Computational Problems in Low-dimensional Topology II

Topology and Geometry of Manifolds Unit, OIST
April 16 – April 18, 2019

Schedule:

Time	Tuesday	Wednesday	Thursday
9:40	Pick up at hotel	Pick up at hotel	Pick up at hotel
10:00 – 10:50	Purcell	Sakasai	Tsvietkova
11:10 – 12:00	Futer	Owad	Rieck
12:00 – 13:20	Lunch	Lunch	Lunch
13:20 – 14:10	Licata	Kofman	Samperton
14:30 – 15:20	Maher	Yamashita	Bell
15:30	Tea	Tour and photo	Cape Manzamo
16:00	Return to hotel		Return to hotel
16:15		Return to hotel	
17:15		Pick up at hotel for dinner	
17:30			Walk to Dinner
19:00		Return to hotel	

Notes for participants: The taxi will leave at 9:40 AM every morning, sharp. If you are planning on taking the taxi, please show up in the lobby by 9:30. If you plan to walk to campus, please let Yukiko know, at yukiko.nakagawa@oist.jp. The talks will take place in the conference center, room 1, and lunches are provided in room 2. Taxis for returning to the hotel will be parked right outside the conference center. There will be a group of taxis that will take you to dinner on Wednesday night to Nakadomori and Thursday night, Onna Tsubaki, is within walking distance of the hotel. More details will be provided later.

Nakadomori: `https://goo.gl/maps/hun3nn741LB2`
Onna Tsubaki: `https://goo.gl/maps/sPJCf2xDMs72`

Organizers:
Tirasan Khandawit, Dale Koenig, Nicholas Owad, Robert Tang, Anastasiia Tsvietkova

Titles and Abstracts:

Mark Bell, Independent, *The Nielsen realisation problem and the conjugacy problem*

Abstract: We will discuss a polynomial-time algorithm that solves the conjugacy problem for the mapping class group of a surface. We will look at tools and techniques for implementing this, in particular for one case which uses a new implementation of the Nielsen realisation problem.

David Futer, Temple University, *Effective theorems in hyperbolic Dehn surgery*

Abstract: The cosmetic surgery conjecture says that distinct Dehn fillings on a nontrivial knot complement cannot yield the same closed, oriented 3-manifold. I will describe a theorem that reduces the verification of this conjecture for any given hyperbolic knot to a finite computer search. This follows as a consequence of a more general result that gives effective control on the change in geometry under Dehn filling. The method has other applications, e.g. to Margulis numbers of closed hyperbolic manifolds. This is joint work with Jessica Purcell and Saul Schleimer.

Ilya Kofman, CUNY, *Right-angled volume of alternating links.*

Abstract: Menasco and Thistlethwaite showed that an alternating link diagram can be decomposed only in limited ways along invariant Conway spheres into alternating tangles. We extract from their decomposition a new geometric link invariant, which is the sum of hyperbolic volumes of right-angled checkerboard polyhedra associated to the alternating tangles. We prove that this invariant can be explicitly computed from an alternating link diagram, and that it is a new lower bound for the hyperbolic volume of the link.

Joint work with Abhijit Champanerkar and Jessica Purcell

Joan Licata, Australian National University, *Unifying Exceptional Dehn Surgeries*

Abstract: In 2015, a computer search was used to find hyperbolic manifolds with two distinct lens space fillings. These manifolds exhibited new properties: distinct Dehn fillings dropped the Heegaard genus by more than one; and these were the first known L-spaces that weren't (branched) covers. I'll discuss recent work which gives a Dehn surgery explanation for the existence of these sporadic examples, as well as generating vastly more. This is joint work with Ken Baker, Nathan Dunfield, and Neil Hoffman.

Joseph Maher, CUNY, *The compression body graph has infinite diameter*

Abstract: The compression body graph is the graph whose vertices consist of non-trivial compression bodies, in which two are connected by an edge if one is contained in the other. This graph is quasi-isometric to the electrification of the curve complex along disc sets, and so is Gromov hyperbolic. We show that it has infinite diameter, and give some applications to random Heegaard splittings. This is joint work with Saul Schleimer.

Nicholas Owad, OIST, *Straight number and snail links*

Abstract: We introduce a new elementary invariant, the straight number of a knot, and discuss the computation of the straight number of all knots with 10 or less crossings. We give some relations to crossing number and discuss ways to build families of knots with straight number greater than crossing number. Then we present a new family of links that were found in the work done for the computation of straight number and some of their interesting properties.

Jessica Purcell, Monash University, *Treewidth, crushing, and hyperbolic volume*

Abstract: For computational purposes, a 3-manifold is often represented by a triangulation. The complexity of that triangulation can be measured by the treewidth of its dual graph. The treewidth plays an important role in algorithmic 3-manifold theory, and so it is useful to find bounds on the treewidth in terms of other properties of the manifold. In this talk, we consider hyperbolic volume, and show that there exists a universal constant c such that any closed hyperbolic 3-manifold admits a triangulation of treewidth at most the product of c and the volume. The converse is not true: we show there exists a sequence of hyperbolic 3-manifolds of bounded treewidth but volume approaching infinity. This is joint with Clement Maria.

Yo'av Rieck, University of Arkansas, *Hardness of embedding and untangling*

Abstract: While much is known about existence of algorithms in the study of 3-manifolds and knot theory, much less is known about lower bounds on their complexity (hardness results). A few computations for links (such as computing the coefficients of the Jones polynomial) were known to be very hard for a long time. Recent progress includes hardness results regarding 3-manifold and links by Lackenby and by Koenig – Tsvietkova. Kuperberg – Samperton proved the first hardness result for knots in the 3-sphere (and not just links).

In this talk we will discuss hardness of several problems. We prove:

Theorem 1: given a 2- or 3-dimensional complex X, deciding if X embed in $\mathbb{R}^3$ in NP-hard.

In particular, taking X to be a manifold with boundary tori, we see that recognizing link exteriors in the 3-sphere is NP-hard.

We also prove that certain link invariants that are defined using 4-dimensional topology give rise to NP-hard problems; for example:

Theorem 2: deciding if a link bounds a smooth surface of non-negative Euler characteristic in the 4-ball is NP-hard.

Next we turn our attention to knots. The unknot recognition problem is known to be in NP and co-NP, and as such, is not expected to be hard. Lackenby proved a polynomial bound on the number of Reidemeister moves needed to untangle an unknot diagram. In light of these two facts, one might hope for an efficient algorithm that find this optimal untangling. Unfortunately this is unlikely to happen since we prove:

Theorem 3: given an unknot diagram D and a positive integer n, deciding if D can be untangled using n Reidemeister moves is NP-hard.

This is joint work with Arnaud de Mesmay, Eric Sedgwick, and Martin Tancer.

Takuya Sakasai, University of Tokyo, *Computations on Johnson homomorphisms and their applications*

Abstract: There are two filtrations of the Torelli group: One is the lower central series and the other is the Johnson filtration. They are closely related to Johnson homomorphisms as well as finite type invariants of homology 3-spheres. We compare the associated graded Lie algebras of the filtrations and report our explicit computational results. Then we discuss some applications of our computations. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

Eric Samperton, UC Santa Barbara, *Some quantum perspectives on the computational complexity of 3-manifold invariants*

Abstract: Ill outline two or three broad perspectives on the computational complexity of 3-manifold invariants. First, well first consider the quantum complexity of quantum invariants from modular tensor categories, and the implications for the classical complexity of these invariants. Second, well consider the classical complexity of classical invariants, but take the previous quantum-quantum case as a starting point for an analogy. Finally, if theres enough time, Ill ruminate on the complexity of homological link invariants, where much less is known. Even if you are skeptical of the realizability of quantum computers, one takeaway from all this is the undeniable theoretical usefulness of the quantum perspective on purely classical questions.

We note that all three types of invariants we shall consider share a common theme: the complexity of the invariant is controlled by the properties of various types of mapping class group actions, including unitary representations, permutation representations, and higher-categorical actions.

Anastasiia Tsvietkova, OIST/Rutgers-Newark, *NP-hard problems naturally arising in knot theory*

Abstract: Many problems that lie at the heart of classical knot theory can be formulated as decision problems, with an algorithm being a solution. Despite the lack of polynomial algorithms, few problems in knot theory were previously known to be NP-hard or NP-complete. We consider decision problems related to Reidemeister moves, to unlinking and splitting by crossing changes, and to detecting alternating links and sublinks. We prove that many of these problems are NP-hard. This is a joint work with Dale Koenig.

Yasushi Yamashita, Nara Women's University, *The diagonal slice of Schottky space*

Abstract: An irreducible representation of the free group on two generators X,Y into SL(2,C) is determined up to conjugation by the traces of X,Y and XY. We study the diagonal slice of representations for which X,Y and XY have equal trace. We describe how to compute the discreteness locus of the diagonal slice. Then, we present some applications of our computation.