Higher Spin Interactions: From Classical to Quantum

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Higher Spin Gravity Online Club
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Part I

- Motivation
- Classical Interactions: Cubic and Higher Order
- Quantum Consistency

Reviews

- A.Fotopoulos, M.T., arXiv 0805.1346 (+ M.T., arXiv 1202.6303)
- A.Sagnotti., arXiv 1112.4285
- R.Rahman, M.Taronna., arXiv 1512.07932

Part II

- Classical Interactions: purely cubic Lagrangians
- Quantum Interactions: Quantum Higher Spin Gravity,
  $D = 4$, massless
- A string-like model,
  $D = 3$, massive
- Conclusions

Based on

- Evgeny Skvortsov, Tung Tran, M.T.,
  arXiv 2006.05809, arXiv 2002.08487, arXiv 1805.00048
- also Angelos Fotopoulos, M.T.,
  arXiv 0907.4061, arXiv 0705. 2939

- For Higher Spin Theories free Lagrangians, equations of motion, are already nontrivial.

Different formulations:

- Metric –like formulation ("Fronsdal formulation");
  Higher Spin Fields are higher rank tensors
- Frame-like formulation ("Vasiliev Formulation") :
  Higher Spin fields form a generalised frame-field and spin connection

Connection with other theories?

- String Theory
- Holography
- $Sp(2n)$ formulation
- Supersymmetry ("Metric–like", "Frame–like")

- Originally considered on $D = 4$ flat background.
- One theory of interacting massless and massive Higher Spin fields is known: the Superstring Theory.
- What happens on $(A)dS_D$?
- "Frame–like" formulation on AdS_D; known as Vasiliev's nonlinear equations
- More recent studies of interacting "Metric-like" fields on AdS_D
- Applications to Holography

- Problems for massless higher spin fields on a flat background:
- Classical: Nonlocalities at the level of quartic interactions
- Quantum: Inconsitency of the S–matrix
- Possible answers :
  a) An unpleasant one: massless Higher Spins can not interact
  b) More pleasant one: massless Higher Spins can not exist as asymptotic states
- One more: there are some indications massless Higher Spins are a part of the bigger picture. a complete theory should contain massive fields and/or some nonlocal objects (like in String Theory), that can make a whole theory local and consistent
- THE GOAL OF THE TALK: SHOW THAT ONE CAN HAVE A MODEL FREE FROM THESE TROUBLES

- Light cone approach (R.Metsaev's clasification for massless/massive fields):
  Invariance under the Dynamical generators implies equations on the vertices
- Noether procedure:
  Interaction vertices are gauge invariant
- BRST antifield formalism
- String - derived:
  Consider the perturbative string theory in $\alpha' \to \infty$ limit
- String - derived BRST methods: (derived from OSFT, more below)
- They are interconnected

- Fock space state

$$|\Phi\rangle = \frac{1}{s!}\Phi_{\mu_1\mu_2,\ldots\mu_s}\alpha^+_{\mu_1}\alpha^+_{\mu_2}\ldots\alpha^+_{\mu_s}|0\rangle, \quad [\alpha_\mu, \alpha^+_\nu] = \eta_{\mu\nu}$$

satisfies mass-shell and transversality conditions

$$l_0|\varphi\rangle = 0, \quad l|\varphi\rangle = 0$$

with

$$l_0 = p\cdot p, \quad l = p\cdot\alpha, \quad l^+ = p\cdot\alpha^+, \quad p_\mu = -i\partial_\mu$$

- Introduce ghost variables

$$\{c_0, b_0\} = \{c, b^+\} = \{c^+, b\} = 1$$

$$|\Phi\rangle = |\varphi\rangle + c^+b^+|D\rangle + c_0b^+|C\rangle$$

- The nilpotent BRST charge

$$Q = c_0l_0 + c^+l + cl^+ - c^+cb_0,$$

- Taking three copies of these fields we get for the Lagrangian

$$\mathcal{L}_{cub.} \sim \sum_{i=1}^{3} \langle \Phi_i | Q_i | \Phi_i \rangle + g \langle \Phi_3 | \langle \Phi_2 | \langle \Phi_1 || V_3 \rangle$$

- Nonlinear gauge transformations

$$\delta_{cub.} | \Phi_1 \rangle \sim Q_1 | \Lambda_1 \rangle - g (\langle \Phi_2 | \langle \Lambda_3 | + \langle \Phi_3 | \langle \Lambda_2 |) | V_3 \rangle)$$

- The invariance of $\mathcal{L}_{cub.}$:

$$\begin{aligned} g^0 : &\qquad Q_1^2 = Q_2^2 = Q_3^2 = 0 \\ g^1 : &\qquad (Q_1 + Q_2 + Q_3) | V_3 \rangle = 0 \end{aligned}$$

Some modifications, namely

- For massive Higher Spin fields

$$L_0 = p \cdot p + m^2, \quad L^{\pm} = p \cdot \alpha^{\pm} + m \; \alpha_D^{\pm}$$

Dimensional reduction: the same BRST charge and extra oscillators

$$[\alpha_D, \alpha_D^+] = 1$$

- For the massless fields on AdS_D:

$$p_\mu = -i(\partial_\mu + \omega_\mu^{ab} \alpha_a^+ \alpha_b), \quad l_0 = p \cdot p + m^2(s, D)$$

- The equation $\sum_{i=1}^{3} Q_i |V\rangle_{AdS} = 0$ is solved order by order in $\frac{1}{L^2}$

$$|V_3\rangle_{AdS} = |V_3\rangle_{Flat} + \frac{1}{L^2} |V'\rangle + \ldots$$

- Alternatively: dimensional reduction, Holography, Noether procedure

- Take four copies of $|\Phi\rangle$ and consider a modified Lagrangian

$$\mathcal{L}_{quart.} \sim \mathcal{L}_{cub.} + g^2 \langle\Phi_1|\langle\Phi_2|\langle\Phi_3|\langle\Phi_4||V_4\rangle$$

and modified gauge transformations

$$\delta_{quart.}|\Phi_i\rangle = \delta_{cub.}|\Phi_i\rangle - g^2(\langle\Phi_{i+1}|\langle\Phi_{i+2}|\langle|\Lambda_{i+3}| + ...)|V_4\rangle$$

- Solutions of the equation for $|V_4\rangle$

$$\sum_{i=1}^{4} Q_i|V_4\rangle \sim \langle V_3||V_3\rangle$$

have the form

$$|V_4\rangle \sim \frac{1}{p^2}\langle V_3||V_3\rangle$$

Nonlocality. Apparently the same for AdS_D

- Is it possible from the knowledge of the cubic interactions to have a control of higher order interactions? Named "constructable theories"
- BCFW shift of external on-shell momenta

$$\hat{p}_i(z) = p_i - qz, \quad \hat{p}_j(z) = p_j + qz$$

  higher point amplitudes can be constructed from the lower point ones
- Four point amplitude $\mathcal{M}_4(z)$ can be constructed as sum over two channels:
  $\mathcal{M}^{(1,2)}(z)$: poles from t,u channels
  $\mathcal{M}^{(1,4)}(z)$: poles from s,u channels

- Agreement after shift: Benincasa-Cachazo four particle test

$$\mathcal{M}^{(1,2)}(0) = \mathcal{M}^{(1,4)}(0)$$

- Taking an interaction a Higher Spin fields with two scalars

$$\mathcal{L}^{0,0,s} = \kappa^{1-h} N_h \Phi^{\mu_1,,\dots,\mu_s} J_{\mu_1,\dots,\mu_s}$$

  and computing $\mathcal{M}_4$ for four scalars: does not work
- In string theory: if one adds extra nonlocal objects, then in Regge limit

$$M_4^t \sim s^{1-\kappa^{-2}t}$$

- A good behaviour under BCFW shifts
- A possibility for massless Higher Spins

- Recall: Bosonic OSFT is cubic. Can we do similar for HS?
- Consider a BRST invariant vertex

$$
\begin{aligned}
|V_{cub.}\rangle &\sim (V^1 \times V^{M.})|0\rangle_{1,2,3} \\
V^1 &= \exp\left(Y_{ij}(p^i \cdot \alpha^{j,+} + b_0^i c^{j+,})\right), \\
V^{M.} &= \exp\left(\frac{S}{2}\alpha^{i,+} \cdot \alpha^{i,+} + S c^{i,+} b^{i+,}\right)
\end{aligned}
$$

- The quartic vertex is zero for $|S|^2 = 1$
- This makes the model purely cubic, but $\mathcal{M}_4$ vanishes too
- Dimensional reduction to massive HS fields,

$$m_1 + m_2 + m_3 = 0$$

- Another example of classically purely cubic model (D.Ponomarev, E.Skvortsov., 16), derived from the results in (R.Metsaev., 91)
- In the light front approach the Poincare generators are dynamical (**D**) or kinematical (**K**). Interactions are "encoded " in (**D**)
- In $D = 4$: $\boldsymbol{p} = (\beta, \gamma, p_a)$, where $a = 1, 2$ and the fields split into self-dual and anti self dual parts
- The generators **D** are

$$P^-, \quad J^{a,-} \ : 3$$

- Expanding

$$P^- = H \sim H_2 + h^{q_1 q_2 q_3}_{\lambda_1 \lambda_2 \lambda_3} \Phi^{q_1}_{\lambda_1} \Phi^{q_2}_{\lambda_2} \Phi^{q_3}_{\lambda_3} + ...$$

and similar for $J^{a,-}$, then from the equations

$$[J^{a-}, J^{b-}] = [J^{a-}, P^-] = 0$$

one gets equations for the vertices h

- A solution for three massless Higher Spin fields in $D = 4$ (A.Bengtsson, I.Bengtsson, N.Linden., 87)

$$h_{\lambda_1,\lambda_2,\lambda_3,} = C^{\lambda_1\lambda_2\lambda_3}\frac{\overline{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + \overline{C}^{-\lambda_1,-\lambda_2,-\lambda_3,}\frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}$$

with

$$\mathbb{P} = \mathbb{P}_{km} = p_k\beta_m - p_m\beta_k, \quad k = 1, 2, 3$$

- Choosing the coupling constant to be

$$C^{\lambda_1\lambda_2\lambda_3} \sim \frac{1}{\Gamma(\lambda_1 + \lambda_2 + \lambda_3)}$$

and putting $\bar{C} = 0$, thus considering a chiral theory, one obtains that the quartic vertex vanishes

- Some comments on the vertex

$$h_{\lambda_1,\lambda_2,\lambda_3,} = C^{\lambda_1\lambda_2\lambda_3} \frac{\overline{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} + \overline{C}^{-\lambda_1,-\lambda_2,-\lambda_3,} \frac{\mathbb{P}^{-\lambda_1-\lambda_2-\lambda_3}}{\beta_1^{-\lambda_1}\beta_2^{-\lambda_2}\beta_3^{-\lambda_3}}$$

- Not all of the cubic vertices in the light -cone approach can be seen in the covariant approach
- The power of $\mathbb{P}$ is a number of space-time derivatives

Examples:

- $\lambda_1 = \lambda_2 = -\lambda_3 = 1$ corresponds to YM
- $\lambda_1 = \lambda_2 = -\lambda_3 = 2$ gravitational self-interactions
- $\lambda_1 = -\lambda_2 = s$ and $\lambda_3 = 2$ has no covariant analog
  Not in conflict with Aragone-Deser argument

- The action for the chiral theory

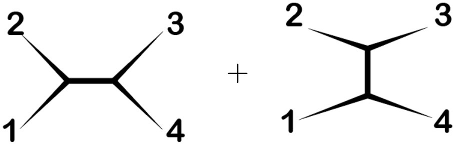
$$S = -\sum_{\lambda} \int (\boldsymbol{p}^2) \mathrm{Tr}[\Phi^{-\lambda}(-\boldsymbol{p})\Phi^{\lambda}(\boldsymbol{p})] + \sum_{\lambda_{1,2,3}} \int C_{\lambda_1,\lambda_2,\lambda_3} \frac{\overline{\mathbb{P}}^{\lambda_1+\lambda_2+\lambda_3}}{\beta_1^{\lambda_1}\beta_2^{\lambda_2}\beta_3^{\lambda_3}} [\Phi^{\lambda_1}_{\boldsymbol{p}_1}\Phi^{\lambda_2}_{\boldsymbol{p}_2}\Phi^{\lambda_3}_{\boldsymbol{p}_3}]\delta^4(\boldsymbol{p}_1+\boldsymbol{p}_2+\boldsymbol{p}_3)$$

- The helicities in the vertex are restricted as $\lambda_1 + \lambda_2 + \lambda_3 \geq 0$
- Reminds of self-dual Yang-Mills (G.Chalmers, W.Siegel., 96)
- One can include "Chan-Patton" factors. They are

$$U(N), \qquad SO(N), \quad USp(N)$$

- The constant l_p can be associated with the Plank length

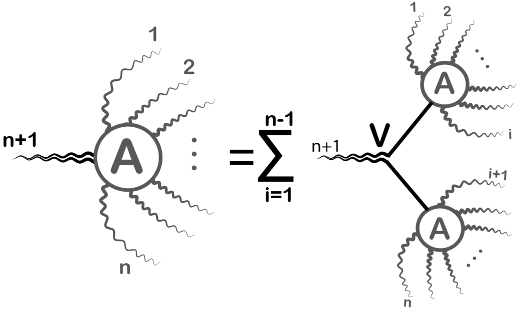
- Easier with "colored" theory; need "color ordered" amplitudes
- Four point tree amplitude



$$A_4 = \frac{\alpha_4^{\Lambda_4 - 2}}{\Gamma(\Lambda_4 - 1) \prod_{i=1}^{4} \beta_i^{\lambda_i - 1}} \frac{\beta_3 \, \boldsymbol{p}_1^2}{4\beta_1 \mathbb{P}_{23} \mathbb{P}_{34}}$$

$$\alpha_4 = \overline{\mathbb{P}}_{12} + \overline{\mathbb{P}}_{34}, \quad \Lambda_4 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$$

- Consistent with $S = 1$, yet having a very nontrivial structure
- All spins must be present and a specific form of the coupling constants are needed "coupling conspiracy"



- Recursive technique: A_n with "1" particle off-shell is proportional to $\mathbf{p}_1^2$. It becomes the propagator to compute A_{n+1}

$$A_n = \frac{(-)^n\, \alpha_n^{\Lambda_n-(n-2)} \beta_3 \ldots \beta_{n-1}\, \boldsymbol{p}_1^2}{2^{n-2}\Gamma(\Lambda_n - (n-3)) \prod_{i=1}^{n} \beta_i^{\lambda_i - 1} \beta_1 \mathbb{P}_{23} \ldots \mathbb{P}_{n-1,n}},$$

$$\alpha_n = \sum_{i<j}^{n-2} \overline{\mathbb{P}}_{ij} + \overline{\mathbb{P}}_{n-1,n}, \quad \Lambda_n = \lambda_1 + \ldots + \lambda_n$$

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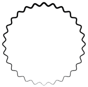
- Vacuum (bubble) diagrams



$$Z_{\text{1-loop}} = \frac{1}{(z_0)^{1/2}} \prod_{s>0} \frac{(z_{s-1})^{1/2}}{(z_s)^{1/2}},$$

- The partition function $Z_{\text{1-loop}} \sim (z_0)^{\nu_0/2}$
- The total number of degrees of freedom ν_0

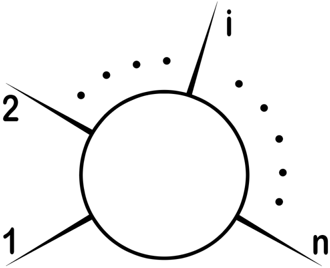
$$\nu_0 = \sum_{\lambda} 1 = 1 + 2\sum_{s=1}^{\infty} 1 = 1 + 2\zeta(0) = 0$$

can be regularized to zero (M.Beccaria, A.Tseytlin., 15)

- Explicitly one loop correction to self-energy, to three vertex, to the four point function, (E.Skvortsov, T.Tran, M.T., 20)
- Generic 1- loop amplitude factorizes as (E.Skvortsov, T.Tran., 20)

$$A_{\text{1-loop}} = A^{++\cdot,,\cdot+}_{\text{1-loop, QCD}} \times D^{HSGR}_{\lambda_1,\dots\lambda_n} \times \nu_0$$

- Multi loop amplitudes



$$\Gamma_n = \nu_0 \frac{(l_p)^{\Lambda_n - n} \alpha_n^{\Lambda_n - n}}{\Gamma(\Lambda_n - (n-1)) \prod_{i=1}^{n} \beta_i^{\lambda_i}} \int \frac{d^4 p}{(2\pi)^4} \frac{\mathcal{K}_n(\overline{\mathbb{P}})}{\boldsymbol{p}^2 (\boldsymbol{p} + \boldsymbol{p}_1)^2 \ldots (\boldsymbol{p} - \boldsymbol{p}_n)^2}$$

- The nonvanishing ones are proportional to the total number of degrees of freedom

- Dimensionally reduce Chiral HSGRA to $D = 3$ (R.Metsaev., 20; E.Skvortsov, T.Tran, M.T., 20)

$$h_3 = \sum_{\lambda_i=-\infty}^{+\infty} \sum_{k_i} C(k_i, \lambda_i) V(\mathbb{P}, \beta_i, k_i, \lambda_i) \,,$$

- The coupling constants and the vertex are

$$C = \frac{\delta_{\sum_i k_i \epsilon_i, 0}}{\Gamma[\lambda_1 + \lambda_2 + \lambda_3]} \,, \qquad V = (\mathbb{P} + \mathbb{P}_\lambda)^{\lambda_1+\lambda_2+\lambda_3} \prod_i \beta_i^{-\lambda_i} \,.$$

$$\mathbb{P}_\lambda = \frac{i}{3} m \sum \check{\beta}_j \epsilon_j k_j \,, \qquad \epsilon_i = \mathrm{sign}(\lambda_i)$$

- Keeps the theory purely cubic

- Masses belong to the “lattice”

$$\sum_j \epsilon_j m_j = 0$$

- Three amplitudes vanish
- UV behaviour of loops expected to soften
- One can consider the “coloured” version

- Quantum Higher Spin Gravity is a Consistent Quantum Theory
- It must be a building block for consistent quantum HS theories
- Includes (self dual) Gravity and Yang Mills vertices
- Its AdS_4 deformation (R.Metsaev., 18; E.Skvortsov., 18)
- Its consistent massive three dimensional "descendant" -a string like model
- More detailed studies

THANK YOU!!!