



Supercaloric functions for the parabolic p -Laplace equation in the fast diffusion case

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This talk discusses a generalized class of supersolutions, so-called p -supercaloric functions, to the parabolic p -Laplace equation. This class of functions is defined as lower semicontinuous functions that are finite in a dense set and satisfy the parabolic comparison principle. Their properties are relatively well understood in the slow diffusion case $p > 2$, but little is known in the fast diffusion case $1 < p < 2$. For $p = 2$ we have supercaloric functions for the heat equation.

Every bounded p -supercaloric function belongs to the natural Sobolev space and is a weak supersolution to the parabolic p -Laplace equation for the entire range $1 < p < \infty$. In the slow diffusion case every unbounded p -supercaloric function has either a Barenblatt type behavior or blows up at least with the rate given by the friendly giant. Our main result is the corresponding result in the supercritical case $\frac{2n}{n+1} < p < 2$. The Barenblatt solution and the infinite point source solution show that both alternatives occur and that the obtained estimates are sharp. The theory is not yet well understood in the subcritical case $1 < p \leq \frac{2n}{n+1}$.
