Reviving Horndeski gravity using Teleparallel gravity

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Abstract

General Relativity (GR) is based on a manifold with curvature and zero torsion and on the contrary, Teleparallel gravity (TG) is a theory which assumes a **non-zero torsion with zero curvature**. It turns out that it is possible to write down a theory in Teleparallel gravity that **is equivalent to GR in terms of the field equations**. Even though these theories are equivalent in field equations, **their modifications are different**. Horndeski gravity which is based from GR **was highly constraint** from the recent gravitational waves observations due to $|c_g/c-1| \gtrsim 10^{-15}$. We constructed an **analogue version of Horndenki gravity** which is based in Teleparallel gravity and showed that in this context, it is possible to **construct a theory satisfying** $c_T = c_g/c = 1$ without eliminating the coupling functions $G_5(\phi, X)$ and $G_4(\phi, X)$ that were highly constraint in standard Horndeski theory. Hence, in the Teleparallel approach, one is **able to restore these terms** creating an interesting way to revive Horndeski gravity.

1. What is Teleparallel gravity?

- It is constructed by the **Weitzenböck connection** $\Gamma^a{}_{\mu\nu}$ which is **curvatureless**, contains **torsion** and **satisfies the metric compatibility condition** $\nabla_\lambda g_{\mu\nu} = 0$.
- Our dynamical variables are the **tetrad fields** $h^a{}_\mu$ where Latin indices indicate tangents space coordinates and Greek indices correspond to spacetime coordinates. The metric can be reconstructed using the following relationship

$$\boxed{g_{\mu\nu} = h^a{}_\mu h^b{}_\nu \eta_{ab}\,,} \tag{1}$$

where η_{ab} is the Minkowski metric $(-+++)$.

- The field strength of the theory is the **torsion tensor** which is defined as the anti-symmetric part of the Weitzenböck connection, namely,

$$T^a{}_{\mu\nu} := 2\Gamma^a{}_{[\mu\nu]} = \partial_\mu h^a{}_\nu - \partial_\nu h^a{}_\mu + \omega^a{}_{b\mu} h^b{}_\nu - \omega^a{}_{b\nu} h^b{}_\mu\,, \tag{2}$$

where $\omega^a{}_{b\nu}$ is the spin connection. This quantity is generally non-vanishing and transforms covariantly under both diffeomorphisms and local Lorentz transformations.

- The torsion tensor can be **decomposed** as follows

$$T_{\lambda\mu\nu} = \frac{2}{3}(t_{\lambda\mu\nu} - t_{\lambda\nu\mu}) + \frac{1}{3}(g_{\lambda\mu} v_\nu - g_{\lambda\nu} v_\mu) + \epsilon_{\lambda\mu\nu\rho} a^\rho\,, \tag{3}$$

where

$$v_\mu = T^\lambda{}_{\lambda\mu}\,, \quad a_\mu = \frac{1}{6}\epsilon_{\mu\nu\sigma\rho} T^{\nu\sigma\rho}\,, \tag{4}$$

$$t_{\lambda\mu\nu} = \frac{1}{2}(T_{\lambda\mu\nu} + T_{\mu\lambda\nu}) + \frac{1}{6}(g_{\nu\lambda} v_\mu + g_{\nu\mu} v_\lambda) - \frac{1}{3} g_{\lambda\mu} v_\nu\,, \tag{5}$$

are three irreducible parts with respect to the local Lorentz group, known as the **vector, axial, and purely tensorial**, torsions, respectively.

- The most studied teleparallel model is the **Teleparallel equivalent of General Relativity** (TEGR) where the Lagrangian is assumed to take the form

$$\boxed{\mathcal{L}_{\rm TEGR} = \frac{1}{4} T^\rho{}_{\mu\nu} T_\rho{}^{\mu\nu} + \frac{1}{2} T^\rho{}_{\mu\nu} T^{\nu\mu}{}_\rho - T^\lambda{}_{\lambda\mu} T_\nu{}^{\nu\mu} = \frac{3}{2} T_{\rm ax} + \frac{2}{3} T_{\rm ten} - \frac{2}{3} T_{\rm vec} \equiv T\,,} \tag{6}$$

where we have defined **three invariants** $T_{\rm ten} = t_{\lambda\mu\nu} t^{\lambda\mu\nu}$, $T_{\rm ax} = a_\mu a^\mu$ and $T_{\rm vec} = v_\mu v^\mu$ and the **scalar torsion** T which is a linear combination of them.

- For a manifold with zero curvature satisfying the metric compatibility condition and non-zero torsion, one can get the following relationship

$$\boxed{R = \mathring{R} + T - \frac{2}{h}\partial_\mu\left(h T^\sigma{}_\sigma{}^\mu\right) = 0 \quad \Rightarrow \quad \mathring{R} = -T + \frac{2}{h}\partial_\mu\left(h T^\sigma{}_\sigma{}^\mu\right) := -T + B\,.} \tag{7}$$

Here, $\mathring{R}$ is the Ricci scalar as determined using the Levi-Civita connection, R is the Ricci scalar as calculated with the Weitzenböck connection which vanishes, and h is the determinant of the tetrad field, $h = \det\left(h^a{}_\mu\right) = \sqrt{-g}$. Thus, **the scalar torsion T differs only by a boundary term B with respect to $\mathring{R}$** and then, since the TEGR Lagrangian is given by (6), the **TEGR field equations are the same as the Einstein field equations**.

- In analogy to the well-known $f(\mathring{R})$ gravity, one can consider $f(T)$ **gravity** in the TEGR framework which is constructed with the Lagrangian $\mathcal{L}_{f(T)} = f(T)$. Since the torsion scalar T only depends on the first derivatives of the tetrads, this theory is a **second-order theory**. $f(T)$ gravity does not differ from $f(\mathring{R})$ by a total derivative term, so that, these theories are no longer equivalent.
- Then, in general, **even though GR and TEGR are equivalent, when one starts modifying them, one gets different modified theories of gravity.**

2. Teleparallel Horndeski gravity

- We construct an **analogue** version of **Horndeski gravity theory** in the **Teleparallel** framework with the following **conditions**:
  1. The resulting field equations **must be at most second-order in terms of derivatives of the tetrad fields** (or equivalently second order in terms of metric tensor derivatives)
  2. The scalar invariants **should not be parity violating**.
  3. The field equations **must be covariant under local Lorentz transformations**.
  4. **Contractions of the torsion tensor can at most be quadratic**. Any number of contractions of the irreducible parts of the torsion tensor will result in second order field equations. This means that an infinite number of terms can be formed in Teleparallel gravity that gives rise to second-order field equations. However, it is unclear how physical such higher order contributions will be.
- The **most general Lagrangian** that has the above properties and gives second order field equations in **Minkowski space-time** is

$$L = c_1\phi + c_2 X - c_3 X\bar{\Box}\phi + c_4 X\left[(\bar{\Box}\phi)^2 - \partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi\right] - \\ - c_5 X\left[(\bar{\Box}\phi)^3 - 3(\bar{\Box}\phi)\,\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi + 2\partial_\mu\partial_\nu\phi\partial^\nu\partial^\lambda\phi\partial_\lambda\partial^\mu\phi\right]\,, \tag{8}$$

where $X = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi$ is the kinetic term, c_i are constants (that can be functions of ϕ and X to be more general) and $\bar{\Box} = \partial_\mu\partial^\mu$.

- In order to **introduce gravity**, one needs to replace quantities according to the table:

GR	Teleparallel
$\eta_{\mu\nu} \to g_{\mu\nu}$	$e^a{}_\mu \to h^a{}_\mu$
$\partial_\mu \to \mathring{\nabla}_\mu$	$\partial_\mu \to \mathcal{D}_\mu = \partial_\mu + h^c{}_\mu w^a{}_{bc} S_a^b$

Here $e^a{}_\mu$ represents tetrad in trivial frames and $S_c{}^b$ is a representation of the Lorentz generators. Hence, **standard Horndeski** is constructed by using the **covariant coupling prescription of GR** and **Teleparallel Horndeski** needs to be constructed by using the **Teleparallel coupling prescription.**

- **The Teleparallel prescription** $\mathcal{D}_\mu$, in the end, **coincides** with the **GR coupling prescription** $\mathring{\nabla}_\mu$ which is a **covariant derivative** with respect to the **Levi-Civita connection**. Then, the Teleparallel Lagrangians $\sum_{i=3}^{5}\mathcal{L}$ **are identical** to the **last three terms** of the standard Horndeski gravity Lagrangian $\sum_{i=3}^{5}\mathring{L}_i$.
- However, when one is considering Teleparallel gravity, $\mathcal{L}_2$ **would be different** to $\mathring{L}_2$ since there are more scalars that one can construct which satisfies the conditions.
- Taking **quadratic contractions** of the torsion tensor, **the most general Lagrangian** of Teleparallel gravity satisfying the conditions turns out to be $f(T_{\rm ax}, T_{\rm vec}, T_{\rm ten})$ (without a scalar field). If one **adds the scalar field**, one can construct the following **7 extra independent scalars**:

$$I_2 = v^\mu\phi_{;\mu}\,, \quad J_1 = a^\mu a^\nu\phi_{;\mu}\phi_{;\nu}\,, \quad J_3 = v_\sigma t^{\sigma\mu\nu}\phi_{;\mu}\phi_{;\nu}\,, \quad J_5 = t^{\sigma\mu\nu} t_\sigma{}^{\bar{\mu}}{}_\nu\phi_{;\mu}\phi_{;\bar{\mu}}\,, \tag{9}$$

$$J_6 = t^{\sigma\mu\nu} t_\sigma{}^{\bar{\mu}\bar{\nu}}\phi_{;\mu}\phi_{;\nu}\phi_{;\bar{\mu}}\phi_{;\bar{\nu}}\,, \quad J_8 = t^{\sigma\mu\nu} t_{\sigma\mu}{}^{\bar{\nu}}\phi_{;\nu}\phi_{;\bar{\nu}}\,, \quad J_{10} = \epsilon^\mu{}_{\nu\rho\sigma} a^\nu t^{\alpha\rho\sigma}\phi_{;\mu}\phi_{;\alpha}\,, \tag{10}$$

therefore, the **extra Lagrangian term** $\mathcal{L}_{\rm Tele} = G_{\rm Tele}(\phi, X, T, T_{\rm ax}, T_{\rm vec}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})$ is needed to be introduced in Teleparallel Horndeski.

- Considering all the possible terms, the final **Lagrangian of Teleparallel Horndeski** is a linear combination of the **standard Horndeski theory plus a new contribution**, namely,

$$\boxed{\mathcal{L}_{\rm TeleDeski} = \sum_{i=2}^{5}\mathcal{L}_i + \mathcal{L}_{\rm Tele} = \text{Horndeski} + \mathcal{L}_{\rm Tele}\,,} \tag{11}$$

where

$$\boxed{\begin{aligned}
\mathcal{L}_{\rm Tele} &= G_{\rm Tele}(\phi, X, T, T_{\rm ax}, T_{\rm vec}, I_2, J_1, J_3, J_5, J_6, J_8, J_{10})\,,\\
\mathcal{L}_2 &= G_2(\phi, X)\,, \quad \mathcal{L}_3 = G_3(\phi, X)\Box\phi\,,\\
\mathcal{L}_4 &= G_4(\phi, X)\,(-T + B) + G_{4,X}(\phi, X)\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]\,,\\
\mathcal{L}_5 &= G_5(\phi, X)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6}G_{5,X}(\phi, X)\left[(\Box\phi)^3 + 2\phi_{;\mu}{}^\nu\phi_{;\nu}{}^\alpha\phi_{;\alpha}{}^\mu - 3\phi_{;\mu\nu}\phi^{\mu\nu}(\Box\phi)\right].
\end{aligned}}$$

Here, semicolon represents differentiation with respect to the Levi-Civita connection and $\Box\phi = \phi_{;\mu}{}^\mu$.

- In Horndeski gravity, $f(\mathring{R})$ does not appear directly since it is a **4th order theory**. In Teleparallel Horndeski, $f(T)$ appears in the Lagrangian since it is a **2nd order theory**.
- Teleparallel Horndeski **has a richer structure** since standard Horndeski is contained on it (setting $G_{\rm Tele} = 0$). See **Figure to see how the theories are connected**.

3. How are all the theories connected?

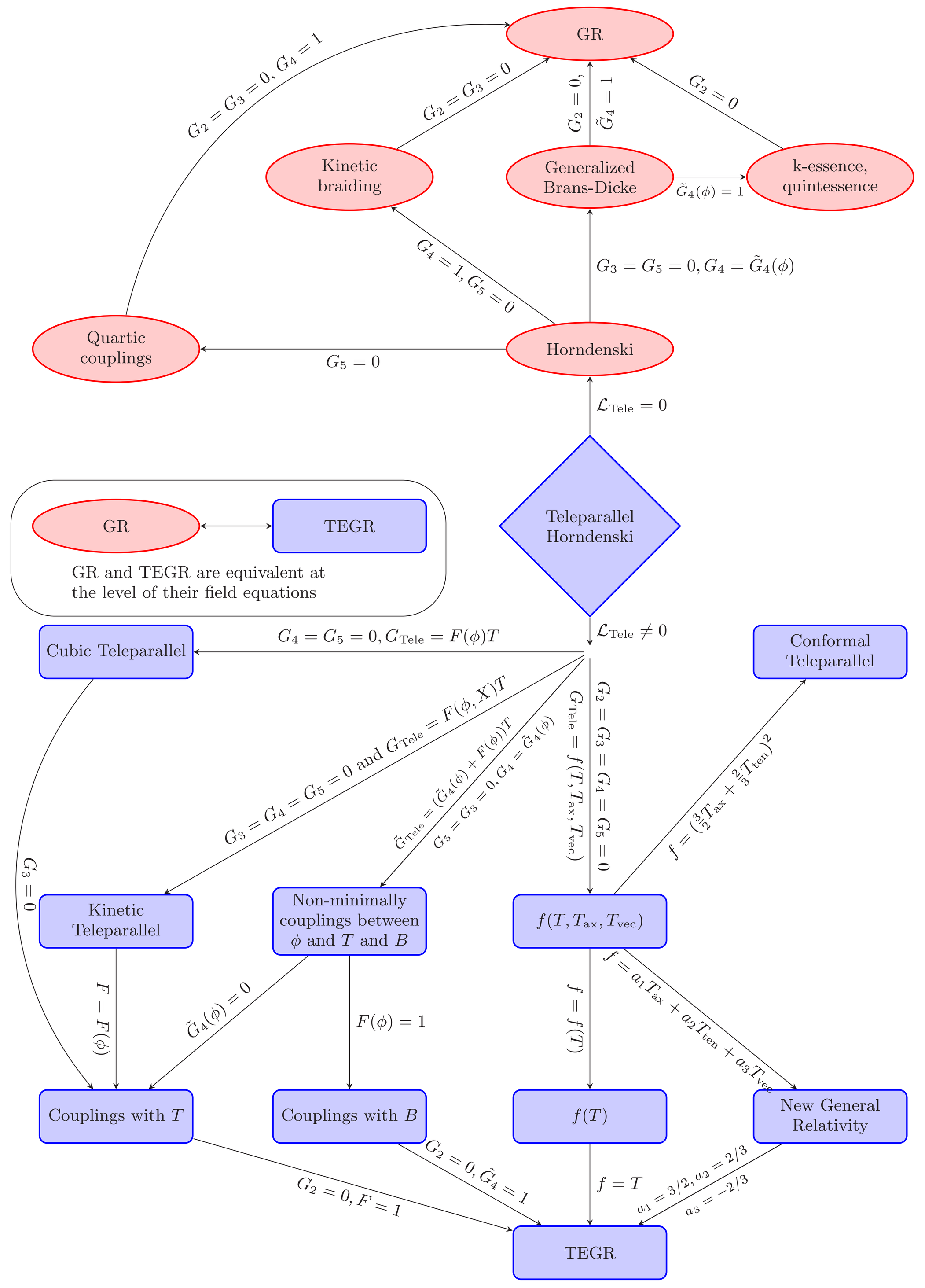

Figure: Relationship between Teleparallel Horndeski and various theories

4. Reviving Horndeski gravity using Teleparallel gravity

- By taking a **flat FLRW background** $ds^2 = -N(t)^2dt^2 + a(t)^2(dx^2 + dy^2 + dz^2)$ which is reproduced by $h^a{}_\mu = \mathrm{diag}(N(t), a(t), a(t), a(t))$ and then by performing **cosmological perturbations** $\bar{h}^a{}_\mu$, the perturbed tetrad becomes

$$\bar{h}^{(0)}{}_\mu = \delta^0_\mu(1 + \Psi) + a\delta^i_\mu(G_i + \partial_i F)\,, \tag{12}$$

$$\bar{h}^{(k)}{}_\mu = a\left[\delta^k_\mu(1 - \Phi) + \frac{1}{2}\delta^i_\mu\delta^{kj}\left(h_{ij} + \partial_i\partial_j B_s + \partial_j C_i + \partial_i C_j\right) + \delta^k_0\delta^j_\mu\partial_j\bar{F}\right]\,, \tag{13}$$

where G_i and C_i are vectorial perturbations, $F, \bar{F}, \psi, \Phi$ and B_s are the scalar perturbations and h_{ij} is the tensorial perturbation.

- If one considers **tensorial perturbations** only, it is possible to find that the **propagation of the gravitational waves for Teleparallel Horndeski** is given by

$$\boxed{c_T^2 = \frac{G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi}) - G_{\rm Tele,T}}{G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi}) + 2XG_{\rm Tele,J_8} + \frac{1}{2}XG_{\rm Tele,J_5} - G_{\rm Tele,T}}\,.} \tag{14}$$

- For standard Horndeski ($G_{\rm Tele} = 0$), one finds that $c_T = 1$ only if $G_5(\phi, X) = \mathrm{constant}$ and $G_4(\phi, X) = G_4(\phi)$. This condition **highly constrains Horndeski gravity**.
- When the Teleparallel term is switch on, from (14) one finds that it is possible **to have a theory respecting** $c_T = 1$ with **non-trivial coupling functions** $G_5 = G_5(\phi)$ and also $G_4(\phi, X)$ depending on X too. The following **Lagrangian respecting** $c_T = 1$ is found:

$$\mathcal{L} = \tilde{G}_{\rm tele}(\phi, X, T, T_{\rm vec}, I_2) + \sum_{i=2}^{4}\mathcal{L}_i + G_5(\phi)\mathcal{G}_{\mu\nu}\phi^{;\mu\nu}\,, \tag{15}$$

which revives $G_5(\phi)$ **and** $G_4(\phi, X)$. **Therefore, one can restore the coupling functions that were ruled out in Horndeski due to GW observations by considering Teleparallel Horndeski gravity**.

Is really General Relativity the correct starting point for quantum gravity?

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1. Metric-Affine theories of gravity

- In metric-affine theories, both the **metric** $g_{\mu\nu}$ and the **connection** $\Gamma^{\alpha}{}_{\mu\nu}$ are the dynamical variables and *a priori* they **do not depend** on each other.
- One can define the failure of the connection to be metric by the so-called **non-metricity tensor** given by
$$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha} g_{\mu\nu}\,. \tag{1}$$
In addition, one can define the antisymmetric part of this connection with the **torsion tensor**:
$$T^{\alpha}{}_{\mu\nu} \equiv 2\Gamma^{\alpha}{}_{[\mu\nu]}\,. \tag{2}$$
The unique connection which is metric compatible and is symmetric is the **Levi-Civita** connection which is based on the Christoffel symbols,
$$\left\{ {\alpha \atop \mu\nu} \right\} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right)\,. \tag{3}$$
- A general connection $\Gamma^{\alpha}{}_{\mu\nu}$ can be decomposed into three different pieces:
$$\Gamma^{\alpha}{}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\} + K^{\alpha}{}_{\mu\nu} + L^{\alpha}{}_{\mu\nu}\,, \tag{4}$$
where $K^{\alpha}{}_{\mu\nu} = \frac{1}{2}T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}$ depend on torsion and it is called the **contortion tensor** and $L^{\alpha}{}_{\mu\nu} = \frac{1}{2}Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)}$ depend on the non-metricity tensor and is called **disformation tensor**.
- The **curvature tensor** can be then defined for a general connection $\Gamma^{\alpha}{}_{\mu\nu}$ as
$$R^{\alpha}{}_{\beta\mu\nu}(\Gamma) = \partial_{\mu}\Gamma^{\alpha}{}_{\nu\beta} - \partial_{\nu}\Gamma^{\alpha}{}_{\mu\beta} + \Gamma^{\alpha}{}_{\mu\lambda}\Gamma^{\lambda}{}_{\nu\beta} - \Gamma^{\alpha}{}_{\nu\lambda}\Gamma^{\lambda}{}_{\mu\beta}\,, \tag{5}$$
and then corresponding Ricci tensor and Ricci scalar are constructed from it.

2. What do curvature, torsion and non-metricity represent geometrically?

- **Curvature** $R^{\alpha}{}_{\beta\mu\nu}$: rotation experienced by a vector when it is parallel transported along a closed curve
- **Torsion** $T^{\alpha}{}_{\mu\nu}$: non-closure of the parallelogram formed when two infinitesimal vectors are parallel transported along each other.
- **Non-metricity** $Q_{\alpha\mu\nu}$: measures how much the length and angle of vectors change as we parallel transport them, so in metric spaces the length of vectors is conserve

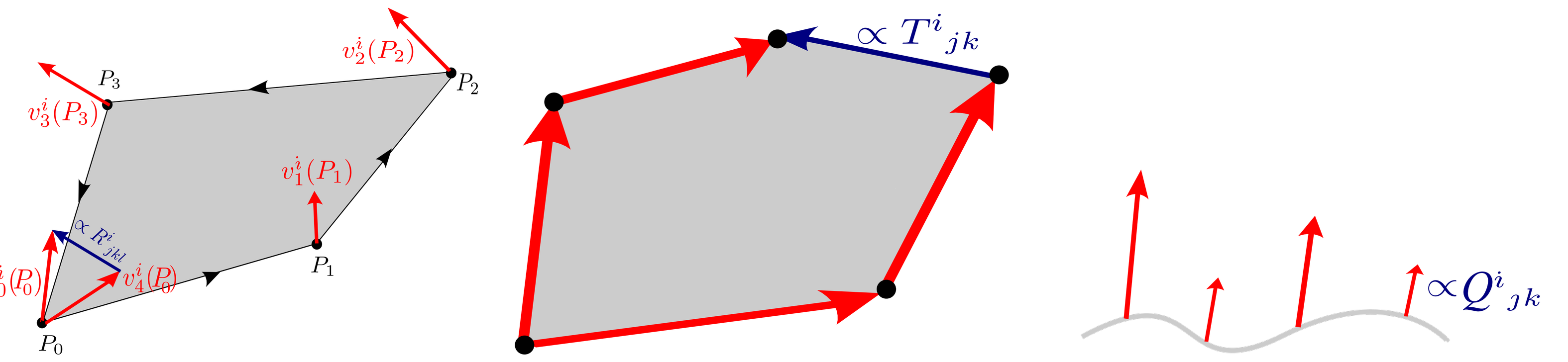

Figure: Geometrical representation of curvature, torsion and non-metricity for a 3-dimensional space

3. Trinity of gravity I: General Relativity (GR), Teleparallel equivalent of GR and Coincident GR

General Relativity (GR):

- GR is the geometrical theory of gravity which assumes a manifold with **non-vanishing curvature** $R^{\alpha}{}_{\beta\mu\nu} \neq 0$ whilst both the **non-metricity tensor** and the **torsion tensor are zero**: $Q_{\alpha\mu\nu} = \nabla_{\alpha} g_{\mu\nu} = 0,\ T^{\alpha}{}_{\mu\nu} = 0$.
- According to (4), the connection is equal to $\mathring{\Gamma}^{\alpha}{}_{\mu\nu} = \left\{ {\alpha \atop \mu\nu} \right\}$, which is the standard Levi-Civita connection, and the contraction of the curvature $\mathcal{R}^{\alpha}{}_{\beta\alpha\nu}$ becomes symmetric.
- The **action of this theory** is constructed from the contraction of the curvature tensor with the metric, known as the Ricci scalar $\mathcal{R} = \mathcal{R}^{\alpha}{}_{\beta\alpha\nu}g^{\beta\nu}$, and reads
$$\mathcal{S}_{\rm GR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathcal{R}(g)\,. \tag{6}$$
Here, $\kappa^2 = 8\pi G$ and this action only depends on the metric, hence, by varying this action with respect to the metric, one gets the standard **Einstein field equations**.
- Matter fields follow the geodesic equation.

4. Trinity of gravity II: General Relativity (GR), Teleparallel equivalent of GR and Coincident GR

Teleparallel equivalent of General Relativity (TEGR):

- TEGR is an alternative theory which assumes the Weitzenböck connection which has a **non-vanishing torsion, zero curvature** $R^{\alpha}{}_{\beta\mu\nu} = 0$ (flat spacetime) and that the metric **satisfies the compatibility condition** $Q_{\alpha\mu\nu} = \nabla_{\alpha} g_{\mu\nu} = 0$.
- The fundamental variables for this theory are the **tetrads** $h^{a}{}_{\nu}$ and the **purely inertial spin connection** $\Gamma^{\alpha}{}_{\mu\nu} = (\Lambda^{-1})^{\alpha}{}_{\lambda}\partial_{\mu}\Lambda^{\lambda}{}_{\nu}$ with Λ being the Lorentz matrix.
- One can then consider **contractions of torsion** to get an alternative theory of gravity. Since torsion is antisymmetric one can only have **three independent contractions** of it
$$\mathbb{T} = c_1 T^{\rho}{}_{\mu\nu}T_{\rho}{}^{\mu\nu} + c_2 T^{\rho}{}_{\mu\nu}T^{\nu\mu}{}_{\rho} + c_3 T^{\lambda}{}_{\lambda\mu}T_{\nu}{}^{\nu\mu}\,, \tag{7}$$
with c_i being coupling constants. The theory constructed from $\mathbb{T}$ is called **New General Relativity**.
- If one assumes zero curvature and zero non-metricity in Eq. (5), and then computes the Ricci scalar, one gets
$$R = \mathcal{R} + \mathring{\mathbb{T}} - \frac{2}{\sqrt{-g}}\partial_{\mu}\left(\sqrt{-g}T^{\sigma}{}_{\sigma}{}^{\mu}\right) = 0 \quad \Rightarrow \quad \mathcal{R} = -\mathring{\mathbb{T}} + 2\mathcal{D}_{\mu}\left(T^{\sigma}{}_{\sigma}{}^{\mu}\right) := -\mathring{\mathbb{T}} + B\,, \tag{8}$$
where $\mathcal{R}$ is the curvature computed with the Levi-Civita connection and we have defined the **torsion scalar** as $\mathring{\mathbb{T}} = \mathbb{T}$ with the coefficient constants $c_1 = \frac{1}{4},\ c_2 = \frac{1}{2},\ c_3 = -1$.
- The **TEGR action** is defined from the torsion scalar as follows
$$\mathcal{S}_{\rm TEGR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathring{\mathbb{T}}(h,\Lambda)\,, \tag{9}$$
and then, since the Einstein-Hilbert action (6) **differs only by a boundary term** B with respect to the above action (see (8)), by varying the TEGR action with respect to the tetrads, one also gets the Einstein field equations. Then, TEGR is **classically equivalent to GR at the level of the field equations**.

5. Trinity of gravity III: General Relativity (GR), Teleparallel equivalent of GR and Coincident GR

Coincident General Relativity (CGR):

- CGR in not a geometrical theory of gravity since assumes that both the **curvature and torsion tensor are zero** $R^{\alpha}{}_{\beta\mu\nu} = T^{\rho}{}_{\mu\nu} = 0$. In this theory, the **non-metricity condition is not satisfied** $Q_{\alpha\mu\nu} = \nabla_{\alpha} g_{\mu\nu} \neq 0$, therefore the **non-metricity tensor** plays the role of being the field strength.
- Similarly as in TEGR, one can then construct theories with the **contractions of the non-metricity tensor**. There are now five possible non-trivial contractions that one can construct
$$\mathcal{Q} = \frac{c_1}{4}Q_{\alpha\beta\gamma}Q^{\alpha\beta\gamma} - \frac{c_2}{2}Q_{\alpha\beta\gamma}Q^{\beta\alpha\gamma} - \frac{c_3}{4}Q_{\alpha}Q^{\alpha} + (c_4 - 1)\tilde{Q}_{\alpha}\tilde{Q}^{\alpha} + \frac{c_5}{2}Q_{\alpha}\tilde{Q}^{\alpha}\,, \tag{10}$$
with c_i being coupling constants, $Q_{\alpha} = Q_{\alpha\lambda}{}^{\lambda}$ and $\tilde{Q}_{\alpha} = Q^{\lambda}{}_{\lambda\alpha}$.
- By setting that both curvature and torsion are equal to zero in (5) and then doing the correct contractions, one finds that the Ricci scalar computed with the Levi-Civita connection $\mathcal{R}$ is related to a specific scalar constructed from the non-metricity tensor, namely
$$R = \mathcal{R} + \mathring{\mathcal{Q}} + \mathcal{D}_{\alpha}(Q^{\alpha} - \tilde{Q}^{\alpha}) = 0 \rightarrow \mathcal{R} = -\mathring{\mathcal{Q}} + \mathcal{D}_{\alpha}(\tilde{Q}^{\alpha} - Q^{\alpha})\,. \tag{11}$$
Here, $\mathring{\mathcal{Q}}$ is the scalar constructed from (10) with $c_i = 1$ $(i = 1,..,5)$. One can notice that $\mathring{\mathcal{Q}}$ and $\mathcal{R}$ **differs only by a boundary term**.
- Thus, similarly as before, one can formulate a **classically equivalent theory of gravity to GR** from the following action
$$\mathcal{S}_{\rm CGR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathring{\mathcal{Q}}(g,\xi)\,, \tag{12}$$
since again this action is **equivalent** up to a total derivative term with respect to the Einstein-Hilbert action.
- The flatness condition again restricts the connection to be purely inertial but now since torsion is also zero, the connection is parametrised by a set of functions ξ^{k} such that $\Gamma^{\alpha}{}_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial \xi^{\lambda}}\partial_{\mu}\partial_{\nu}\xi^{\lambda}$. Thus, the connection can be trivialised by a coordinate transformation $\xi^{\alpha} = x^{\alpha}$ which is known as the **coincident gauge**.
- In the coincident gauge, it is possible to rewrite the action (12) containing only **first derivatives of the metric**
$$\mathcal{S}_{\rm CGR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,g^{\mu\nu}\left(\left\{ {\alpha \atop \beta\mu} \right\}\left\{ {\beta \atop \nu\alpha} \right\} - \left\{ {\alpha \atop \beta\alpha} \right\}\left\{ {\beta \atop \mu\nu} \right\}\right)\,, \tag{13}$$
then, leading to a well-posed variational principle **without any GHY boundary terms**.

6. How are the theories related?

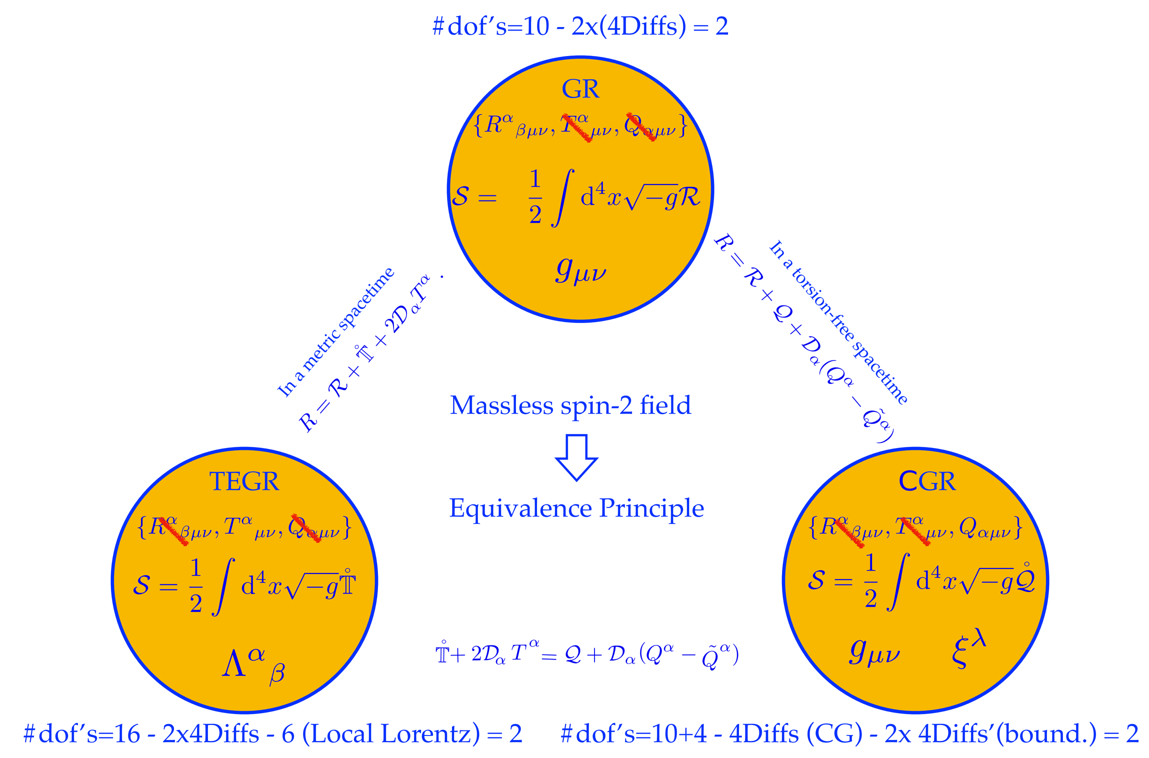

Figure: Trinity of gravity (taken from [1]).

7. Why TEGR or CGR could be good for quantum gravity?

- Since GR, TEGR and CGR have the **same classical field equations**, and all the quantum approaches have been made in GR, it would be interesting to see what happens in quantum approaches in the other two theories.
- Since these three theories are classically equivalent in field equations, it is then a **matter of convention** to say if gravity is the **curvature of a torsionless spacetime**, or **torsion of a curvatureless spacetime**, or if it occurs due to the **non-metricity of a curvatureless and torsionless spacetime**.
- In GR, one needs to introduce the **Gibbons-Hawking-York boundary term**, otherwise the action is not regular. On the other hand in both TEGR and CGR, one **does not need to introduce this boundary term**. Hence, both the TEGR and CGR actions **exhibit a well-posed action principle** and readily admits the **path integral approach to quantization**. As an example, the computation of the **Schwarzschild black hole entropy** in each theory is:
$$\mathcal{S}_{GR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathcal{R} + \int_{\partial M} d^3x\mathcal{K} + \int_{\partial M} d^3x\mathcal{K}_0 = 0 + \infty + \infty = 4\pi GM^2\,, \tag{14}$$
$$\mathcal{S}_{TEGR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathring{\mathbb{T}} = -\frac{1}{2\kappa^2}\int_{\partial M}\sqrt{-g}B = 4\pi GM^2\,, \tag{15}$$
$$\mathcal{S}_{CGR} = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\,\mathring{\mathcal{Q}}\,|_{\xi^{\alpha}=x^{\alpha}} = -\frac{1}{2\kappa^2}\int_{\partial M} d^3x\tilde{Q}^{\mu}n_{\nu} = 4\pi GM^2\,. \tag{16}$$
- GR is based on the **equivalence principle** whose strong version establishes the **local** equivalence between gravitation and inertia. The fundamental asset of quantum mechanics, on the other hand, is the **uncertainty principle**, which is essentially **nonlocal**. It seems that both approaches are not consistent. Further, at quantum scales up to now, it has not been discovered yet if the equivalence principle is always valid or not. In TEGR, one **can** formulate the theory **without the equivalence principle**. Hence, it can **comply with universality but remains a consistent theory in its absence**.
- In CGR (in the coincidence gauge) and TEGR (in the Weitzenböck gauge), there is a way to **define the gravitational energy-momentum tensor** (as a tensor locally conserved) and in GR, this is not possible. Moreover, in both CGR and TEGR one can **separate gravity from inertia**.
- CGR can be written as the **canonical framework for a gauge theory** of the Abelian group of **translations** since $[\nabla_{\mu}, \nabla_{\nu}] = 0$. All the other **forces** are written in the **gauge language** so this is a good hint to explore quantum approaches in this framework.

References

- J. Beltrán Jiménez, L. Heisenberg and T. S. Koivisto, arXiv:1903.06830 [hep-th].
- J. B. Jiménez, L. Heisenberg and T. S. Koivisto, arXiv:1903.12072 [gr-qc].
- S. Bahamonde and M. Faizal, arXiv:1905.04372 [gr-qc].

Wicked metrics vs path integral in Lorentzian spacetimes

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Abstract

There are various ways of defining the Wick rotation in a gravitational context. In order to preserve the manifold structure, it would be preferable to view it as an analytic continuation of the metric, instead of the coordinates. We focus on one very general definition and show that it is not always compatible with the additional requirements of preserving the field equations and the symmetries at global level. Then we consider another approach based not on the deformation of the time or the metric, but of the integration contour of the fields. In particular we discuss the calculation of one-loop effective actions in Lorentzian spacetimes, based on a very simple application of the method of steepest descent to the integral over the field. We show that for static spacetimes this procedure agrees with the analytic continuation of Euclidean calculations. When applied to quantum gravity on static backgrounds, our procedure is equivalent to analytically continuing time and the integral over the conformal factor.

References:
A. Baldazzi, R. Percacci and V. Skrinjar, "Quantum fields without Wick rotation," Symmetry **11** (2019) no.3, 373 doi:10.3390/sym11030373 [arXiv:1901.01891 [gr-qc]]
A. Baldazzi, R. Percacci and V. Skrinjar, "Wicked metrics," Class. Quant. Grav. **36** (2019) no.10, 105008 doi:10.1088/1361-6382/ab187d [arXiv:1811.03369 [gr-qc]]

Wicked metrics

The standard prescription for Wick rotation is $\boxed{iS_L\Big|_{t\to -it_E} = -S_E}$.

We note that time has no physical meaning in GR: if the Wick rotation is performed on time, one immediately finds that the result depends very strongly on the coordinate system. For example for de Sitter space

$$ds^2 = -dt^2 + H^{-2}e^{2Ht}\left(dr^2 + r^2 d\Omega_2^2\right)$$
$$ds^2 = -d\tau^2 + H^{-2}\cosh^2(H\tau)\left(\frac{dr^2}{1-r^2} + r^2 d\Omega_2^2\right)$$
$$ds^2 = -d\bar\tau^2 + H^{-2}\sinh^2(H\bar\tau)\left(\frac{dr^2}{1+r^2} + r^2 d\Omega_2^2\right)$$

the prescription $t \to -it_E$ leads to a metric that is either complex, or positive definite, or again Lorentzian but with opposite signature.

Define Wick rotation in such a way that

(1) it does not depend on the coordinates;

(2) causality is taken into account;

(3) it leaves the differentiable structure of the manifold fixed.

Possible solution: analytic continuation of the metric [M. Visser, arXiv:1702.05572 [gr-qc]]

$$\boxed{g_{(\sigma)\mu\nu} = g_{(L)\mu\nu} + (1+\sigma)X_\mu X_\nu} \ ,$$

where X is a unit time-like vector and σ varies between -1 and 1. Clearly $g_{(-1)} = g_{(L)}$ and $g_{(1)} = g_{(E)}$, while for $\sigma = 0$ the metric is degenerate.

However, the choice of X *is arbitrary* , so let's put additional requirements

(4) a solution of the Lorentzian field equations should map to a solution of the Euclidean field equations;

(5) it should preserve the number of Killing vectors;

(6) it should preserve the maximal symmetry.

These three properties are satisfied by continuations of the coordinates.

Examples

REGULAR EXAMPLES: there exist X such that all properties are satisfied.

- **Minkowski space**: the isometry group changes from $SO(1,3)$ to $SO(4)$;
- **Anti de Sitter space**: the isometry group changes from $SO(2,3)$ to $SO(1,4)$;
- **pp-wave spacetime**.

IRREGULAR EXAMPLES

- **de Sitter space**

(a) $ds^2(\sigma) = \sigma d\tau^2 + H^{-2}\cosh^2(H\tau)(d\chi^2 + \sin\chi^2 d\Omega_2^2)$, $R_{(\sigma)\mu\nu} = -\frac{3H^2}{\sigma} g_{(\sigma)\mu\nu} + 2H^2\frac{(1+\sigma)}{\sigma}P_{\mu\nu}$
$X = d\tau$ violates condition (4-5-6);

(b) $ds^2(\sigma) = \sigma d\bar\tau^2 + H^{-2}\sinh^2 H\bar\tau\ (d\chi^2 + \sinh^2\chi d\Omega_2^2)$
$X = d\bar\tau$ violates condition (4-5-6) and is not globally defined;

(c) $ds^2(\sigma) = \sigma dt^2 + H^{-2}e^{2Ht}\left(dx^2 + dy^2 + dz^2\right)$, $R_{(\sigma)\mu\nu\rho\sigma} = \frac{H^2}{-\sigma}\left[g_{(\sigma)\mu\rho}\, g_{(\sigma)\nu\sigma} - g_{(\sigma)\mu\sigma}\, g_{(\sigma)\nu\rho}\right]$
The isometry group remains $SO(1,4)$, but $X = dt$ is not globally defined;

(d) $ds^2(\sigma) = H^{-2}\left[\sigma\cos^2\zeta\ dt^2 + d\zeta^2 + \sin^2\sigma\ d\Omega_2^2\right]$, $R_{(\sigma)\mu\nu\rho\sigma} = H^2\left[g_{(\sigma)\mu\rho}\, g_{(\sigma)\nu\sigma} - g_{(\sigma)\mu\sigma}\, g_{(\sigma)\nu\rho}\right]$
$SO(1,4)$ for $\sigma < 0$ and $SO(5)$ for $\sigma > 0$, but $X = H^{-1}\cos\zeta dt$ is not globally defined.

- **Schwarzschild spacetime**

(a) $ds^2 = \sigma\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2$
Ricci-flat and all Killing vectors for all σ , but $X = \sqrt{1 - \frac{2M}{r}}dt$ is not globally defined;

(b) $ds^2(\sigma) = \frac{16M^2}{X^2-T^2}\frac{W(z)}{W(z)+1}\left[\sigma dT^2 + dX^2\right] + 4M^2\left(W(z)+1\right)^2 d\Omega_2^2$, where $z \equiv \frac{X^2-T^2}{e}$.
$X = 4M\sqrt{\frac{W(z)}{(X^2-T^2)(W(z)+1)}}\,dT$ is globally defined but the space is not Ricci-flat.

LESSON

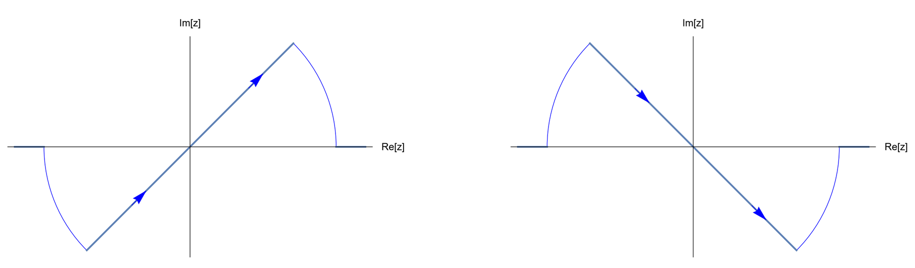
The irregular examples are characterized by the presence of horizons. One can demand that *local* solutions of the Lorentzian EOM go to *local* solutions of the Euclidean EOM, but in the presence of horizons, such continuations cannot be extended to the whole manifold. This was true for analytic continuations of the coordinates, but it seems to be true also when one continues the metric.

Path integrals in Lorentzian spacetimes

Can we compute Z_L directly? Instead of deforming time or the metric, deform the integration contour of the field and use Picard-Lefschetz theory.

Decomposing $\phi = \sum_n z_n\phi_n$ with ϕ_n eigenvectors of the kinetic operator, the path integral is a product of integrals of the form $I(\lambda) = \int_{-\infty}^{\infty} dz\, e^{i\lambda z^2}$.

This kind of integral is **conditionally convergent**, in particular it is convergent choosing the steepest descent contour. In our case the contour depends on the sign of λ (which is the eigenvalue of the kinetic operator): we choose the path on the left if $\lambda > 0$ and the path on the right if $\lambda < 0$



and the result is : $I(\lambda) = e^{\mathrm{sign}(\lambda)i\pi/4}\sqrt{\frac{\pi}{|\lambda|}}$.

Absorbing the phases in normalization

$$Z_L = \int(d\phi)\exp\left[i\frac{1}{2}\int d^dx\sqrt{-g}\,\phi(\Box - m^2)\phi\right] = \left[\det\left(\frac{\Box - m^2 + i\zeta}{\mu^2}\right)\right]^{-1/2} .$$

So the ill-definiteness of the Lorentzian functional integral is due not to oscillatory character of integrals but to the sum over the field modes, like the Euclidean case.

Calculation of the determinant using Heat Kernel For the Euclidean case: $\Gamma_E = -\log Z_E$

$$\Gamma_E = -\frac{1}{2}\int_0^\infty \frac{ds}{s}\mathrm{Tr}K_{-\Box+m^2}(s)\ ,$$

while for the Lorentzian case: $\Gamma_L = -i\log Z_L$

$$\Gamma_L = -\frac{i}{2}\int_0^\infty \frac{dis}{is}\mathrm{Tr}\exp\left[is(\Box - m^2 + i\zeta)\right]\ .$$

Euclidean vs Lorentzian effective actions

Examples of calculations of one-loop effective actions for static spacetimes with Euclidean signature ($\sigma = 1$) and Lorentzian signature ($\sigma = -1$)

- Massless free scalar field on $\Sigma \times S^1$ with time period T
  Eigenvalues of kinetic operator $\lambda_{n,i} = \sigma\left(\frac{2\pi n}{T}\right)^2 + \omega_i^2$

$$\Gamma(\sigma) = -\sigma^{1-\frac{d}{2}}\left(\frac{\sqrt{\pi}}{T}\right)^{d-1}\sum_{j=0}^{\infty}\sigma^j\,\zeta_R[1-d+2j]\,\Gamma\left(\frac{1-d}{2}+j\right)\left(\frac{T}{2\pi}\right)^{2j}B_{2j}(\Delta_\Sigma)\ .$$

- Massive free scalar field on $\Sigma \times R$
  Eigenvalues of the kinetic operator $\lambda_{n,i} = \sigma E^2 + \omega_i^2 + m^2$

$$\Gamma(\sigma) = -\frac{\sigma}{2}\frac{Tm^d}{(4\pi)^{d/2}}\sum_{j=0}^{\infty}\Gamma\left(j - \frac{d}{2}\right)m^{-2j}\,B_{2j}(\Delta_\Sigma)\ ,$$

in both cases this procedure agrees with the standard prescription:

$$\boxed{i\Gamma_L(V, -iT) = -\Gamma_E(V,T)}\ .$$

Interacting theories: for *perturbative expansion* Gaussian integrals are all one needs

$$Z_\sigma[j] = e^{\sqrt{\sigma}S_{\mathrm{int}}\left(\frac{1}{\sqrt{\sigma}}\frac{\delta}{\delta j}\right)}\int(d\phi)e^{\sqrt{\sigma}\int d^dx\sqrt{g}\left(\frac{1}{2}\phi\Delta_\sigma\phi + j\phi\right)} = Z_\sigma[0]e^{\sqrt{\sigma}S_{\mathrm{int}}\left(\frac{1}{\sqrt{\sigma}}\frac{\delta}{\delta j}\right)}e^{-\frac{\sqrt{\sigma}}{2}j\cdot\Delta_\sigma^{-1}\cdot j}\ .$$

Only for *non-perturbative calculations* does one need to use more complicated steepest descent contours. [J.Feldbrugge, J.-L. Lehners, N. Turok, Phys.Rev. D95 (2017) no.10, 103508].

The conformal factor problem

$$S^{(2)}_{on-shell}(h,\bar g) = \frac{1}{4\kappa^2}\int d^4x\sqrt{\bar g}\left[\frac{1}{2}h^{TT}_{\mu\nu}\left(\triangle_2 - 2\Lambda\right)h^{TT\mu\nu} - \frac{3}{16}s\left(\triangle_0 - \frac{4}{3}\Lambda\right)s\right]$$

Choosing path of steepest descent for Euclidean GR, we obtain the same result as Cambridge prescription.

Gravitation as blackbody radiation and black hole thermodynamics

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Abstract

We provide a heuristic way to consider gravity as blackbody radiation. Our formulation involves two basic elements. A quantum spacetime, described as the quantum excitations of a new kind of fields, which live in momentum space, named as accelerated fields. And a massive object defined as a collection of spacetime quanta in thermodynamic equilibrium. This object behaves as a blackbody, emitting quantum lengths and time intervals, at temperature inversely proportional to its mass. Having established a correspondence between quantum and Rindler spacetime (PLB 781, 611, 2018), equivalence principle allows us to identify gravity with the massive object's blackbody radiation, since the last affects the corresponding geometry of spacetime. At Planck scale, our argument directly leads to the Hawking temperature of a black hole (up to a constant) demonstrating that a black hole can actually reach mass equilibrium if spacetime comes in discrete units.

Introduction

Gravity is the oldest known fundamental force. However, we are still unable to reach consensus on its true nature. Einstein's general relativity was a huge step forward, but certain results (e.g. singularity theorems, black hole thermodynamics, and Hawking radiation) indicate that we must modify our understanding of gravity (and spacetime) once again, in order to take these properties into account.
This work provides a new theory of quantum gravity. In this theory spacetime arises as quantum excitations of a new kind of quantum fields, named as accelerated fields. Matter is designated as the simplest macroscopic object made out of spacetime quanta in thermal equilibrium. Finally, gravitation, instead of curvature of spacetime continuum, is identified as blackbody radiation of these spacetime quanta [2].
The theory stems from a recently established correspondence between Rindler and quantum spacetimes [1]. In that work we show that an appropriately quantized spacetime redefines classical accelerated reference frames in a way that if one consider matter fields in that spacetime, Unruh effect comes up. Throughout the text, the units are chosen such that $c = \hbar = 1$.

Matter in quantum spacetime

- "massless" space: $ds^2 = dE^2 - dp^2$
- "massive" space: $ds^2 = dm^2 - m^2 ds^2$ ($s = \sigma/m$ and σ a parameter)

- A relativistic invariant way to implement mass is the energy momentum relation $E^2 - p^2 = m^2$. In momentum space, this relation modifies its metric.
- The situation resembles Unruh effect in matter fields, with an essential difference: the role of acceleration is played now by mass.

Matter via statistical mechanics

- Partition function: $Z(T) = \sum_{N_t} e^{-\frac{N_t x_t}{k'T}}$

k', has dimensions of length divided by temperature, keeps Z dimensionless.

- Average quanta number: $\langle N_t \rangle = \left(e^{\frac{x_t}{k'T}} - 1 \right)^{-1}$ (single-partite state)
- Temperature: $T = \frac{1}{2\pi k' m}$

The chemical potential is zero, $\mu = 0$.
With matter the number of quanta is not conserved.

Matter is made up of spacetime quanta in thermal equilibrium, with temperature be a function of mass.

Spacetime as a quantum object

The process of spacetime quantization requires to generalize the existed notion of quantum fields by introducing quantum fields in momentum space. If matter fields represent all the fields defined in spacetime, accelerated fields stand for the fields in momentum space. Then quantum spacetime indicates quantum excitations of accelerated fields, in the same sense in which a particles are quantum excitation of matter fields.

Basic formulas in accelerated field theory

- Field equation: $\left(\widetilde{\partial}_E^2 - \widetilde{\partial}_p^2 - \frac{1}{\alpha^2} \right) \widetilde{\Psi}(E,p) = 0$

based on the kinematic relation $x^2 - t^2 = 1/\alpha^2$

- $x_t = \hbar\, \widetilde{\omega}$ and $t = \hbar\, \widetilde{k}$

for $\hbar = 1$, we obtain the dispersion relation $x_t^2 - t^2 = 1/\alpha^2$

- Field operator: $\widetilde{\Psi}(E,p) = \int dt \left(\widetilde{a}_t \widetilde{u}_t(E,p) + \widetilde{a}_t^\dagger \widetilde{u}_t^*(E,p) \right)$

$\widetilde{a}_t$ ($\widetilde{a}_t^\dagger$) the annihilation (creation) operator for $\widetilde{u}_t = e^{i(tE - x_t p)}/\sqrt{4\pi x_t}$

- Hamiltonian: $\widetilde{X} = \int \frac{dt}{\sqrt{4\pi x_t}} x_t\, \widetilde{a}_t^\dagger\, \widetilde{a}_t$ with dimensions of length.

operator $\widetilde{T} = \int \frac{dt}{\sqrt{4\pi x_t}} t\, \widetilde{a}_t^\dagger\, \widetilde{a}_t$ has dimensions of time.

- Eigenstates: $|N_1, N_2, \cdots\rangle = \frac{1}{\sqrt{N_1! N_2! \cdots}} \left(\widetilde{a}_1^\dagger \right)^{N_1} \left(\widetilde{a}_2^\dagger \right)^{N_2} \cdots |0\rangle$

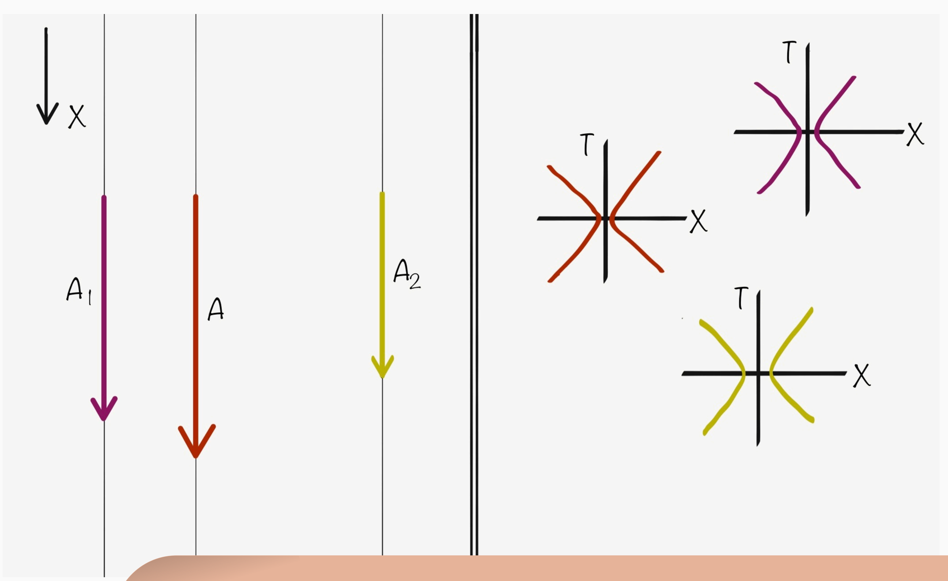
- number operator: $\widetilde{N}_t := \widetilde{a}_t^\dagger\, \widetilde{a}_t$
- for total number operator $[\widetilde{N}, \widetilde{X}] = 0$ i.e. the number of quanta is conserved.

Summary of blackbody/matter analogy

Blackbody in thermal equilibrium	Matter in quantum spacetime
Boltzman cons.: k_B	k'
Temperature: T	$T = \frac{1}{2\pi k' m}$
Photons energy: $E_{ph} = \hbar\omega$	$x_t = \hbar\widetilde{\omega}$
It takes up space, having volume	It takes up momentum space, having energy and momentum
Energy distribution: $\langle N_{ph} \rangle = \left(e^{\frac{E_{ph}}{k_B T}} - 1 \right)^{-1}$	Length distribution: $\langle N_t \rangle = \left(e^{\frac{x_t}{k'T}} - 1 \right)^{-1}$

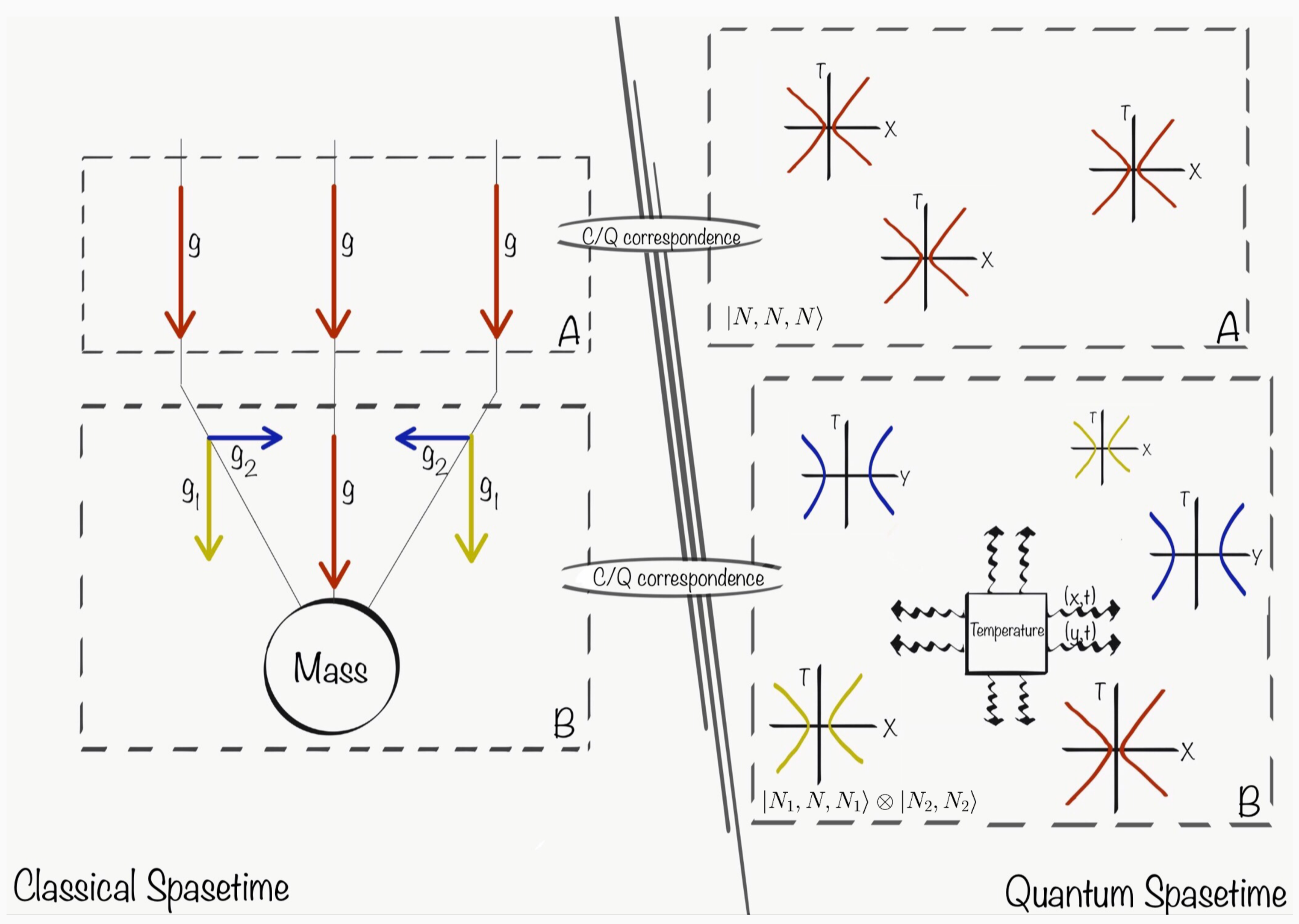
Rindler/quantum spacetime correspondence

Rindler spacetime	Quantum spacetime
$X^\mu(\tau) = (X(\tau), T(\tau))$	$X_{N_t} = x_t N_t$ $T_{N_t} = t N_t$
$X^2 - T^2 = 1/\mathsf{A}^2$	$X_{N_t}^2 - T_{N_t}^2 = N_t^2/\alpha^2$
$A^\mu = d^2 X^\mu / d\tau$	$\mathsf{A}_{N_t} = \alpha / N_t$



The classical spacetime (left) corresponds to the quantum state $|N_1, N, N_2\rangle$ (right), with $N < N_1 < N_2$.

Illustration of gravitation/blackbody radiation analogy



Mass equilibrium of black hole

- Hawking radiation combines matter quantum fields and general relativity: a black hole of mass M radiates energy at temperature $T_H(M) = 1/8\pi G k_B M$.
- In Planck scale semi-classical approach breaks down.

In Planck scale accelerated field theory can take over:

- $k' = l_P/T = k_B G$ (Newton cons. G just a conversion parameter)
- $T = 4\, T_H(M) \frac{M}{m_P}$, $[T_H(m_P) = T/4]$

Results:

- black hole cannot disappear because at Planck scale it is described as a blackbody in *mass*-equilibrium
- matter radiation is statistical in nature and contains all the information about the black hole (no information-loss problem).

Summary

We presented a new quantum description of gravity in which an equivalent resolution to blackbodies for black holes is achieved. In this approach to gravity we derived the Hawking expression for the temperature of a black hole without any reference to classical relativistic physics and showed that a black hole can actually reach mass equilibrium if spacetime comes in quanta of accelerated fields.

References

[1] L. C. Céleri and V.I.K., Phys. Lett. B **781**, 611 (2018).

[2] V.I.K., arXiv:1807.06451, (2018).

DOS of GR

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Based on *Class. Quant. Grav. 36 (2019) 155003 (arXiv: 1906.11838)*

SUMMARY

① We investigate the Density of States(DOS) of GR with $S^2 \times \mathbb{R}$ Lorentzian boundary.

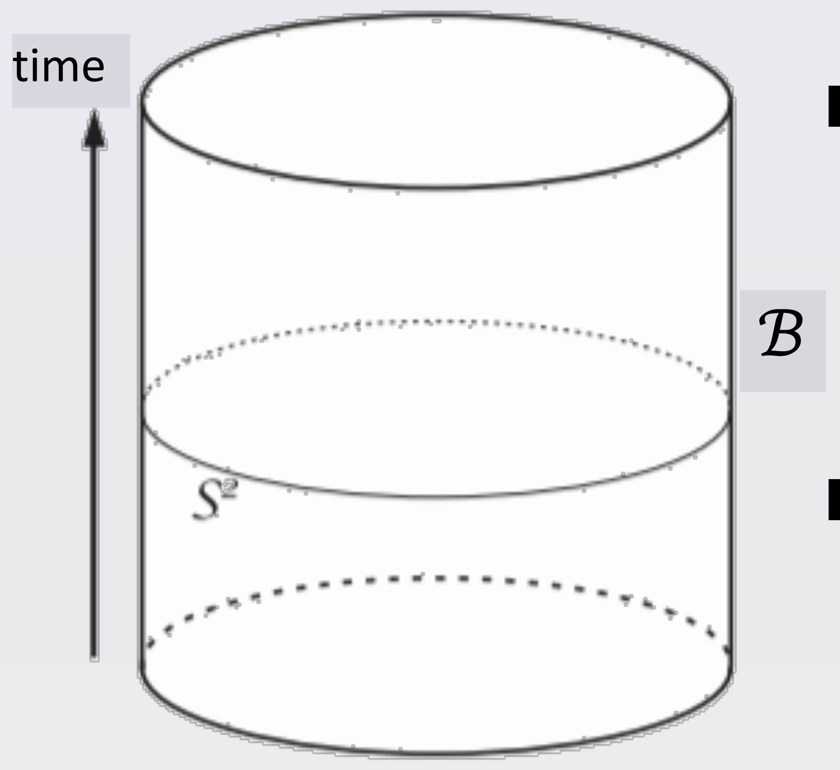
② In order to obtain the DOS, Brown-York microcanonical path integral is used.

③ There exist two convergent integration contours.

④ One of the DOSs exhibits good behavior.

① Thermodynamics and statistical mechanics of Gravity

Thermodynamics of a quantum spacetime



■Brown-York tensor

$$\tau^{ij}(y) \equiv \frac{2}{\sqrt{-\gamma}} \frac{\delta I_c[g]}{\delta \gamma_{ij}(y)}$$

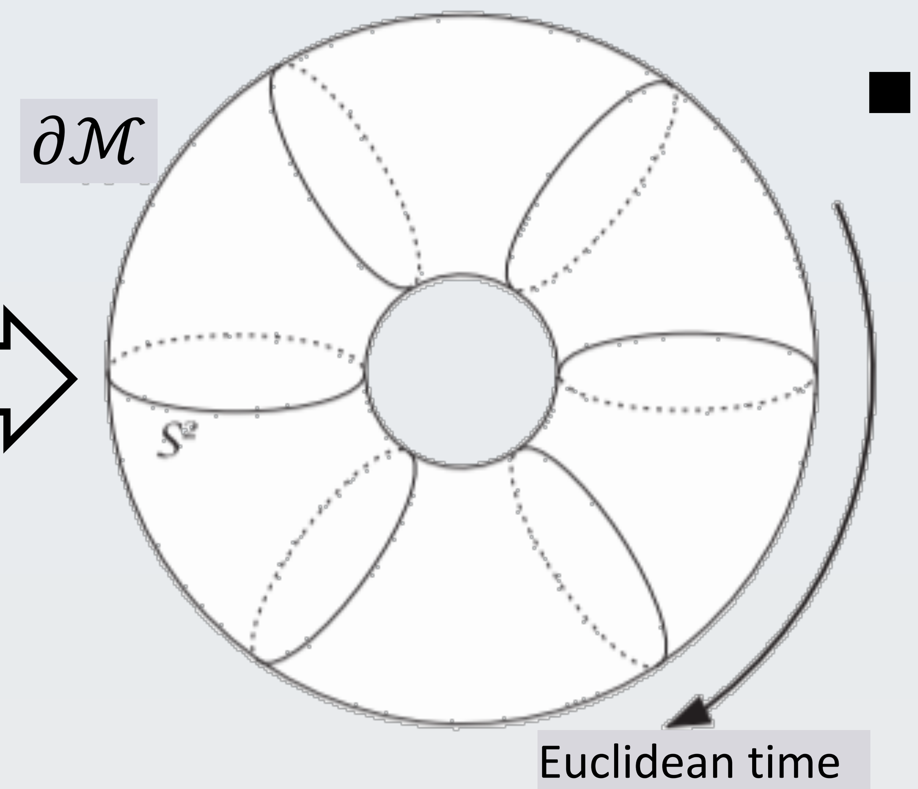
■Thermodynamical variables

E, β, P, V

are defined on $\mathcal{B} = S^2 \times \mathbb{R}$

$\Leftrightarrow$

Statistical mechanics of a quantum spacetime



■Partition function

$$Z_{\mathfrak{E}}[\mathfrak{Q}, \mathfrak{W}] = \int_{\Gamma} \mathcal{D}g \; e^{-I^E_{\mathfrak{E}}}$$

- Measure
- Action functional
  → (EH term) + (Ensemble deapendent boundary term)
- Integration Contour
  → genuinely complex

[Gibbons, Hawking, Perry, 1978]

② DOSs in the previous works and this work

Previous works

DOSs are obtained by Inverse Laplace Transformation(ILT) from various canonical partition functions

$$\boxed{Z_c = \int_\Gamma \mathcal{D}g \; e^{-I^E_c}} \xrightarrow{\text{minisuperspace \& saddle point method}} \boxed{Z^Y_c(\beta, V)} \xrightarrow{\text{ILT}} \boxed{Z^{BWY}_{mc}(E, V)}$$

[York, 1986] [Braden, Whiting, York, 1987]

$$\boxed{Z_c} \longrightarrow \boxed{Z^{LW}_c(\beta, V)} \longrightarrow \boxed{Z^{LW}_{mc}(E, V)}$$

[Louko, Whiting, 1992]

$$\boxed{Z_c} \longrightarrow \boxed{Z^{MW}_c(\beta, V)} \longrightarrow \boxed{Z^{MW}_{mc}(E, V)}$$

[Melmed, Whiting, 1994]

This work

We directly derive DOS from the microcanonical path integral

$$\boxed{Z_{mc} = \int_\Gamma \mathcal{D}g \; e^{-I^E_{mc}}} \xrightarrow{\text{minisuperspace \& saddle point method}} \boxed{Z_{mc}(E, V)}$$

[Brown, York, 1993]

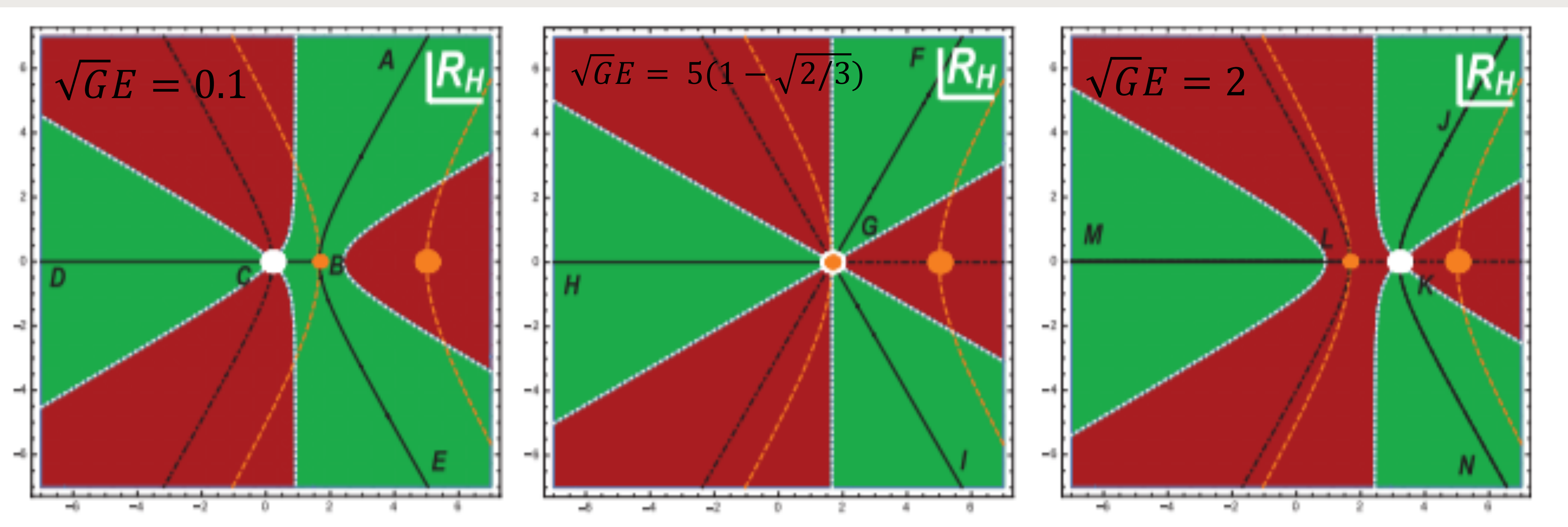
③ Lapse integral & its contour

We concentrate of $Disc \times S^2$ topology sector of the path integral

$$Z_{mc} = \int_{\Gamma} \mathcal{D}g \; e^{-I^E_{mc}} \xrightarrow{\text{Minisuperspace method}} \int_{\Gamma} dR_H \exp\left(-\frac{\pi}{G}\left[2R_b\left(1 - \frac{1}{\eta^2}\right) R_H + \left(\frac{4}{\eta^2} - 3\right) R_H^2 - \frac{2}{R_b \eta^2} R_H^3 \right] \right)$$

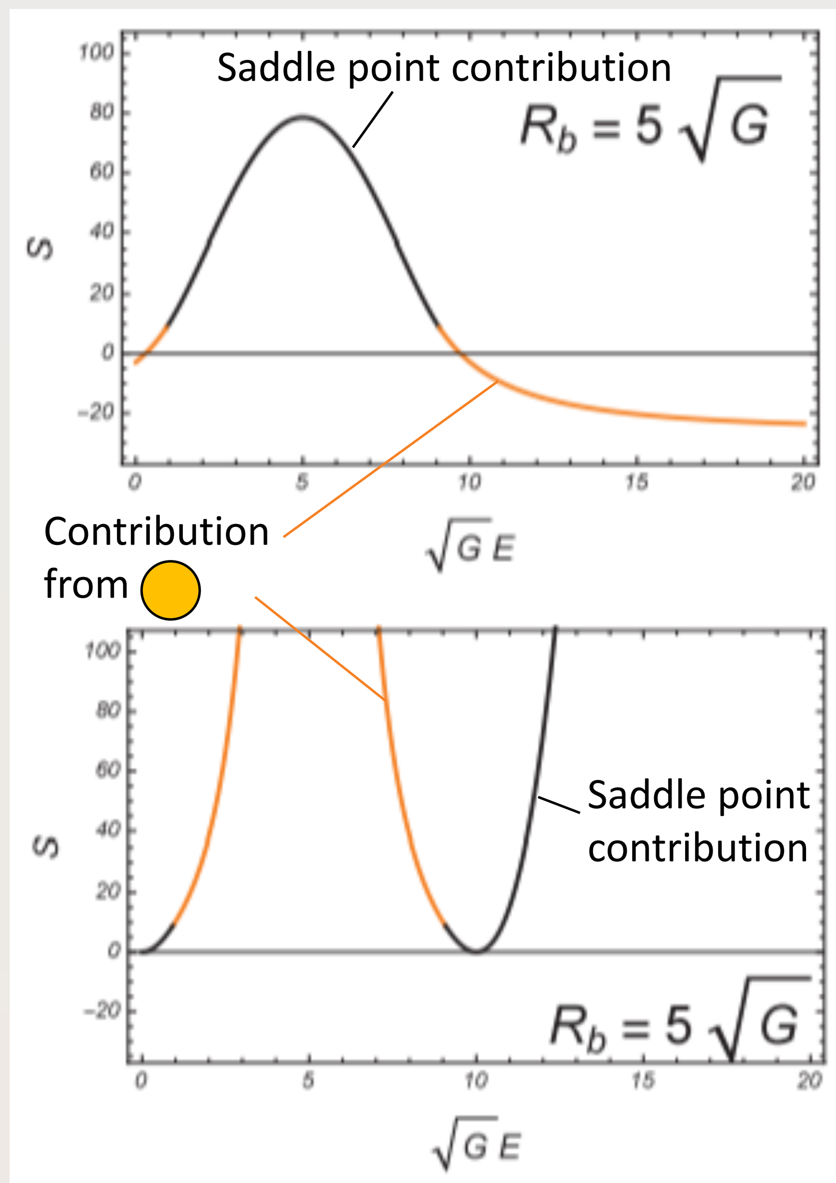
$$I^E_{mc} = \frac{-1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{g} R + \frac{-1}{8\pi G} \int_{\partial\mathcal{M}} d^3y \sqrt{\gamma}\, \tau_\mu \Theta^{\mu\nu} \partial_\nu \tau \qquad \eta \equiv \left(\frac{GE}{R_b} - 1\right)$$

The location of the saddle point, and its steepest descent (ascent) contour



○ : the saddle point — : steepest descent / - · - : steepest ascent contour

④ Behavior of entropy



Questions

- ■ For high energy, the saddle point no longer dominates→ Contribution from the other sectors become important ?
- ■ Relation to ill-definedness of Halliwell-Louko canonical partition function

Space-time foliations, doubled aspects of Vaisman algebroid and gauge symmetry in double field theory

Haruka Mori (Kitasato U.)
based on **arXiv:1901.04777[hep-th][math-DG]** with **Shin Sasaki** and **Kenta Shiozawa** (Kitasato U.)

1. Motivation

Double Field Theory (DFT): [Hull-Zwiebach '09]

- a gravity theory with manifest T-duality
- space-time coordinates are "doubled" $X^M = (x^\mu, \tilde{x}_\mu)$
  - DFT geometry is the doubled geometry
  - x^μ is Fourier dual of KK-mode, $\tilde{x}_\mu$ is Fourier dual of winding mode
- DFT has the physically condition called "strong constraint"
  - this is to drop the $\tilde{x}_\mu$ coordinate
  - a way to solve the strong constraint trivially is to make $\partial_* = 0$
  - DFT with the strong constraint is reduced to a supergravity

How to interpreted the "strong constraint" in mathematically?

Our results

- the Vaisman algebroid is composed of two Lie algebroids
- the DFT geometry is an example in which the Vaisman algebroid appears
- "strong constraint" in DFT is interpreted mathematically

2. Gauge symmetry in DFT

- it is described by C-bracket which defined on the doubled space

$$[\Xi_1, \Xi_2]_C = [X_1, X_2]_L + \mathcal{L}_{\xi_1} X_2 - \mathcal{L}_{\xi_2} X_1 - \frac{1}{2}\tilde{\mathrm{d}}\left(\iota_{X_2}\xi_1 - \iota_{X_1}\xi_2\right)$$
$$+ [\xi_1, \xi_2]_{\tilde{L}} + \mathcal{L}_{X_1}\xi_2 - \mathcal{L}_{X_2}\xi_1 - \frac{1}{2}\mathrm{d}(\iota_{\xi_2}X_1 - \iota_{\xi_1}X_2)$$

- C-bracket defines the structure called the Vaisman (metric) algebroid [Vaisman '13]
- C-bracket with "strong constraint" becomes a exact Courant bracket which defines Courant algebroid on non-doubled space-time
- we show the Vaisman algebroid is obtained by "double of Lie algebroids $L, \tilde{L}$"
- this is the generalization of the "Drinfel'd double of Lie bialgebroid".
- the key is the "derivation condition" $\mathrm{d}_*[X,Y]_S = [\mathrm{d}_* X, Y]_S + [X \mathrm{d}_* Y]_S$
  - $X, Y \in \Gamma(L)$, d_* is exterior derivative, $[\cdot,\cdot]_S$ is Schouten bracket
- this condition needs to define a Lie bialgebroid
- we find the Vaisman algebroid doesn't request the derivation condition

[Liu-Weinstein-Xu '96] [Mori-Sasaki-Shiozawa '19]

Courant algebroid		Vaisman algebroid
$\mathcal{C} = L \oplus L^*$		$\mathcal{V} = L \oplus \tilde{L}$
↑ Drinfel'd double	← + derivation condition	↑ double
Lie bialgebroid (L, L^*)		Lie algebroids $L, \tilde{L}$

3. Doubled space-time geometry

Doubled space-time: described by para-Hermitian manifold [Vaisman '13]

Para-Hermitian manifold $(\mathcal{M}, K, \eta)$:

- differentiable manifold $\mathcal{M}$
- K: almost para-complex structure $K^2 = 1$, $K \in \mathrm{End}(T\mathcal{M})$
- η: neutral metric $T\mathcal{M} \times T\mathcal{M} \to \mathbb{R}$
  - compatibility condition: $\eta(K(X), K(Y)) = -\eta(X, Y)$
- integrability condition: $N_K(X,Y) = 0$
  - N_K is the Nijenhuis tensor

$$N_K(X,Y) = \frac{1}{4}\Big\{[K(X), K(Y)] + [X,Y] - K\big([K(X), Y] + [X, K(Y)]\big)\Big\}.$$

Using the almost para-complex structure K,
we obtain the decomposition $T\mathcal{M} = L \oplus \tilde{L}$

- which is performed via the projection operators

$$P = \frac{1}{2}(1 + K), \qquad \tilde{P} = \frac{1}{2}(1 - K)$$

- L ($\tilde{L}$) is the eigenbundle associated with the eigenvalue $K = +1$ ($K = -1$)
- Nijenhuis tensor is decomposed by $P, \tilde{P}$

$$N_K(X,Y) = N_P(X,Y) + N_{\tilde{P}}(X,Y)$$
$$N_P(X,Y) = \tilde{P}[P(X), P(Y)], \qquad N_{\tilde{P}}(X,Y) = P[\tilde{P}(X), \tilde{P}(Y)]$$

- $L, \tilde{L}$ are distributions of $T\mathcal{M}$
  - a distribution is maximally isotropic $\eta(X,Y) = 0$ and rank $L = 1/2$ dim $\mathcal{M}$

4. Foliation structure

Integrability of distributions:

- it is represented by the Frobenius theorem.

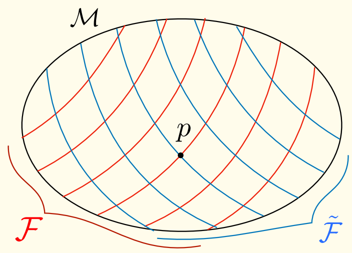
  a distribution L ($\tilde{L}$) is completely integrable iff L ($\tilde{L}$) is involutive (Their Lie bracket $[X,Y]_L$ belongs to L, then the distribution L is called involutive)
- if N_P ($N_{\tilde{P}}$) vanishes, the distribution L ($\tilde{L}$) is involutive
- the integrability of $L, \tilde{L}$ is independent of each other

Foliation structure:

- Frobenius theorem (alternative rep.)

  a subbundle $E \subset T\mathcal{M}$ is integrable iff it is defined by a regular foliation of $\mathcal{M}$

  then, L and $\tilde{L}$ are integrable, they have foliation structures $L = T\mathcal{F}, \tilde{L} = T\tilde{\mathcal{F}}$
- the $\mathcal{F}$ is given by the union of leaves
- a leaf M_p is a subspace of $\mathcal{F}$ that path through a point p
- for $\mathcal{F}$, the local coodinate x^μ is given along a leaf M_p while the one for the transverse directions to leaves is $\tilde{x}$
  - this means that $\tilde{x}$ is a constant on a leaf M_p in $\mathcal{F}$
  - a physical space-time is identified as the leaf



5. Para-Dolbeault operator in $L, \tilde{L}$

We define natural exterior algebra on tangent bundle over $\mathcal{M}$:

- a set of doubled multi-vectors $\hat{\mathcal{A}} = \Gamma(\wedge^k T\mathcal{M})$
- we define $\mathcal{A}^{r,s}(\mathcal{M})$ as the section of $(\wedge^r L) \wedge (\wedge^s \tilde{L})$
  - we obtain the decomposition $\hat{\mathcal{A}}^k(\mathcal{M}) = \bigoplus_{k=r+s} \mathcal{A}^{r,s}(\mathcal{M})$
- the canonical projection operator $\pi^{r,s} : \hat{\mathcal{A}}^{r+s}(\mathcal{M}) \to \mathcal{A}^{r,s}(\mathcal{M})$ induced by $P, \tilde{P}$
- the exterior derivatives acting on L and $\tilde{L}$

$$\tilde{\mathrm{d}} : \mathcal{A}^{r,s}(\mathcal{M}) \to \mathcal{A}^{r+1,s}(\mathcal{M}) \qquad \mathrm{d} : \mathcal{A}^{r,s}(\mathcal{M}) \to \mathcal{A}^{r,s+1}(\mathcal{M})$$

- $\tilde{\mathrm{d}}$ and d have the following properties: $\mathrm{d}^2 = 0, \quad \tilde{\mathrm{d}}^2 = 0, \quad \mathrm{d}\tilde{\mathrm{d}} + \tilde{\mathrm{d}}\mathrm{d} = 0$
- we also define interior products and Lie derivatives

6. Algebroid structure in DFT

- we focus on the DFT on the flat para-Hermitian manifold
- the tangent bundle $T\mathcal{M}$ is spanned by $\partial_M (M = 1, \ldots, 2D)$
- vector fields on $T\mathcal{M}$ are decomposed by projector $P, \tilde{P}$

$$\Xi^M \partial_M = A^\mu(x, \tilde{x})\partial_\mu + \alpha_\mu(x,\tilde{x})\tilde{\partial}^\mu \qquad A \in \Gamma(L), \alpha \in \Gamma(\tilde{L})$$

- we explicitly showed that the exterior algebras of DFT are incompatible with the derivation condition

In DFT realization,

- we find that $\tilde{d}$ on L and Schouten bracket $[\cdot,\cdot]^*_S$ on $\tilde{L}$ is compatible
- the same disscution holds for the operator d on $\tilde{L}$
- we have defined the Lie algebroid and its dual Lie algrboid in DFT

then, we examine the derivation condition in DFT by explicit calculation, we obtain

$$\tilde{\mathrm{d}}[A,B]_S = [\tilde{\mathrm{d}}A, B]_S + [A, \tilde{\mathrm{d}}B]_S + (\tilde{\partial}^\rho A^\mu \partial_\rho B^\nu + \tilde{\partial}^\rho B^\nu \partial_\rho A^\mu)\partial_\mu \wedge \partial_\nu$$

- the last term is the violation of derivation condition
- the last term vanished when the "strong constraint" is imposed
- this is a part of the algebraic origin of the "strong constraint"

7. Conclusion

- we show that the Vaisman algebroid composed of two Lie algebroids
- we discuss algebraic origin of "strong constraint" in DFT, this is closely related to derivation condition

Future direction

- the structure of the global symmetry in DFT
  - the gloval symmetry is like "Finite transformation"
  - we need exponential map of Vaisman algebroid, which is related to "*coquecigrue problems*"
  - we expect that we obtain some groupoid structure from Vaisman algebroid

NON-GAUSSIANITY IN EARLY UNIVERSE PARTICLE PRODUCTION

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Abstract

Correspondence between the conduction phenomena in electrical wires with impurity and the scattering events responsible for particle production during stochastic inflation and reheating implemented under a closed quantum mechanical system in early universe cosmology.Derivating the quantum corrected version of the Fokker–Planck equation without dissipation for the probability distribution profile can be used to study the dynamical features of the particle creation event.With difussion effect Ito, Stratonovich prescription and the explicit role of finite temperature effective potential for probability distribution profile gives quantum nature. With random polynomial interaction potential we formulate this in RMT background and found the bound on Spectral form factor for such a system.

Introduction

- Quantum fields in an inflationary background or during reheating gives rise to the burst of particle production, which has been extensively studied in Primordial Cosmology.
- Such particle production events are completely random (or chaotic) when the evolution is non-adiabatic in nature.
- A non-adiabatic change in the time dependent coupling of the fields as the background evolution of the fields passes through special points in field space producing these burst of particles.
- There lies a one-to-one correspondence between such cosmological events to that of the stochastic random phenomena occurring in mesoscopic systems where fluctuations in physical quantities play a significant role of producing stochastic randomness.

Non-Gaussianity

- For a electron in wire with impurity(we choose this impurity to be dirac-delta scatrrer) the dynamical evolution can be represented as Fokker-Planck equation. (Fokker planck equation for this case can be derived from Schrodinger equation)
- This particle production event is same as electron getting scattered by dirac delta imputiry.Solving the transfer matrix we get total occupation number and lyapunov exponents for large number of impurities. This establish the randomness in such a phenomena(→ Anderson Localization length is directly related in this matter.) So this establish that particle production event can be considered to be random Brownian motion type.
- The probability density for particle position of Brownian motion in a random system can be expressed in terms of Smoluchowski equation.

$$P(M;\tau+\partial\tau) = \int_{-\infty}^{\infty} P(M_1.\tau)P(M_2,\partial\tau)dM_2 = \langle P(M_1,\tau) \tag{4}$$

For Markovian process, Smoluchowski equation describes a two point conditional probability distribution.And time evolution of the probability density function can be expressed as:-

$$\partial_\tau P(M,\tau) = \frac{\langle \delta M\rangle_{M_2}}{\delta\tau}\partial_M P(M,\tau) + \frac{\langle \delta M\delta M\rangle_{M_2}}{\delta\tau}\partial_M P(M,\tau) + \dots \tag{5}$$

- conditional probability of getting Y at time t $+\tau$ in terms of probability of getting nearby to Y -ξ at time t and then to Y in time τ

$$P_2(Y_0|Y,t+\tau) = \int_{-\infty}^{\infty} d\xi P_2(Y_0|Y-\xi,t)P_2(Y-\xi|Y,\tau) \tag{6}$$

Taylor expansion around $\tau = 0$

$$\frac{\partial P_2(Y_0|Y,t)}{\partial t}\tau = -\int_{-\infty}^{\infty} P_2(Y_0|Y,t)P_2(Y|Y-\xi,\tau)d\xi + \int_{-\infty}^{\infty} P_2(Y_0|Y-\xi,t)P_2(Y-\xi|Y,\tau)d\xi$$

- Now we choose maximum entropy ansatz for shannon entropy and get the constraints:- normalization, fix local-mean particle production rate, and infinitesimal interval doesn't corresponds to a finite significant change in transfer matrix This reduces the ϕ dependence of probability density which comes from tranfer matrix approach. Then we taylor expand on each side with one side w.r.t δn and other side w.r.t $\delta\tau$

$$\langle P(n+\delta n;\tau)\rangle_{\delta\tau} = \langle P(n;\tau)\rangle_{\delta\tau} + \sum_{q=1}^{\infty}\frac{1}{q!}\frac{\delta^q P(n;\tau)}{\delta n^q} \times \langle(\delta n)^q\rangle_{\delta\tau}$$

$$P(n;\tau+\delta\tau) = P(n;\tau) + \sum_{q=1}^{\infty}\frac{1}{q!}\frac{\partial^q P(n;\tau)}{\delta\tau^q}(\delta\tau)^q$$

References

- S. Choudhury, A. Mukherjee, P. Chauhan and S. Bhattacherjee,"Quantum Out-of-Equilibrium Cosmology,",Eur. Phys. J. C **79** (2019) no.4, 320
- S.Choudhury , A. Mukherjee A bound on quantum chaos from RMT with GUE J. High Energ. Phys. (2019) 2019: 149.
- S.Choudhury, A. Mukherjee Quantum randomness in the Sky Eur. Phys. J. C (2019) 79: 554

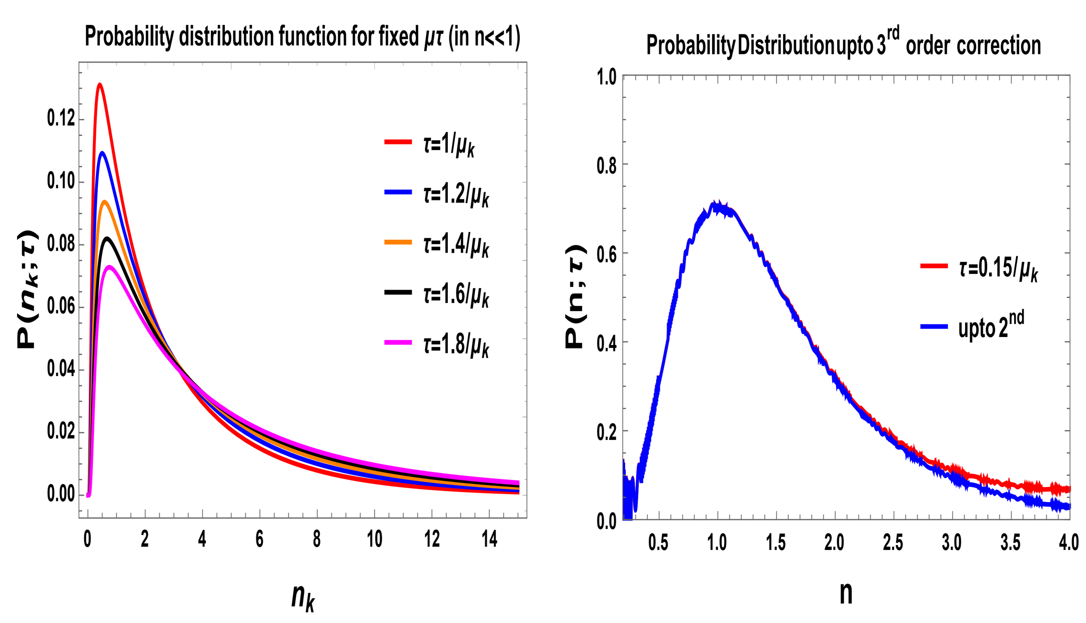
Questions

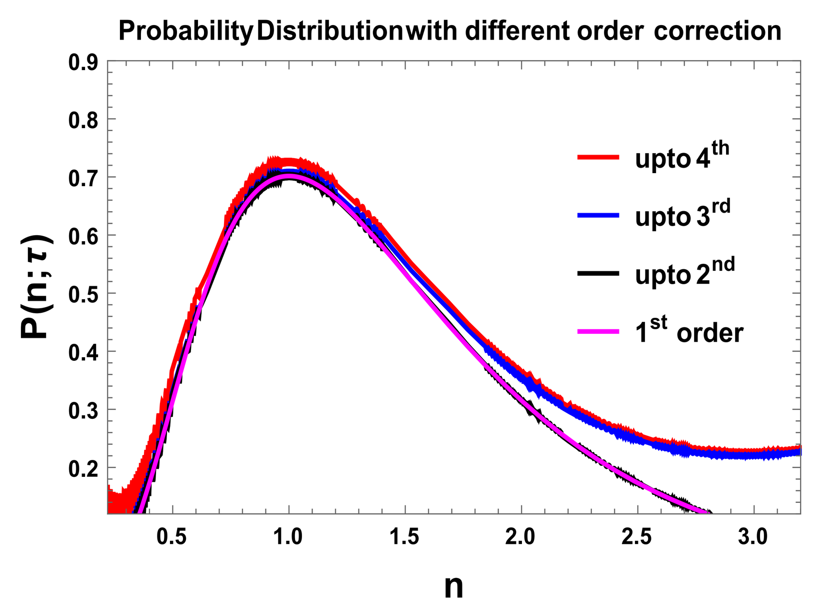
- How exactly conduction wire– cosmology correspondence can be built?
- How can we show Non-Gaussianity in particle production?
- If we don't know anything about the effective interactions (time dependent couplings in QFT) then how one can quantify randomness?

First order Fokker Planck

$$\frac{1}{\mu_k}\frac{\partial P(n;\tau)}{\partial\tau} = (1+2n)\frac{\partial P(n;\tau)}{\partial n} + n(1+n)\frac{\partial^2 P(n;\tau)}{\partial n^2} \tag{9}$$

Corrected FokkerPlanck





Itô-Stratnovich

In presence of diffusion Fokker planck equation come from Langevian equation with Einstein diffusion coefficient[D(n)] and zero gaussian white noise[$b(\tau)$].

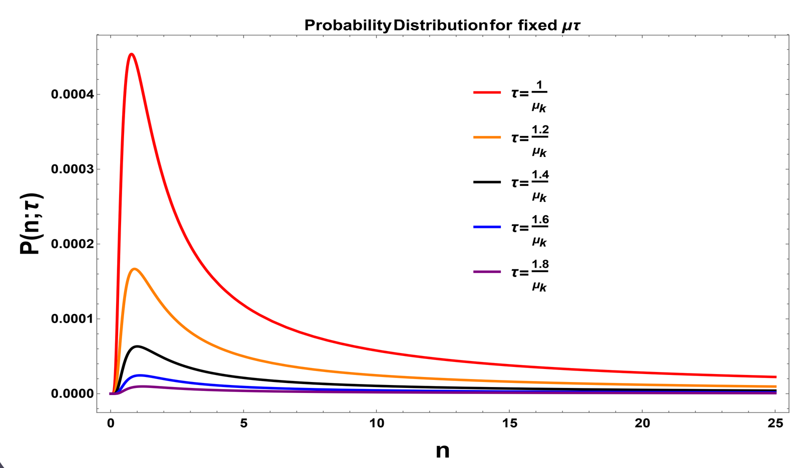
$$\frac{dn(\tau)}{d\tau} = a(n) + \sqrt{D(n_0)}b(\tau) \rightarrow Langevian\ equation$$

$ITÔ$

$$\frac{\partial P(n;\tau)}{\partial\tau} = -\frac{\partial}{\partial n}(a(n)P(n;\tau)) + \frac{\partial^2}{\partial n^2}(D(n)P(n;\tau)) \tag{10}$$

$Stratnovich$

$$\frac{\partial P(n;\tau)}{\partial\tau} = -\frac{\partial[a(n)P(n;\tau)]}{\partial n} + \frac{\partial[\sqrt{D(n)}\frac{\partial[\sqrt{D(n)}P(n;\tau)]}{\partial n}]}{\partial n}$$



SFF bound

$$\mathbf{S}_2(t) = \frac{1}{|Z(\beta)|^2}\sum_{m,n,m\neq n} e^{-\beta(E_m+E_n)}e^{-it(E_m-E_n)} = \frac{|Z(\beta+it)|^2}{|Z(\beta)|^2} \tag{12}$$

SFF can be written as summation of two point green's function (connected and disconnected part) and for large time the disconnected part having hypergeomtric PFQ regularized function goes to zero.This gives us the bound on spectral form factor

Wires to Cosmology

- Time independent one dimensional Schrödinger equation appearing in the context of quantum mechanical system which describes the space evolution of electron inside a wire in presence of impurity

$$[\frac{d^2}{dx^2} + E - V(x)]\psi(x) = 0 \tag{1}$$

- Dynamics of this fluctuating scalar field in FLRW cosmological background with a time-dependent coupling. Consider the action for massive scaler field with time-dependent mass profile-

$$S = -\frac{1}{2}\int d^4x\sqrt{-g}(g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - m^2(\tau)\chi^2) \tag{2}$$

$\rightarrow Constraint\ :\ -\chi(-k,\tau) = \chi^*(k,\tau)$,After fourier transform and considering (quassi)de sitter background with scale factor $a(\tau)$.
redefintion of fields as $\phi_k(\tau) = a(\tau)\chi_k(\tau)$

- Reheating region this scale factor is constant, gives us the klein-gordon equation describing patricle production. equation

$$[\frac{d^2}{d\tau^2} + (k^2 + m^2(\tau)]\phi(\tau) = 0 \tag{3}$$

- This two equation has very similar sturucture and so the one-one correspondence.

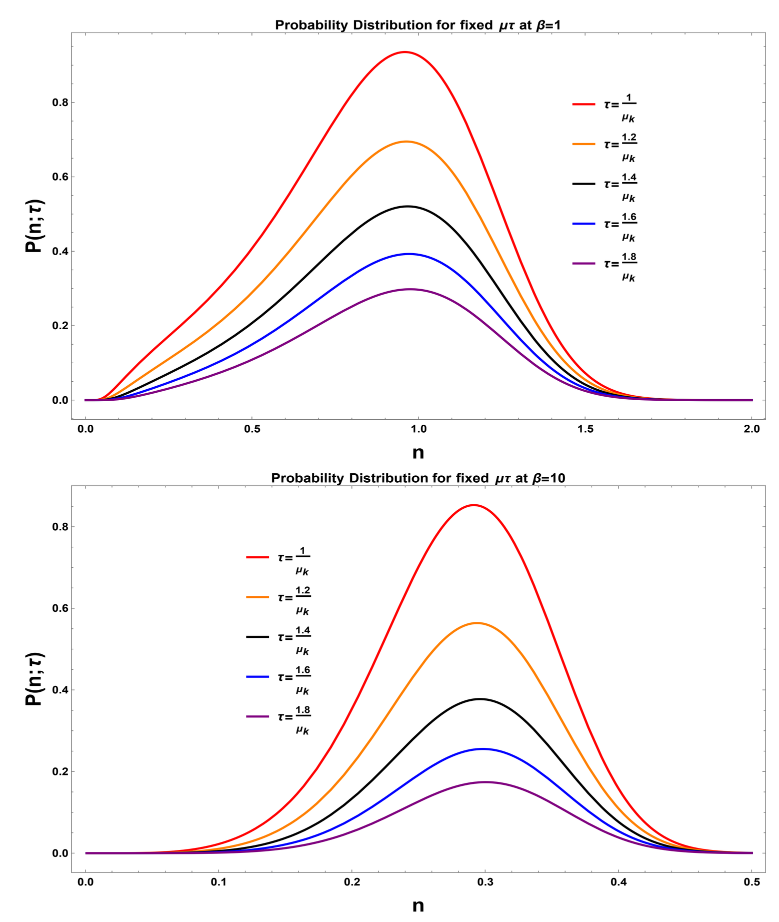
Gen. FP at equilibrium,$\beta \neq 0$

For equilibrium Fokker Planck current(J(n;τ)) is zero and at finite temperature

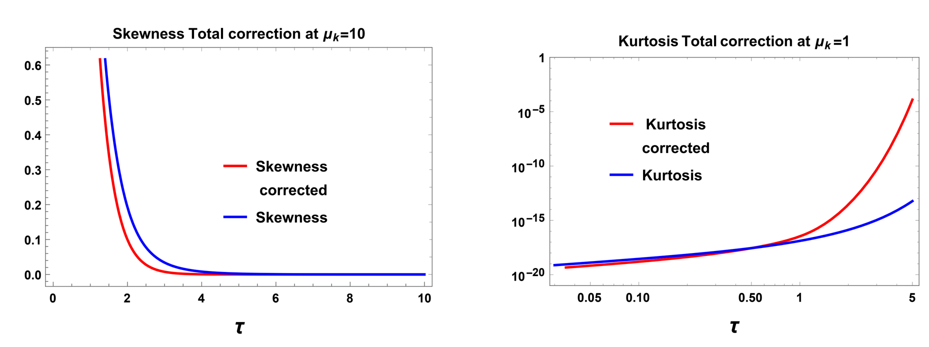
$$\frac{\partial}{\partial n}\left(n(n+1)\frac{\partial W(n;\tau)}{\partial n}\right) - U(n)W(n;\tau) = \frac{\partial W(n;\tau)}{\partial\tau}$$

$Effective\ potential$ (11)

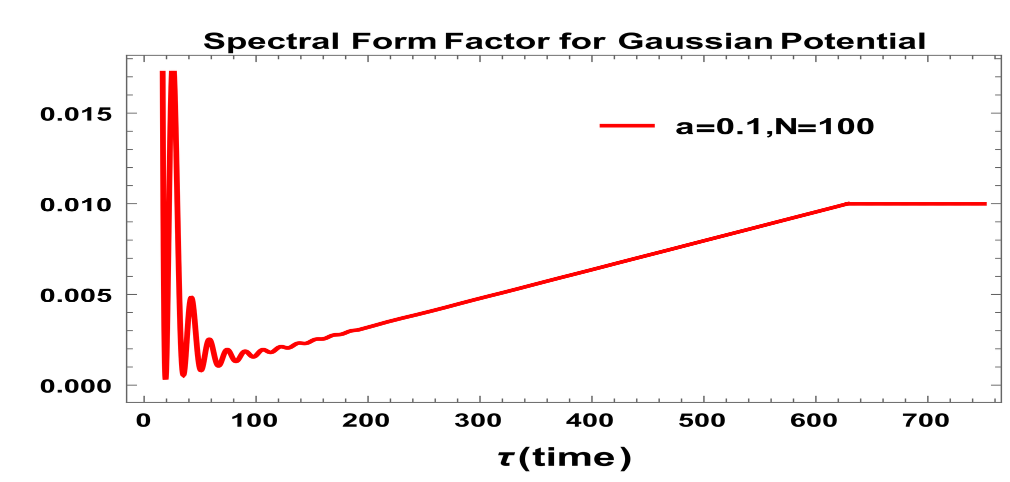
$$U(n) = [\frac{\beta^2}{4}n(n+1)(\frac{\partial V(n)}{\partial n})^2 - \frac{\beta}{2}n(n+1)(\frac{\partial^2 V(n)}{\partial n^2}) - \frac{\beta}{2}(\frac{\partial(n(n+1))}{\partial n})(\frac{\partial V(n)}{\partial n})]$$



Kurtosis,Skewness



SFF



Collisional Penrose process on ISCO

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Kei-ichi Maeda (Waseda Uni.)

Based on arXiv: 1907.07126

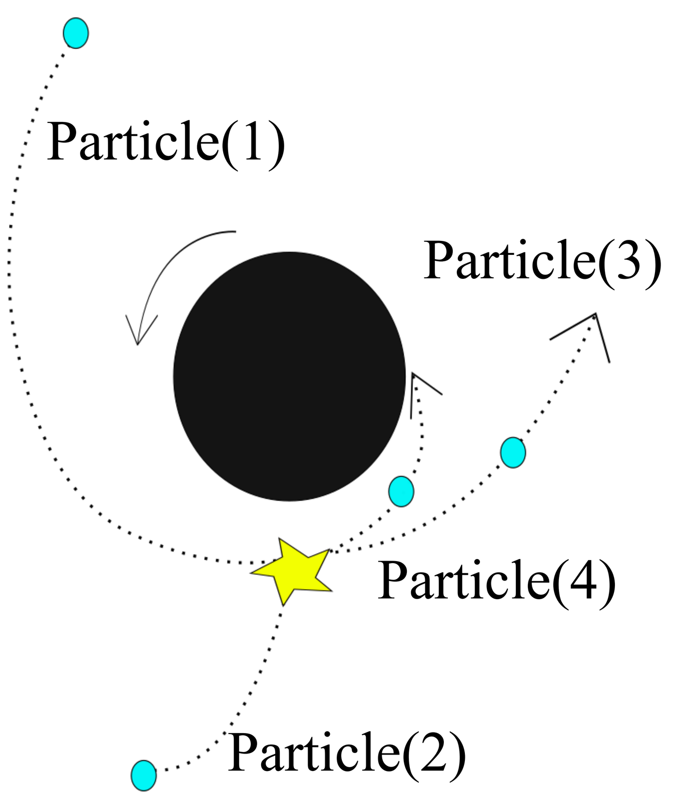
Introduction

In 2009, it is pointed out that the center of mass energy can take arbitrary value when two particles (1),(2) collide near the horizon of an extreme Kerr BH. (Banados, Silk and West '09)

$$E_{cm} \equiv -(P_1^\mu + P_2^\mu)(P_{1\mu} + P_{2\mu}) \to \infty$$

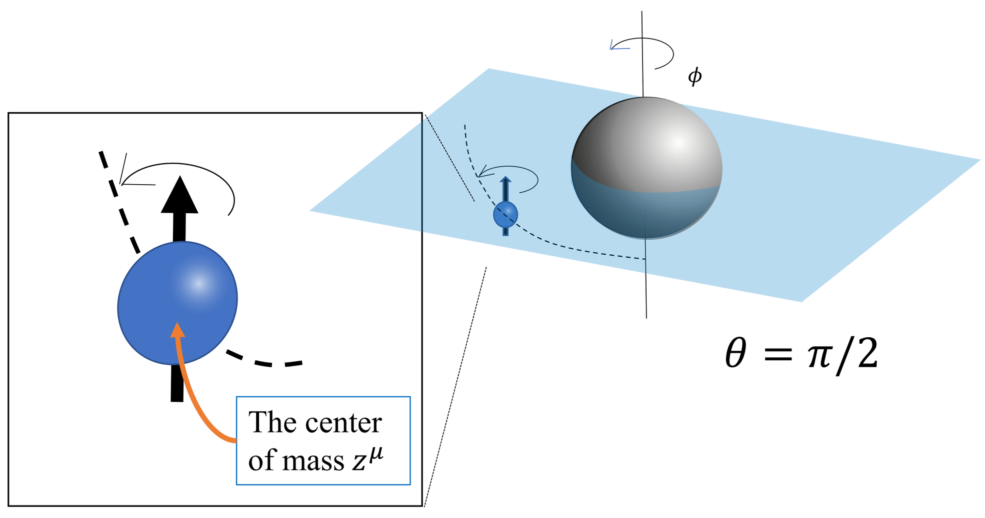
(at the horizon of an extreme BH)

- Due to the red shift, it is not trivial that the energy at infinity (of a resulting particle (3)) also diverges.
- In previous works, the collision between particles impinging from infinity is considered.



The collision between a particle on its ISCO and a particle impinging from infinity

Spinning particle



- E.O.M

$$\frac{Dp^\mu}{d\tau} = -\frac{1}{2}R^\mu{}_{\nu\rho\sigma}v^\nu S^{\rho\sigma}$$

$$\frac{DS^{\mu\nu}}{d\tau} = p^\mu v^\nu - p^\nu v^\mu \qquad S^{\mu\nu}p_\nu = 0$$

- Conserved quantity by Killing vector

$$Q_\xi = p^\mu \xi_\mu + \frac{1}{2}S^{\mu\nu}\nabla_\mu \xi_\nu$$

- Conserved quantity

$$S^{\mu\nu}S_{\mu\nu} = 2\mu^2 s^2$$

$$-p^\mu p_\mu = \mu^2$$

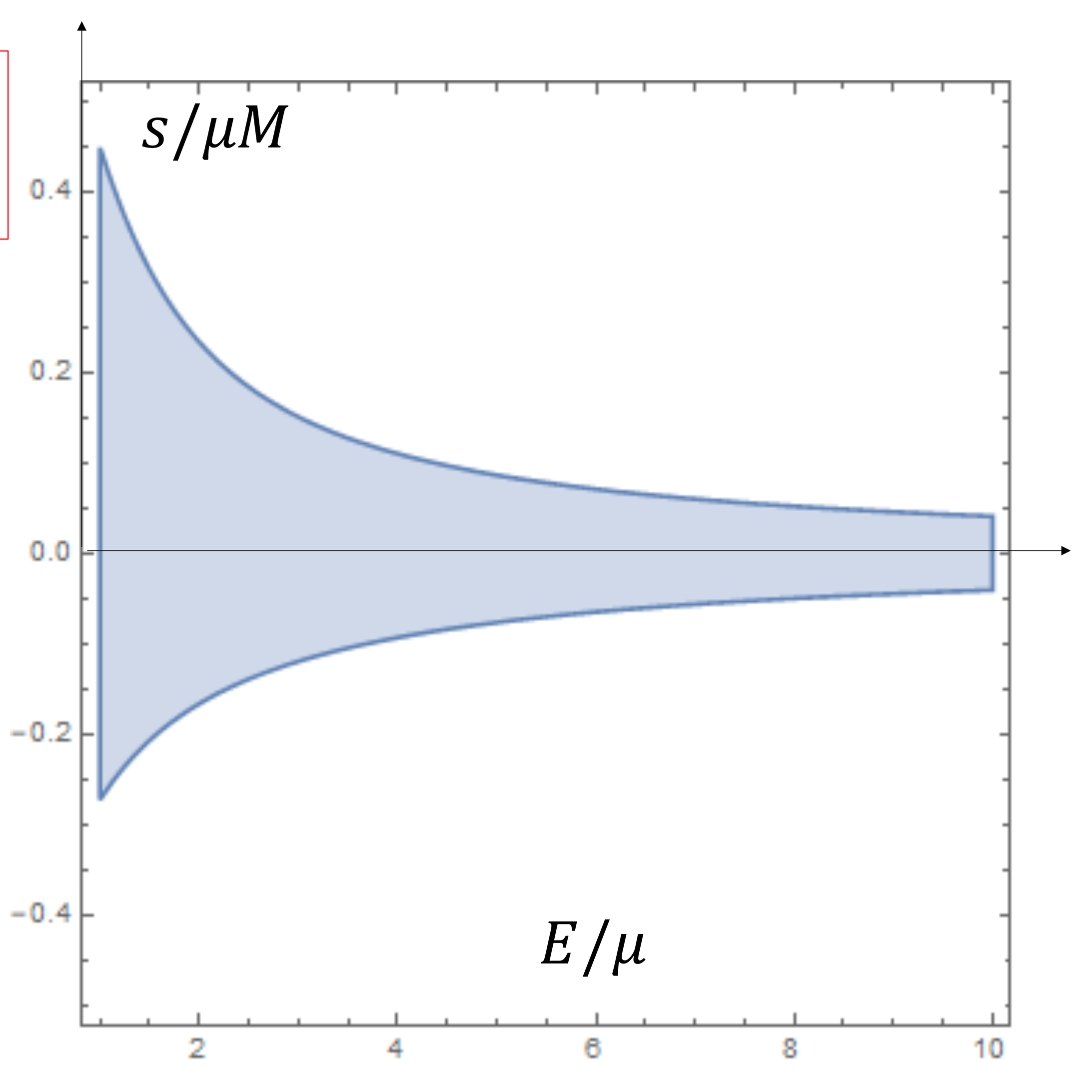
- Energy $E \equiv -Q_{\xi_t}$ and Total angular momentum $J \equiv Q_{\xi_\phi}$
- Assuming spinning particles move on the equatorial plane, momentums, 4 velocity and spin can be described by using E and J
- Note that the momentum and the 4-velocity is not parallel.

$$p^\mu = \mu v^\mu + v_\nu \frac{DS^{\mu\nu}}{d\tau}$$

$$\mu^2 = -p^\mu p_\mu$$

The signature $v^\mu v_\mu$ is non trivial.

$$v^\mu = \frac{dz^\mu}{d\tau}$$



The relation between E and s for a particle to reach the horizon with satisfying $v^\mu v_\mu < 0$.

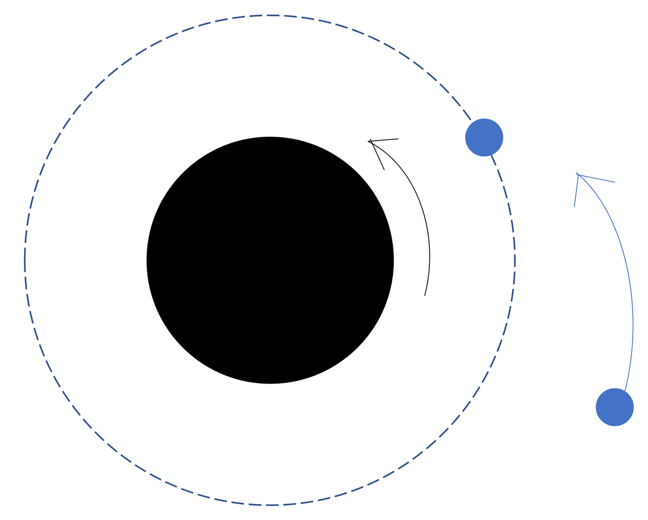
Innermost stable circular orbit

1. Circular Orbit → $(p_1)^2 = 0,\ \frac{d}{dr}(p_1)^2 = 0$

   → E and J are determined.

2. Innermost Stable circular orbit → $\frac{d^2}{dr^2}(p_1)^2 = 0$

   → the radius of the circular orbit is determined



Collisional Penrose process

- Conservation laws

$$E_1 + E_2 = E_3 + E_4 \qquad P_1^r + P_2^r = P_3^r + P_4^r$$

$$J_1 + J_2 = J_3 + J_4 \qquad s_1 + s_2 = s_3 + s_4$$

$$J_1 = 2E_1 M \ (\text{ISCO}),\quad J_2 = 2E_2 M(1+\zeta)\ (\zeta < 0)$$

- The collision near the horizon: $r_c = \frac{M}{1-\epsilon}$

$$J_3 = 2E_3 M(1 + \alpha_3\epsilon + \beta_3\epsilon^2) \qquad s_3 = s_1, s_4 = s_2$$

1. Expand the radial momentum about ϵ and check the conservation law order by order.
2. Solve in terms of E_2 and E_3
3. Analyze the energy efficiency: $\eta = \frac{E_3}{E_1+E_2}$

Conclusion and discussion

Collisional process	spin(s_1, s_2)	Input energy (E_1, E_2)	Output energy(E_3)	Maximal energy efficiency
Elastic collision	Non-spinning $(0.0319\mu M, -0.270\mu M)$	$(0.5773\mu, \mu)$ $(0.5591\mu, \mu)$	4.041μ 13.16μ	2.562 8.442
Inverse Compton scattering	Non-spinning $(0.0636\mu M, 0)$	$(0.5773\mu, 0)$ $(0.5416\mu, \mu)$	4.041μ 6.791μ	7 12.54

- The maximal efficiency in both case is increasing when the spin is taken into account.
- In the elastic collision, the efficiency becomes three times larger than the non-spining case.
- The spin plays an important role in the collision with the ISCO particle.

- ☐ The case of non-extreme BH
- ☐ Quantum signatures through a collision around a black hole.

Worldsheet Instanton Corrections to Five-branes and Waves in Double Field Theory

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Based on **JHEP 07(2018)001** [arXiv:**1803.11087**] and **JHEP 12(2018)095** [arXiv:**1810.02169**]

1. Introduction

- Double Field Theory (DFT) is a formalism where the T-duality group is realized as a manifest symmetry. [Hull-Zwiebach '09]
- DFT monopole solution has KK-monopole localized in winding (T-dualized) space. [Berman-Rudolph '14]
- Instanton corrections break the isometry in winding space. [Harvey-Jensen '05]

Our Results

- Obtained various five-branes with winding dependence from DFT monopole
- Interpretation of winding dependence as instanton corrections to 5^2_2-branes

2. Five-branes in DFT

DFT is defined in the doubled space $X^M = (x^\mu, \tilde{x}_\mu)$.

- Note that the coordinates $\tilde{x}_\mu$ simply stand for the doubled pairs but not the winding coordinates. In the following, we will assign the roles of geometrical and winding coordinates to each coordinate.

The ansatz for the localized DFT monopole (codim 4) [Berman-Rudolph '14]

$$\begin{aligned} ds^2_{\text{DFT}} &= H(\delta_{ab} - H^{-2} b_{ac} b^c{}_b) dy^a dy^b + H^{-1} \delta^{ab} d\bar{y}_a d\bar{y}_b + 2H^{-1} b_a{}^b dy^a d\bar{y}_b \\ &\quad + \eta_{mn} dx^m dx^n + \eta^{mn} d\tilde{x}_m d\tilde{x}_n, \\ d &= \text{const.} - \tfrac{1}{2} \log H \qquad\qquad (m,n = 0,\ldots,5), (a,b = 6,\ldots,9) \end{aligned}$$

In order for the DFT EoMs to vanish, we impose the condition

$$3\partial_{[a} b_{bc]} = \varepsilon_{abcd}\, \partial^d H(y).$$

By assigning geometrical coordinates to a half of X^M and comparing the codim 4 ansatz with the parametrization of the DFT "line element"

$$ds^2_{\text{DFT}} = (g_{\mu\nu} - B_{\mu\rho} g^{\rho\sigma} B_{\sigma\nu}) dx^\mu dx^\nu + 2(B_{\mu\rho} g^{\rho\nu}) dx^\mu d\tilde{x}_\nu + g^{\mu\nu} d\tilde{x}_\mu d\tilde{x}_\nu,$$

we can write down explicit solutions for conventional supergravity fields.

- NS5-brane: Choosing the geom. coord. $x^\mu = (x^m, y^i, y^9)$ $(i = 6,7,8)$

$$\begin{aligned} ds^2 &= \eta_{mn} dx^m dx^n + H(\delta_{ij} dy^i dy^j + (dy^9)^2), \\ B &= (b_{ij} dy^i - b_{j9} dy^9) \wedge dy^j, \qquad e^{2\phi} = H. \end{aligned}$$

- KK-monopole (with w^1): Choosing $x^\mu = (x^m, y^i, \bar{y}_9)$

$$\begin{aligned} ds^2 &= \eta_{mn} dx^m dx^n + H^{-1}(d\bar{y}_9 + b_{i9} dy^i)^2 + H\delta_{ij} dy^i dy^j, \\ B &= b_{ij} dy^i \wedge dy^j, \qquad e^{2\phi} = \text{const.} \end{aligned}$$

- 5^2_2-brane (with w^2): Choosing $x^\mu = (x^m, y^\alpha, \bar{y}_8, \bar{y}_9)$ $(\alpha = 6,7)$

$$\begin{aligned} ds^2 &= \eta_{mn}\, dx^m dx^n + H\delta_{\alpha\beta}\, dy^\alpha dy^\beta \\ &\quad + HK^{-1}\big[(d\bar{y}_9 + b_{\alpha 9} dy^\alpha)^2 + (d\bar{y}_8 + b_{\alpha 8} dy^\alpha)^2\big], \\ B &= b_{\alpha\beta}\, dy^\alpha \wedge dy^\beta - b_{89} K^{-1}\Big[(d\bar{y}_8 + b_{\alpha 8} dy^\alpha) \wedge (d\bar{y}_9 + b_{\beta 9} dy^\beta)\Big], \\ e^{2\phi} &= HK^{-1}, \qquad K := H^2 + b_{89}^2. \end{aligned}$$

- 5^3_2-brane (with w^3): Choosing $x^\mu = (x^m, y^6, \bar{y}_{\hat{\imath}})$ $(\hat{\imath} = 7,8,9)$

$$\begin{aligned} ds^2 &= \eta_{mn}\, dx^m dx^n + H(dy^6)^2 \\ &\quad + K_2^{-1}\big[H^2(d\bar{y}_{\hat{k}} + b_{6\hat{k}} dy^6)^2 + \big(\tfrac{1}{2}\varepsilon^{\hat{\imath}\hat{\jmath}\hat{k}} b_{\hat{\imath}\hat{\jmath}}(d\bar{y}_{\hat{k}} + b_{6\hat{k}} dy^6)\big)^2\big], \\ B &= -Hb_{\hat{\imath}\hat{\jmath}} K_2^{-1}\, (d\bar{y}_{\hat{\imath}} + b_{6\hat{\imath}} dy^6) \wedge d\bar{y}_{\hat{\jmath}}, \\ e^{2\phi} &= HK_2^{-1}, \qquad K_2 := H(H^2 + b_{89}^2 + b_{79}^2 + b_{78}^2). \end{aligned}$$

- Space-filling 5^4_2-brane (with w^4): Choosing $x^\mu = (x^m, \bar{y}_a)$ $(a = 6,7,8,9)$

$$\begin{aligned} ds^2 &= \eta_{mn}\, dx^m dx^n + HK_3^{-1}\big[\big(H^2 + \tfrac{1}{2} b_{cd} b^{cd}\big)\delta^{ab} - \delta_{cd}\, b^{ca} b^{db}\big] d\bar{y}_a d\bar{y}_b, \\ B &= K_3^{-1}\big[b^{cd} b_{ca} b_{db} - \big(H^2 + \tfrac{1}{2} b_{cd} b^{cd}\big) b_{ab}\big] d\bar{y}_a \wedge d\bar{y}_b, \\ e^{2\phi} &= HK_3^{-1}, \qquad K_3 := H^4 + \tfrac{1}{2} H^2\, b_{ab} b^{ab} + \big(\tfrac{1}{8}\varepsilon^{abcd} b_{ab} b_{cd}\big)^2. \end{aligned}$$

The solutions we obtained are listed in the table.

	5_2	5^1_2	5^2_2	5^3_2	5^4_2
codim 4	NS5	KKM + w^1	$5^2_2 + w^2$	$5^3_2 + w^3$	$5^4_2 + w^4$
codim 3	HM	KKM	$5^2_2 + w^1$	$5^3_2 + w^2$	$5^4_2 + w^3$
defect	sHM	sKKM	5^2_2	$5^3_2 + w^1$	$5^4_2 + w^2$
domain-wall	dsHM	dsKKM	$s5^2_2$	$5^3_2(+w^1)$	$5^4_2 + w^2$
space-filling	tsHM	tsKKM	$ds5^2_2$	$s5^3_2$	5^4_2

- "HM", "KKM": H-monopole, KK-monopole
- "s", "ds", "ts": smeared, double smeared, triple smeared

3. Winding dependence

The function H is a harmonic function: $\Box H = 0$, where $\Box = \sum_{a=6}^{9} (\partial_a)^2$.

We focus on the 5^2_2-brane with 2-winding dependence.

- Double periodic array of NS5-brane:

$$H(\rho, x^8, x^9) = c + \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{Q}{\rho^2 + (x^8 - 2\pi R_8 m)^2 + (x^9 - 2\pi R_9 n)^2},$$

where $\rho^2 = (x^6)^2 + (x^7)^2$.

- Using Poisson resummation formula:

$$\sum_{n=-\infty}^{\infty} f(2\pi n) = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} dt\ f(t) e^{int}.$$

- Regularize the divergence and separate the zero winding sector
- The formal T-duality along (x^8, x^9)-direction: replacement $(x, R) \to (\tilde{x}, \tilde{R})$

$$H = \frac{Q}{2\pi \tilde{R}_8 \tilde{R}_9} \left[\log \frac{\mu}{\rho} + \sum_{n,m \neq (0,0)} e^{im\frac{\tilde{x}_8}{\tilde{R}_8}} e^{in\frac{\tilde{x}_9}{\tilde{R}_9}} K_0 \left(\rho \sqrt{\left(\frac{m}{\tilde{R}_8}\right)^2 + \left(\frac{n}{\tilde{R}_9}\right)^2} \right) \right].$$

The others H of five-branes can be obtained by the same procedure.

4. Worldsheet Instanton

- Dependence on winding space has been related to worldsheet instanton effects. [Gregory-Harvey-Moore '97]
- A number of calculations have supported this view. [Tong '02, Harvey-Jensen '05, Okuyama '05, Witten '09, Kimura-Sasaki '13]

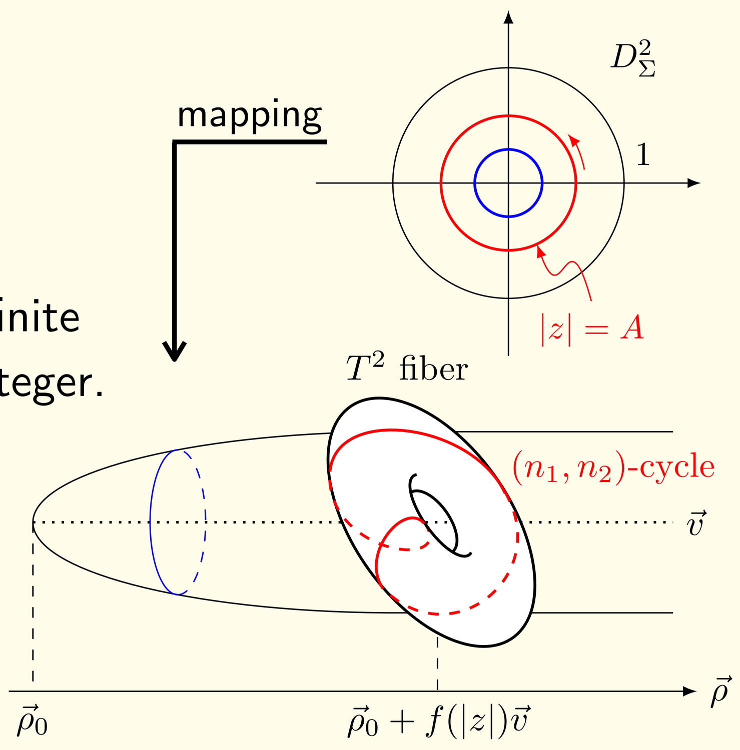
The worldsheet instantons:

- a map ϕ from the tree-level worldsheet $\Sigma = S^2$ to a 2-cycle K in the target space which minimizes the Polyakov action. [Wen-Witten '86]

Disk instanton: can define non-trivial 2-cycles in an appropriate limit of parameters (e.g. single centered KK-monopole) [Okuyama '05]

Worldsheet instanton corrections to the 5^2_2-brane geometry

- This geometry is torus fibration over $\mathbb{R}^2$.
- Physical radius of each S^1 is given by HK^{-1}.
- at origin: $HK^{-1} \to 0$, at cutoff μ: HK^{-1} is finite
- A general 1-cycle in T^2 is generated by two integer.
- (n_1, n_2)-cycle: fibered over segment $\rho \in [0, \mu]$ and defines an open cigar.
  - A generalization of the **disk instanton** $\to$
- in an appropriate limit of parameter, the two open cigar are closed at $\rho \sim \mu$.



GLSM describing geometries with T^N-fibration: Gauge group $U(1)^N$

$$[\text{GLSM}] \xrightarrow{\text{IR limit}} [\text{NLSM}]$$

- Truncated model (in an appropriate limit of parameter): 2D Abelian-Higgs

$$\mathcal{L}_{\text{E}} = \sum_{a=1}^{N} \left[\frac{1}{2e_a^2} (F_{12,a})^2 + |D_m q_a|^2 + \frac{e_a^2}{2} \left(|q_a|^2 - \sqrt{2}\, \zeta_a \right)^2 + i\sqrt{2}\, \vartheta_a F_{12,a} \right].$$

- The most stringent bound of the Euclidean action

$$S_{\text{E}} \geq \sqrt{2} |\vec{\zeta}| \sqrt{\textstyle\sum_a Q_a^2} - i\sqrt{2}\, \vec{\vartheta} \cdot \vec{Q}, \qquad (\vec{Q} = -\tfrac{1}{2\pi} \textstyle\int d^2 x \vec{F}_{12}).$$

- Path integral of S_{E} is related to exp. factor (from K_0) appearing in H of 5^2_2.
- We focused only on topological structure $\to$ neglected any F-terms
- We construct the semi-doubled GLSM for five-branes of codim. 2
  More detail $\to$ [**JHEP 12(2018)095**]

5. Conclusion

- We have explored classical DFT solution of five-branes
- found new solutions that depend on multiple winding coordinates
- exp. behaviours of the winding coordinate dependence
  $\to$ suggest that they originate from instanton effects
- Generalization of Disk instanton: to the 5^2_2-brane geometry
- Discussion on the instanton effects based on the GLSM language

Geometry of small causal diamonds

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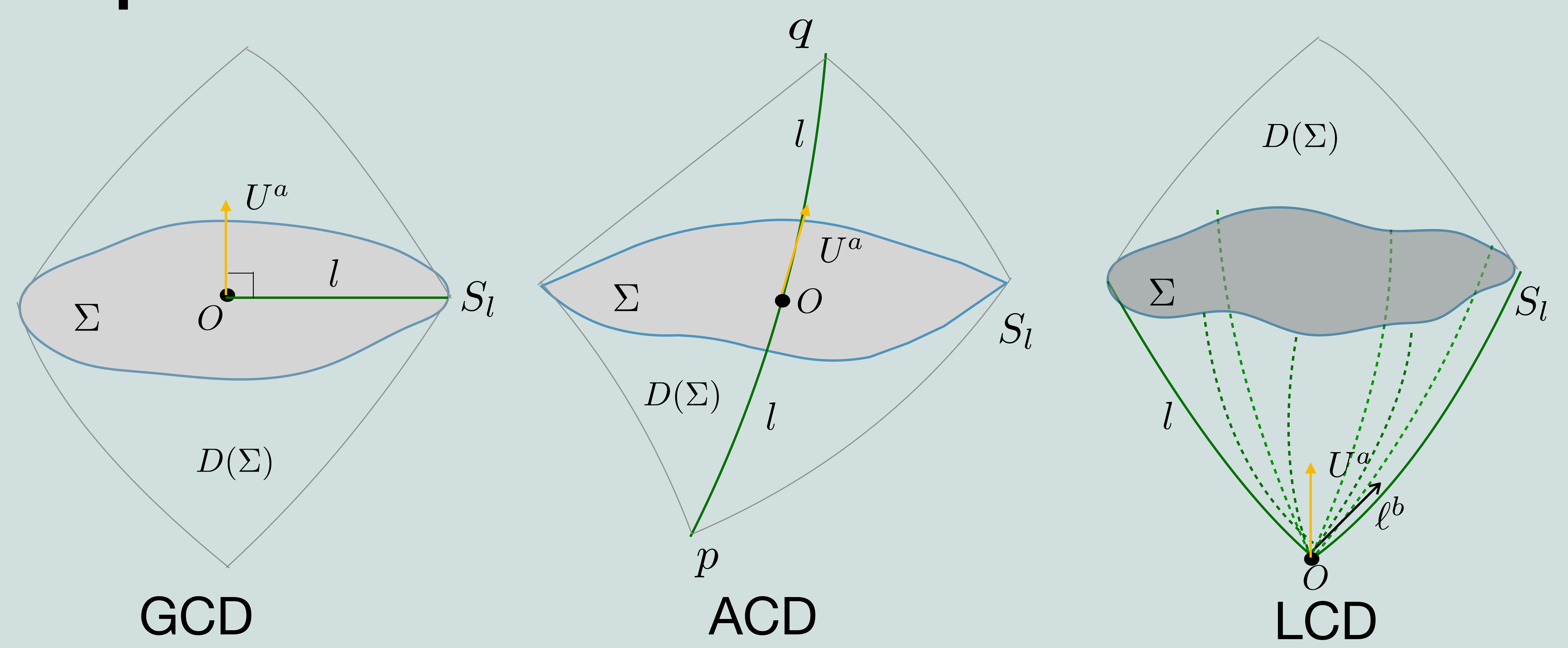
arXiv 1904.01034

Introduction

- Feynman interpreted the Einstein field equation as directly relating radius excess of some small spatial ball with the matter energy contained within, while holding the area same as in flat Minkowski space. It suggests the essence of spacetime dynamics is captured by the geometry of a small causal diamond.
- Jacobson demonstrated that the Einstein field equation can be derived from the entanglement equilibrium using the causal diamond setup [1].
- The causal diamond setup also plays an important role in the study of quantum gravity. It has been used in causal set theory, holography and cosmology.
- In this work, the geometry of small causal diamonds is systematically studied, based on three distinct constructions.Our work complements and extends the earlier works on the causal diamond geometry by Gibbons and Solodukhin [2], Jacobson, Senovilla and Speranza [3] and others [4,5].

Setup



The size parameter l (green), orientation U^a (yellow), diamond edge S_l (blue) and diamond origin O are indicated for each causal diamond. Σ specifies the spacelike hypersurface with maximal volume and its domain of dependence $D(\Sigma)$ defines the causal diamond D_{S_l}.

- We define a causal diamond as the domain of dependence of any spacelike hypersurface bounded by a given S, and we denote it by $D(S)$.
- It resembles the notion of causally complete set in AQFT and the definition of the entanglement wedge in AdS/CFT. One can bound the entropy within the diamond by the Area of S.
- Our definition allows different constructions of causal diamonds built from different edges S.
- We study three constructions that are common in the literature. They are the **geodesic ball,** the **Alexandrov Interval** and the **lightcone cut**.

Results

Background	Edge area A	Maximal hypersurface volume V	Isoperimetric ratio I
$\mathbb{M}^d$	$\Omega_{d-2}l^{d-2}$	$\frac{\Omega_{d-2}l^{d-1}}{d-1}$	1
GCD	$-\frac{\Omega_{d-2}l^{d}G_{00}}{3(d-1)}$	$-\frac{\Omega_{d-2}l^{d+1}G_{00}}{3(d-1)(d+1)}$	$\frac{G_{00}l^2}{(d-2)(d+1)}$
ACD	$-\frac{\Omega_{d-2}l^{d}(R+(d-4)R_{00})}{6(d-1)}$	$-\frac{\Omega_{d-2}l^{d+1}(R+(d-1)R_{00})}{6(d^2-1)}$	$\frac{G_{00}l^2}{(d-2)(d+1)}$
LCD	$-\frac{\Omega_{d-2}l^{d}(dR_{00}+R)}{6(d-1)}$	$-\frac{\Omega_{d-2}l^{d+1}G_{00}}{3(d-1)(d+1)}$	/

TABLE I: **The leading-order geometry of small causal diamonds in non-vacuum.** The first row shows the vacuum in flat Minkowski background and the rows below give the leading order deivations of the edge area, the maximal hypersurface volume and the isoperimetric ratio for each diamond in non-vacuum.

Background	Edge area A	Maximal hypersurface volume V	Isoperimetric ratio I
$\mathbb{M}^d$	$\Omega_{d-2}l^{d-2}$	$\frac{\Omega_{d-2}l^{d-1}}{d-1}$	1
GCD	$\Omega_{d-2}l^{d+2}\frac{\left(-\frac{D^2}{8}-\frac{H^2}{2}+\frac{E^2}{3}\right)}{15(d^2-1)}$	$\Omega_{d-2}l^{d+3}\frac{\left(-\frac{D^2}{8}-\frac{H^2}{2}+\frac{E^2}{3}\right)}{15(d^2-1)(d+3)}$	$\frac{\left(\frac{D^2}{8}+\frac{H^2}{2}-\frac{E^2}{3}\right)l^4}{3(d+3)(d+1)(d-2)}$
ACD	$\Omega_{d-2}l^{d+2}\frac{(14d^2-32d-4)E^2+6(d-4)H^2-3D^2}{360(d-1)(d+1)}$	$\Omega_{d-2}l^{d+3}\frac{(14d^2+28d-34)E^2+6(d+1)H^2-3D^2}{360(d-1)(d+1)(d+3)}$	$\frac{\left((-2d^2+2d+16)E^2+12H^2+3D^2\right)l^4}{72(d+3)(d+1)(d-2)}$
LCD	$-\Omega_{d-2}l^{d+2}\frac{\left(2(d^2+2)E^2+3D^2+6dH^2\right)}{360(d^2-1)}$	$-\Omega_{d-2}l^{d+3}\frac{\left((40d+112)E^2-12(d+2)H^2+3D^2\right)}{360(d^2-1)(d+3)}$	/

TABLE II: **leading order geometry of small causal diamonds in vacuum.** The first row shows the vacuum in flat Minkowski background and the rows below give the leading order deivations of the edge area, the maximal hypersurface volume and the isoperimetric ratio for each diamond in vacuum.

- We focus on the leading order variations of the edge geometry due to curvature in non-vacuum and vacuum.
- The variations are described by Ricci curvature, and the electromagnetic components (E,H,D) of the Weyl tensor.
- In non-vacuum, many variations are directly proportional to the Einstein/stress tensor, which leads to several derivations of the Einstein equation using the causal diamond cf Ted Jacobson.
- In vacuum, most results have no direct geometric interpretations.
- However, the LCD area deficit in 4D is proportional the Bel-Robinson tensor in particular.
- The spacetime volume of ACD is also computed in vacuum:

$$V^{(d)} = \frac{2\Omega_{d-2}l^d}{d(d-1)} + \Omega_{d-2}l^{d+4}\frac{(7d^3+58d^2+146d+108)E^2+6(d+2)(d+6)H^2-3(d+2)D^2/2}{90(d-1)(d+1)(d+2)(d+3)(d+4)} + O(l^{d+5}).$$

Applications

- Our results can be applied whenever the perturbative geometry of causal diamonds is relevant in both classical and quantum gravity.
- Our method is systematic and one can in principle compute up to any perturbation order.
- One can compute other integral quantities evaluated in the causal diamond, such as the gravitational action in a WdW patch, which is used in the holographic complexity (CA).
- In particular, our ACD results have direct applications in causal set theory.
- Our LCD results could be applied in studying the small sphere limit of the quasilocal mass in arbitrary dimensions.
- Our vacuum results suggest it might be difficult to generalise Jacobson's arguments to higher orders.

[1] T. Jacobson, Physical Review Letters 116 201101 (2016).
[2] G. Gibbons and S. Solodukhin, Physics Letters B 649, 317 (2007).
[3] T. Jacobson, J. M. Senovilla, and A. J. Speranza, Classical and Quantum Gravity 35, 085005 (2018).
[4] J. Myrheim, CERN-TH-2538 preprint (1978).
[5] M. Roy, D. Sinha, and S. Surya, Physical Review D 87, 044046 (2013).

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On the Hawking-Hayward mass in arbitrary dimensions

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Introduction

The equivalence principle forbids a covariant tensor characterising the gravitational energy. One can only resort to the notion of quasilocal mass (QLM), which is a mass associated to a codimension-2 closed spacelike submanifold S using only the geometric data on S. Physically, a QLM should capture the total energy bounded in S. There are many proposals for a good QLM over the past half-century [2]. Many of them are defined particularly for four dimensional spacetime. QLM in higher dimensions, however, is rarely studied. We study here a canonical proposal by Hawking and later refined by Hayward, known as the Hawking-Hayward (HH) mass, that admits a natural generalisation to higher dimensions. We prove a result that distinguishes out the QLM in four dimensions.

Criteria for QLM [1,2]

- Positivity and rigidity
- Reduce to Misner-Sharp mass for round spheres.
- Appropriate monotonicity.
- Globally asymptote to ADM mass or Bondi mass
- Locally converge to the stress energy T_{00} or Bel-Robinson (BR) superenergy Q_{0000} in vacuum.

Definition of Hawking-Hayward mass [3,4]

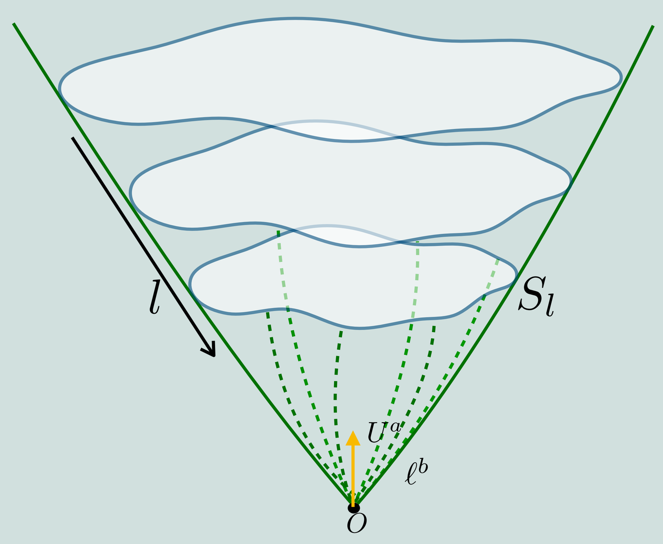
For a closed, spacelike 2-surface S, the Hawking-Hayward mass in n dimensions is defined as:

$$M_\alpha(S) = \frac{\left(\frac{Area(S)}{\Omega}\right)^{\frac{1}{n-2}}}{8\pi(n-3)} \int_S \left(\frac{\mathcal{R}}{2} + \frac{n-3}{n-2}\theta^-\theta^+ - \alpha\sigma^-_{ab}\sigma^{+ab}\right) \mathrm{d}\sigma,$$

where $\mathcal{R}, \theta^\pm, \sigma^\pm$ are the intrinsic scalar curvature of S, expansions and shears of in/outgoing null generators respectively.

This is the natural generalisation of HH mass in 4D. It satisfies the global asymptotics [5] and reduces to the Misner-Sharp mass for round spheres. It is unique up to α. However, it is in general not positive as in 4D.

***Question**: what about the small sphere limits?*



Evaluating the small sphere limits serves as a sanity check for a QLM. The limit is taken along shrinking lightcone cuts.

Results

We compute the small sphere limits in the standard tensorial formalism rather than the generalised GHP. The results are in general characterised by the Ricci-related tensors in non-vacuum and the electromagnetic components of the Weyl tensor, E, H, D in vacuum. We are looking for traces of the BR superenergy which is given by

$$W = Q_{0000} = \frac{1}{2}\left(E^2 + H^2 + \frac{1}{4}D^2\right)$$

***Theorem.** Let S_l be the one-parameter family of lightcone cuts with respect to U^a approaching o, and the small sphere limits of the Hawking-Hayward mass are given by*

$$\lim_{l\to 0} \frac{M_\alpha(S_l)}{l^{n-1}} = \frac{\Omega\, T_{00}}{n-1},$$

$$\lim_{l\to 0} \frac{M_\alpha(S_l)}{l^{n+1}} = \frac{\Omega\left[(6n^2-20n+8)E^2 + 6(n-3)H^2 - 3D^2\right]}{288\pi(n-3)(n^2-1)} - \alpha\frac{\Omega\left[(6n^2-8n-4)E^2 + 6nH^2 - 3D^2\right]}{288\pi(n-3)(n^2-1)}.$$

in non-vacuum and vacuum respectively, where the tensors T, E, H, D are evaluated at o.

- In non-vacuum, the result matches our expectation, and the limit gives the total energy = volume of the spatial ball times T_{00}.
- In vacuum, however, the small sphere limit fails to connect to the BR superenergy in $n > 4$ for any α. The vacuum small sphere limit is **only** proportional to BR superenergy in 4D.
- In 4D, our results match with the result by Horowitz and Schmidt [6] for the original definition of the Hawking mass.

Conclusions and further directions

- The result is intriguing as both HH mass and BR superenergy are uniquely generalised [7] to higher dimensions.
- It resembles the result for the area deficit, which is also proportional to BR superenergy only in 4D.
- This shows 4D is somehow special when considering QLM.
- One might need to revisit the notion of QLM, in particular its local limits, in higher dimensions.
- Such tension cannot be resolved until we find better definitions of QLM in arbitrary dimensions.
- Ultimately, can quantum gravity give us some hints? Does the small sphere limits of QLM or BR superenergy agree with the expectation value of some graviton excitation energy?

[1] S.-T. Christodoulou, D.and Yau, Contemp. Math 71 (1988).
[2] L. B. Szabados, Living reviews in relativity 12, 4 (2009).
[3] Hawking, S. W. J. Math Phys 9, 598 (1968).
[4] S. A. Hayward, Physical Review D 49, 831 (1994).
[5] P. Miao, L.-F. Tam, and N. Xie, The Journal of Geometric Analysis 27, 1323 (2017).
[6] G. Horowitz and B. Schmidt, Proc. R. Soc. Lond. A 381, 215 (1982).
[7] J. M. Senovilla, Classical and Quantum Gravity 17, 2799 (2000).