LECTURE 7. CIRCLE COMPACTIFICATIONS. D-BRANES. PART II

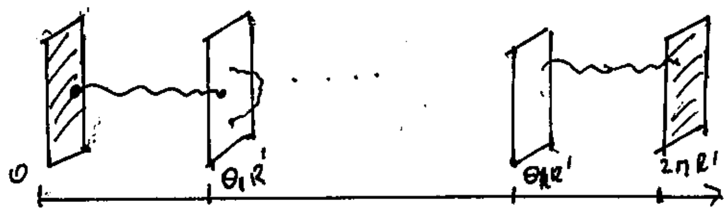
1. Some properties of D-branse

Let us state, without proof, some properties of D- branes. Since the gauge field "lives" on the endpoints of the open string, a D- brane contains a gauge field on its world- volume. If we have only one D-brane then the field is a Maxwell vector field and the corresponding gauge group is $U(1)$.

We can have a configuration of a few parallel Dp-branes. If this is the case, then the analogous equation of the previous lecture changes to

$$\bar{x}^{25}(\tau,\pi) - \bar{x}^{25}(\tau,0) = (2\pi n + \theta_j - \theta_i)R' \tag{1}$$

Here the variables θ_i describe the position of the brane in the space, $\theta_i R'$ is a coordinate of the Dp brane.



An equation which describes the masses of the fields is

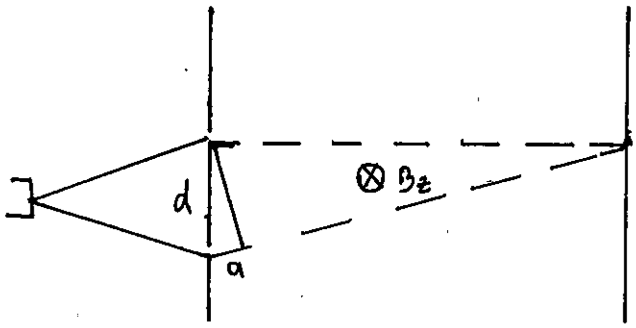
$$m^2 = \left(\frac{(2\pi n + \theta_j - \theta_i)R'}{2\pi\alpha'}\right)^2 + \frac{1}{\alpha'}(N-1) \tag{2}$$

In this case the gauge group is $(U(1))^k$, where k is a number of the parallel Dp branes. When $\theta_i \to \theta_j$, i.e., when the branes are located at the same point, we have an enhancement of the gauge group, since extra massless states appear. If all Dp- branes (k of them) have the same position, the gauge group increases to $U(k)$.

2. Origin of the θ_i variable

2.1. Aharonov - Bohm effect. In order to understand the origin of θ_i variables, it is useful to recall the Aharonov -Bohm effect.

Consider a double slit experiment,



The difference a between the trajectories 1 and 2 of the electron, causes the phase difference

$$\delta = \frac{2\pi a}{\lambda}$$

we have an interference picture on the screen, where λ is a wavelength.

Let us place a thin solenoid with a magnetic flux going through it. Inside the solenoid we have a constant magnetic field, with a nonzero component $B_z = B$. Outside the solenoid the magnetic field is zero. This configuration can be described by the potential

$$A_r = A_z = 0, \quad A_\phi = \frac{Br}{2}$$

inside the solenoid and

$$A_r = A_z = 0, \quad A_\phi = \frac{BR^2}{2r}$$

outside the solenoid. Here R is a radius of the solenoid. The electromagnetic field outside the solenoid is the pure gauge (recall, this means $A_\mu = \partial_\mu \chi$) with

$$\chi = \frac{BR^2}{2}\phi$$

Let us consider Quantum Mechanics of the problem. The free electron is described by the wave function $\psi(r) = \psi \exp(i\vec{p}\cdot\vec{r})$. Now, since we have a nonzero potential, the phase in the wave function changes to

$$\vec{p}\cdot\vec{r} \to \vec{p}\cdot\vec{r} - e\vec{A}\cdot\vec{r}$$

As a result a total change of the phase gets an additional shift

$$\Delta\delta = e\int_1 \vec{A}\cdot\vec{r} - e\int_2 \vec{A}\cdot\vec{r} = e\oint \vec{A}\cdot\vec{r} = e\int \vec{B}\cdot d\vec{S} = e\Phi = e(\chi(2\pi) - \chi(0))$$

Here Φ is the flux through the solenoid. Therefore, the interference picture gets shifted by

$$\delta X \sim \Delta\delta$$

An expression

$$W = \exp\left(i\int d\vec{x}\cdot\vec{A}\right)$$

is called a Wilson line. Therefore we have a situation, when the electron moves in the zero electromagnetic field background, but it still has a nonzero physical effect (an observable phase shift).

The Aharonov-Bohm effect happens because of the following reason. We have an $U(1)$ symmetry, with the corresponding group element $U = e^{ie\chi}$. The group space of $U(1)$ is a circle. The electron moves on a plane with a point pinched out - also a circle. The function χ is a map from the group space to a configuration space. This map defines the group $\pi_1(S^1) = \mathbb{Z}$ which is nontrivial, and therefore the Aharonov-Bohm effect takes place

2.2. **D-branes.** Coming back to D - branes, let us compactify the 25-th dimension on a circle of a radius R. D-branes have a gauge field on their world volume. This can be found by quantizing an Open String with Dirichle boundary conditions. Recall, that the Open String endpoint can be endowed with Chan-Paton factors and consider a case of the group $U(k)$. A constant field configuration

$$A_{25} = diag\left(\frac{\theta_1}{2\pi R}, \frac{\theta_2}{2\pi R}, ..., \frac{\theta_k}{2\pi R}\right)$$

breaks the $U(N)$ symmetry down to $(U(1))^N$. Each $U(1)$ field is locally a pure gauge, with

$$U = diag\left(e^{i\frac{\theta_1 x^{25}}{2\pi R}}, ..., e^{i\frac{\theta_k x^{25}}{2\pi R}}\right)$$

Therefore this picture is completely analogous to the Aharonov - Bohm effect. Notice, that under $x^{25} + 2\pi R$ the group element U acquires the factor (Wilson loop)

$$W = diag\left(e^{i\theta_1}, ..., e^{i\theta_k}\right)$$

Similarly to how we had for the Aharonov-Bohm effect, the open string wavefunction $|p, ij\rangle$ gets a factor $e^{i(\theta_j-\theta_i)}$under the periodic shift. Since the momentum p_{25} is now shifted to

$$p_{25} = \frac{n}{R} + \frac{\theta_j - \theta_i}{2\pi R}$$

we can repeat our previous analysis and get (1) and (2).

2.3. **Action for D-branes.** Since D branes are dynamical objects, there is an action for them. It is called Dirac -Born -Infeld action

$$S_{DBI} = -\frac{1}{(2\pi)^p g_s(\sqrt{\alpha'})^{p+1}}\sqrt{-det(\eta_{ab} + \partial_a x^m \partial_b x^m + 2\pi\alpha' F_{ab})}$$

and $a, b = 0, ...p$ and $m = p+1, ..., 25$. Notice, that when $F_{ab} = 0$ we obtain a generalization of the Nambu-Goto action for a p-brane.