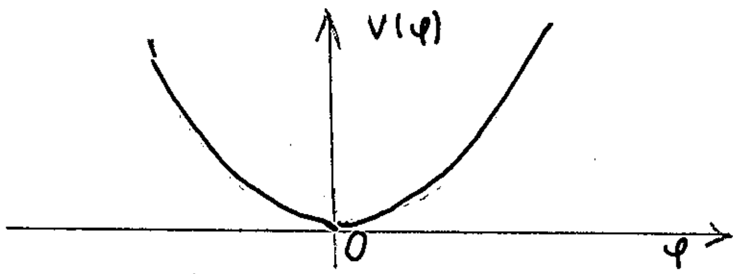
### 1. Spontaneous SUSY breaking

Let us recall how a global symmetry breaking happens in the Field Theory.

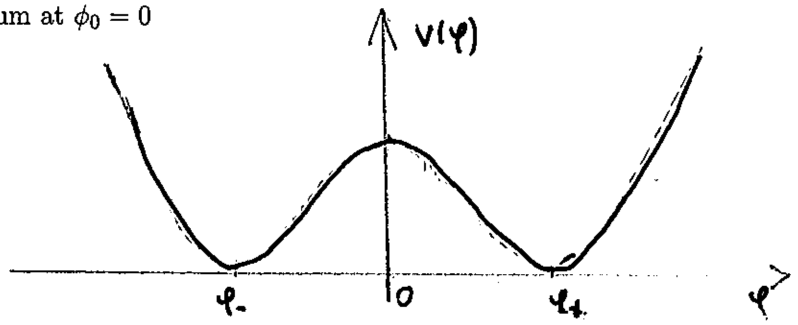
If a Lagrangian has some global symmetry, it can be broken by a vacuum configuration. In other words, the corresponding solution of the equations of motion is not invariant under the symmetry transformations. For example, consider a theory with a real scalar field

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4}\phi^4 \tag{1}$$

The Lagrangian is invariant under the discrete $Z_2$ symmetry $\phi(x) \to -\phi(x)$. If the mass parameter $m^2 \geq 0$, the potential has a minimum at $\phi_0 = 0$




If the mass parameter $m^2 \leq 0$, the potential has two minima at $\phi_\pm = \pm\sqrt{-\frac{m^2}{\lambda}}$ and a local maximum at $\phi_0 = 0$




Let us note, that the extrema of the potential are solutions of the equations of motion

$$\partial_\mu \frac{\partial L}{\partial(\partial_\mu \phi)} = \frac{\partial V}{\partial \phi}$$

The solution $\phi_0 = 0$ in the second model is not stable (it is tachyonic) and the stable solutions are $\phi_\pm$ (there is no tachyon produced when considering the vacuum excitations are around these minima). On the other hand, the $Z_2$ symmetry transforms $\phi_+$ into $\phi_-$ and vice versa. Therefore, in this model we have a spontaneous breaking of the global $Z_2$ symmetry.

Let us go back to SUSY and consider the transformation rules for the component fields in chiral and vector superfields. SUSY will be spontaneously broken, if there is a vacuum configuration such that the variations of the component fields will be different from zero.

Let us notice, that the vacuum expectation value for fermionic fields is zero[1] since a vacuum can not have a spin. Therefore, a variation of the bosonic component fields is zero.

Considering again static configurations $\partial_\mu \phi(x) = F_{\mu\nu}(x) = 0$, we can see that supersymmetry is spontaneously broken if either $F(x) \neq 0$ or $D(x) \neq 0$.

From the SUSY algebra one can conclude, that the energy is greater or equal to zero, since the Hamiltonian is a square of hermitian operators

$$H = \frac{1}{4}(\bar{Q}_{\dot{1}} Q_1 + \bar{Q}_{\dot{2}} Q_2 + Q_1 Q_{\dot{1}} + Q_2 Q_{\dot{2}})$$

On the other hand $Q_\alpha$ and $\bar{Q}_{\dot{\alpha}}$ are generators of supersymmetry. As we discussed above, a symmetry is spontaneously broken, if for the corresponding generator $Q$, one has $Q|vac\rangle \neq 0$. Therefore, if

$$\langle vac|H|vac\rangle > 0$$

SUSY is spontaneously broken.

In general, if we have several (i=1,...,k) chiral superfields, the corresponding scalar potential is

$$V = \sum_{i=1}^{k} \frac{\partial \bar{W}}{\partial A_i^\star} \frac{\partial W}{\partial A_i}$$

Therefore, in order SUSY to be spontaneously broken, it is enough that one $F$- term is not zero.

1.1. **An example.** Consider four chiral superfields $X, Y, \Phi_+$ and $\Phi_-$. Let the superpotential be

$$W = \lambda_1 X(\Phi_+ \Phi_- - m^2) + \lambda_2 Y \Phi_+ \Phi_-$$

where $\lambda_1$ and $\lambda_2$ are coupling constants. Corresponding F-terms are

$$F_X^\star = \lambda_1(A_+ A_- - m^2), \quad F_Y^\star = \lambda_2 A_+ A_-,$$

$$F_+^\star = (\lambda_1 A_X + \lambda_2 A_Y) A_-, \quad F_-^\star = (\lambda_1 A_X + \lambda_2 A_Y) A_+ \tag{2}$$

and the scalar potential is

$$V = F_X^\star F_X + F_Y^\star F_Y + F_+^\star F_+ + F_-^\star F_-$$

Apparently there are no such values for $A_X, A_Y, A_+$ and $A_-$ that make all $F$-terms in (2) to be equal to zero. Therefore, in this model SUSY is spontaneously broken.

[1] One can have in principle a nonzero bilinnear of fermions, so called gaugino condensate. We shall not consider it here

**1.2. Non-renormalization Theorem.** Since we did not have time to go through the perturbation theory at all, we simply state the non-renormalization theorem.

The nonrenormalization theorem states that the perturbative expansion give the effective action, which has the following form

$$\int d^2\theta d^2\bar{\theta} \int d^4x_1, .., d^4x_n \, G(x_1, ..., x_n) \ \cdot F(\Phi(x_1, \theta, \bar{\theta}), \, V(x_1, \theta, \bar{\theta}),$$
$$\Phi(x_2, \theta, \bar{\theta}), \, ..., \, D_\alpha \Phi(x_n, \theta, \bar{\theta}), D_\alpha V(x_n, \theta, \bar{\theta}), \, ..).$$

Here $F$ is a function of superfields and their supercovariant derivatives, and $G(x_1, ..., x_n)$ is a translation invariant function. The important point is that the integral is *only* over entire superspace $d^2\theta d^2\bar{\theta}$ and not over its subspaces, such as $d^2\theta$ or $d^2\bar{\theta}$.

Nonperturbative effects can also generate an $N = 1$ superpotential.